Cambridge International AS & A Level Physics

Giáo trình luyện thi chuyên sâu

Song ngữ Anh–Việt

8020 ENGLISH

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9.0 IELTS cao nhất	**1570** SAT cao nhất
5/5 AP — điểm tuyệt đối	**90/100** IB Diploma
100% GV $\geq$ 8.0 IELTS / 1500 SAT	**CMU** Tiến sĩ ĐH #1 Hoa Kỳ về CS

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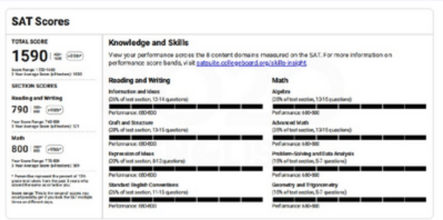

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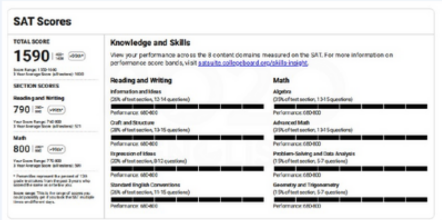

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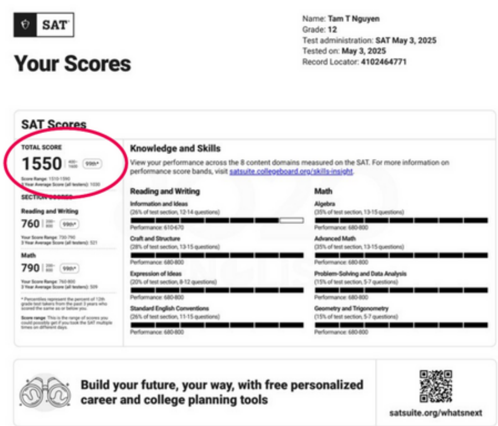

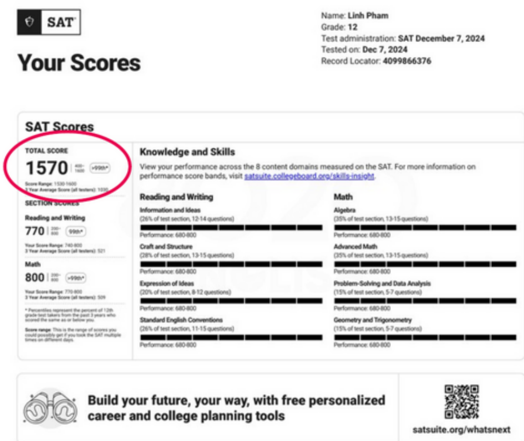

Học viên 8020 — SAT 1550 & 1570 (College Board)

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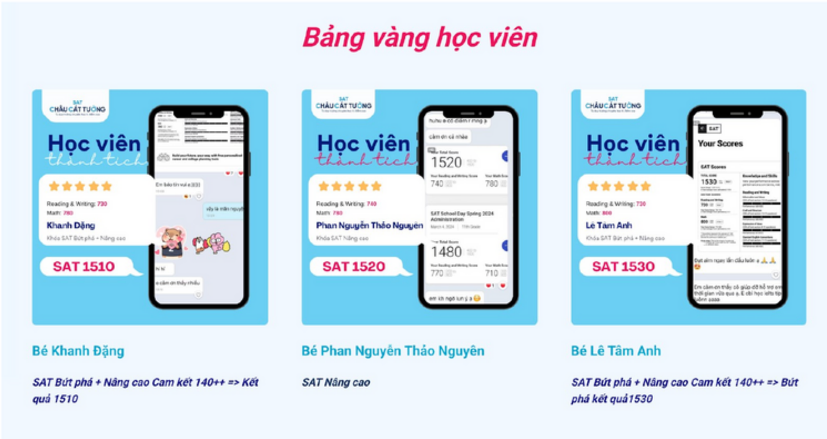
SAT CHÂU CÁT TƯỜNG



KHÁNH ĐẶNG – SAT 1510
"SAT Bứt phá + Nâng cao cam kết 140++ → 1510"

THẢO NGUYÊN – SAT 1520
"SAT Nâng cao → 1520"

TÂM ANH – SAT 1530
"SAT Bứt phá + Nâng cao cam kết 140++ → 1530"

Bảng vàng SAT — Học viên đạt 1510 · 1520 · 1530

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AP Calculus AB: 5

AP Calculus BC: 5

AP Chemistry: 5

Bảng điểm AP thật của học viên – nhiều môn đạt điểm 4 & 5

Điểm AP thật — Calculus AB/BC 5/5 · Biology & Chemistry 4–5 (College Board)

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Physical Quantities & Units

Mục tiêu Unit: Đại lượng vật lý = độ lớn số $\times$ đơn vị; bảy đại lượng cơ bản SI và đơn vị dẫn xuất; kiểm tra tính đồng nhất thứ nguyên (homogeneity) và dùng nó để tìm số mũ chưa biết trong công thức; tiền tố SI và cách đổi đơn vị; đại lượng vô hướng và vector – phân giải, cộng, trừ vector, hợp lực của nhiều lực đồng phẳng và điều kiện cân bằng; độ chính xác (accuracy), độ chuẩn xác (precision), sai số hệ thống và sai số ngẫu nhiên; dụng cụ đo (thước cặp, panme); chữ số có nghĩa; các quy tắc kết hợp độ không đảm bảo đo (uncertainty) – nền tảng bắt buộc cho toàn bộ chương trình 9702.

1.1 Physical Quantities and SI Units

Every physical quantity is defined as the product of a **numerical magnitude** and a **unit**: for example, a length of 2.5 m means "2.5 times the length called one metre". Stating a number without its unit is meaningless in physics, and mixing units within a single calculation (e.g. adding a length in cm to one in m without converting) is one of the most common sources of error at this level.

1.1.1 SI base quantities and units

The Système International (SI) defines seven base quantities. Every other physical quantity in the Cambridge 9702 syllabus is a combination of these seven.

Base quantity	SI base unit	Symbol
Mass	kilogram	kg
Length	metre	m
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

The seven SI base quantities and units. At AS & A Level, mass, length, time, current, and temperature are used constantly; amount of substance and luminous intensity appear rarely but complete the set.

Khái niệm cốt lõi

Derived units are formed by multiplying or dividing combinations of base units — no new base unit is needed. Examples: speed $= \dfrac{\text{length}}{\text{time}}$ has unit m s^{-1}; acceleration $= \dfrac{\text{speed}}{\text{time}}$ has unit m s^{-2}; force $=$ mass $\times$ acceleration has unit kg m s^{-2}, given the special name **newton** (N); pressure $= \dfrac{\text{force}}{\text{area}}$ has unit N m^{-2} $=$ kg m^{-1} s^{-2}, given the special name **pascal** (Pa); energy (work) $=$ force $\times$ distance has unit N m $=$ kg m^{2} s^{-2}, given the special name **joule** (J); power $= \dfrac{\text{energy}}{\text{time}}$ has unit J s^{-1} $=$ kg m^{2} s^{-3}, given the special name **watt** (W); electric charge $=$ current $\times$ time has unit A s, given the special name **coulomb** (C); frequency $= \dfrac{1}{\text{period}}$ has unit s^{-1}, given the special name **hertz** (Hz).

VN **GIẢI THÍCH TIẾNG VIỆT**

Mỗi đại lượng vật lý là tích của một **số đo** và một **đơn vị**. Hệ SI có **bảy đại lượng cơ bản**: khối lượng (kg), chiều dài (m), thời gian (s), cường độ dòng điện (A), nhiệt độ nhiệt động (K), lượng chất (mol), cường độ sáng (cd). Mọi đơn vị khác đều là **đơn vị dẫn xuất**, được tạo thành bằng cách nhân/chia các đơn vị cơ bản (ví dụ: N = kg m s^{-2}, Pa = kg m^{-1} s^{-2}, J = kg m^2 s^{-2}, W = kg m^2 s^{-3}). Học sinh cần thuộc lòng cách phân tích các đơn vị dẫn xuất này về đơn vị cơ bản, vì đây là nền tảng của phần kiểm tra thứ nguyên (homogeneity) tiếp theo.

1.1.2 Homogeneity of equations and dimensional analysis

An equation is **homogeneous** (dimensionally consistent) only if *every term* on both sides, when reduced to base units, gives exactly the same combination of base units. Homogeneity is a *necessary* condition for an equation to be physically correct, but it is *not sufficient* — it cannot detect an error in a purely numerical (dimensionless) factor such as 2, π, or $\frac{1}{2}$.

Ví dụ 1 · Checking equations by homogeneity

Check whether each equation is homogeneous (i.e. has the same base units on both sides).
(a) Pressure at depth in a fluid: $p = \rho g h$.
(b) The period of a simple pendulum: $T = 2\pi\sqrt{\dfrac{L}{g}}$.
(c) Kinetic energy: $E_k = \frac{1}{2}mv^2$.
(d) Stokes' law for the viscous drag on a small sphere: $F = 6\pi\eta r v$, where the viscosity η has unit Pa s = kg m^{-1} s^{-1}.
(a) RHS units = (kg m^{-3})(m s^{-2})(m) = kg m^{-1} s^{-2} = Pa — matches p. ✓
(b) Inside the root: $\dfrac{[\mathrm{m}]}{[\mathrm{m\,s^{-2}}]} = \mathrm{s}^2$, so $\sqrt{\mathrm{s}^2} = \mathrm{s}$ — matches T (the factor 2π is dimensionless and plays no part). ✓
(c) RHS units = (kg)(m s^{-1})^2 = kg m^2 s^{-2} = J — matches E_k. ✓
(d) RHS units = (kg m^{-1} s^{-1})(m)(m s^{-1}) = kg m^{-1+1+1} s^{-1-1} = kg m s^{-2} = N — matches F. ✓
All four equations are dimensionally homogeneous.

Using homogeneity to find an unknown power

Khái niệm cốt lõi

When a formula is proposed in the form (quantity) $= k \times (\text{variable}_1)^a \times (\text{variable}_2)^b \times \ldots$, with k an unknown dimensionless constant, homogeneity can be used to find the unknown powers $a, b, \ldots$ — even though k itself can never be found this way. Write every quantity in base units, collect the total power of each base unit on the right-hand side, and equate it to the power of that base unit on the left-hand side; this gives one simultaneous equation per base unit involved.

Ví dụ 2 · Finding an unknown power: period of a mass–spring system

It is proposed that the period T of a mass m oscillating on a spring of spring constant k (unit $\mathrm{N\,m^{-1}} = \mathrm{kg\,s^{-2}}$) is given by $T = C\,m^a k^b$, where C is a dimensionless constant. Use homogeneity to find a and b.

Base units: $[\mathrm{s}] = [\mathrm{kg}]^a\,[\mathrm{kg\,s^{-2}}]^b = \mathrm{kg}^{a+b}\,\mathrm{s}^{-2b}$.
Comparing powers of kg: $a + b = 0$. Comparing powers of s: $-2b = 1 \Rightarrow b = -\frac{1}{2}$.
So $a = \frac{1}{2}$, giving $T = C\sqrt{\dfrac{m}{k}}$ — consistent with the known result $T = 2\pi\sqrt{m/k}$ (homogeneity correctly predicts the powers, but not the constant 2π).

Ví dụ 3 · Finding three unknown powers: drag force on a sphere

It is suggested that the drag force F on a sphere moving through a fluid depends on the fluid density ρ ($\mathrm{kg\,m^{-3}}$), the sphere's speed v ($\mathrm{m\,s^{-1}}$), and its cross-sectional area A ($\mathrm{m^2}$), such that $F = k\,\rho^a v^b A^c$, where k is dimensionless. Find a, b and c.

$[\mathrm{kg\,m\,s^{-2}}] = [\mathrm{kg\,m^{-3}}]^a\,[\mathrm{m\,s^{-1}}]^b\,[\mathrm{m^2}]^c = \mathrm{kg}^a\,\mathrm{m}^{-3a+b+2c}\,\mathrm{s}^{-b}$.
kg: $a = 1$. s: $-b = -2 \Rightarrow b = 2$. m: $-3(1) + 2 + 2c = 1 \Rightarrow 2c = 2 \Rightarrow c = 1$.
So $F = k\rho v^2 A$ — the same form as the standard drag equation $F = \frac{1}{2}C_D\rho A v^2$, confirming the powers (but again not the numerical constant).

(!) Lỗi thường gặp: Homogeneity can never determine a purely **numerical** (dimensionless) constant such as 2, π, $\frac{1}{2}$, or a dimensionless drag/friction coefficient — these constants have no base units at all, so they cancel out of every comparison of powers. An equation can therefore be dimensionally homogeneous and still be numerically wrong (e.g. if the constant should be $\frac{1}{2}$ but is missing entirely). Also remember that angles (in radians), and ratios of two identical quantities (such as $\sin\theta$, efficiency, or refractive index), are **dimensionless** and contribute nothing to a homogeneity check.

GIẢI THÍCH TIẾNG VIỆT

VN

Một phương trình **đồng nhất về thứ nguyên** (homogeneous) khi mọi số hạng ở hai vế, quy về đơn vị cơ bản, đều giống hệt nhau. Đây là điều kiện **cần nhưng chưa đủ**: kiểm tra thứ nguyên không thể phát hiện sai số ở một hằng số thuần túy (như 2, π, $\frac{1}{2}$) vì các hằng số này không mang thứ nguyên. Một ứng dụng quan trọng là dùng tính đồng nhất để **tìm số mũ chưa biết** trong một công thức đề xuất dạng $y = k\,x^a z^b$: quy mọi đại lượng về đơn vị cơ bản, so sánh số mũ của từng đơn vị cơ bản (kg, m, s) ở hai vế để lập hệ phương trình tìm $a, b, \ldots$ — như trong Ví dụ 2 (con lắc lò xo) và Ví dụ 3 (lực cản của chất lưu). Góc (radian) và các tỉ số như $\sin\theta$, hiệu suất, chiết suất đều **không có thứ nguyên**.

1.1.3 Prefixes and standard form

Very large or very small quantities are usually written in **standard form** ($a \times 10^n$, with $1 \le a < 10$) together with an SI prefix, rather than with many zeros.

Prefix	Symbol	Factor	Example
tera	T	10^{12}	3.2 THz
giga	G	10^{9}	3.2 GHz
mega	M	10^{6}	5.0 MW
kilo	k	10^{3}	12 kN
(none)	–	10^{0}	base unit
deci	d	10^{-1}	2.0 dm
centi	c	10^{-2}	5.0 cm
milli	m	10^{-3}	0.50 mA
micro	μ	10^{-6}	$2.2\,\mu$F
nano	n	10^{-9}	750 nA
pico	p	10^{-12}	4.7 pF

Standard SI prefixes. Always convert every quantity in a calculation to *base* SI units (no prefix) before substituting into a formula, then convert the final answer back to a sensible prefix if required.

Ví dụ 4 · Converting between several prefixes in one problem

(a) A leakage current of 750 nA flows through a sensor. Express this current in mA and in A.
(b) A processor's clock runs at 3.2 GHz. Express this frequency in Hz and in MHz.
(c) A digital signal has a period of 4.0 ns. Find its frequency, in GHz.

(a) $I = 750\,\text{nA} = 750 \times 10^{-9}\,\text{A} = 7.5 \times 10^{-7}$ A. In mA: $\dfrac{7.5 \times 10^{-7}}{10^{-3}} = 7.5 \times 10^{-4}$ mA.

(b) $f = 3.2\,\text{GHz} = 3.2 \times 10^{9}$ Hz. In MHz: $\dfrac{3.2 \times 10^{9}}{10^{6}} = 3200$ MHz.

(c) $T = 4.0\,\text{ns} = 4.0 \times 10^{-9}$ s. $f = \dfrac{1}{T} = \dfrac{1}{4.0 \times 10^{-9}} = 2.5 \times 10^{8}\,\text{Hz} = \dfrac{2.5 \times 10^{8}}{10^{9}} = 0.25$ GHz.

(!) Lỗi thường gặp: When converting a unit that is **squared or cubed** (area, volume), the linear conversion factor must also be squared or cubed — it is *not* enough to convert as if the unit were linear. For example, converting 5.4×10^{4} mm^2 to m^2: since $1\,\text{mm} = 10^{-3}$ m, then $1\,\text{mm}^2 = (10^{-3})^2\,\text{m}^2 = 10^{-6}\,\text{m}^2$, so $5.4 \times 10^{4}\,\text{mm}^2 = 5.4 \times 10^{4} \times 10^{-6}\,\text{m}^2 = 5.4 \times 10^{-2}\,\text{m}^2 = 0.054\,\text{m}^2$ — *not* 5.4×10^{1} m^2, a very common error.

VN GIẢI THÍCH TIẾNG VIỆT

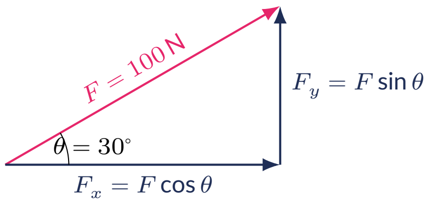
Tiền tố SI (nano, micro, milli, kilo, mega, giga, ...) dùng để viết gọn các số rất lớn hoặc rất nhỏ; luôn đổi mọi đại lượng về đơn vị cơ bản (không tiền tố) trước khi thay vào công thức. Lỗi phổ biến nhất: khi đổi đơn vị diện tích (mm^2 → m^2) hoặc thể tích (cm^3 → m^3), phải **bình phương hoặc lập phương** hệ số chuyển đổi chiều dài, chứ không chỉ nhân một lần — ví dụ $1\,\text{mm}^2 = 10^{-6}\,\text{m}^2$ (không phải 10^{-3} m^2).

1.2 Scalars and Vectors

A **scalar** quantity has magnitude only (e.g. mass, distance, speed, time, energy, work, power, pressure, density, temperature). A **vector** quantity has both magnitude *and* direction (e.g. displacement, velocity, acceleration, force, weight, momentum, electric field strength). Two vectors of the same type can only be combined correctly by taking their directions into account; scalars are simply added or subtracted arithmetically.

1.2.1 Resolving a vector into components

Any vector can be resolved into two perpendicular **components**. For a vector of magnitude F at angle θ to a reference (usually horizontal) direction: $F_x = F\cos\theta$ (component along the reference direction) and $F_y = F\sin\theta$ (component perpendicular to it). This is the reverse of finding a resultant, and it is essential whenever a force acts at an angle to the direction of motion or to another force.



Resolving a force F into perpendicular components: $F_x = F\cos\theta$ along the reference direction, $F_y = F\sin\theta$ perpendicular to it. The two components, added tip-to-tail, reconstruct the original vector F.

Ví dụ 5 · Resolving a force into components

A rope pulls a sledge with a force of $100\,\text{N}$ at 30° above the horizontal ground. Find the horizontal and vertical components of the force.

$F_x = F\cos\theta = 100\cos 30^\circ = 100(0.866) = 86.6\,\text{N}$ (horizontal).
$F_y = F\sin\theta = 100\sin 30^\circ = 100(0.500) = 50.0\,\text{N}$ (vertical, lifting the sledge slightly).

Ví dụ mẫu · Resolving a force at a general angle

A wind force of $40.0\,\text{N}$ acts on a sail at 25° above the horizontal. Find its horizontal and vertical components.

$F_x = 40.0\cos 25^\circ = 40.0(0.9063) = 36.3\,\text{N}$. $F_y = 40.0\sin 25^\circ = 40.0(0.4226) = 16.9\,\text{N}$.

1.2.2 Adding and subtracting vectors

Khái niệm cốt lõi

To **add** two vectors, draw them tip-to-tail (or complete the parallelogram from a common starting point); the vector from the very first tail to the very last tip is the resultant, or equivalently add their components: $R_x = \sum F_x$, $R_y = \sum F_y$, then $R = \sqrt{R_x^2 + R_y^2}$ and $\theta = \tan^{-1}(R_y/R_x)$. To **subtract** vector **u** from vector **v**, add **v** to $(-\mathbf{u})$, i.e. reverse the direction of **u** (keep its magnitude) and add it to **v** tip-to-tail, or subtract components directly: $(\mathbf{v}-\mathbf{u})_x = v_x - u_x$, $(\mathbf{v}-\mathbf{u})_y = v_y - u_y$.

Ví dụ 6 · Subtracting two vectors: change in velocity

A cyclist's velocity changes from $u = 8.0\,\text{m}\,\text{s}^{-1}$ due east to $v = 6.0\,\text{m}\,\text{s}^{-1}$ due north. Find the magnitude and direction of the change in velocity, $\Delta v = v - u$.

Take east as $+x$, north as $+y$: $\mathbf{u} = (8.0,\ 0)$, $\mathbf{v} = (0,\ 6.0)$.
$\Delta\mathbf{v} = \mathbf{v} - \mathbf{u} = (0 - 8.0,\ 6.0 - 0) = (-8.0,\ 6.0)$.
Magnitude: $|\Delta v| = \sqrt{8.0^2 + 6.0^2} = \sqrt{64 + 36} = \sqrt{100} = 10.0\,\text{m}\,\text{s}^{-1}$.
Direction: the components are $8.0\,\text{m}\,\text{s}^{-1}$ west and $6.0\,\text{m}\,\text{s}^{-1}$ north, so $\theta = \tan^{-1}(6.0/8.0) =$

$36.9°$ north of west.

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Cộng vector: vẽ hai vector nối đuôi nhau (hoặc hoàn thành hình bình hành); vector từ điểm đầu tới điểm cuối là hợp vector, hoặc cộng các thành phần $R_x = \sum F_x$, $R_y = \sum F_y$. **Trừ vector:** $\mathbf{v} - \mathbf{u} = \mathbf{v} + (-\mathbf{u})$, tức là đảo ngược chiều của **u** (giữ nguyên độ lớn) rồi cộng vào **v**, hoặc trừ trực tiếp từng thành phần. Đây là kỹ năng quan trọng khi tính **độ biến thiên vận tốc** $\Delta v = v - u$ (khác với độ biến thiên tốc độ, vốn chỉ là hiệu độ lớn) — một lỗi rất phổ biến là học sinh trừ trực tiếp độ lớn của hai vector mà quên xét hướng.

1.2.3 Resultant of three or more coplanar forces

When three or more coplanar forces act on a point, resolve *each* force into x- and y-components (choosing consistent positive directions), then sum all the x-components and all the y-components separately: $R_x = \sum F_x$, $R_y = \sum F_y$. The resultant then follows from Pythagoras' theorem, $R = \sqrt{R_x^2 + R_y^2}$, at angle $\theta = \tan^{-1}(R_y/R_x)$ from the reference axis. This component method works for *any* number of coplanar forces at any angles, unlike a scale diagram.

Ví dụ 7 · Resultant of three coplanar forces

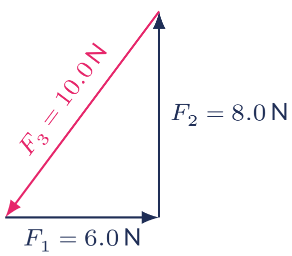
Three forces act on a point: $F_1 = 10.0\,\text{N}$ at $0°$, $F_2 = 6.0\,\text{N}$ at $60°$, and $F_3 = 8.0\,\text{N}$ at $150°$ (all angles measured anticlockwise from the $+x$ axis). Find the magnitude and direction of the resultant.

$F_{1x} = 10.0,\ F_{1y} = 0.$
$F_{2x} = 6.0\cos 60° = 3.00,\ F_{2y} = 6.0\sin 60° = 5.20.$
$F_{3x} = 8.0\cos 150° = -6.93,\ F_{3y} = 8.0\sin 150° = 4.00.$
$R_x = 10.0 + 3.00 - 6.93 = 6.07\,\text{N}.\quad R_y = 0 + 5.20 + 4.00 = 9.20\,\text{N}.$

$R = \sqrt{6.07^2 + 9.20^2} = \sqrt{36.8 + 84.6} = \sqrt{121.4} = 11.0\,\text{N}.\quad \theta = \tan^{-1}\left(\frac{9.20}{6.07}\right) = 56.6°$ above the $+x$ axis.

1.2.4 Equilibrium of coplanar forces

A point object is in **equilibrium** under coplanar forces when the resultant force is exactly zero, which requires *both* $\sum F_x = 0$ *and* $\sum F_y = 0$ simultaneously. Equivalently, if the forces are drawn tip-to-tail in order, they form a **closed polygon** (the last tip meets the first tail) — this is the "triangle of forces" rule for three coplanar forces in equilibrium. If two of three forces in equilibrium are known, the third (the **equilibrant**) has the same magnitude as the resultant of the other two, but acts in exactly the opposite direction.



Closed triangle of forces: F_1 (east), F_2 (north), and F_3 (equilibrant) drawn tip-to-tail return exactly to the starting point, confirming $F_1 + F_2 + F_3 = 0$, i.e. the point is in equilibrium.

Ví dụ 8 · Finding the unknown third force for equilibrium

A point object is held in equilibrium by three coplanar forces: $F_1 = 6.0\,\text{N}$ due east, $F_2 = 8.0\,\text{N}$ due north, and an unknown force F_3. Find the magnitude and direction of F_3.

First find the resultant of F_1 and F_2 (perpendicular, so Pythagoras applies directly):

$$R = \sqrt{F_1^2 + F_2^2} = \sqrt{6.0^2 + 8.0^2} = \sqrt{36 + 64} = \sqrt{100} = 10.0\,\text{N}, \quad \theta = \tan^{-1}\left(\frac{8.0}{6.0}\right) = 53.1^\circ$$

north of east.

For equilibrium, F_3 must be equal in magnitude to R but exactly opposite in direction: $F_3 = 10.0\,\text{N}$, directed at 53.1° south of west (i.e. $180^\circ + 53.1^\circ = 233.1^\circ$ from the $+x$ axis).

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VN

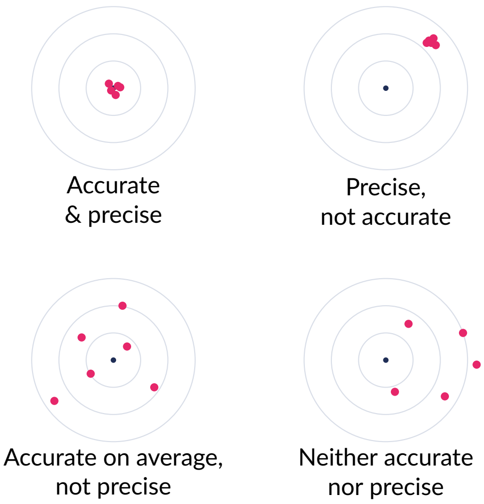
Khi ba hoặc nhiều lực đồng phẳng tác dụng lên một điểm, phân giải **mỗi** lực thành thành phần x, y rồi cộng riêng từng thành phần: $R_x = \sum F_x$, $R_y = \sum F_y$, sau đó dùng Pythagoras để tìm độ lớn và $\tan\theta$ để tìm hướng hợp lực. Vật ở trạng thái **cân bằng** dưới tác dụng của các lực đồng phẳng khi $\sum F_x = 0$ **và** $\sum F_y = 0$ đồng thời — tương đương với việc các vector lực nối đuôi nhau tạo thành một **đa giác khép kín**. Nếu biết hai trong ba lực cân bằng, lực thứ ba (gọi là **lực cân bằng** hay equilibrant) có độ lớn bằng hợp lực của hai lực kia nhưng ngược hướng hoàn toàn (180°).

1.3 Measurement: Precision, Accuracy and Uncertainty

1.3.1 Accuracy and precision

Khái niệm cốt lõi

An **accurate** measurement (or set of measurements) is close to the true (accepted) value. A **precise** measurement (or set of repeated measurements) has readings that are closely clustered together, with little spread — precision describes the *consistency* of repeated readings, regardless of whether they are close to the true value. It is entirely possible for a set of measurements to be precise but not accurate (e.g. a systematic error shifts every reading consistently away from the true value), or accurate on average but not precise (large random scatter that happens to average close to the true value).



Repeated measurements shown as dots on a "target", with the true value at the centre. Tightly clustered dots are **precise**; dots close to the centre are **accurate**. The two properties are independent of each other.

1.3.2 Systematic and random errors

A **systematic error** shifts every reading by the same amount, in the same direction (e.g. a zero error on an instrument, an incorrectly calibrated scale, or a consistent parallax viewing angle). Repeating the measurement and taking a mean **does not reduce** a systematic error, because every repeat is displaced by the same fixed amount — it can only be removed by correcting or recalibrating the instrument, or by identifying and subtracting a known zero error. A **random error** causes readings to scatter unpredictably above and below the true value (e.g. the observer's varying reaction time, small variations in judging when a bubble meniscus aligns with a scale mark). Taking **repeat readings and calculating a mean** reduces the effect of random error, since the scatter tends to average out; using an instrument with finer graduations can also reduce random error but has no effect on a systematic error.

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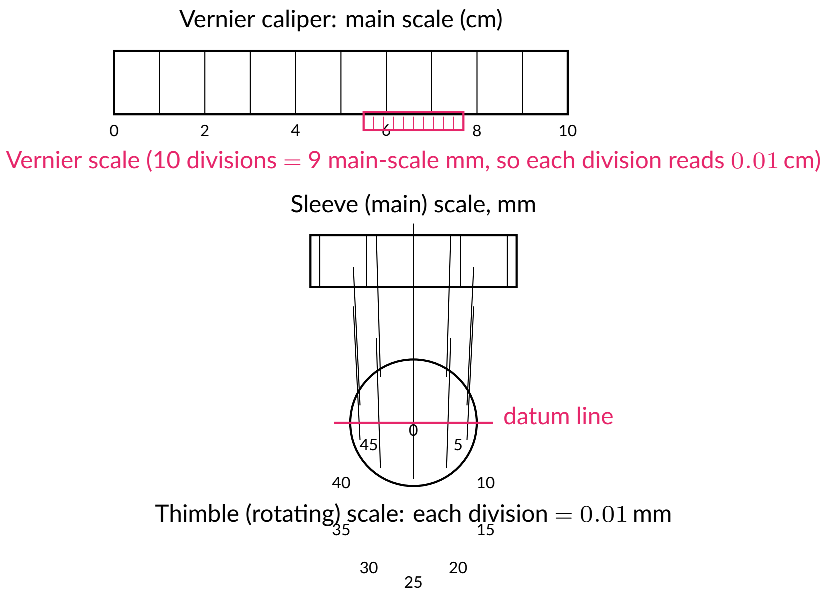
Sai số hệ thống (systematic error) làm mọi lần đo lệch cùng một lượng, cùng một hướng (ví dụ: dụng cụ có **sai số không** — zero error, hoặc luôn nhìn thước ở một góc thị sai cố định) — lấy trung bình nhiều lần đo **không** khắc phục được sai số này; chỉ có thể loại bỏ bằng cách hiệu chỉnh lại dụng cụ hoặc trừ đi giá trị sai số không đã biết. Sai số ngẫu nhiên (random error) làm các lần đo dao động không dự đoán được quanh giá trị thật (ví dụ: thời gian phản xạ khi bấm đồng hồ bấm giờ) — đo lặp lại nhiều lần rồi lấy **giá trị trung bình** sẽ làm giảm ảnh hưởng của sai số ngẫu nhiên. Độ chuẩn xác (precision) mô tả độ tập trung của các lần đo lặp lại; độ chính xác (accuracy) mô tả mức độ gần với giá trị thật — hai khái niệm này **độc lập** với nhau.

1.3.3 Common measuring instruments

A metre rule reads to the nearest millimetre; for smaller lengths, two instruments are standard at this level: the **vernier caliper** and the **micrometer screw gauge**.



Vernier caliper (top): reads to 0.1 mm (0.01 cm) — used for external/internal dimensions of small objects. **Micrometer screw gauge** (bottom): a sleeve (main) scale in mm/half-mm plus a rotating thimble scale, each thimble division $= 0.01$ mm — used for very small diameters such as wires or ball bearings.

Ví dụ 9 · Reading a vernier caliper and a micrometer

(a) A vernier caliper's main scale reads 2.30 cm just before the zero of the vernier scale, and the 6th vernier division (each $= 0.01$ cm) coincides exactly with a main-scale mark. Find the reading.
(b) A micrometer's sleeve (main) scale reads 4.0 mm and the thimble scale shows 34 divisions, each worth 0.01 mm. Find the reading.

(a) Reading $= 2.30 + (6 \times 0.01) = 2.30 + 0.06 = 2.36$ cm.
(b) Reading $= 4.0 + (34 \times 0.01) = 4.0 + 0.34 = 4.34$ mm.

1.3.4 Significant figures

A calculated answer should never be quoted to **more significant figures** than the least precise piece of data used to obtain it — a calculator display of 8–10 digits does not make a result any more precise than the data allow. If a length measured to 2 significant figures is used to calculate an area or a volume, the final answer should also be quoted to about 2 significant figures, not to every digit the calculator shows.

VN **GIẢI THÍCH TIẾNG VIỆT**

Thước cặp (vernier caliper) đọc chính xác tới 0.1 mm (0.01 cm): đọc vạch chính ngay trước vạch 0 của du xích, sau đó tìm vạch du xích trùng khít với một vạch trên thước chính. Panme (micrometer screw gauge) đọc chính xác tới 0.01 mm: cộng số đọc trên thang ống (mm) với số đọc trên thang tròn nhân 0.01 mm — dùng cho các vật rất nhỏ như dây kim loại. Kết quả tính toán **không nên** có nhiều chữ số có nghĩa hơn số liệu đo ban đầu ít chính xác nhất, dù máy tính hiển thị rất nhiều chữ số thập phân.

1.3.5 Combining uncertainties

Addition or subtraction ($y = a + b$ or $y = a - b$): the *absolute* uncertainties are added, regardless of whether the quantities themselves are added or subtracted: $\Delta y = \Delta a + \Delta b$. **Multiplication or division** ($y = ab$ or $y = a/b$): the *percentage* uncertainties are added: $\frac{\Delta y}{y} \times 100\% = \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right) \times 100\%$. **Powers** ($y = a^n$): the percentage uncertainty is multiplied by $|n|$: $\frac{\Delta y}{y} \times 100\% = |n| \times \frac{\Delta a}{a} \times 100\%$.

Ví dụ 10 · Combining uncertainties: multiplication/division

A resistor has $V = (6.0 \pm 0.1)$ V and $I = (0.50 \pm 0.02)$ A. Find $R = V/I$ with its absolute uncertainty.

$R = \frac{6.0}{0.50} = 12.0\,\Omega$. % uncertainty in $V = \frac{0.1}{6.0} \times 100\% = 1.67\%$; % uncertainty in $I = \frac{0.02}{0.50} \times 100\% = 4.0\%$.
Total % uncertainty $= 1.67\% + 4.0\% = 5.7\%$. Absolute uncertainty $= 0.057 \times 12.0 = 0.7\,\Omega$.
$R = (12.0 \pm 0.7)\,\Omega$.

Ví dụ 11 · Combining uncertainties: powers

A sphere has measured radius $r = (2.0 \pm 0.1)$ cm. Find its volume $V = \frac{4}{3}\pi r^3$ with its absolute uncertainty.

$V = \frac{4}{3}\pi(2.0)^3 = \frac{4}{3}\pi(8.0) = 33.5\,\text{cm}^3$. % uncertainty in $r = \dfrac{0.1}{2.0} \times 100\% = 5.0\%$.
Since $V \propto r^3$: % uncertainty in $V = 3 \times 5.0\% = 15\%$. Absolute uncertainty $= 0.15 \times 33.5 = 5.0\,\text{cm}^3$.
$V = (33.5 \pm 5.0)\,\text{cm}^3$.

Ví dụ 12 · Combining uncertainties: addition/subtraction

A temperature rise is found from two thermometer readings: $\theta_1 = (45.2 \pm 0.5)\,^\circ$C and $\theta_2 = (23.6 \pm 0.5)\,^\circ$C. Find the temperature rise $\Delta\theta = \theta_1 - \theta_2$ with its absolute and percentage uncertainty.

$\Delta\theta = 45.2 - 23.6 = 21.6\,^\circ$C. Absolute uncertainties *always add*, even for a subtraction: $\Delta(\Delta\theta) = 0.5 + 0.5 = 1.0\,^\circ$C.

$\Delta\theta = (21.6 \pm 1.0)\,^\circ$C. Percentage uncertainty $= \dfrac{1.0}{21.6} \times 100\% = 4.6\%$ — notice this is much larger than the $\sim 2\%$ percentage uncertainty of either individual reading, because subtracting two similar quantities amplifies the relative effect of a fixed absolute uncertainty.

(!) Lỗi thường gặp: Never mix **absolute** and **percentage** uncertainty in the same step of a calculation — convert consistently before combining. Also, when a quantity $y = a - b$ is calculated from two measured values that are *close in size* to each other ($a \approx b$), the result y is only **poorly determined**: the absolute uncertainty in y stays the same size as before, but because y itself is now small, the *percentage* uncertainty in y becomes very large (as seen in Ví dụ 12). Where possible, design an experiment to measure such a small difference *directly*, rather than as the difference of two much larger separately-measured quantities.

Ví dụ 13 · Combining all three rules: finding g from a pendulum experiment

A student measures the length of a simple pendulum as $L = (0.850 \pm 0.005)$ m. She times 20 complete oscillations and obtains a total time of (37.0 ± 0.4) s. Using $T = 2\pi\sqrt{L/g}$, rearranged as $g = \dfrac{4\pi^2 L}{T^2}$, find g with its absolute uncertainty.

Step 1 (division rule, reducing uncertainty): $T = \dfrac{37.0}{20} = 1.85$ s; the absolute timing uncertainty also divides by 20: $\Delta T = \dfrac{0.4}{20} = 0.02$ s. So $T = (1.85 \pm 0.02)$ s.

Step 2 (calculate g): $g = \dfrac{4\pi^2 L}{T^2} = \dfrac{4\pi^2(0.850)}{(1.85)^2} = \dfrac{33.56}{3.4225} = 9.80\,\text{m s}^{-2}$.

Step 3 (percentage uncertainties): % uncertainty in $L = \dfrac{0.005}{0.850} \times 100\% = 0.59\%$. % uncertainty in $T = \dfrac{0.02}{1.85} \times 100\% = 1.08\%$.

Step 4 (multiplication/division rule + power rule together, since $g \propto L^1 T^{-2}$):
% uncertainty in $g = (1 \times 0.59\%) + (2 \times 1.08\%) = 0.59\% + 2.16\% = 2.75\%$.
Step 5 (absolute uncertainty): $\Delta g = 0.0275 \times 9.80 = 0.27\,\text{m s}^{-2}$.
$g = (9.80 \pm 0.27)\,\text{m s}^{-2}$, i.e. $g = (9.8 \pm 0.3)\,\text{m s}^{-2}$ to appropriate precision.

VN GIẢI THÍCH TIẾNG VIỆT

Quy tắc kết hợp độ không đảm bảo đo: **cộng/trừ** → cộng độ không đảm bảo **tuyệt đối**; **nhân/chia** → cộng độ không đảm bảo **phần trăm**; **lũy thừa** $y = a^n$ → nhân độ không đảm bảo phần trăm với $|n|$. Ví dụ 13 kết hợp **cả ba quy tắc** trong một bài toán: chia độ không đảm bảo tuyệt đối của thời gian cho số dao động (quy tắc chia, giảm sai số phần trăm), sau đó dùng quy tắc nhân/chia và quy tắc lũy thừa (vì $g \propto L^1T^{-2}$, số mũ của T là -2 nên nhân đôi phần trăm sai số của T) để tìm sai số phần trăm của g, rồi đổi ngược ra sai số tuyệt đối. Đây là dạng bài kết hợp nhiều bước rất hay gặp trong đề thi thực hành (Paper 5) và đề lý thuyết.

1.4 Practice Bank

Câu 1

Which of the following is a *derived* SI unit?

(A) kg

(B) mol

(C) Pa

(D) A

Lời giải chi tiết

(C). The pascal is built from base units: Pa $=$ N m^{-2} $=$ kg m^{-1} s^{-2}. kg, mol and A are all SI *base* units.

Câu 2

The combination of base units kg m^2 s^{-3} is equivalent to:

(A) the newton (N)

(B) the joule (J)

(C) the watt (W)

(D) the pascal (Pa)

Lời giải chi tiết

(C). Power $P = \dfrac{E}{t} = \dfrac{\text{kg m}^2\,\text{s}^{-2}}{\text{s}} = \text{kg m}^2\,\text{s}^{-3} = \text{W}$.

Câu 3

Show, using base units, that Einstein's equation $E = mc^2$ is dimensionally consistent with E being measured in joules, given that c (the speed of light) has unit m s^{-1}.

Lời giải chi tiết

RHS units $= (\text{kg})(\text{m s}^{-1})^2 = \text{kg m}^2\,\text{s}^{-2}$. Since J $=$ N $\cdot$ m $= (\text{kg m s}^{-2})(\text{m}) = \text{kg m}^2\,\text{s}^{-2}$, the two sides match — the equation is homogeneous.

Câu 4

It is suggested that the speed v of sound in a gas depends on the gas pressure p (kg m^{-1} s^{-2}) and density ρ (kg m^{-3}), such that $v = k\,p^a\rho^b$, with k dimensionless. Use homogeneity to find a and b.

Lời giải chi tiết

$[\text{m s}^{-1}] = [\text{kg m}^{-1}\,\text{s}^{-2}]^a[\text{kg m}^{-3}]^b = \text{kg}^{a+b}\,\text{m}^{-a-3b}\,\text{s}^{-2a}$.
s: $-2a = -1 \Rightarrow a = 0.5$. kg: $a+b = 0 \Rightarrow b = -0.5$. Check m: $-0.5-3(-0.5) = -0.5+1.5 = 1.0$ ✓.
So $v = k\sqrt{p/\rho}$ — consistent with the real result $v = \sqrt{\gamma p/\rho}$.

Câu 5

A capacitance is quoted as 4.7×10^{-9} F. Which SI-prefixed value below is equivalent?

(A) 4.7 mF
(B) $4.7\,\mu$F
(C) 4.7 nF
(D) 4.7 pF

Lời giải chi tiết

(C). Since nano $= 10^{-9}$, 4.7×10^{-9} F $= 4.7$ nF.

Câu 6

Convert a pressure of $250\,000$ Pa into (a) kPa and (b) MPa.

Lời giải chi tiết

(a) $250\,000$ Pa $= 250$ kPa (divide by 10^3). (b) $250\,000$ Pa $= 0.25$ MPa (divide by 10^6).

Câu 7

A capacitor is rated at 2.2 nF. Express this in (a) pF, (b) μF, (c) F.

Lời giải chi tiết

2.2 nF $= 2.2 \times 10^{-9}$ F.
(a) $\dfrac{2.2 \times 10^{-9}}{10^{-12}} = 2200$ pF. (b) $\dfrac{2.2 \times 10^{-9}}{10^{-6}} = 2.2 \times 10^{-3}\,\mu\text{F} = 0.0022\,\mu$F. (c) 2.2×10^{-9} F.

Câu 8

An area is measured as 3.6×10^4 mm^2. Expressed in m^2, this is:

(A) 3.6×10^{-2} m^2
(B) 3.6×10^{2} m^2
(C) 3.6×10^{-8} m^2
(D) 3.6×10^{4} m^2

Lời giải chi tiết

(A). $1\,\text{mm}^2 = (10^{-3})^2\,\text{m}^2 = 10^{-6}\,\text{m}^2$, so $3.6 \times 10^4 \times 10^{-6} = 3.6 \times 10^{-2}$ m^2.

Câu 9

A cube has edge length 5.0 cm. Calculate its volume in (a) cm^3, (b) m^3.

Lời giải chi tiết

(a) $V = 5.0^3 = 125\,\text{cm}^3$. (b) $1\,\text{cm}^3 = 10^{-6}\,\text{m}^3$, so $V = 125 \times 10^{-6} = 1.25 \times 10^{-4}\,\text{m}^3$.

Câu 10

Which of the following is a vector quantity?

(A) energy
(B) power
(C) momentum
(D) time

Lời giải chi tiết

(C). Momentum has both magnitude and direction. Energy, power and time are all scalars.

Câu 11

A force of 40.0 N acts at 25° above the horizontal. Find its horizontal and vertical components.

Lời giải chi tiết

$F_x = 40.0 \cos 25^\circ = 40.0(0.9063) = 36.3\,\text{N}$. $F_y = 40.0 \sin 25^\circ = 40.0(0.4226) = 16.9\,\text{N}$.

Câu 12

A cyclist's velocity changes from $5.0\,\text{m s}^{-1}$ due north to $5.0\,\text{m s}^{-1}$ due east. Find the magnitude and direction of the change in velocity.

Lời giải chi tiết

Taking east as $+x$, north as $+y$: $\mathbf{u} = (0, 5.0)$, $\mathbf{v} = (5.0, 0)$.
$\Delta\mathbf{v} = \mathbf{v} - \mathbf{u} = (5.0, -5.0)$. $|\Delta v| = \sqrt{5.0^2 + 5.0^2} = \sqrt{50} = 7.1\,\text{m s}^{-1}$.
Direction: components are $5.0\,\text{m s}^{-1}$ east and $5.0\,\text{m s}^{-1}$ south $\Rightarrow 45^\circ$ south of east (south-east).

Câu 13

Two forces of equal magnitude F act on an object in exactly opposite directions (180° apart). The magnitude of their resultant is:

(A) $2F$
(B) F
(C) 0
(D) $F\sqrt{2}$

Lời giải chi tiết

(C). Opposite vectors of equal magnitude cancel exactly: $R = F + (-F) = 0$.

Câu 14

Three coplanar forces of equal magnitude 10.0 N act on a point at 0°, 120° and 240° (measured anticlockwise from the $+x$ axis). Show, by resolving into components, that the resultant is zero.

Lời giải chi tiết

$F_1 = (10.0,\ 0)$.
$F_2 = (10.0\cos 120^\circ,\ 10.0\sin 120^\circ) = (-5.0,\ 8.66)$.
$F_3 = (10.0\cos 240^\circ,\ 10.0\sin 240^\circ) = (-5.0,\ -8.66)$.
$R_x = 10.0 - 5.0 - 5.0 = 0$. $R_y = 0 + 8.66 - 8.66 = 0$. Resultant $= 0$ — the point is in equilibrium.

Câu 15

Three coplanar forces act on a point in equilibrium: $F_1 = 15.0\,\text{N}$ due east, $F_2 = 20.0\,\text{N}$ due north, and an unknown force F_3. Find the magnitude and direction of F_3.

Lời giải chi tiết

Resultant of F_1, F_2: $R = \sqrt{15.0^2 + 20.0^2} = \sqrt{225 + 400} = \sqrt{625} = 25.0\,\text{N}$, at $\theta = \tan^{-1}(20.0/15.0) = 53.1^\circ$ north of east.
For equilibrium, $F_3 = 25.0\,\text{N}$, directed exactly opposite: 53.1° south of west.

Câu 16

Two forces of magnitude $8.0\,\text{N}$ and $5.0\,\text{N}$ act on an object. Which of the following could *not* be the magnitude of their resultant?

(A) $3.0\,\text{N}$
(B) $6.0\,\text{N}$
(C) $13.0\,\text{N}$
(D) $15.0\,\text{N}$

Lời giải chi tiết

(D). The resultant of two forces F_1, F_2 can range only from $|F_1 - F_2| = 3.0\,\text{N}$ (opposite directions) to $F_1 + F_2 = 13.0\,\text{N}$ (same direction). $15.0\,\text{N}$ lies outside this range.

Câu 17

A student always starts a stopwatch $0.15\,\text{s}$ late because of a slow reaction, but stops it correctly. State whether this is a systematic or random error, and explain whether repeating the timing and averaging would reduce its effect on a measured time interval.

Lời giải chi tiết

This is a **systematic error** — every reading is shifted by the same fixed amount, in the same direction. Repeating the measurement and averaging would **not** reduce it, since every repeat is affected by exactly the same $0.15\,\text{s}$ late start; the error can only be removed by correcting the method (e.g. using an electronic light gate instead of manual timing).

Câu 18

Which of the following is an example of a *random* error?

(A) A metre rule with a worn zero end, used for every measurement
(B) A top-pan balance that was never zeroed before use
(C) Slightly different by-eye judgements of a pointer's position on each reading
(D) A voltmeter with a permanent zero offset

Lời giải chi tiết

(C). Small, unpredictable variations in judgement scatter the readings randomly. The other three options all bias every reading in the same fixed direction — they are systematic errors.

Câu 19

A vernier caliper's main scale reads 3.10 cm just before the zero of the vernier scale, and the 7th vernier division (each $= 0.01$ cm) coincides with a main-scale mark. Find the reading.

Lời giải chi tiết

Reading $= 3.10 + (7 \times 0.01) = 3.10 + 0.07 = 3.17$ cm.

Câu 20

A micrometer's sleeve scale reads 6.5 mm and the thimble scale shows 42 divisions (each 0.01 mm). The instrument has a zero error of $+0.02$ mm (it reads 0.02 mm when the jaws are fully closed). Find the corrected diameter.

Lời giải chi tiết

Raw reading $= 6.5 + (42 \times 0.01) = 6.5 + 0.42 = 6.92$ mm. Corrected reading $= 6.92 - 0.02 = 6.90$ mm (subtract the positive zero error).

Câu 21

A calculation based on data measured to only 2 significant figures gives a calculator display of 14.7326 m. To how many significant figures should the final answer be quoted?

(A) 6

(B) 4

(C) 2

(D) as many as the calculator shows

Lời giải chi tiết

(C). The result cannot be more precise than the least precise input data (2 s.f.), so the answer should be quoted as 15 m (2 s.f.).

Câu 22

The length of a room is found from two tape measurements placed end to end: (4.25 ± 0.02) m and (2.18 ± 0.02) m. Find the total length with its absolute uncertainty.

Lời giải chi tiết

Total length $= 4.25 + 2.18 = 6.43$ m. Absolute uncertainties add: $\Delta L = 0.02 + 0.02 = 0.04$ m. $L = (6.43 \pm 0.04)$ m.

Câu 23

A rectangular plate has sides $a = (12.0 \pm 0.2)$ cm and $b = (8.0 \pm 0.1)$ cm. Find its area with its absolute uncertainty.

Lời giải chi tiết

$A = 12.0 \times 8.0 = 96.0\,\text{cm}^2$. % uncertainty in $a = \dfrac{0.2}{12.0} \times 100\% = 1.67\%$; in $b = \dfrac{0.1}{8.0} \times 100\% = 1.25\%$.
Total % uncertainty $= 1.67\% + 1.25\% = 2.9\%$. Absolute uncertainty $= 0.029 \times 96.0 = 2.8\,\text{cm}^2$.
$A = (96.0 \pm 2.8)\,\text{cm}^2$.

Câu 24

The radius of a circle is measured as $r = (5.00 \pm 0.05)$ cm. Find the percentage uncertainty and the absolute uncertainty in the calculated area $A = \pi r^2$.

Lời giải chi tiết

% uncertainty in $r = \dfrac{0.05}{5.00} \times 100\% = 1.0\%$. Since $A \propto r^2$: % uncertainty in $A = 2 \times 1.0\% = 2.0\%$.
$A = \pi(5.00)^2 = 78.5\,\text{cm}^2$. Absolute uncertainty $= 0.020 \times 78.5 = 1.6\,\text{cm}^2$. $A = (78.5 \pm 1.6)\,\text{cm}^2$.

Câu 25

A hollow sphere's mass is found by weighing an empty container, $m_1 = (85.4 \pm 0.2)$ g, then the container with the sphere inside, $m_2 = (132.6 \pm 0.2)$ g. The sphere's radius is measured as $r = (1.80 \pm 0.03)$ cm. Find, with absolute uncertainties: (a) the mass of the sphere, (b) its volume ($V = \frac{4}{3}\pi r^3$), (c) its density.

Lời giải chi tiết

(a) Mass $= m_2 - m_1 = 132.6 - 85.4 = 47.2\,\text{g}$. Absolute uncertainties add (subtraction rule): $\Delta m = 0.2 + 0.2 = 0.4\,\text{g}$. Mass $= (47.2 \pm 0.4)\,\text{g} \Rightarrow$ % uncertainty $= \dfrac{0.4}{47.2} \times 100\% = 0.85\%$.

(b) $V = \frac{4}{3}\pi(1.80)^3 = \frac{4}{3}\pi(5.832) = 24.4\,\text{cm}^3$. % uncertainty in $r = \dfrac{0.03}{1.80} \times 100\% = 1.67\%$; since $V \propto r^3$ (power rule): % uncertainty in $V = 3 \times 1.67\% = 5.0\%$. Absolute uncertainty $= 0.050 \times 24.4 = 1.2\,\text{cm}^3$. $V = (24.4 \pm 1.2)\,\text{cm}^3$.

(c) $\rho = \dfrac{\text{mass}}{V} = \dfrac{47.2}{24.4} = 1.93\,\text{g cm}^{-3}$. % uncertainty in ρ (division rule) $= 0.85\% + 5.0\% = 5.8\%$. Absolute uncertainty $= 0.058 \times 1.93 = 0.11\,\text{g cm}^{-3}$. $\rho = (1.93 \pm 0.11)\,\text{g cm}^{-3}$ — this problem combines the subtraction, power, and division rules in a single calculation.

Câu 26

A set of repeated measurements of a fixed quantity are clustered tightly together, but far from the accepted true value. This set of measurements is:

(A) accurate and precise

(B) accurate but not precise

(C) precise but not accurate

(D) neither accurate nor precise

Lời giải chi tiết

(C). Tight clustering means high precision; being far from the true value means low accuracy — most likely caused by a systematic error.

Câu 27

A student must measure the diameter of a thin copper wire, approximately 0.8 mm. State, with a reason, which instrument (metre rule, vernier caliper, or micrometer screw gauge) is most appropriate, and estimate the percentage uncertainty if the reading obtained is 0.78 mm (taking the instrument's precision as the absolute uncertainty).

Lời giải chi tiết

The **micrometer screw gauge** is most appropriate: it reads to 0.01 mm, matching the size of the wire, whereas a metre rule (1 mm precision) or a vernier caliper (0.1 mm precision) would give a far larger, unacceptable percentage uncertainty on such a small diameter.

% uncertainty $= \dfrac{0.01}{0.78} \times 100\% = 1.3\%$.

Câu 28

A proposed equation for the range R of a projectile launched at speed v and angle θ is $R = \dfrac{v^2}{g} \sin\theta$, where g is the gravitational field strength ($\mathrm{m\,s^{-2}}$). Which statement is correct?

(A) It is not homogeneous, because $\sin\theta$ is dimensionless

(B) It is homogeneous, since v^2/g has units of length and $\sin\theta$ is dimensionless

(C) It is not homogeneous, because R must have units of time

(D) It cannot be checked without knowing the numerical value of θ

Lời giải chi tiết

(B). $\dfrac{v^2}{g}$ has units $\dfrac{\mathrm{m^2\,s^{-2}}}{\mathrm{m\,s^{-2}}} = \mathrm{m}$, a length, matching R; $\sin\theta$ is dimensionless (a pure ratio) and does not affect the check. The equation is homogeneous.

Kinematics

Mục tiêu Unit: Chuyển động thẳng: quãng đường, độ dịch chuyển, tốc độ, vận tốc, gia tốc; đồ thị độ dịch chuyển - thời gian và vận tốc - thời gian (bao gồm cách dùng tiếp tuyến với đồ thị cong để tìm vận tốc/gia tốc tức thời); các phương trình chuyển động thẳng biến đổi đều (SUVAT); rơi tự do; chuyển động ném ngang và ném xiên; vận tốc tương đối theo một phương.

2.1 Distance, Displacement, Speed and Velocity

Distance is the total length of the path travelled, measured in metres. It is a scalar quantity: it has magnitude only, and it can never decrease while an object is moving. Displacement is the change in position of an object, measured in a straight line from the starting point to the finishing point, together with a direction. Displacement is a vector quantity.

Speed is the rate at which distance is covered: speed $= \dfrac{\text{distance}}{\text{time}}$. Velocity is the rate of change of displacement: velocity $= \dfrac{\text{displacement}}{\text{time}}$, and, being derived from a vector, velocity is itself a vector. Average speed and average velocity are calculated over a finite time interval; instantaneous speed and instantaneous velocity refer to values at one particular instant.

Average velocity $= \dfrac{\text{total displacement}}{\text{total time}}$, Average speed $= \dfrac{\text{total distance}}{\text{total time}}$.

For a body moving in a straight line without reversing direction, distance travelled and the magnitude of displacement are equal, so average speed and the magnitude of average velocity are also equal. If the body reverses direction at any point, the two values differ.

VN GIẢI THÍCH TIẾNG VIỆT

Quãng đường (distance) là tổng chiều dài đường đi thực tế, luôn dương và không giảm — đây là đại lượng vô hướng. Độ dịch chuyển (displacement) là khoảng cách theo đường thẳng nối điểm đầu và điểm cuối, kèm theo hướng — đây là đại lượng vectơ. Nếu vật đi rồi quay lại, quãng đường luôn lớn hơn độ lớn của độ dịch chuyển. Tốc độ trung bình dùng quãng đường; vận tốc trung bình dùng độ dịch chuyển.

Ví dụ mẫu · Distance and displacement on a two-leg walk

A hiker walks 8.0 m due east, and then turns and walks 3.0 m due west, arriving back at a point 5.0 m east of the start. The whole walk takes 5.0 s. Find (a) the total distance walked, (b) the displacement, (c) the average speed, (d) the average velocity.

Solution. (a) Distance $= 8.0 + 3.0 = 11.0$ m.

(b) Displacement $= 8.0 - 3.0 = 5.0$ m east (taking east as positive).

(c) Average speed $= \dfrac{11.0}{5.0} = 2.2\,\text{m s}^{-1}$.

(d) Average velocity $= \dfrac{5.0}{5.0} = 1.0\,\text{m s}^{-1}$ east.

Notice that the average speed (2.2 m s^{-1}) is larger than the magnitude of the average velocity (1.0 m s^{-1}), precisely because the hiker reversed direction partway through the walk.

2.2 Acceleration

Acceleration is the rate of change of velocity:

$$a = \frac{\Delta v}{\Delta t} = \frac{v - u}{t}$$

where u is the initial velocity, v is the final velocity and t is the time taken. Acceleration is a vector, measured in m s^{-2}. A body slowing down has an acceleration directed opposite to its velocity (this is often called deceleration or retardation), which in a one-dimensional problem simply means a carries a negative sign relative to the chosen positive direction.

$a = \dfrac{v-u}{t}$. A positive value of a means the velocity is increasing in the positive direction; a negative value means the velocity is decreasing (or increasing in the negative direction). Always state a positive direction before assigning signs.

VN **GIẢI THÍCH TIẾNG VIỆT**

Gia tốc là tốc độ biến thiên của vận tốc theo thời gian, $a = (v - u)/t$, đơn vị m s^{-2}. Gia tốc là vectơ: nếu vật đang chuyển động theo chiều dương mà tốc độ giảm dần (vật đang "phanh"), gia tốc mang giá trị âm. Luôn quy ước chiều dương trước khi gán dấu cho các đại lượng.

Ví dụ mẫu · Calculating acceleration

A car's velocity increases from 5.0 m s^{-1} to 17.0 m s^{-1} in 4.0 s, travelling in a straight line. Find its acceleration.

Solution. $a = \dfrac{v-u}{t} = \dfrac{17.0 - 5.0}{4.0} = \dfrac{12.0}{4.0} = 3.0\,\mathrm{m\,s^{-2}}$.

2.3 Displacement-Time and Velocity-Time Graphs

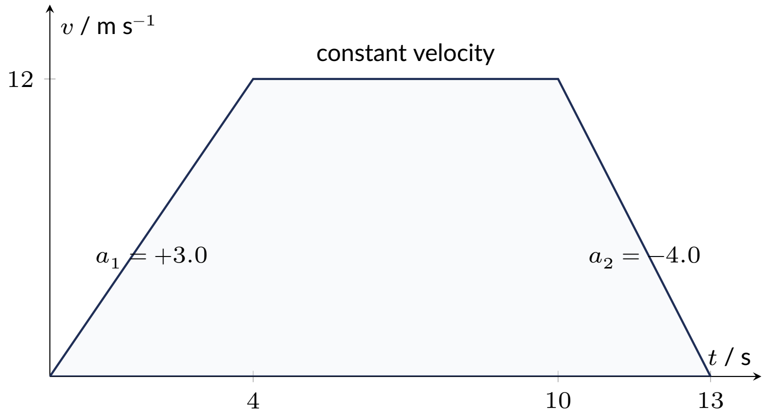
2.3.1 Uniform motion: straight-line graphs

For motion in a straight line, plotting displacement s against time t, or velocity v against time t, reveals the whole history of the motion.

On a displacement-time (s-t) graph: the gradient at any point equals the velocity at that instant. On a velocity-time (v-t) graph: the gradient at any point equals the acceleration at that instant, and the area between the graph and the time axis (over some interval) equals the displacement over that interval. Area below the axis represents negative (backward) displacement.

VN **GIẢI THÍCH TIẾNG VIỆT**

Trên đồ thị s-t: độ dốc (gradient) tại một điểm chính là vận tốc tức thời tại thời điểm đó. Trên đồ thị v-t: độ dốc là gia tốc tức thời, còn diện tích giữa đồ thị và trục thời gian chính là độ dịch chuyển. Phần diện tích nằm dưới trục thời gian ứng với độ dịch chuyển âm (đi ngược chiều dương).



A velocity-time graph for a three-stage journey: uniform acceleration from rest to $12\,\mathrm{m\,s^{-1}}$ in $4.0\,\mathrm{s}$, constant velocity for $6.0\,\mathrm{s}$, then uniform deceleration to rest in $3.0\,\mathrm{s}$. The shaded area equals the total displacement.

Ví dụ mẫu · Area under a v-t graph

For the journey shown above, find (a) both accelerations, (b) the total displacement, (c) the average velocity for the whole journey.

Solution. (a) Stage 1: $a_1 = \dfrac{12-0}{4.0} = 3.0\,\mathrm{m\,s^{-2}}$. Stage 3: $a_2 = \dfrac{0-12}{3.0} = -4.0\,\mathrm{m\,s^{-2}}$.

(b) The area is made of two triangles and a rectangle:

Triangle 1 $= \frac{1}{2}(4.0)(12) = 24$ m, Rectangle $= (6.0)(12) = 72$ m, Triangle 2 $= \frac{1}{2}(3.0)(12) = 18$ m

Total displacement $= 24 + 72 + 18 = 114\,\mathrm{m}$.

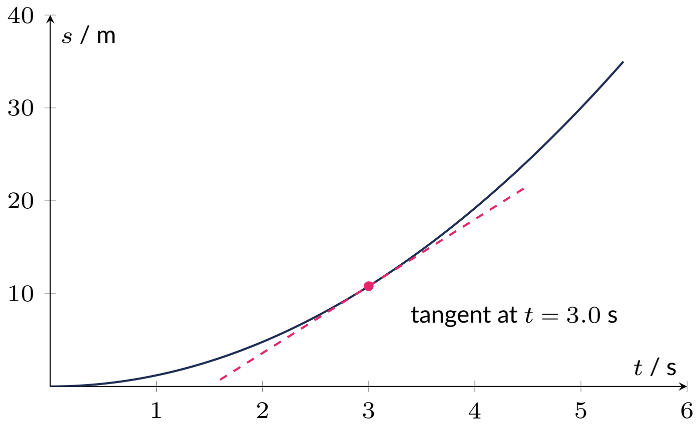
(c) Total time $= 4.0 + 6.0 + 3.0 = 13.0$ s. Average velocity $= \dfrac{114}{13.0} = 8.77\,\mathrm{m\,s^{-1}}$ (to 3 s.f.).

2.3.2 Non-uniform motion: using tangents

When acceleration is not constant, the s-t and v-t graphs are curved rather than straight. The gradient still gives velocity (on an s-t graph) or acceleration (on a v-t graph), but since the gradient itself is changing, it must be found at each instant by drawing a **tangent** to the curve at the point of interest and measuring the gradient of that tangent line — not the gradient of a chord joining two separated points, which only gives an average value over that interval.

VN **GIẢI THÍCH TIẾNG VIỆT**

Khi gia tốc không còn không đổi, đồ thị s-t hay v-t trở thành đường cong. Muốn tìm vận tốc (hoặc gia tốc) tức thời tại một điểm, ta phải vẽ **tiếp tuyến** với đường cong tại đúng điểm đó rồi tính độ dốc của tiếp tuyến — chứ không phải nối hai điểm cách xa nhau trên đồ thị (đó chỉ cho giá trị trung bình trong khoảng đó).



A curved displacement-time graph, $s = 1.2t^2$. The gradient of the tangent drawn at $t = 3.0$ s gives the instantaneous velocity at that instant.

Ví dụ mẫu · Instantaneous velocity from a tangent

A trolley's displacement follows $s = 1.2t^2$ (with s in metres, t in seconds). Find the instantaneous velocity at $t = 3.0$ s.

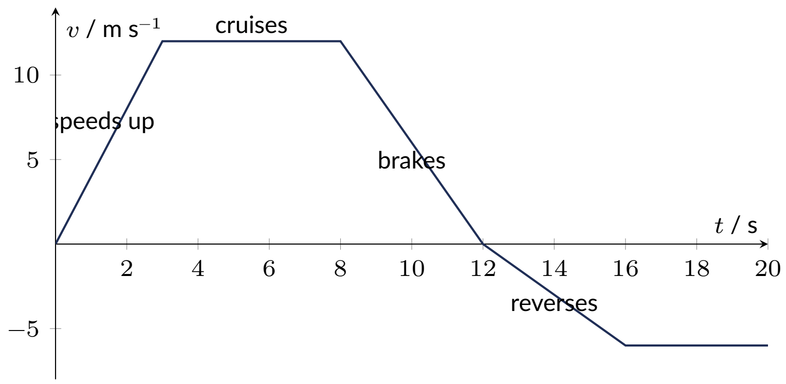
Solution. The gradient of the curve at any instant is $\dfrac{ds}{dt} = 2.4t$. At $t = 3.0$ s: $v = 2.4 \times 3.0 = 7.2\,\text{m s}^{-1}$.

Check by comparing with a symmetric chord: $s(2.0) = 1.2(4) = 4.8$ m, $s(4.0) = 1.2(16) = 19.2$ m, so the average velocity over 2.0 s to 4.0 s is $(19.2 - 4.8)/2.0 = 7.2\,\text{m s}^{-1}$ — this matches the tangent gradient exactly because the interval is symmetric about $t = 3.0$ s and s is quadratic in t; for an asymmetric interval the chord gradient would *not* equal the true instantaneous value.

(!) Lỗi thường gặp: Do not estimate an instantaneous velocity (or acceleration) by joining two points that are far apart on a curved graph — that gradient is only an *average* over the interval between them. A true instantaneous value requires a tangent drawn at a single point.

2.3.3 Qualitative sketches of motion

Examiners frequently ask for a sketch or a verbal description of s-t and v-t graphs for a described scenario, without any numbers at all. The key is to translate each phase of the motion into a gradient (for s-t) or a straight/curved segment (for v-t).



A qualitative velocity-time sketch for a car that accelerates from rest, cruises at constant velocity, brakes to rest, and then accelerates backwards into reverse. The graph crosses the time axis (at $t = 12$ s) exactly when the car is momentarily stationary.

For a ball dropped onto a hard floor and left to bounce repeatedly, each free-fall stage (whether falling or rising) has exactly the same magnitude of gradient on a v-t graph, namely g, because gravity alone acts on the ball in the air. At each bounce the velocity reverses direction almost instantaneously (a near-vertical jump on the graph) and its magnitude is reduced, since some kinetic energy is lost on impact; consequently, the time between successive bounces gets shorter, and the maximum height reached after each bounce (the peak on the corresponding s-t graph) gets smaller.

VN GIẢI THÍCH TIẾNG VIỆT

Với bóng nảy trên sàn: mỗi lần bóng đang bay tự do (đi lên hoặc rơi xuống), độ dốc của đồ thị v-t luôn có độ lớn bằng g vì chỉ có trọng lực tác dụng. Ngay tại thời điểm chạm sàn, vận tốc đổi dấu gần như tức thời và độ lớn giảm đi (mất năng lượng khi va chạm), nên độ cao nảy lên sau mỗi lần và khoảng thời gian giữa hai lần chạm sàn liên tiếp đều giảm dần.

2.4 Equations of Uniformly Accelerated Motion (SUVAT)

When acceleration is constant, four equations link displacement s, initial velocity u, final velocity v, acceleration a and time t. They follow directly from the definition of acceleration and the fact that, for constant acceleration, average velocity $= \frac{1}{2}(u + v)$.

$$v = u + at$$
$$s = ut + \tfrac{1}{2}at^2$$
$$v^2 = u^2 + 2as$$
$$s = \tfrac{1}{2}(u + v)t$$

Each equation involves four of the five quantities s, u, v, a, t — choose the equation that contains the three known quantities and the one required unknown.

VN GIẢI THÍCH TIẾNG VIỆT

Bốn phương trình SUVAT áp dụng khi gia tốc không đổi. Mỗi phương trình chỉ chứa 4 trong 5 đại lượng s, u, v, a, t; bí quyết làm bài là xác định đại lượng nào đã biết (3 đại lượng) và đại lượng cần tìm, rồi chọn đúng phương trình chứa đủ các đại lượng đó — tránh phải giải hệ nhiều bước không cần thiết.

Ví dụ mẫu · Trolley accelerating from rest

A trolley starts from rest and accelerates uniformly at $2.0\,\mathrm{m\,s^{-2}}$ for $5.0\,\mathrm{s}$. Find its final velocity and the distance travelled.

Solution. $u = 0$, $a = 2.0$ m s^{-2}, $t = 5.0$ s.

$$v = u + at = 0 + (2.0)(5.0) = 10.0\,\mathrm{m\,s^{-1}}$$

$$s = ut + \tfrac{1}{2}at^2 = 0 + \tfrac{1}{2}(2.0)(5.0)^2 = 25.0\,\mathrm{m}$$

Ví dụ mẫu · Car braking to rest

A car travelling at $24\,\mathrm{m\,s^{-1}}$ brakes and comes to rest in $6.0\,\mathrm{s}$. Find the deceleration and the braking distance.

Solution. $u = 24$ m s^{-1}, $v = 0$, $t = 6.0$ s.

$$a = \frac{v-u}{t} = \frac{0-24}{6.0} = -4.0\,\text{m}\,\text{s}^{-2}$$

$$s = \tfrac{1}{2}(u+v)t = \tfrac{1}{2}(24+0)(6.0) = 72\,\text{m}$$

Ví dụ mẫu · Finding acceleration from s, u and t

A cyclist moving at $4.0\,\text{m}\,\text{s}^{-1}$ accelerates uniformly and covers $20\,\text{m}$ in the next $4.0\,\text{s}$. Find the acceleration and the velocity at the end of this interval.
Solution. $u = 4.0$ m s^{-1}, $s = 20$ m, $t = 4.0$ s.

$$s = ut + \tfrac{1}{2}at^2 \;\Rightarrow\; 20 = (4.0)(4.0) + \tfrac{1}{2}a(4.0)^2 = 16 + 8a \;\Rightarrow\; a = \frac{4}{8} = 0.50\,\text{m}\,\text{s}^{-2}$$

$$v = u + at = 4.0 + (0.50)(4.0) = 6.0\,\text{m}\,\text{s}^{-1}$$

Check with $v^2 = u^2 + 2as$: $6.0^2 = 36$, and $4.0^2 + 2(0.50)(20) = 16 + 20 = 36$. ✓

Ví dụ mẫu · Finding initial velocity from v, a and s

A car decelerates uniformly at $5.0\,\text{m}\,\text{s}^{-2}$ and comes to rest after travelling $40\,\text{m}$. Find its initial velocity and the time taken to stop.
Solution. $v = 0$, $a = -5.0$ m s^{-2}, $s = 40$ m.

$$v^2 = u^2 + 2as \;\Rightarrow\; 0 = u^2 + 2(-5.0)(40) = u^2 - 400 \;\Rightarrow\; u = \sqrt{400} = 20.0\,\text{m}\,\text{s}^{-1}$$

$$t = \frac{v-u}{a} = \frac{0-20.0}{-5.0} = 4.0\,\text{s}$$

(!) Lỗi thường gặp: Deceleration must be entered as a *negative* value of a in every SUVAT equation once a positive direction has been chosen — never substitute a "positive deceleration" into the formulas, or the signs of s, v and t will come out wrong. Also remember that $v^2 = u^2 + 2as$ hides two possible signs for v (or u) when taking a square root; discard the physically impossible root.

2.5 Free Fall and Vertical Motion

An object falling freely under gravity, with no air resistance, has a constant downward acceleration $g = 9.81\,\text{m}\,\text{s}^{-2}$. Taking "upward" as positive, $a = -g$ for any object in flight near the Earth's surface, whether it is rising, falling, or momentarily at its highest point (where $v = 0$ but a is still $-g$).

For an object dropped from rest: $v = gt$ and $s = \frac{1}{2}gt^2$ (taking downward as positive).
For an object thrown vertically upward with speed u: time to reach maximum height $= u/g$; maximum height $= u^2/(2g)$; by symmetry, the object returns to its starting level after a total time $2u/g$, with a downward speed equal in magnitude to u (if air resistance is negligible).

VN GIẢI THÍCH TIẾNG VIỆT

Vật rơi tự do (bỏ qua sức cản không khí) có gia tốc không đổi hướng xuống, $g = 9.81\,\mathrm{m\,s^{-2}}$. Nếu vật được ném thẳng đứng lên trên với tốc độ u, thời gian lên tới đỉnh là u/g, độ cao cực đại là $u^2/(2g)$, và do tính đối xứng, tổng thời gian bay (đi lên rồi rơi về đúng vị trí ban đầu) là $2u/g$, với tốc độ lúc chạm đất bằng đúng u nhưng hướng xuống.

Ví dụ mẫu · Dropped and thrown-up objects

(a) A stone is dropped from rest. Find its velocity and the distance it has fallen after 2.50 s.
(b) A ball is thrown vertically upward at $20.0\,\mathrm{m\,s^{-1}}$ from ground level. Find the time to reach maximum height, the maximum height, the total flight time back to the ground, and the velocity on landing.
Solution. (a) $v = gt = (9.81)(2.50) = 24.5\,\mathrm{m\,s^{-1}}$ (3 s.f.). $s = \frac{1}{2}gt^2 = \frac{1}{2}(9.81)(2.50)^2 = 30.7$ m (3 s.f.).
(b) Time to top: $t_{\text{up}} = \dfrac{u}{g} = \dfrac{20.0}{9.81} = 2.04$ s.
Maximum height: $h_{\text{max}} = \dfrac{u^2}{2g} = \dfrac{20.0^2}{2(9.81)} = \dfrac{400}{19.62} = 20.4$ m.
Total flight time (up and back down to the same level) $= 2t_{\text{up}} = 4.08$ s.
By symmetry, the ball lands with speed $20.0\,\mathrm{m\,s^{-1}}$, directed downward.

2.6 Projectile Motion

2.6.1 Horizontal projectile motion

A projectile launched horizontally (or undergoing horizontal projectile motion after being dropped from a moving carrier) has two independent components of motion: a horizontal component with *constant velocity* (no horizontal force, air resistance neglected), and a vertical component with *constant acceleration* g downward, exactly as in free fall. The two components share only one thing: time.

Horizontal: $x = u_x t$ (constant velocity u_x).
Vertical (taking downward as positive, starting from rest vertically): $y = \frac{1}{2}gt^2$, $\quad v_y = gt$.
Resultant speed at any instant: $v = \sqrt{u_x^2 + v_y^2}$, at an angle $\theta = \arctan(v_y/u_x)$ below the horizontal.

VN GIẢI THÍCH TIẾNG VIỆT

Với chuyển động ném ngang: theo phương ngang, vận tốc không đổi (không có lực nào tác dụng theo phương ngang); theo phương thẳng đứng, vật rơi tự do với gia tốc g. Hai chuyển động thành phần độc lập với nhau, chỉ liên hệ qua đại lượng thời gian t chung. Tốc độ tổng hợp tại một thời điểm được tính bằng cách cộng vectơ hai thành phần vuông góc.

(!) Lỗi thường gặp: Never combine the horizontal and vertical motions into a single SUVAT calculation with one "resultant" velocity substituted throughout — the horizontal motion is uniform (constant velocity, $a = 0$) while the vertical motion is uniformly accelerated ($a = g$). Treat them as two completely separate SUVAT problems and link them only through the common time t.

Ví dụ mẫu · Ball thrown horizontally off a cliff

A ball is thrown horizontally at $15.0\,\text{m s}^{-1}$ from the top of a cliff 19.6 m high. Find (a) the time of flight, (b) the horizontal distance travelled (the range), (c) the speed and direction just before landing.

Solution. (a) Vertically: $19.6 = \frac{1}{2}(9.81)t^2 \Rightarrow t^2 = \dfrac{39.2}{9.81} = 3.996 \Rightarrow t = 2.00$ s.
(b) Horizontally: $x = u_x t = (15.0)(2.00) = 30.0$ m.
(c) $v_y = gt = (9.81)(2.00) = 19.6\,\text{m s}^{-1}$.

$$v = \sqrt{15.0^2 + 19.6^2} = \sqrt{225 + 384} = \sqrt{609} = 24.7\,\text{m s}^{-1}$$

$$\theta = \arctan\left(\frac{19.6}{15.0}\right) = \arctan(1.307) = 52.6^\circ \text{ below the horizontal}$$

2.6.2 Projectile motion at an angle

When a projectile is launched with speed u at an angle θ above the horizontal, resolve the initial velocity into horizontal and vertical components, $u_x = u\cos\theta$ and $u_y = u\sin\theta$, and treat each component exactly as before.

$$u_x = u\cos\theta, \qquad u_y = u\sin\theta$$

Time of flight (returning to launch height): $T = \dfrac{2u\sin\theta}{g}$

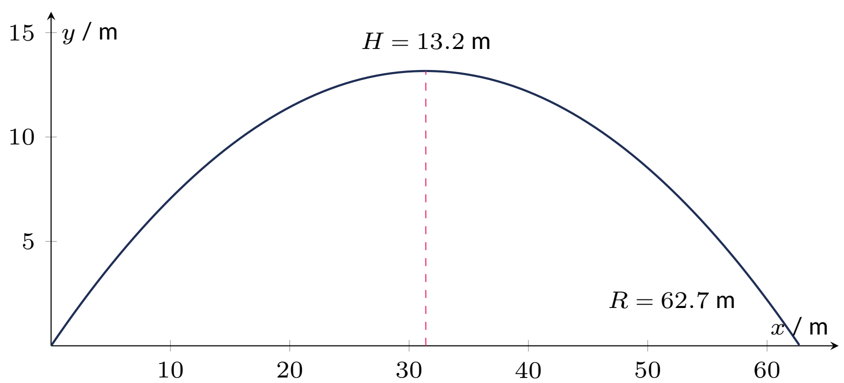
Maximum height: $H = \dfrac{u^2\sin^2\theta}{2g}$

Horizontal range (launch and landing at the same height): $R = u_x T = \dfrac{u^2\sin(2\theta)}{g}$

GIẢI THÍCH TIẾNG VIỆT

VN

Với ném xiên góc θ so với phương ngang: phân tích vận tốc ban đầu thành hai thành phần $u_x = u\cos\theta$ (không đổi) và $u_y = u\sin\theta$ (chịu gia tốc $-g$). Thời gian bay $T = 2u\sin\theta/g$, độ cao cực đại $H = u^2\sin^2\theta/(2g)$, tầm xa $R = u^2\sin(2\theta)/g$ — công thức tầm xa này chỉ áp dụng khi điểm phóng và điểm rơi có cùng độ cao.



Trajectory of a projectile launched at $25\,\text{m s}^{-1}$ at 40° above the horizontal, showing the maximum height H and the range R.

Ví dụ mẫu · Full trajectory of an angled projectile

A ball is launched at $25\,\text{m}\,\text{s}^{-1}$ at 40° above the horizontal, from level ground. Find the time of flight, the maximum height, and the range.
Solution. $u_x = 25\cos 40^\circ = 25(0.766) = 19.15$ m s^{-1}; $u_y = 25\sin 40^\circ = 25(0.643) = 16.07$ m s^{-1}.

$$T = \frac{2u_y}{g} = \frac{2(16.07)}{9.81} = 3.28\,\text{s}$$
$$H = \frac{u_y^2}{2g} = \frac{16.07^2}{2(9.81)} = \frac{258.2}{19.62} = 13.2\,\text{m}$$
$$R = u_xT = (19.15)(3.28) = 62.7\,\text{m}$$

As a check, $R = \dfrac{u^2\sin(2\theta)}{g} = \dfrac{625\sin 80^\circ}{9.81} = \dfrac{625(0.985)}{9.81} = 62.7\,\text{m}$. ✓

Ví dụ mẫu · Angled projectile clearing an obstacle

A stone is thrown from ground level at $18.0\,\text{m}\,\text{s}^{-1}$ at 50° above the horizontal, towards a wall 12.0 m away that is 6.0 m tall. Does the stone clear the wall, and if so, by how much?
Solution. $u_x = 18.0\cos 50^\circ = 18.0(0.643) = 11.57$ m s^{-1}; $u_y = 18.0\sin 50^\circ = 18.0(0.766) = 13.79$ m s^{-1}.
Time to reach the wall: $t = \dfrac{12.0}{11.57} = 1.037\,\text{s}$.
Height of the stone at that instant:

$$y = u_yt - \tfrac{1}{2}gt^2 = (13.79)(1.037) - \tfrac{1}{2}(9.81)(1.037)^2 = 14.30 - 5.28 = 9.02\,\text{m}$$

Since 9.02 m > 6.0 m, the stone clears the wall, by about $9.02 - 6.0 = 3.0$ m.

2.7 Relative Velocity in One Dimension

The velocity of object A *relative to* object B is the velocity that an observer moving with B would measure A to have: $v_{AB} = v_A - v_B$, with signs taken along a single chosen positive direction. This is especially useful for "closing speed" or "catch-up" problems.

> Same direction: $v_{AB} = v_A - v_B$ (the relative velocity is the difference of the two velocities). Opposite directions (approaching each other): the closing speed equals the *sum* of the two speeds, since one velocity is negative relative to the other's direction.

GIẢI THÍCH TIẾNG VIỆT

VN

Vận tốc tương đối của A so với B là $v_{AB} = v_A - v_B$ (lấy theo một chiều dương thống nhất). Nếu hai vật chuyển động cùng chiều, vận tốc tương đối là hiệu hai vận tốc (dùng để tính thời gian đuổi kịp); nếu chuyển động ngược chiều (tiến lại gần nhau), tốc độ tiến lại gần bằng tổng hai tốc độ.

Ví dụ mẫu · Catching up and approaching

(a) Car A travels at $26\,\text{m}\,\text{s}^{-1}$; car B, 180 m ahead of A and travelling in the same direction, moves at $20\,\text{m}\,\text{s}^{-1}$. How long does A take to catch up to B, assuming both velocities stay constant?

(b) Two cars approach each other on a straight road, one at $18\,\mathrm{m\,s^{-1}}$ and the other at $12\,\mathrm{m\,s^{-1}}$, initially 450 m apart. After how long do they meet?
Solution. (a) Relative velocity of A with respect to B $= 26 - 20 = 6.0\,\mathrm{m\,s^{-1}}$ (A closes the gap at this rate).

$$t = \frac{180}{6.0} = 30\,\mathrm{s}$$

(b) Since the cars move towards each other, the closing speed is the sum: $18 + 12 = 30\,\mathrm{m\,s^{-1}}$.

$$t = \frac{450}{30} = 15\,\mathrm{s}$$

2.8 Multi-Stage Journeys

Many exam questions describe a journey in several distinct stages of uniform acceleration (or constant velocity). The total displacement is found by treating the v-t graph as a set of triangles and trapezia (or rectangles) and adding their areas; the average velocity for the whole journey is then the total displacement divided by the total time — which is generally *not* the same as the average of the individual stage velocities.

VN GIẢI THÍCH TIẾNG VIỆT

Với hành trình gồm nhiều giai đoạn (tăng tốc - đều - giảm tốc), tổng độ dịch chuyển bằng tổng diện tích các hình (tam giác, hình chữ nhật, hình thang) dưới đồ thị v-t. Vận tốc trung bình của cả hành trình bằng tổng độ dịch chuyển chia cho tổng thời gian — nói chung **khác** với trung bình cộng đơn giản của vận tốc từng giai đoạn.

Ví dụ mẫu · Three-stage delivery van journey

A delivery van starts from rest and accelerates uniformly to $15\,\mathrm{m\,s^{-1}}$ in 10 s, travels at this constant velocity for 20 s, and finally decelerates uniformly to rest in 5.0 s. Find (a) both accelerations, (b) the total distance travelled, (c) the average velocity for the whole journey.
Solution. (a) $a_1 = \dfrac{15-0}{10} = 1.5\,\mathrm{m\,s^{-2}}$; $a_2 = \dfrac{0-15}{5.0} = -3.0\,\mathrm{m\,s^{-2}}$.
(b) Stage 1 (triangle): $\frac{1}{2}(10)(15) = 75$ m. Stage 2 (rectangle): $(20)(15) = 300$ m. Stage 3 (triangle): $\frac{1}{2}(5.0)(15) = 37.5$ m.

$$\text{Total distance} = 75 + 300 + 37.5 = 412.5\,\mathrm{m}$$

(c) Total time $= 10 + 20 + 5.0 = 35$ s.

$$\text{Average velocity} = \frac{412.5}{35} = 11.8\,\mathrm{m\,s^{-1}}$$

2.9 Practice Questions

Bài tập luyện tập

Which one of the following quantities is a vector?

(A) Distance
(B) Speed
(C) Displacement
(D) Time

Lời giải chi tiết

Displacement has both a magnitude and a direction, so it is a vector; distance, speed and time are all scalars. Answer: C.

Bài tập luyện tập

A hiker walks 6.0 km due north, then 8.0 km due east, in a total time of 3.5 h. Find (a) the total distance walked, (b) the magnitude of the displacement, (c) the average speed, (d) the magnitude of the average velocity.

Lời giải chi tiết

(a) Distance $= 6.0 + 8.0 = 14.0$ km.
(b) The displacement is the hypotenuse of a right-angled triangle with legs 6.0 km and 8.0 km:

$$\sqrt{6.0^2 + 8.0^2} = \sqrt{36 + 64} = \sqrt{100} = 10.0\,\text{km}$$

(c) Average speed $= \dfrac{14.0}{3.5} = 4.0\,\text{km}\,\text{h}^{-1}$.
(d) Average velocity magnitude $= \dfrac{10.0}{3.5} = 2.9\,\text{km}\,\text{h}^{-1}$ (3 s.f.).

Bài tập luyện tập

The gradient of a displacement-time graph at a given instant represents:

(A) Acceleration
(B) Velocity
(C) Displacement
(D) Distance

Lời giải chi tiết

By definition, velocity is the rate of change of displacement, so the gradient of an s-t graph is the velocity. Answer: B.

Bài tập luyện tập

A particle's displacement is given by $s = 0.6t^2$ (with s in metres and t in seconds). Find the instantaneous velocity at $t = 4.0$ s, and verify your answer using the average velocity between $t = 3.0$ s and $t = 5.0$ s.

Lời giải chi tiết

The gradient of the curve is $\dfrac{ds}{dt} = 1.2t$. At $t = 4.0$ s: $v = 1.2(4.0) = 4.8\,\text{m}\,\text{s}^{-1}$.
Check: $s(3.0) = 0.6(9) = 5.4$ m; $s(5.0) = 0.6(25) = 15.0$ m.

$$\text{Average velocity} = \frac{15.0 - 5.4}{5.0 - 3.0} = \frac{9.6}{2.0} = 4.8\,\text{m}\,\text{s}^{-1}$$

This matches exactly because the interval 3.0 s to 5.0 s is symmetric about $t = 4.0$ s.

Bài tập luyện tập

The area between a velocity-time graph and the time axis represents:

(A) Velocity
(B) Acceleration
(C) Displacement
(D) Speed, but only if the velocity is constant

Lời giải chi tiết

The area under a v-t graph (with areas below the axis counted as negative) always equals the displacement, whether or not the velocity is constant. Answer: C.

Bài tập luyện tập

A car accelerates uniformly from rest to $20\,\mathrm{m\,s^{-1}}$ in $5.0\,\mathrm{s}$, travels at this velocity for a further $10\,\mathrm{s}$, and then decelerates uniformly to rest in $5.0\,\mathrm{s}$. Find (a) both accelerations, (b) the total distance travelled, (c) the average speed for the whole journey.

Lời giải chi tiết

(a) $a_1 = \dfrac{20-0}{5.0} = 4.0\,\mathrm{m\,s^{-2}}$; $a_2 = \dfrac{0-20}{5.0} = -4.0\,\mathrm{m\,s^{-2}}$.
(b) Triangle 1: $\frac{1}{2}(5.0)(20) = 50$ m. Rectangle: $(10)(20) = 200$ m. Triangle 2: $\frac{1}{2}(5.0)(20) = 50$ m.

$$\text{Total distance} = 50 + 200 + 50 = 300\,\mathrm{m}$$

(c) Total time $= 5.0 + 10 + 5.0 = 20$ s.

$$\text{Average speed} = \frac{300}{20} = 15.0\,\mathrm{m\,s^{-1}}$$

Bài tập luyện tập

A ball is dropped onto a hard floor and bounces repeatedly, each bounce reaching a smaller height than the previous one. Which description best matches the velocity-time graph of the ball's motion?

(A) A single straight line of constant negative gradient throughout
(B) A series of straight-line segments, each of the same gradient magnitude g, with sudden reversals of sign at each bounce, decreasing peak speeds, and progressively shorter time intervals between bounces
(C) A smooth curve that gradually flattens towards zero velocity
(D) A horizontal line at $v = 0$ throughout

Lời giải chi tiết

Between bounces the ball is in free fall (rising or falling), so each segment has gradient of magnitude g; at each impact the velocity reverses direction almost instantaneously and loses magnitude (energy is lost), so peaks get smaller and successive intervals shorten. Answer: B.

Bài tập luyện tập

Describe, in words, the shape of the velocity-time graph for a car that accelerates uniformly from rest, travels at constant velocity, brakes uniformly to rest, waits briefly, and then accelerates uniformly in reverse back along the same road.

Lời giải chi tiết

The graph starts at the origin and rises with constant positive gradient (uniform acceleration); it then becomes a horizontal line above the axis (constant velocity); it then falls with constant negative gradient back down to the time axis (braking, decelerating to rest); it then lies exactly on the time axis for the duration of the wait ($v = 0$); finally it falls below the axis with a constant (negative) gradient, becoming a horizontal line below the axis once the car reaches a steady reverse speed. The corresponding s-t graph would rise with increasing then constant gradient, flatten as the car brakes and stops, remain flat during the wait, and then fall back towards the starting displacement as the car reverses.

Bài tập luyện tập

A train's velocity increases uniformly from $12\,\mathrm{m\,s^{-1}}$ to $28\,\mathrm{m\,s^{-1}}$ while covering $200\,\mathrm{m}$. Find (a) the acceleration, (b) the time taken.

Lời giải chi tiết

(a) Using $v^2 = u^2 + 2as$:

$$28^2 = 12^2 + 2a(200) \;\Rightarrow\; 784 = 144 + 400a \;\Rightarrow\; a = \frac{640}{400} = 1.6\,\mathrm{m\,s^{-2}}$$

(b) Using $v = u + at$:

$$t = \frac{28 - 12}{1.6} = 10.0\,\mathrm{s}$$

Bài tập luyện tập

An object starts from rest and accelerates uniformly at $2.5\,\mathrm{m\,s^{-2}}$. What distance does it cover in the first $4.0\,\mathrm{s}$?

(A) 10.0 m

(B) 20.0 m

(C) 40.0 m

(D) 5.0 m

Lời giải chi tiết

$$s = ut + \tfrac{1}{2}at^2 = 0 + \tfrac{1}{2}(2.5)(4.0)^2 = \tfrac{1}{2}(2.5)(16) = 20.0\,\mathrm{m}$$

Answer: B.

Bài tập luyện tập

A sprinter starts from rest and, accelerating uniformly, covers $18.0\,\mathrm{m}$ in the first $3.0\,\mathrm{s}$. Find (a) her acceleration, (b) her velocity at $t = 3.0\,\mathrm{s}$.

Lời giải chi tiết

(a) $s = ut + \frac{1}{2}at^2$: $18.0 = 0 + \frac{1}{2}a(3.0)^2 = 4.5a \Rightarrow a = 4.0\,\mathrm{m\,s^{-2}}$.
(b) $v = u + at = 0 + (4.0)(3.0) = 12.0\,\mathrm{m\,s^{-1}}$.
Check: $v^2 = u^2 + 2as$: $12.0^2 = 144$, and $0 + 2(4.0)(18.0) = 144$. ✓

Bài tập luyện tập

A car travelling at $27\,\mathrm{m\,s^{-1}}$ brakes with a uniform deceleration of $6.0\,\mathrm{m\,s^{-2}}$. Find (a) the stopping distance, (b) the time taken to stop.

Lời giải chi tiết

(a) $v^2 = u^2 + 2as$: $0 = 27^2 + 2(-6.0)s = 729 - 12s \Rightarrow s = \dfrac{729}{12} = 60.8\,\mathrm{m}$ (3 s.f.).

(b) $t = \dfrac{v-u}{a} = \dfrac{0-27}{-6.0} = 4.5\,\mathrm{s}$.

Bài tập luyện tập

An object is dropped from rest and falls freely. How long does it take to fall $45.0\,\mathrm{m}$?

(A) 2.14 s
(B) 3.03 s
(C) 4.59 s
(D) 9.17 s

Lời giải chi tiết

$$s = \tfrac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2s}{g}} = \sqrt{\frac{2(45.0)}{9.81}} = \sqrt{9.174} = 3.03\,\mathrm{s}$$

Answer: B.

Bài tập luyện tập

A stone is dropped from rest down a well. Find (a) its velocity, (b) the distance it has fallen, after $2.50\,\mathrm{s}$.

Lời giải chi tiết

(a) $v = gt = (9.81)(2.50) = 24.5\,\mathrm{m\,s^{-1}}$ (3 s.f.).
(b) $s = \frac{1}{2}gt^2 = \frac{1}{2}(9.81)(2.50)^2 = 30.7\,\mathrm{m}$ (3 s.f.).

Bài tập luyện tập

A ball is thrown vertically upward at $18.0\,\mathrm{m\,s^{-1}}$ from ground level. Find (a) the time to reach maximum height, (b) the maximum height, (c) the total time of flight until it returns to the ground, (d) its speed when it lands.

Lời giải chi tiết

(a) $t_{\text{up}} = \dfrac{u}{g} = \dfrac{18.0}{9.81} = 1.83\,\mathrm{s}$.

(b) $h_{\text{max}} = \dfrac{u^2}{2g} = \dfrac{18.0^2}{2(9.81)} = \dfrac{324}{19.62} = 16.5\,\text{m}$.
(c) Total flight time $= 2t_{\text{up}} = 3.67\,\text{s}$.
(d) By symmetry, the ball lands with speed $18.0\,\text{m}\,\text{s}^{-1}$ (directed downward).

Bài tập luyện tập

A ball thrown vertically upward returns to the thrower's hand after a total time of $3.20\,\text{s}$. What was its initial speed?

(A) 7.85 m s^{-1}
(B) 15.7 m s^{-1}
(C) 31.4 m s^{-1}
(D) 3.20 m s^{-1}

Lời giải chi tiết

Total flight time $T = \dfrac{2u}{g} \Rightarrow u = \dfrac{gT}{2} = \dfrac{(9.81)(3.20)}{2} = 15.7\,\text{m}\,\text{s}^{-1}$. Answer: B.

Bài tập luyện tập

A ball is thrown horizontally at $12.0\,\text{m}\,\text{s}^{-1}$ from a cliff and lands $36.0\,\text{m}$ from the base of the cliff. Find (a) the time of flight, (b) the height of the cliff, (c) the speed and direction of the ball just before landing.

Lời giải chi tiết

(a) $t = \dfrac{x}{u_x} = \dfrac{36.0}{12.0} = 3.00\,\text{s}$.
(b) $h = \frac{1}{2}gt^2 = \frac{1}{2}(9.81)(3.00)^2 = 44.1\,\text{m}$.
(c) $v_y = gt = (9.81)(3.00) = 29.4\,\text{m}\,\text{s}^{-1}$.

$$v = \sqrt{12.0^2 + 29.4^2} = \sqrt{144 + 865} = \sqrt{1009} = 31.8\,\text{m}\,\text{s}^{-1}$$

$$\theta = \arctan\left(\frac{29.4}{12.0}\right) = \arctan(2.45) = 67.8^\circ \text{ below the horizontal}$$

Bài tập luyện tập

A stone is thrown horizontally at $25.0\,\text{m}\,\text{s}^{-1}$ from the top of a cliff $78.4\,\text{m}$ high. Find (a) the time to reach the ground, (b) the horizontal distance travelled, (c) the speed just before impact.

Lời giải chi tiết

(a) $t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2(78.4)}{9.81}} = \sqrt{15.98} = 4.00\,\text{s}$.
(b) $x = u_x t = (25.0)(4.00) = 100\,\text{m}$.
(c) $v_y = gt = (9.81)(4.00) = 39.2\,\text{m}\,\text{s}^{-1}$.

$$v = \sqrt{25.0^2 + 39.2^2} = \sqrt{625 + 1537} = \sqrt{2162} = 46.5\,\text{m}\,\text{s}^{-1}$$

Bài tập luyện tập

A ball is kicked at $22.0\,\text{m s}^{-1}$ at 35° above the horizontal, from level ground. Find (a) the time of flight, (b) the maximum height reached, (c) the horizontal range.

Lời giải chi tiết

$u_x = 22.0\cos 35^\circ = 22.0(0.819) = 18.0$ m s^{-1}; $u_y = 22.0\sin 35^\circ = 22.0(0.574) = 12.6$ m s^{-1}.

(a) $T = \dfrac{2u_y}{g} = \dfrac{2(12.6)}{9.81} = 2.57\,\text{s}$.

(b) $H = \dfrac{u_y^2}{2g} = \dfrac{12.6^2}{19.62} = 8.12\,\text{m}$.

(c) $R = u_xT = (18.0)(2.57) = 46.4\,\text{m}$. (Check: $R = u^2\sin(2\theta)/g = 484\sin 70^\circ/9.81 = 484(0.940)/9.81 = 46.4\,\text{m}$.) ✓

Bài tập luyện tập

A golfer strikes a ball at $30.0\,\text{m s}^{-1}$ at 45° above the horizontal on level ground. A tree $80.0\,\text{m}$ away is $10.0\,\text{m}$ tall, directly in the ball's path. Does the ball clear the tree, and if so, by how much?

Lời giải chi tiết

$u_x = u_y = 30.0\cos 45^\circ = 30.0(0.7071) = 21.21$ m s^{-1} (since $\theta = 45^\circ$).

Time to reach the tree: $t = \dfrac{80.0}{21.21} = 3.772\,\text{s}$.

Height at that instant:

$$y = u_yt - \tfrac{1}{2}gt^2 = (21.21)(3.772) - \tfrac{1}{2}(9.81)(3.772)^2 = 80.02 - 17.44 = 22.56\,\text{m}$$

Wait — check against the full range first: $R = u^2/g = 900/9.81 = 91.7\,\text{m}$, so the ball is still airborne at $x = 80.0$ m. Since 22.56 m (recomputed precisely as $y = 80.0 - \frac{(9.81)(80.0)^2}{900} = 80.0 - 69.8 = 10.2$ m using the trajectory equation $y = x - \dfrac{gx^2}{2u^2\cos^2\theta}$) gives $y = 10.2\,\text{m}$.

The ball's height at $x = 80.0$ m is $10.2\,\text{m}$, which is greater than the tree's height of $10.0\,\text{m}$, so the ball just clears the tree, by about $0.2\,\text{m}$.

Bài tập luyện tập

Two trains move towards each other on adjacent parallel tracks, one at $25\,\text{m s}^{-1}$ and the other at $15\,\text{m s}^{-1}$. What is their closing speed?

(A) 10 m s^{-1}

(B) 20 m s^{-1}

(C) 40 m s^{-1}

(D) 25 m s^{-1}

Lời giải chi tiết

Since the trains move in opposite directions, the closing speed is the sum of their speeds: $25 + 15 = 40\,\text{m s}^{-1}$. Answer: C.

Bài tập luyện tập

Car A travels at $24\,\mathrm{m\,s^{-1}}$. Car B, travelling in the same direction at $18\,\mathrm{m\,s^{-1}}$, is initially $150\,\mathrm{m}$ ahead of A. Assuming both velocities remain constant, find the time for A to catch up with B.

Lời giải chi tiết

Relative velocity of A with respect to B $= 24 - 18 = 6.0\,\mathrm{m\,s^{-1}}$.

$$t = \frac{150}{6.0} = 25\,\mathrm{s}$$

Bài tập luyện tập

A lift starts from rest and accelerates uniformly at $1.2\,\mathrm{m\,s^{-2}}$ for $2.5\,\mathrm{s}$, then moves at constant velocity for a further $6.0\,\mathrm{s}$, and finally decelerates uniformly to rest in $2.0\,\mathrm{s}$. Find (a) the constant velocity reached, (b) the deceleration in the final stage, (c) the total distance travelled, (d) the average velocity for the whole trip.

Lời giải chi tiết

(a) $v = a_1 t_1 = (1.2)(2.5) = 3.0\,\mathrm{m\,s^{-1}}$.
(b) $a_3 = \dfrac{0 - 3.0}{2.0} = -1.5\,\mathrm{m\,s^{-2}}$.
(c) Stage 1 (triangle): $\frac{1}{2}(2.5)(3.0) = 3.75$ m. Stage 2 (rectangle): $(6.0)(3.0) = 18.0$ m. Stage 3 (triangle): $\frac{1}{2}(2.0)(3.0) = 3.00$ m.

$$\text{Total distance} = 3.75 + 18.0 + 3.00 = 24.75\,\mathrm{m}$$

(d) Total time $= 2.5 + 6.0 + 2.0 = 10.5$ s.

$$\text{Average velocity} = \frac{24.75}{10.5} = 2.36\,\mathrm{m\,s^{-1}}$$

Bài tập luyện tập

A particle accelerates uniformly at $2.0\,\mathrm{m\,s^{-2}}$, with its velocity increasing from $3.0\,\mathrm{m\,s^{-1}}$ to $11.0\,\mathrm{m\,s^{-1}}$. What distance does it travel?

(A) 14.0 m
(B) 28.0 m
(C) 56.0 m
(D) 7.0 m

Lời giải chi tiết

$$v^2 = u^2 + 2as \;\Rightarrow\; 11.0^2 = 3.0^2 + 2(2.0)s \;\Rightarrow\; 121 = 9 + 4s \;\Rightarrow\; s = \frac{112}{4} = 28.0\,\mathrm{m}$$

Answer: B.

Bài tập luyện tập

A ball is thrown vertically upward at $15.0\,\mathrm{m\,s^{-1}}$ from the top of a tower $25.0\,\mathrm{m}$ high, and falls all the way to the ground at the base of the tower. Taking upward as positive, find the total time taken to reach the ground.

Lời giải chi tiết

Taking the top of the tower as the origin, the ground is at displacement $s = -25.0$ m, with $u = +15.0$ m s^{-1} and $a = -9.81$ m s^{-2}.

$$s = ut + \frac{1}{2}at^2 \;\Rightarrow\; -25.0 = 15.0t - 4.905t^2 \;\Rightarrow\; 4.905t^2 - 15.0t - 25.0 = 0$$

Using the quadratic formula:

$$t = \frac{15.0 \pm \sqrt{15.0^2 + 4(4.905)(25.0)}}{2(4.905)} = \frac{15.0 \pm \sqrt{225 + 490.5}}{9.81} = \frac{15.0 \pm \sqrt{715.5}}{9.81} = \frac{15.0 \pm 26.75}{9.81}$$

Taking the positive root (the negative root is not physically meaningful):

$$t = \frac{15.0 + 26.75}{9.81} = \frac{41.75}{9.81} = 4.26\,\text{s}$$

Bài tập luyện tập

The gradient of the tangent to a displacement-time graph at a particular instant represents:

(A) The average velocity over the whole journey

(B) The instantaneous velocity at that instant

(C) The instantaneous acceleration at that instant

(D) The average acceleration over the whole journey

Lời giải chi tiết

A tangent gives the gradient at a single point on the curve, which is the instantaneous velocity at that instant (not an average, and not an acceleration, since acceleration comes from the gradient of a v-t graph). Answer: B.

Dynamics: Newton's Laws, Momentum & Collisions

Mục tiêu Unit: Ba định luật Newton và biểu đồ lực tự do (free-body diagram); cân bằng của một chất điểm dưới tác dụng của nhiều lực; hệ vật nối với nhau qua ròng rọc hoặc tiếp xúc trực tiếp; ma sát tĩnh và ma sát động ($F = \mu N$); vận tốc giới hạn (terminal velocity); động lượng, xung lực và đồ thị lực–thời gian; định luật bảo toàn động lượng; va chạm đàn hồi và không đàn hồi, kể cả bài toán động lượng hai chiều.

Dynamics is the study of the forces that cause (or fail to cause) changes in motion. Everything in this unit follows from just three statements made by Newton in 1687, together with a fourth idea (conservation of momentum) that follows directly from the third law. Mastering how to draw a clean **free-body diagram** and apply Newton's second law is the single most useful skill in the whole of A Level mechanics.

3.1 Newton's Laws of Motion

Newton's First Law: an object remains at rest, or continues to move with constant velocity (constant speed in a straight line), unless it is acted on by a resultant (net) external force. **Newton's Second Law:** the resultant force acting on an object is equal to the rate of change of its momentum, $F = \dfrac{\Delta p}{\Delta t}$; for an object of **constant mass** this reduces to the familiar $F = ma$. **Newton's Third Law:** if object A exerts a force on object B, then object B exerts an equal and opposite force, of the *same type*, on object A, along the same line of action.

3.1.1 Newton's First Law: Equilibrium

Newton's first law is really a statement about **equilibrium**: an object at rest, or moving with constant velocity, has **zero resultant force** acting on it — the forces on it are perfectly balanced. Conversely, if the resultant force on an object is zero, its velocity cannot change (this is the definition of *inertia*: the natural tendency of any mass to resist a change in its state of motion). A resultant force is required not to keep something moving, but to change its velocity — speed it up, slow it down, or change its direction.

VN GIẢI THÍCH TIẾNG VIỆT

Định luật 1 Newton: một vật đứng yên hoặc chuyển động thẳng đều sẽ giữ nguyên trạng thái đó **trừ khi** có một lực hợp (lực không cân bằng) tác dụng lên nó. Nói cách khác, nếu hợp lực bằng không thì vận tốc không đổi – đây chính là **quán tính** của vật.

3.1.2 Newton's Second Law

$$F_{\text{net}} = \frac{\Delta p}{\Delta t} = \frac{\Delta(mv)}{\Delta t} \quad \xrightarrow{m \text{ constant}} \quad F_{\text{net}} = ma$$

where F_{net} is the resultant (net) force in newtons (N), m is mass in kilograms (kg), and a is acceleration in m s^{-2}. One newton is defined as the resultant force that gives a mass of 1.0 kg an acceleration of $1.0\,\text{m s}^{-2}$: $1\,\text{N} = 1\,\text{kg m s}^{-2}$.

Mass and weight are easily confused. **Mass** m (kg) is a fixed amount of matter in an object, the same everywhere in the universe. **Weight** $W = mg$ (N) is the gravitational force on that mass, and depends on the local gravitational field strength g — it is therefore *not* constant if an object is moved to a different planet, though on Earth's surface we take $g = 9.81\,\text{m s}^{-2}$ unless told otherwise (a fuller treatment of weight, and of forces such as upthrust and tension in static situations, is given in the *Forces, Density & Pressure* chapter — this chapter focuses on what happens when the forces on an object do *not* balance).

Ví dụ mẫu 1 · Newton's second law: car acceleration

A car of mass 1200 kg accelerates uniformly from rest to $25\,\text{m s}^{-1}$ in 8.0 s. Find (a) its acceleration and (b) the resultant force required.

(a) $a = \dfrac{\Delta v}{\Delta t} = \dfrac{25-0}{8.0} = \boxed{3.125 \approx 3.1\,\text{m s}^{-2}}$.

(b) $F = ma = (1200)(3.125) = \boxed{3750\,\text{N}}$.

VN **GIẢI THÍCH TIẾNG VIỆT**

Định luật 2 Newton: $F = \Delta p/\Delta t$, và khi khối lượng không đổi thì rút gọn thành $F = ma$. **Khối lượng** m là lượng vật chất, không đổi ở mọi nơi; **trọng lượng** $W = mg$ là lực hấp dẫn, phụ thuộc vào g tại nơi đó (Trái Đất lấy $g = 9.81\,\text{m/s}^2$).

3.1.3 Newton's Third Law

Newton's third law pairs always: (1) act on **two different objects** (never both forces on the same object); (2) are the **same type** of force (e.g. both gravitational, or both a contact/normal force); (3) are **equal in magnitude** and **opposite in direction**; (4) act along the **same line of action**. A common trap is to pair an object's weight with the normal contact force supporting it — these *cannot* be a third-law pair, since they act on the *same* object and are *different* types of force (one gravitational, one a contact/electromagnetic force); they merely happen to be equal in magnitude when that object is in vertical equilibrium.

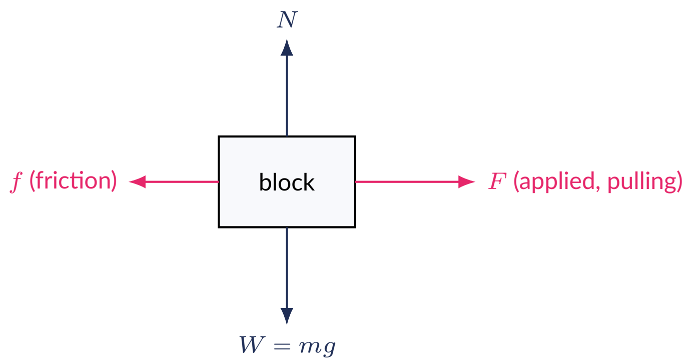
(!) Lỗi thường gặp: Newton's third law pairs act on **two different bodies** and are the **same type** of force. The classic "horse and cart" puzzle asks: if the cart pulls back on the horse exactly as hard as the horse pulls forward on the cart, how does anything ever move? The resolution is that these two forces act on *different* objects (horse and cart), so they do not cancel for either object individually. Whether the horse–cart system accelerates forward depends on the *other* forces present — chiefly the friction of the ground pushing forward on the horse's hooves (the reaction to the horse pushing backward on the ground). As long as this forward friction from the ground exceeds the backward pull of the cart on the horse, the whole system accelerates forward.

VN **GIẢI THÍCH TIẾNG VIỆT**

Cặp lực định luật 3 Newton luôn tác dụng lên **hai vật khác nhau**, cùng loại lực, cùng độ lớn, ngược chiều, cùng phương tác dụng. Lỗi thường gặp: nhầm trọng lực và phản lực pháp tuyến (cùng tác dụng lên MỘT vật, khác loại lực) là một cặp định luật 3 – thực ra chúng chỉ tình cờ bằng nhau về độ lớn khi vật cân bằng theo phương thẳng đứng.

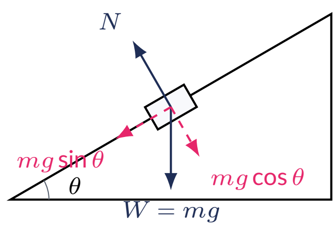
3.1.4 Free-Body Diagrams

A **free-body diagram (FBD)** isolates a single object and shows *only* the forces acting directly *on* that object (never the forces it exerts on other objects), each drawn as an arrow from the object's centre, roughly proportional in length to its magnitude. Constructing a correct FBD is the essential first step before applying $F_{\text{net}} = ma$ to any dynamics problem.



Free-body diagram of a block being pulled across a rough horizontal surface: weight $W = mg$ (down), normal reaction N (up), applied force F (in the direction of intended motion) and friction f (always opposing relative sliding, so acting backward here). Vertically, $N = W$ (no vertical acceleration); horizontally, the resultant force is $F - f$, giving acceleration $a = (F - f)/m$.

A second standard scenario is an object on a **frictionless inclined plane**. Here it is essential to resolve the weight mg (which always acts vertically downward) into two **perpendicular components relative to the slope**: one component *along* the slope, $mg\sin\theta$, and one component *into* the slope, $mg\cos\theta$, where θ is the angle of the incline to the horizontal.



Free-body diagram: block of weight $W = mg$ on a **frictionless** incline at angle θ. The component $mg\sin\theta$ acts down the slope and is unbalanced (there is nothing to oppose it), so it is the resultant force producing acceleration $a = g\sin\theta$ down the slope. The perpendicular component $mg\cos\theta$ is exactly balanced by the normal reaction, $N = mg\cos\theta$, since the block has zero acceleration perpendicular to the slope.

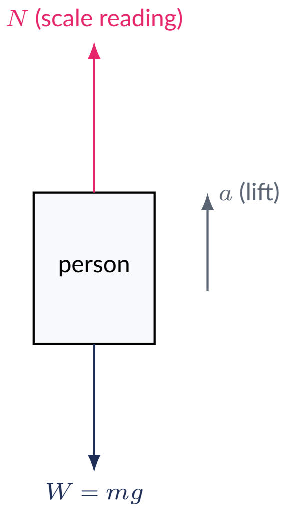
Ví dụ mẫu 2 · Frictionless inclined plane

A block of mass 5.0 kg is released from rest on a frictionless plane inclined at 30° to the horizontal. Find (a) its acceleration down the slope and (b) the normal reaction force.

(a) Along the slope, the only force is the unbalanced component of weight: $a = g\sin\theta = (9.81)(\sin 30^\circ) = (9.81)(0.500) = \boxed{4.9\,\text{m}\,\text{s}^{-2}}$ (independent of mass — a heavier block acceler-

ates down a frictionless slope at exactly the same rate).
(b) Perpendicular to the slope: $N = mg\cos\theta = (5.0)(9.81)(\cos 30°) = (49.05)(0.866) = \boxed{42.5\,\text{N}}$.

A third classic scenario is a person standing on a set of bathroom scales inside a **lift (elevator)**. Only two forces act on the person: weight $W = mg$ down, and the normal contact force N from the scale/floor up (which is exactly what the scale reads — the person's *apparent weight*).



Free-body diagram of a passenger in a lift accelerating **upward**. Taking up as positive: $N - W = ma \Rightarrow N = m(g+a) > W$, so the scale reads *more* than the true weight. If the lift instead accelerates downward, $N = m(g-a) < W$, and the scale reads *less* than the true weight; if the lift moves at constant velocity (or is stationary), $a = 0$ and $N = W$ exactly.

Ví dụ mẫu 3 · Apparent weight in an accelerating lift

A passenger of mass $70\,\text{kg}$ stands on scales inside a lift. Find the scale reading when the lift accelerates (a) upward at $2.0\,\text{m}\,\text{s}^{-2}$, and (b) downward at $2.0\,\text{m}\,\text{s}^{-2}$. Compare both with the passenger's true weight.

True weight: $W = mg = (70)(9.81) = 686.7\,\text{N} \approx 690\,\text{N}$.
(a) Upward acceleration: $N = m(g+a) = (70)(9.81+2.0) = (70)(11.81) = \boxed{827\,\text{N}}$ — heavier than true weight.
(b) Downward acceleration: $N = m(g-a) = (70)(9.81-2.0) = (70)(7.81) = \boxed{547\,\text{N}}$ — lighter than true weight.
This is exactly the "heavy/light" feeling in a lift: the scale reading (apparent weight) only ever equals the true weight when the lift's acceleration is zero.

GIẢI THÍCH TIẾNG VIỆT

VN

Sơ đồ lực tự do (FBD) chỉ vẽ các lực tác dụng **lên** một vật đang xét. Với vật trên mặt phẳng nghiêng không ma sát, trọng lực được phân tích thành $mg\sin\theta$ (dọc theo mặt phẳng, gây gia tốc $a = g\sin\theta$) và $mg\cos\theta$ (vuông góc, cân bằng với phản lực N). Với người trong thang máy đang tăng tốc, phản lực $N = m(g \pm a)$ chính là "trọng lượng biểu kiến" mà cân đo được – lớn hơn trọng lượng thật khi thang máy tăng tốc đi lên, nhỏ hơn khi tăng tốc đi xuống.

3.2 Equilibrium of a Particle Under Several Forces

Newton's first law states that an object in equilibrium has zero resultant force. When exactly **three** coplanar forces act on a particle in equilibrium, they must form a closed triangle when drawn tip-to-tail (a vector triangle) — equivalently, their components in *any* two perpendicular directions must each sum to zero. (Static situations of this kind — ladders, beams, objects on strings — are explored further in the *Forces, Density & Pressure* chapter; here the method is developed as a direct application of Newton's first law.)

For a particle in equilibrium under several coplanar forces:

$$\sum F_x = 0 \qquad \text{and} \qquad \sum F_y = 0$$

(resolving into any two convenient perpendicular directions, usually horizontal and vertical). Equivalently, the forces form a closed vector polygon.

Ví dụ mẫu 4 · Equilibrium under three forces

A lamp of weight 40 N hangs in equilibrium from two strings attached to a horizontal ceiling: string A makes 30° with the ceiling, string B makes 60° with the ceiling on the other side. Find the tension in each string.

Resolving horizontally (x) and vertically (y), with T_A, T_B the tensions:

$x:\ T_A \cos 30^\circ = T_B \cos 60^\circ \ \Rightarrow\ T_A(0.866) = T_B(0.500) \ \Rightarrow\ T_B = 1.732\,T_A$

$y:\ T_A \sin 30^\circ + T_B \sin 60^\circ = W \ \Rightarrow\ 0.500\,T_A + 0.866\,T_B = 40$

Substituting $T_B = 1.732\,T_A$: $0.500\,T_A + 0.866(1.732\,T_A) = 0.500\,T_A + 1.500\,T_A = 2.000\,T_A = 40$

$\Rightarrow T_A = \boxed{20.0\,\text{N}}$, and $T_B = 1.732(20.0) = \boxed{34.6\,\text{N}}$.

Check: horizontal components, $20.0(0.866) = 17.3$ N and $34.6(0.500) = 17.3$ N ✓; vertical components, $20.0(0.500) + 34.6(0.866) = 10.0 + 30.0 = 40.0$ N $= W$ ✓.

VN GIẢI THÍCH TIẾNG VIỆT

Khi ba lực đồng phẳng tác dụng lên một chất điểm đang cân bằng (định luật 1 Newton), tổng các thành phần lực theo phương ngang bằng 0 và theo phương thẳng đứng cũng bằng 0. Phương pháp giải: phân tích từng lực thành hai thành phần vuông góc, lập hai phương trình cân bằng, rồi giải hệ để tìm lực chưa biết.

3.3 Connected Particles and Systems

Many dynamics problems involve two or more objects connected by an **inextensible string** or in direct **contact**, forced to move with the **same magnitude of acceleration** at all times. The standard method has two stages: (1) treat the whole system together to find the common acceleration, then (2) isolate one object on its own to find an internal force (tension or contact force).

3.3.1 Objects Connected by a String Over a Pulley

Consider two masses $m_1 > m_2$ connected by a light, inextensible string passing over a smooth, light (frictionless, massless) pulley, hanging freely on either side. The heavier mass accelerates downward and the lighter mass accelerates upward, both with the same magnitude a, and the tension T is the same throughout the string.

For two masses $m_1 > m_2$ connected over a smooth pulley, treating the system as a whole (driving force is the weight *difference*, total mass being moved is $m_1 + m_2$):

$$a = \frac{(m_1 - m_2)g}{m_1 + m_2} \qquad T = m_2(g + a) = m_1(g - a)$$

Ví dụ mẫu 5 · Pulley system (Atwood machine)

Masses of $3.0\,\text{kg}$ and $5.0\,\text{kg}$ are connected by a light inextensible string over a smooth, light pulley and released from rest. Find (a) the acceleration of the system and (b) the tension in the string.

(a) $a = \frac{(m_1 - m_2)g}{m_1 + m_2} = \frac{(5.0 - 3.0)(9.81)}{5.0 + 3.0} = \frac{(2.0)(9.81)}{8.0} = \frac{19.62}{8.0} = \boxed{2.45\,\text{m s}^{-2}}$.

(b) Isolating the lighter (accelerating upward) mass: $T - m_2 g = m_2 a \Rightarrow T = m_2(g + a) = (3.0)(9.81 + 2.45) = (3.0)(12.26) = \boxed{36.8\,\text{N}}$.

Check using the heavier mass (accelerating downward): $T = m_1(g - a) = (5.0)(9.81 - 2.45) = (5.0)(7.36) = 36.8\,\text{N}$ ✓.

3.3.2 Objects in Contact

When one object pushes another directly (e.g. two blocks in contact on a frictionless table, pushed by a single applied force), both move with the same acceleration, and the **contact (normal) force** between them can be found by isolating either block.

Ví dụ mẫu 6 · Connected blocks in direct contact

Block A ($4.0\,\text{kg}$) is in contact with block B ($2.0\,\text{kg}$) ahead of it on a frictionless horizontal surface. A horizontal force of $18\,\text{N}$ pushes block A (and hence B) forward. Find (a) the common acceleration and (b) the contact force between the blocks.

(a) Treating both blocks as one system of total mass $6.0\,\text{kg}$: $a = \frac{F}{m_A + m_B} = \frac{18}{6.0} = \boxed{3.0\,\text{m s}^{-2}}$.

(b) Isolating block B (only the contact force N from A acts on it horizontally): $N = m_B a = (2.0)(3.0) = \boxed{6.0\,\text{N}}$.

Check using block A: $F - N = m_A a \Rightarrow 18 - 6.0 = 12.0 = (4.0)(3.0) = 12.0\,\text{N}$ ✓.

GIẢI THÍCH TIẾNG VIỆT

VN

Với hệ vật nối nhau (qua dây và ròng rọc, hoặc tiếp xúc trực tiếp), phương pháp chuẩn là: (1) xét **cả hệ** để tìm gia tốc chung a (lực gây gia tốc là hiệu trọng lượng hai vật trong bài toán ròng rọc, hoặc lực đẩy trong bài toán tiếp xúc, chia cho tổng khối lượng); (2) tách riêng **một vật** để tìm lực bên trong hệ (lực căng dây T hoặc lực tiếp xúc N).

3.4 Friction

Friction is the force that opposes relative sliding (or the tendency to slide) between two surfaces in contact, acting parallel to the surfaces.

Static friction acts when there is no relative sliding: it adjusts automatically, from zero up to a maximum value $F_{s,\text{max}} = \mu_s N$, to exactly match whatever force is trying to cause sliding — it only reaches its maximum value at the instant sliding is about to begin. **Kinetic (dynamic) friction** acts once sliding has actually started, with an approximately **constant** magnitude

$$F_k = \mu_k N$$

where N is the normal contact force between the surfaces and μ_k is the (dimensionless) **coefficient of kinetic friction** for that pair of surfaces. For most surface pairs, $\mu_s > \mu_k$ — it typically takes more force to *start* an object sliding than to *keep* it sliding.

Ví dụ mẫu 7 · Friction on a rough horizontal surface

A block of mass 10 kg rests on a rough horizontal surface, coefficient of kinetic friction $\mu = 0.25$. A horizontal force of 40 N is applied, and the block is already sliding. Find its acceleration.

Normal force: $N = mg = (10)(9.81) = 98.1$ N (vertical equilibrium — no vertical acceleration).
Friction: $f = \mu N = (0.25)(98.1) = 24.5$ N (opposing the motion).
Resultant horizontal force: $F_{\text{net}} = F - f = 40 - 24.5 = 15.5$ N.
$a = \dfrac{F_{\text{net}}}{m} = \dfrac{15.5}{10} = \boxed{1.5\,\text{m}\,\text{s}^{-2}}$.

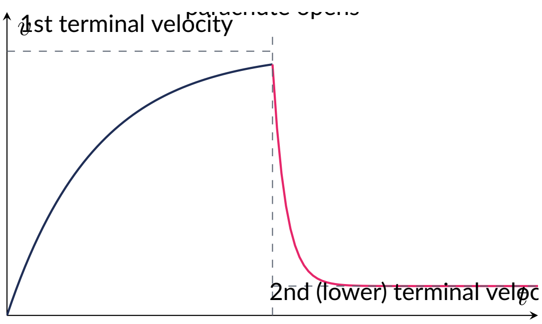
(!) Lỗi thường gặp: The normal force N is **not always equal to the object's weight** — only when the surface is horizontal *and* there is no other force with a vertical component. On a slope, $N = mg\cos\theta$ (not mg); if a rope pulls at an angle above the horizontal, its vertical component *reduces* N below mg. Always find N from vertical equilibrium (or Newton's second law in the perpendicular direction) for the specific situation, *before* calculating friction from $F = \mu N$.

GIẢI THÍCH TIẾNG VIỆT

VN Ma sát tĩnh (static) tự điều chỉnh từ 0 đến giá trị tối đa $\mu_s N$ để cân bằng lực tác dụng khi vật chưa trượt; ma sát động/trượt (kinetic) có độ lớn gần như không đổi $F = \mu_k N$ khi vật đã trượt. Lưu ý: phản lực pháp tuyến N **không phải lúc nào cũng bằng trọng lượng** – trên mặt phẳng nghiêng $N = mg\cos\theta$, cần luôn xét cân bằng theo phương vuông góc với bề mặt trước khi tính lực ma sát.

3.5 Terminal Velocity

When an object falls through a fluid (air or liquid), it experiences its weight (constant, downward) and a resistive **drag force** (upward, opposing motion) which increases as speed increases. Immediately after being released, drag is negligible and the object accelerates at close to g; as speed builds up, drag grows and the resultant force (and hence acceleration) **decreases**, even though the object is still speeding up. Eventually drag becomes exactly equal in magnitude to the weight, the resultant force becomes zero, and the object continues at a constant **terminal velocity** — it keeps moving, but no longer accelerates (Newton's first law again: zero resultant force means constant velocity).



Sketch v–t graph for a skydiver: the curve rises steeply at first (acceleration $\approx g$, drag small), then levels off as drag grows to balance weight, reaching a constant terminal velocity. When the parachute opens, drag increases sharply (larger surface area), producing a large upward resultant force that rapidly decelerates the skydiver down to a new, much lower, terminal velocity, which is then maintained until landing.

Ví dụ mẫu 8 · Explaining terminal velocity qualitatively

A skydiver jumps from a stationary balloon. Explain, in terms of the forces acting, the shape of the skydiver's speed–time graph before the parachute opens.

Immediately after jumping, speed is (nearly) zero, so air resistance (drag) is negligible; the only significant force is weight, so the skydiver accelerates at close to g. As speed increases, drag (which increases with speed) grows, so the resultant force $(W - \text{drag})$ decreases — the skydiver keeps speeding up, but at a **decreasing rate** (the graph curves over, becoming less steep). Eventually drag becomes equal in magnitude to weight; the resultant force is zero, so acceleration is zero, and the skydiver falls at a **constant terminal velocity** from then on, until the parachute opens (and the drag force suddenly increases due to the far larger surface area, producing a net upward force that rapidly reduces speed to a new, much smaller, terminal velocity for landing).

VN **GIẢI THÍCH TIẾNG VIỆT**

Khi vật rơi trong chất lưu (không khí), lực cản (drag) tăng dần theo tốc độ trong khi trọng lượng không đổi; khi lực cản bằng trọng lượng, hợp lực bằng 0 và vật đạt **vận tốc giới hạn** (terminal velocity) – tốc độ không đổi (định luật 1 Newton). Khi vận động viên nhảy dù bung dù, lực cản tăng đột ngột làm xuất hiện hợp lực hướng lên, giảm tốc nhanh xuống một vận tốc giới hạn mới thấp hơn.

3.6 Momentum and Impulse

3.6.1 Linear Momentum

The **linear momentum** of an object is

$$p = mv$$

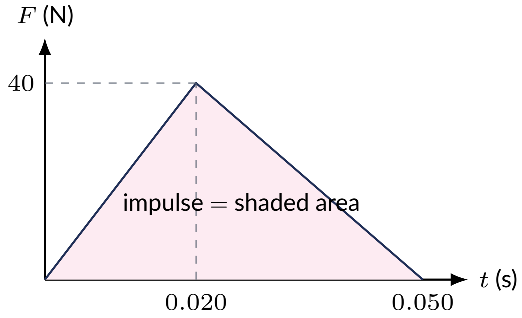
where m is mass (kg) and v is velocity (m s^{-1}); p is measured in kg m s^{-1} (equivalently N s), and — like velocity — is a **vector** quantity, with the same direction as v.

3.6.2 Impulse

Newton's second law in its general form, $F = \Delta p / \Delta t$, rearranges to define **impulse**:

$$\text{Impulse } J = F\Delta t = \Delta p = mv - mu$$

where F is a *constant* resultant force acting for time Δt. If the force varies with time, the impulse is instead equal to the **area under a force–time graph**, whatever the shape of that graph.



A non-uniform force acting over time: impulse $J = \Delta p$ equals the area under the F–t graph. Here a triangular force pulse rises to a peak of 40 N at $t = 0.020$ s and falls back to zero by $t = 0.050$ s.

Ví dụ mẫu 9 · Impulse from a non-uniform force–time graph

A force acts on a stationary ball of mass 0.50 kg, rising linearly from zero to a peak of 40 N at $t = 0.020$ s, then falling linearly back to zero by $t = 0.050$ s (as shown above). Find (a) the impulse delivered and (b) the ball's speed immediately afterward.

(a) Impulse = area of the triangle $= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2}(0.050)(40) = \boxed{1.0\,\text{N s}}$.

(b) $J = \Delta p = m\Delta v \Rightarrow \Delta v = \frac{J}{m} = \frac{1.0}{0.50} = \boxed{2.0\,\text{m s}^{-1}}$ (from rest, so this is the ball's final speed).

(!) Lỗi thường gặp: Momentum (and impulse) is a **vector** — always assign a positive direction first and be consistent. A velocity in the *opposite* direction to the one chosen as positive must be substituted as a **negative** number, even though ”speed” itself is never negative. Forgetting this sign is the single most common source of error in momentum and collision calculations.

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Động lượng $p = mv$ là đại lượng vector, đơn vị kg · m/s. Xung lực $J = F\Delta t = \Delta p$; nếu lực thay đổi theo thời gian, xung lực bằng **diện tích dưới đồ thị lực–thời gian**. Lỗi thường gặp nhất: động lượng là vector – phải chọn chiều dương và gán dấu âm cho vận tốc ngược chiều.

3.7 Conservation of Momentum and Collisions

3.7.1 Conservation of Linear Momentum

For any system of objects with **no net external force** acting (e.g. during a collision or explosion, where internal forces are far larger than any external force during the brief interaction), the **total momentum before** equals the **total momentum after**:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

This follows directly from Newton's third law: during a collision, the force each object exerts on the other is equal and opposite at every instant, so their impulses (and hence changes in momentum) are equal and opposite, and the total momentum of the system is unchanged.

Ví dụ mẫu 10 · Perfectly inelastic collision

A trolley of mass 2.0 kg moving at $6.0\,\mathrm{m\,s^{-1}}$ collides with a stationary trolley of mass 4.0 kg and sticks to it. Find their common velocity after the collision.
$m_1u_1 + m_2u_2 = (m_1 + m_2)v \;\Rightarrow\; (2.0)(6.0) + (4.0)(0) = (2.0 + 4.0)v$
$12.0 = 6.0v \;\Rightarrow\; v = \boxed{2.0\,\mathrm{m\,s^{-1}}}$ (in the direction of the original motion).

Ví dụ mẫu 11 · Explosion (one dimension)

A stationary object of mass 8.0 kg splits into two fragments: a 5.0 kg piece moving off at $6.0\,\mathrm{m\,s^{-1}}$, and a 3.0 kg piece. Find the velocity of the 3.0 kg piece.
Total momentum before $= 0$ (object was at rest), so total momentum after must also be zero:
$(5.0)(6.0) + (3.0)v_3 = 0 \;\Rightarrow\; 30.0 + 3.0v_3 = 0 \;\Rightarrow\; v_3 = \boxed{-10.0\,\mathrm{m\,s^{-1}}}$ — i.e. $10.0\,\mathrm{m\,s^{-1}}$ in the **opposite** direction to the 5.0 kg fragment.

3.7.2 Elastic and Inelastic Collisions

Momentum is conserved in every collision (elastic or inelastic), provided there is no external force. In an **elastic** collision, **kinetic energy is also conserved** (none is converted to heat, sound, or permanent deformation). In an **inelastic** collision, momentum is still conserved, but **total kinetic energy decreases**; in a **perfectly inelastic** collision, the colliding objects stick together and move with a common final velocity (this is the case of *maximum* kinetic energy loss consistent with momentum conservation).

Ví dụ mẫu 12 · Verifying whether a collision is elastic

A trolley of mass 1.0 kg moving at $4.0\,\mathrm{m\,s^{-1}}$ collides with a stationary trolley of mass 3.0 kg. After the collision, the 1.0 kg trolley rebounds at $2.0\,\mathrm{m\,s^{-1}}$ (i.e. velocity -2.0 m/s) and the 3.0 kg trolley moves off at $2.0\,\mathrm{m\,s^{-1}}$ in the original direction. Determine whether the collision is elastic.
Check momentum: before $= (1.0)(4.0) = 4.0\,\mathrm{kg\,m\,s^{-1}}$; after $= (1.0)(-2.0) + (3.0)(2.0) = -2.0 + 6.0 = 4.0\,\mathrm{kg\,m\,s^{-1}}$ ✓ (conserved, as it must always be).
Check kinetic energy: before $= \frac{1}{2}(1.0)(4.0)^2 = 8.0$ J; after $= \frac{1}{2}(1.0)(2.0)^2 + \frac{1}{2}(3.0)(2.0)^2 = 2.0 + 6.0 = \boxed{8.0\,\mathrm{J}}$.
Since kinetic energy after (8.0 J) equals kinetic energy before (8.0 J), **the collision is elastic.**

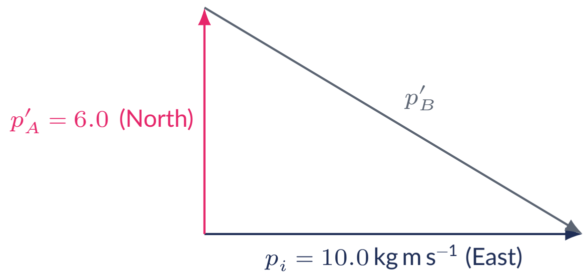
3.7.3 Momentum in Two Dimensions

When objects move (or fly apart) in directions that are not all along the same line, momentum conservation must be applied **separately to each perpendicular component** (e.g. x and y), since momentum is a vector.

Ví dụ mẫu 13 · Two-dimensional collision

A puck of mass 2.0 kg moving at $5.0\,\mathrm{m\,s^{-1}}$ due East collides with a stationary puck of mass 3.0 kg on frictionless ice. After the collision, the 2.0 kg puck moves off at $3.0\,\mathrm{m\,s^{-1}}$ due **North** (perpendicular to its original direction). Find the velocity (magnitude and direction) of the 3.0 kg puck after the collision.

Momentum before: $p_x = (2.0)(5.0) = 10.0\,\mathrm{kg\,m\,s^{-1}}$ (East), $p_y = 0$.
Momentum of the 2.0 kg puck after: $p'_x = 0$ (it now moves due North only), $p'_y = (2.0)(3.0) = 6.0\,\mathrm{kg\,m\,s^{-1}}$ (North).
By conservation, for the 3.0 kg puck: $p_{x,B} = 10.0 - 0 = 10.0\,\mathrm{kg\,m\,s^{-1}}$ (East); $p_{y,B} = 0 - 6.0 = -6.0\,\mathrm{kg\,m\,s^{-1}}$ (i.e. South).
$v_{x,B} = \dfrac{10.0}{3.0} = 3.33$ m/s; $v_{y,B} = \dfrac{-6.0}{3.0} = -2.0$ m/s.
Speed: $v_B = \sqrt{3.33^2 + 2.0^2} = \sqrt{11.1 + 4.0} = \sqrt{15.1} = \boxed{3.9\,\mathrm{m\,s^{-1}}}$.
Direction: $\theta = \arctan\left(\dfrac{2.0}{3.33}\right) = \boxed{31^\circ \text{ South of East}}$.



Vector diagram for the collision above: the initial momentum $\vec{p}_i$ must equal the **vector sum** of the two final momenta, $\vec{p}\,'_A + \vec{p}\,'_B = \vec{p}_i$. Since $\vec{p}_i$ and $\vec{p}\,'_A$ are known, $\vec{p}\,'_B$ is found by closing the triangle — here $|\vec{p}\,'_B| = \sqrt{10.0^2 + 6.0^2} = 11.7\,\mathrm{kg\,m\,s^{-1}}$, directed South of East, matching $v_B = 3.9\,\mathrm{m\,s^{-1}}$ once divided by the 3.0 kg mass.

GIẢI THÍCH TIẾNG VIỆT

VN

Động lượng luôn bảo toàn trong mọi va chạm (không cần lực ngoài); va chạm **đàn hồi** bảo toàn cả động năng, va chạm **không đàn hồi** chỉ bảo toàn động lượng còn động năng giảm (va chạm **hoàn toàn không đàn hồi** là khi hai vật dính vào nhau). Với bài toán hai chiều, phải áp dụng bảo toàn động lượng **riêng biệt theo từng phương** (x và y) vì động lượng là vector.

3.8 Practice Bank

Bài tập 1

A resultant force of 15 N acts on a 3.0 kg object initially at rest. Find its velocity after 4.0 s.

Lời giải chi tiết

$a = F/m = 15/3.0 = 5.0\,\mathrm{m\,s^{-2}}$. $v = u + at = 0 + (5.0)(4.0) = \boxed{20\,\mathrm{m\,s^{-1}}}$.

Bài tập 2

Trolley A (4.0 kg, moving at $3.0\,\mathrm{m\,s^{-1}}$ East) collides with stationary trolley B (2.0 kg) and couples to it. Find their common velocity.

Lời giải chi tiết

$m_A u_A = (m_A + m_B)v \Rightarrow (4.0)(3.0) = (6.0)v \Rightarrow v = 12.0/6.0 = \boxed{2.0\,\mathrm{m\,s^{-1}}\text{ East}}$.

Bài tập 3

A ball of mass 0.20 kg hits a wall at $20\,\mathrm{m\,s^{-1}}$ and rebounds elastically at $20\,\mathrm{m\,s^{-1}}$ (same speed, reversed direction). The ball is in contact with the wall for 0.010 s. Find the average force exerted by the wall on the ball.

Lời giải chi tiết

Taking the initial direction as positive: $\Delta v = (-20) - (20) = -40$ m/s. $\Delta p = m\Delta v = (0.20)(-40) = -8.0$ kg m/s. $F = \dfrac{|\Delta p|}{\Delta t} = \dfrac{8.0}{0.010} = \boxed{800\,\mathrm{N}}$ (directed back into the ball, away from the wall).

Bài tập 4

Using Newton's third law, explain how a horse can accelerate a cart forward, even though the cart pulls back on the horse with a force equal in magnitude to the force the horse exerts on the cart.

Lời giải chi tiết

The horse-on-cart force and the cart-on-horse force are indeed a Newton's third-law pair (equal, opposite, same type, same line of action) — but they act on **different objects** (cart and horse respectively), so they cannot cancel for either one. Whether the system accelerates forward depends on *all* the forces on the horse: the ground exerts a forward friction/reaction force on the horse's hooves (the reaction to the horse pushing backward on the ground) which can exceed the backward pull of the cart, giving a net forward force on the horse (and, through the connecting harness, the whole horse–cart system accelerates forward).

Bài tập 5

Which of the following is the correct statement of Newton's first law?

(A) An object always moves in a straight line at constant speed.

(B) An object continues at rest, or at constant velocity, unless acted on by a resultant external force.

(C) The acceleration of an object is proportional to the resultant force acting on it.

(D) Every action has an equal and opposite reaction.

Lời giải chi tiết

(B). (A) is false (objects can be at rest or moving, and "straight line at constant speed" ignores the "at rest" case, and applies only in the *absence* of a resultant force). (C) is Newton's second law. (D) is Newton's third law.

Bài tập 6

A book of weight 5.0 N rests on a table, which exerts a normal reaction force of 5.0 N upward on the book. Which pair of forces below is a genuine Newton's third-law pair?

(A) The book's weight and the table's normal reaction on the book.

(B) The book's weight (Earth pulling book down) and the book pulling the Earth up.

(C) The table's normal reaction on the book and the book's normal reaction on the table.

(D) Both (B) and (C).

Lời giải chi tiết

(D). Weight and normal reaction on the *same* object [choice (A)] are NOT a third-law pair (same object, different force types). Both (B) — the gravitational pair between Earth and book — and (C) — the contact-force pair between book and table — ARE genuine third-law pairs: each connects two different objects with an equal, opposite, same-type force.

Bài tập 7

A lift decelerates while moving upward, with acceleration of magnitude $1.5\,\mathrm{m\,s^{-2}}$ directed downward. A passenger of mass 60 kg stands on scales inside. Find the scale reading, and state whether it is more or less than the passenger's true weight.

Lời giải chi tiết

Taking up as positive, $a = -1.5\,\mathrm{m/s^2}$ (deceleration while moving up means acceleration points down). $N = m(g + a) = (60)(9.81 - 1.5) = (60)(8.31) = \boxed{499\,\mathrm{N}}$. True weight $= mg = (60)(9.81) = 589$ N. Since $499\,\mathrm{N} < 589\,\mathrm{N}$, the scale reads **less** than the true weight (the passenger feels momentarily lighter).

Bài tập 8

A block of mass 8.0 kg slides down a rough incline at $20\,^\circ$ to the horizontal, coefficient of kinetic friction $\mu = 0.30$. Find its acceleration down the slope.

Lời giải chi tiết

$mg\sin\theta = (8.0)(9.81)(\sin 20^\circ) = (78.5)(0.342) = 26.8\,\mathrm{N}$ (down the slope).
$N = mg\cos\theta = (78.5)(\cos 20^\circ) = (78.5)(0.940) = 73.8\,\mathrm{N}$.
$f = \mu N = (0.30)(73.8) = 22.1\,\mathrm{N}$ (up the slope, opposing the sliding).
$F_{\text{net}} = 26.8 - 22.1 = 4.7\,\mathrm{N}$. $a = F_{\text{net}}/m = 4.7/8.0 = \boxed{0.59\,\mathrm{m\,s^{-2}}}$ down the slope.

Bài tập 9

Masses of 6.0 kg and 4.0 kg are connected over a smooth, light pulley and released from rest. Find (a) the acceleration of the system and (b) the tension in the string.

Lời giải chi tiết

(a) $a = \dfrac{(6.0-4.0)(9.81)}{6.0+4.0} = \dfrac{19.62}{10.0} = \boxed{2.0\,\text{m}\,\text{s}^{-2}}$.
(b) $T = m_2(g+a) = (4.0)(9.81+1.962) = (4.0)(11.77) = \boxed{47.1\,\text{N}}$ (check: $T = m_1(g-a) = (6.0)(9.81-1.962) = (6.0)(7.848) = 47.1$ N ✓).

Bài tập 10

Blocks A (5.0 kg) and B (3.0 kg) are connected by a light string, lying on a rough horizontal surface with coefficient of kinetic friction $\mu = 0.20$ for both blocks. A horizontal force of 32 N is applied to block A, pulling block B behind it via the string. Find (a) the acceleration of the system and (b) the tension in the connecting string.

Lời giải chi tiết

Total mass $= 8.0$ kg; total normal force $= 8.0 \times 9.81 = 78.5$ N; total friction $= \mu \times 78.5 = (0.20)(78.5) = 15.7$ N.
(a) $a = \dfrac{F - f_{\text{total}}}{m_{\text{total}}} = \dfrac{32-15.7}{8.0} = \dfrac{16.3}{8.0} = \boxed{2.0\,\text{m}\,\text{s}^{-2}}$.
(b) Isolate block B (pulled only by tension T, opposed by its own friction $f_B = \mu m_B g = (0.20)(3.0)(9.81) = 5.89$ N): $T - f_B = m_B a \Rightarrow T = (3.0)(2.038) + 5.89 = 6.11 + 5.89 = \boxed{12.0\,\text{N}}$.
Check with block A: $F - T - f_A = m_A a$, where $f_A = (0.20)(5.0)(9.81) = 9.81$ N: $32 - 12.0 - 9.81 = 10.2 \approx (5.0)(2.038) = 10.2$ ✓.

Bài tập 11

Which statement about friction is correct?

(A) The maximum static friction force between two given surfaces is generally greater than the kinetic friction force between the same surfaces.

(B) Kinetic friction increases without limit as speed increases.

(C) The friction force does not depend on the normal contact force.

(D) Static friction is always exactly equal to $\mu_s N$, regardless of the applied force.

Lời giải chi tiết

(A). (B) is false — kinetic friction is approximately constant, $F = \mu_k N$, independent of speed. (C) is false — friction is directly proportional to N. (D) is false — static friction adjusts to match the applied force, reaching its maximum $\mu_s N$ only at the point of slipping.

Bài tập 12

The coefficient of kinetic friction μ between two surfaces is defined as:

(A) the ratio of the friction force to the normal contact force between the surfaces

(B) the ratio of the friction force to the weight of the object

(C) the ratio of the applied force to the normal contact force

(D) the ratio of the normal contact force to the weight of the object

Lời giải chi tiết

(A). By definition, $F = \mu N$, so $\mu = F/N$ — the ratio of friction force to normal contact force (not weight, which only equals N on a horizontal surface with no other vertical forces).

Bài tập 13

A block rests on an incline at $25\,^\circ$ to the horizontal. The coefficient of static friction between block and incline is $\mu_s = 0.50$. Determine, with a calculation, whether the block remains stationary or slides.

Lời giải chi tiết

The block slides only if the required friction exceeds the maximum available, i.e. if $\tan\theta > \mu_s$. Here $\tan 25^\circ = 0.466$, and $\mu_s = 0.50$. Since $0.466 < 0.50$, the maximum available static friction is **more than enough** to balance the down-slope component of weight, so the block **remains stationary** (does not slide).

Bài tập 14

Explain, in terms of the forces acting, why a raindrop falling from a cloud reaches a constant (terminal) velocity rather than continuing to accelerate all the way to the ground.

Lời giải chi tiết

The raindrop's weight is constant, but air resistance (drag) increases as its falling speed increases. Initially weight exceeds drag, so the raindrop accelerates; as speed grows, drag grows too, so the resultant force (and acceleration) decreases. Eventually drag becomes exactly equal to weight, the resultant force is zero, and (by Newton's first law) the raindrop continues at this constant terminal velocity for the remainder of its fall.

Bài tập 15

Which of the following is a correct SI unit for impulse?

(A) N s (equivalent to kg m s^{-1})

(B) N s^{-1}

(C) J s

(D) kg s^{-1}

Lời giải chi tiết

(A). Impulse $= F\Delta t$, so its unit is (force)×(time)=N s; since $\Delta p = m\Delta v$ has unit kg m s^{-1}, and 1 N $= 1$ kg m s^{-2}, these two units are indeed equivalent.

Bài tập 16

A constant force of 6.0 N acts on a 2.0 kg trolley, initially at rest, for 5.0 s. Find its final velocity.

Lời giải chi tiết

Impulse $= F\Delta t = (6.0)(5.0) = 30$ N s $= \Delta p = m\Delta v$. $\Delta v = 30/2.0 = \boxed{15\,\text{m s}^{-1}}$ (final velocity, since it started from rest).

Bài tập 17

A constant force of $25\,\text{N}$ acts for $0.40\,\text{s}$ on a stationary object of mass $5.0\,\text{kg}$. Find (a) the impulse and (b) the object's final speed.

Lời giải chi tiết

(a) $J = F\Delta t = (25)(0.40) = \boxed{10.0\,\text{N s}}$.
(b) $\Delta v = J/m = 10.0/5.0 = \boxed{2.0\,\text{m s}^{-1}}$.

Bài tập 18

Momentum is conserved in a collision between two objects provided that:

(A) the collision is elastic

(B) there is no net external force acting on the system during the collision

(C) the two objects have equal mass

(D) kinetic energy is conserved

Lời giải chi tiết

(B). Conservation of momentum follows from Newton's third law alone and applies to **any** collision (elastic or inelastic), regardless of the masses involved, as long as there is no external resultant force on the system. (A) and (D) describe the extra condition for an *elastic* collision specifically, and (C) is irrelevant.

Bài tập 19

A firework shell of mass $2.0\,\text{kg}$ moving horizontally at $15\,\text{m s}^{-1}$ explodes into two fragments of equal mass ($1.0\,\text{kg}$ each). Immediately after the explosion, one fragment moves vertically upward at $20\,\text{m s}^{-1}$ (perpendicular to the original direction). Find the velocity (magnitude and direction) of the other fragment.

Lời giải chi tiết

Momentum before: $p_x = (2.0)(15) = 30\,\text{kg m s}^{-1}$ (East), $p_y = 0$.
Fragment A (up): $p_{x,A} = 0$, $p_{y,A} = (1.0)(20) = 20\,\text{kg m s}^{-1}$ (up).
Fragment B: $p_{x,B} = 30 - 0 = 30\,\text{kg m s}^{-1}$; $p_{y,B} = 0 - 20 = -20\,\text{kg m s}^{-1}$ (down).
$v_{x,B} = 30/1.0 = 30\,\text{m/s}$; $v_{y,B} = -20/1.0 = -20\,\text{m/s}$.
Speed $= \sqrt{30^2 + 20^2} = \sqrt{900 + 400} = \sqrt{1300} = \boxed{36\,\text{m s}^{-1}}$; direction $= \arctan(20/30) = \boxed{34^\circ \text{ below the original (horizontal) direction}}$.

Bài tập 20

Puck A ($0.50\,\text{kg}$, moving at $4.0\,\text{m s}^{-1}$ East) collides with stationary puck B ($0.50\,\text{kg}$) on frictionless ice. After the collision, puck A moves off at $2.0\,\text{m s}^{-1}$, deflected 60° from its original direction. Find the velocity (magnitude and direction) of puck B.

Lời giải chi tiết

Momentum before: $p_x = (0.50)(4.0) = 2.0\,\mathrm{kg\,m\,s^{-1}}$, $p_y = 0$.
Puck A after: $p'_{x,A} = (0.50)(2.0)\cos 60^\circ = (1.0)(0.500) = 0.50$; $p'_{y,A} = (1.0)(\sin 60^\circ) = (1.0)(0.866) = 0.866$.
Puck B: $p'_{x,B} = 2.0 - 0.50 = 1.50$; $p'_{y,B} = 0 - 0.866 = -0.866$.
$v_{x,B} = 1.50/0.50 = 3.0\,\mathrm{m/s}$; $v_{y,B} = -0.866/0.50 = -1.73\,\mathrm{m/s}$.
Speed $= \sqrt{3.0^2 + 1.73^2} = \sqrt{9.0 + 3.0} = \sqrt{12.0} = \boxed{3.5\,\mathrm{m\,s^{-1}}}$; direction $= \arctan(1.73/3.0) = \boxed{30^\circ \text{ below the original direction of A}}$.

Bài tập 21

A trolley of mass $2.0\,\mathrm{kg}$ moving at $6.0\,\mathrm{m\,s^{-1}}$ collides with a stationary trolley of mass $4.0\,\mathrm{kg}$. After the collision, the $2.0\,\mathrm{kg}$ trolley is at rest and the $4.0\,\mathrm{kg}$ trolley moves off at $3.0\,\mathrm{m\,s^{-1}}$. Determine whether this collision is elastic.

Lời giải chi tiết

Momentum before $= (2.0)(6.0) = 12.0\,\mathrm{kg\,m\,s^{-1}}$; after $= (2.0)(0) + (4.0)(3.0) = 12.0\,\mathrm{kg\,m\,s^{-1}}$ ✓ conserved.
KE before $= \frac{1}{2}(2.0)(6.0)^2 = 36.0\,\mathrm{J}$; KE after $= \frac{1}{2}(2.0)(0)^2 + \frac{1}{2}(4.0)(3.0)^2 = 0 + 18.0 = 18.0\,\mathrm{J}$.
Since KE after ($18.0\,\mathrm{J}$) is **less** than KE before ($36.0\,\mathrm{J}$), the collision is **not elastic** (it is inelastic — half the kinetic energy is lost, converted to heat/sound/deformation).

Bài tập 22

A rifle of mass $4.0\,\mathrm{kg}$, initially at rest, fires a bullet of mass $0.010\,\mathrm{kg}$ ($10\,\mathrm{g}$) forward at $400\,\mathrm{m\,s^{-1}}$. Find the recoil velocity of the rifle.

Lời giải chi tiết

Total momentum before (rifle at rest) $= 0$, so total momentum after must also be zero: $m_{\text{bullet}}v_{\text{bullet}} + m_{\text{rifle}}v_{\text{rifle}} = 0$.
$(0.010)(400) + (4.0)v_{\text{rifle}} = 0 \Rightarrow 4.0 + 4.0v_{\text{rifle}} = 0 \Rightarrow v_{\text{rifle}} = \boxed{-1.0\,\mathrm{m\,s^{-1}}}$ — i.e. the rifle recoils backward at $1.0\,\mathrm{m\,s^{-1}}$.

Bài tập 23

Which quantity is always conserved in any collision (elastic or inelastic), provided there is no external force on the system?

(A) Momentum only
(B) Kinetic energy only
(C) Both momentum and kinetic energy
(D) Neither momentum nor kinetic energy

Lời giải chi tiết

(A). Momentum is conserved in *every* collision as a direct consequence of Newton's third law. Kinetic energy is conserved **only** in elastic collisions — in inelastic collisions some kinetic energy is always lost.

Bài tập 24

Trolley A (3.0 kg, $4.0\,\mathrm{m\,s^{-1}}$ East) and trolley B (2.0 kg, $5.0\,\mathrm{m\,s^{-1}}$ West) move toward each other and collide, sticking together. Find their common velocity (magnitude and direction) after the collision.

Lời giải chi tiết

Taking East as positive: momentum before $= (3.0)(4.0) + (2.0)(-5.0) = 12.0 - 10.0 = 2.0\,\mathrm{kg\,m\,s^{-1}}$.
$v = \dfrac{2.0}{3.0+2.0} = \dfrac{2.0}{5.0} = \boxed{0.40\,\mathrm{m\,s^{-1}}\text{ East}}$ (positive, so the combined wreckage moves slowly in trolley A's original direction).

Bài tập 25

A force rises linearly from zero to a peak of 60 N over the first 0.10 s, then falls linearly back to zero over the next 0.15 s (a triangular pulse, total duration 0.25 s), acting on a stationary ball of mass 1.5 kg. Find (a) the impulse delivered and (b) the ball's final velocity.

Lời giải chi tiết

(a) Impulse $=$ area of triangle $= \dfrac{1}{2} \times$ total base $\times$ height $= \dfrac{1}{2}(0.25)(60) = \boxed{7.5\,\mathrm{N\,s}}$.

(b) $\Delta v = J/m = 7.5/1.5 = \boxed{5.0\,\mathrm{m\,s^{-1}}}$ (final velocity, from rest).

Bài tập 26

An object is in equilibrium under three coplanar forces: 6.0 N due North, 8.0 N due East, and a third force F. What magnitude must F have for the object to remain in equilibrium?

(A) 10.0 N
(B) 14.0 N
(C) 2.0 N
(D) 100 N

Lời giải chi tiết

(A). The resultant of 6.0 N North and 8.0 N East has magnitude $\sqrt{6.0^2+8.0^2} = \sqrt{36+64} = \sqrt{100} = 10.0$ N (a 6–8–10 right triangle), directed arctan$(6/8) = 36.9^\circ$ North of East. For equilibrium, F must be equal in magnitude (10.0 N) and exactly opposite in direction (i.e. 36.9° South of West) to this resultant. Choice (B) incorrectly adds the two forces directly (ignoring their perpendicular directions); choice (C) incorrectly subtracts them; choice (D) forgets to take the square root.

Forces, Density & Pressure

Mục tiêu Unit: Các loại lực (trọng lực, phản lực pháp tuyến, ma sát, lực căng, lực đẩy Archimedes, lực cản nhớt); tổng hợp lực và điều kiện cân bằng của một chất điểm (hợp lực, tam giác lực, phân tích lực theo hai phương vuông góc); mô-men lực, ngẫu lực và nguyên lý mô-men; trọng tâm và hai điều kiện cân bằng của vật rắn (cân bằng lực VÀ cân bằng mô-men); khối lượng riêng; áp suất, áp suất thủy tĩnh và lực đẩy Archimedes (điều kiện nổi/chìm).

4.1 Types of Force

A **force** is a push or a pull, arising from the interaction between two objects, that can change an object's velocity (speed and/or direction), or change its shape. Forces are vector quantities, measured in newtons (N), and every force in mechanics can be classified as either a **contact force** (the two interacting objects are physically touching) or a **long-range (field) force** (the objects interact without touching, across empty space).

Contact forces include: **normal contact force**, **friction**, **tension**, **upthrust**, and **viscous drag/air resistance** — all of these require the surfaces or materials of the two objects to be touching. **Long-range (field) forces** act at a distance, with no contact required: **gravitational force** (weight is a special case — the gravitational pull of the Earth on a mass), **electrostatic force** (between charged objects), and **magnetic force** (between magnets or moving charges).

4.1.1 Weight

The **weight** of an object is the force of gravitational attraction exerted on it (in this course, by the Earth), given by

$$W = mg$$

where W is weight in newtons (N), m is mass in kilograms (kg), and g is the gravitational field strength (numerically equal to the acceleration of free fall), $g = 9.81\,\mathrm{m\,s^{-2}}$ near the Earth's surface, unless a different value is stated. Weight always acts **vertically downwards**, and may be treated as acting at a single point called the **centre of gravity** of the object (defined and located in Section 3).

4.1.2 Normal Contact Force

When two surfaces are pressed together, each exerts a **normal contact force** on the other, directed **perpendicular to the surfaces in contact**, pushing them apart. It is the force that prevents solid objects from passing through one another — for example, the force a table exerts upwards on a book resting on it, or the force a wall exerts horizontally on a ladder leaning against it.

4.1.3 Friction

Friction is a contact force that opposes relative motion (or attempted relative motion) between two surfaces in contact, acting **along** the surfaces (tangential to them), in the direction that resists sliding. Friction is what allows walking, braking a car, and holding an object without it slipping from your grip;

it also causes surfaces to heat up when they slide against one another, converting kinetic energy into thermal energy.

4.1.4 Tension

Tension is the pulling force transmitted through a stretched string, rope, cable, spring, or rod, acting along its length and pulling inwards at each end. A string can only **pull**, never push — this is why tension always acts to pull an object towards the point of attachment of the string, never to push it away.

4.1.5 Upthrust (Buoyancy)

Upthrust is the upward force exerted by a fluid (liquid or gas) on any object that is wholly or partly immersed in it, arising because fluid pressure increases with depth (Section 5 derives this and gives the full quantitative treatment, together with Archimedes' principle).

4.1.6 Viscous Drag and Air Resistance

Viscous drag (in a liquid) and **air resistance** (in a gas) are contact forces that oppose the relative motion of an object through a fluid, acting in the direction opposite to the object's velocity relative to the fluid. Unlike friction between two solid surfaces, these forces generally increase in size as the object's speed through the fluid increases, which is why a falling object (e.g. a skydiver) can reach a **terminal velocity**, where drag has grown large enough to balance the weight.

VN GIẢI THÍCH TIẾNG VIỆT

Lực là một đẩy hoặc kéo, có thể là **lực tiếp xúc** (hai vật chạm nhau: phản lực pháp tuyến, ma sát, lực căng, lực đẩy Archimedes, lực cản) hoặc **lực tác dụng từ xa** (không cần chạm nhau: trọng lực, lực tĩnh điện, lực từ). Trọng lượng $W = mg$ luôn hướng thẳng đứng xuống dưới và coi như tác dụng tại **trọng tâm**. Phản lực pháp tuyến vuông góc với bề mặt tiếp xúc; ma sát tác dụng dọc theo bề mặt, cản trở chuyển động tương đối; lực căng dây chỉ có thể **kéo**, không thể đẩy; lực cản (ma sát nhớt/lực cản không khí) tăng theo tốc độ, cho phép vật đạt **vận tốc giới hạn (terminal velocity)**.

Every force analysis begins with a clear **free-body diagram**, showing an object as a single point or outline with all the forces acting *on* it (never the forces it exerts on other objects) drawn as labelled arrows, starting from the object and pointing in the direction each force acts — the convention introduced in the Dynamics chapter, used throughout this unit and, indeed, throughout the whole of mechanics.

4.2 Combining Forces and Equilibrium of a Particle

When two or more forces act on the same point (or on an object small enough to be treated as a single point — a **particle**), they may be combined into a single equivalent force called the **resultant**, which has exactly the same effect as all the original forces acting together.

4.2.1 Finding a Resultant: Parallelogram / Triangle of Forces

For **two** forces, the resultant can be found by scale drawing: draw the two force vectors tip-to-tail (**triangle of forces**), or from a common point as two adjacent sides of a parallelogram (**parallelogram of forces**) — the resultant is the diagonal of the parallelogram, or equivalently the vector joining the start of the first force to the end of the second when drawn tip-to-tail. This can be done accurately with a scale drawing, or calculated using trigonometry.

For two forces that happen to be **perpendicular** to one another, the resultant's magnitude follows directly from Pythagoras' theorem, and its direction from simple trigonometry (tan $\theta =$ opposite/adjacent). For forces at other angles, the **cosine rule** and **sine rule** may be used on the triangle of forces, or (more commonly, and more generally) the forces can instead be **resolved** into perpendicular (x and y) components and the components summed separately (Section 2.2).

4.2.2 Resolving Forces into Components

Any force F can be replaced by two perpendicular **components** without changing its effect: if F acts at angle θ to a chosen x-axis,

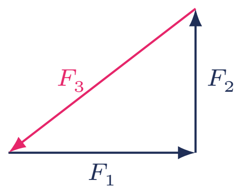
$$F_x = F\cos\theta \qquad F_y = F\sin\theta.$$

To find the resultant of several forces acting on a point, resolve *every* force into its x- and y-components, sum each set of components separately (ΣF_x, ΣF_y), and then recombine:

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}, \qquad \tan\theta = \frac{\Sigma F_y}{\Sigma F_x}.$$

4.2.3 Equilibrium of a Particle

A particle (or point object) is in **equilibrium** when the **vector sum of all the forces acting on it is zero**: $\Sigma F_x = 0$ and $\Sigma F_y = 0$ simultaneously (equivalently, the resultant force $R = 0$). Drawn as a triangle or polygon of forces, this means the vectors form a **closed shape** — there is no gap left over representing an unbalanced resultant.



Triangle of forces: when three forces acting on a point are in equilibrium, drawing them tip-to-tail (F_1, then F_2, then F_3) always produces a **closed** triangle — there is no leftover gap, confirming the resultant is zero.

Ví dụ mẫu · Triangle of forces: finding an unknown third force

A small ring is held in equilibrium on a smooth peg by three coplanar forces. Two of the forces are $F_1 = 6.0$ N, acting horizontally, and $F_2 = 8.0$ N, acting vertically (perpendicular to F_1). Find the magnitude and direction of the third force F_3 required to keep the ring in equilibrium.

Since F_1 and F_2 are perpendicular, their resultant R has magnitude, by Pythagoras,

$$R = \sqrt{F_1^2 + F_2^2} = \sqrt{6.0^2 + 8.0^2} = \sqrt{36 + 64} = \sqrt{100} = 10\,\text{N},$$

at an angle $\theta = \tan^{-1}\left(\frac{8.0}{6.0}\right) = \tan^{-1}(1.333) = 53.1^\circ$ from the direction of F_1.

For equilibrium, F_3 must be equal in magnitude to this resultant but acting in exactly the **opposite** direction (this is the **equilibrant**):

$$F_3 = \boxed{10\,\text{N, at } 53.1^\circ \text{ from the reverse of } F_1\text{'s direction}}$$

(i.e. $180^\circ + 53.1^\circ = 233.1^\circ$ measured from the original direction of F_1).

Ví dụ mẫu · Resolving three coplanar forces into components

A ring is held in equilibrium by three coplanar forces: $P = 10\,\text{N}$ acting at 0° (along the $+x$-axis), $Q = 10\,\text{N}$ acting at 120° (measured anticlockwise from the $+x$-axis), and an unknown third force F. Find the magnitude and direction of F.

Resolve P and Q into components:

$$P_x = 10\cos 0^\circ = 10, \quad P_y = 10\sin 0^\circ = 0.$$

$$Q_x = 10\cos 120^\circ = 10(-0.500) = -5.0, \quad Q_y = 10\sin 120^\circ = 10(0.866) = 8.66.$$

Sum: $\Sigma F_x = 10 - 5.0 = 5.0$; $\quad \Sigma F_y = 0 + 8.66 = 8.66$.

For equilibrium, F must exactly cancel this sum, so $F_x = -5.0\,\text{N}$ and $F_y = -8.66\,\text{N}$. Magnitude:

$$F = \sqrt{5.0^2 + 8.66^2} = \sqrt{25 + 75} = \sqrt{100} = \boxed{10\,\text{N}}.$$

Direction: reference angle $= \tan^{-1}(8.66/5.0) = 60.0^\circ$; since both components are negative, F lies in the third quadrant, at $180^\circ + 60.0^\circ = \boxed{240^\circ}$ from the $+x$-axis. (This is a neat, symmetric result: three equal forces spaced exactly 120° apart always balance exactly — a useful pattern to recognise.)

GIẢI THÍCH TIẾNG VIỆT

VN Có thể tổng hợp hai lực bằng **quy tắc hình bình hành** (hoặc tam giác lực, vẽ nối đuôi các vector) hoặc bằng cách **phân tích lực** thành hai thành phần vuông góc ($F_x = F\cos\theta$, $F_y = F\sin\theta$) rồi cộng riêng từng thành phần. Một **chất điểm** ở trạng thái **cân bằng** khi tổng hợp lực bằng không ($\Sigma F_x = 0$ và $\Sigma F_y = 0$); vẽ thành tam giác/đa giác lực thì hình phải **khép kín**, không còn khoảng hở nào biểu diễn hợp lực dư.

4.3 Turning Effects of Forces

4.3.1 Moment of a Force

A force can cause an object to rotate about a fixed point (a **pivot** or **axis**) even while producing no overall translation. The turning effect of a single force about a given point is called its **moment**.

$$\text{Moment of a force} = F \times d$$

where F is the size of the force (N) and d is the **perpendicular distance** from the pivot to the **line of action** of the force (m) — *not* simply the distance to the point where the force is applied, unless the force happens to act exactly perpendicular to the line joining the pivot to that point. The unit of moment is the **newton metre (N m)**. A moment is described as **clockwise** or **anticlockwise** according to the sense of rotation it would produce.

If a force does *not* act perpendicular to the line from the pivot to its point of application, the perpendicular distance must be found using trigonometry (e.g. if a force F acts at the end of a lever arm of length L, at angle θ to the arm, the perpendicular distance is $L\sin\theta$, so the moment is $M = FL\sin\theta$).

(!) Lỗi thường gặp: The distance used in a moment calculation must always be the **perpendicular** distance from the pivot to the force's line of action — *not* the straight-line distance to the point of application, unless the force happens to act perpendicular to that line. Forgetting to resolve an angled force (or use $d = L\sin\theta$) is one of the most common moment-calculation errors at this level.

Ví dụ mẫu · Moment of a force at an angle

A uniform shop sign of weight $80\,\text{N}$ and length $1.2\,\text{m}$ is hinged to a wall at one end and held horizontal by a cable attached to the other end, making an angle of $40°$ with the sign. Find the tension T in the cable, taking moments about the hinge (ignore the wall's own reaction force, since it passes through the hinge and has zero moment about it).

Moment of the weight about the hinge (weight acts vertically, at the midpoint, so perpendicular distance = horizontal distance from hinge $= \frac{1.2}{2} = 0.60\,\text{m}$):

$$M_W = 80 \times 0.60 = 48\,\text{N m}.$$

Moment of the tension about the hinge: perpendicular component of T is $T \sin 40°$, acting at the full length $1.2\,\text{m}$ from the hinge:

$$M_T = T \sin 40° \times 1.2 = T(0.643)(1.2) = 0.771\,T.$$

By the principle of moments (Section 3.2), $M_T = M_W$:

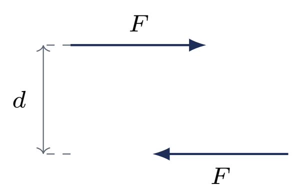
$$0.771\,T = 48 \;\Rightarrow\; T = \boxed{62\,\text{N}}.$$

4.3.2 Couples

A **couple** is a pair of forces that are **equal in magnitude**, **opposite in direction**, and **parallel**, but whose lines of action do **not coincide** (they are offset by some perpendicular distance). A couple produces a pure turning effect (**torque**) with **no resultant force** (since the two forces cancel exactly) — this is why, for example, turning a steering wheel or a screwdriver rotates the object without pushing it sideways.

$$\text{Torque of a couple} = F \times d$$

where F is the size of *one* of the two forces, and d is the perpendicular distance **between** their two lines of action.



A couple: two equal, opposite, parallel forces F, offset by perpendicular distance d. The resultant force is zero, but the torque $F \times d$ produces pure rotation.

Ví dụ mẫu · Torque of a couple

Two forces of $3.0\,\text{N}$ each act in opposite directions on the rim of a steering wheel of diameter $0.36\,\text{m}$, one on each side. Calculate the torque of the couple.

$$\text{Torque} = F \times d = 3.0 \times 0.36 = \boxed{1.08\,\text{N m}}$$

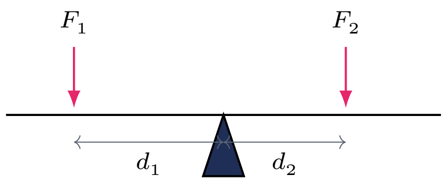
(the perpendicular distance between the two forces' lines of action is the full diameter, since the forces act at opposite ends of a diameter).

VN GIẢI THÍCH TIẾNG VIỆT

Mô-men lực $= F \times d$, với d là **khoảng cách vuông góc** từ trục quay đến đường tác dụng của lực (không phải khoảng cách bất kỳ). **Ngẫu lực (couple)** là một cặp lực bằng nhau, ngược chiều, song song nhưng không cùng đường tác dụng — không tạo hợp lực nhưng tạo ra mô-men xoắn (torque) $= F \times d$, với d là khoảng cách vuông góc giữa hai đường tác dụng của lực.

4.3.3 The Principle of Moments

Principle of moments: for a body in rotational equilibrium, the **sum of the clockwise moments** about any point equals the **sum of the anticlockwise moments** about that same point.



Principle of moments for a beam pivoted at a point: for rotational equilibrium, $F_1 d_1 = F_2 d_2$ (the moment tending to rotate the beam one way about the pivot equals the moment tending to rotate it the other way).

Ví dụ mẫu · Seesaw: finding an unknown distance

A seesaw is pivoted at its centre. A child of weight 300 N sits 1.5 m from the pivot. How far from the pivot, on the other side, must a child of weight 250 N sit for the seesaw to balance? (Assume the seesaw itself is uniform, so its own weight acts exactly at the pivot and has zero moment about it.)

By the principle of moments: $300 \times 1.5 = 250 \times d$

$$d = \frac{300 \times 1.5}{250} = \frac{450}{250} = \boxed{1.8\,\text{m}}.$$

Ví dụ mẫu · Uniform beam on two supports: finding both reactions

A uniform plank AB, length 6.0 m and weight 600 N, rests horizontally on two supports: one at end A, and one at point C, located 5.0 m from A (so the plank overhangs C by 1.0 m). A person of weight 450 N stands at the far end B (on the overhanging section). Find the support reactions R_A and R_C.

Condition 1 (force balance): $R_A + R_C = 600 + 450 = 1050$ N.

Condition 2 (moments): take moments about A (this eliminates R_A, since it passes through A). The plank's weight acts at its midpoint, 3.0 m from A; the person is at 6.0 m from A; both downward weights give clockwise moments about A, balanced by the anticlockwise moment of the upward reaction R_C at 5.0 m:

$$R_C \times 5.0 = 600 \times 3.0 + 450 \times 6.0 = 1800 + 2700 = 4500$$

$$R_C = \frac{4500}{5.0} = \boxed{900\,\text{N}}.$$

Then, from Condition 1:

$$R_A = 1050 - 900 = \boxed{150\,\text{N}}.$$

Check (moments about C instead): clockwise moments about C are $R_A \times 5.0$ (upward force to the left of C) plus 450×1.0 (person, 1.0 m to the right of C); anticlockwise is the plank's weight, 600×2.0 (2.0 m to the left of C):

$$R_A \times 5.0 + 450 \times 1.0 = 600 \times 2.0 \;\Rightarrow\; 5.0\,R_A = 1200 - 450 = 750 \;\Rightarrow\; R_A = 150\,\text{N} \checkmark$$

4.3.4 Centre of Gravity

The **centre of gravity** of an object is the single point at which the *entire weight* of the object may be considered to act, for the purposes of calculating moments (and for translational motion in general). For a uniform, symmetric object, it lies at the geometric centre.

Locating the centre of gravity experimentally (plumb-line method): an irregularly-shaped lamina (a flat sheet) is suspended so it can swing freely from a pin through a hole near its edge; once it hangs at rest, a **plumb line** (a thread with a small mass hanging from it) is hung from the same pin, and a line is drawn on the lamina along the plumb line's path — the centre of gravity must lie somewhere on this line (since the lamina hangs so that its centre of gravity is directly below the point of suspension). The lamina is then suspended from a *second*, different point near its edge, and the process is repeated to draw a second line. The centre of gravity is located at the point where the two lines **intersect**.

4.3.5 Stability and Toppling

An object will **topple over** if it is tilted far enough that the vertical line through its centre of gravity falls **outside its base area** (the region of contact with the ground) — once this happens, the weight's moment about the edge of the base acts to continue the rotation rather than restore the object to its original position. An object is more **stable** (harder to topple) if it has a **low centre of gravity** and/or a **wide base area**, since a greater angle of tilt is then required before the vertical line through the centre of gravity moves outside the base.

4.3.6 Equilibrium of a Rigid (Extended) Body

A rigid body — unlike an idealised particle, which has no physical size — can rotate as well as translate, so its equilibrium requires *two* separate conditions to both be satisfied simultaneously.

For a rigid, extended body to be in **equilibrium**, **both** of the following must hold:
(i) the resultant force acting on it is zero ($\Sigma F_x = 0$ and $\Sigma F_y = 0$); **and**
(ii) the resultant moment about *any* point is zero (equivalently, the sum of clockwise moments equals the sum of anticlockwise moments about any chosen point).
Condition (i) alone is **not sufficient** for an extended body: a couple, for example, gives zero resultant force but a non-zero resultant moment, so the body would still rotate. This is the key difference from the equilibrium of a single particle (Section 2.3), where condition (i) by itself is the complete requirement, since a particle has no physical extent and so cannot experience a net turning effect.

(!) Lỗi thường gặp: A very common exam error is to treat the equilibrium of an extended (rigid) body the same as the equilibrium of a particle, checking *only* that the resultant force is zero. For a rigid body this is not enough: the resultant **moment** about any point must also be zero, otherwise the body has zero net force but is still free to rotate (exactly the situation created by a couple).

GIẢI THÍCH TIẾNG VIỆT

VN

Trọng tâm là điểm mà toàn bộ trọng lượng của vật có thể coi như tác dụng tại đó; xác định bằng **phương pháp dây dọi**: treo tự do vật tại một điểm, đánh dấu đường dây dọi, lặp lại ở điểm treo thứ hai, giao điểm hai đường chính là trọng tâm. Vật **lật đổ (toppling)** khi đường thẳng đứng qua trọng tâm rơi ra ngoài đế đứng (base); trọng tâm càng thấp và đế càng rộng thì vật càng **ổn định**. Đối với **vật rắn (extended body)**, cần **cả hai** điều kiện cân bằng: (i) hợp lực bằng không và (ii) tổng mô-men quanh một điểm bất kỳ bằng không – đây là điểm khác biệt quan trọng so với cân bằng của một chất điểm (chỉ cần điều kiện (i)), vì một ngẫu lực có hợp lực bằng không nhưng vẫn có mô-men khác không, khiến vật vẫn quay được.

4.4 Density

$$\rho = \frac{m}{V}$$

where ρ (rho) is **density**, m is mass (kg), and V is volume (m^3); the SI unit of density is **kilograms per cubic metre** ($kg\,m^{-3}$). Density is a property of the **material** an object is made from, independent of the size or shape of any particular sample of that material.

Material	Typical density
Air (sea level, room temperature)	$\approx 1.2\,kg\,m^{-3}$
Ice	$\approx 920\,kg\,m^{-3}$
Fresh water	$1000\,kg\,m^{-3}$
Seawater	$\approx 1030\,kg\,m^{-3}$
Steel	$\approx 7900\,kg\,m^{-3}$

Typical densities used for calculation in this unit; real samples vary somewhat with temperature, alloy composition, or (for seawater) salinity.

Ví dụ mẫu · Mass from density and volume

A block of steel (density $7900\,kg\,m^{-3}$) has a volume of $1.0 \times 10^{-3}\,m^3$ ($1000\,cm^3$). Find its mass.

$$m = \rho V = 7900 \times 1.0 \times 10^{-3} = \boxed{7.9\,kg}.$$

Ví dụ mẫu · Density of an irregular solid by displacement

A stone of mass $240\,g$ ($= 0.240\,kg$) is lowered into a measuring cylinder of water; the water level rises from $50\,cm^3$ to $130\,cm^3$. Find the density of the stone, in $kg\,m^{-3}$.

Volume of stone = volume of water displaced $= 130 - 50 = 80\,cm^3 = 80 \times 10^{-6}\,m^3 = 8.0 \times 10^{-5}\,m^3$.

$$\rho = \frac{m}{V} = \frac{0.240}{8.0 \times 10^{-5}} = \boxed{3000\,kg\,m^{-3}}.$$

VN **GIẢI THÍCH TIẾNG VIỆT**

Khối lượng riêng $\rho = m/V$ (đơn vị kg m^{-3}) là một tính chất của **vật liệu**, không phụ thuộc kích thước hay hình dạng mẫu vật. Với vật rắn có hình dạng bất định (không đo trực tiếp thể tích được), dùng **phương pháp đo thể tích chất lỏng bị chiếm chỗ** (thả vật vào ống đong nước, đo mực nước dâng lên) để xác định thể tích, sau đó tính khối lượng riêng.

4.5 Pressure

4.5.1 Definition of Pressure

$$p = \frac{F}{A}$$

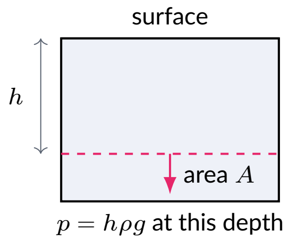
where p is pressure in pascals (Pa), F is the force acting **perpendicular** to a surface (N), and A is the area of that surface (m^2); 1 Pa $= 1$ N m^{-2}.

4.5.2 Pressure in a Fluid: Hydrostatic Pressure

Consider a column of fluid of density ρ, cross-sectional area A, and height (depth) h. The weight of this column is $W = mg = (\rho \times A \times h) \times g$, and this weight is supported by the fluid (or surface) beneath, over the base area A. The pressure at the base of the column is therefore

$$p = \frac{W}{A} = \frac{\rho A h g}{A} = \rho g h \qquad \Longrightarrow \qquad p = h\rho g$$

where h is the depth below the fluid's surface (m), ρ is the fluid's density (kg m^{-3}), and g is the gravitational field strength (m s^{-2}). This pressure due to the fluid acts **equally in all directions** at a given depth (not just downwards), and **increases with depth** — it does *not* depend on the shape or cross-sectional area of the container, only on h, ρ and g.



A column of fluid of depth h and base area A: the weight of the column ($\rho A h g$) divided by the base area gives the pressure at that depth, $p = h\rho g$, independent of the shape of the container.

Atmospheric pressure is a useful reference value: **standard atmospheric pressure** is approximately 1.01×10^5 Pa at sea level; the total pressure at a depth h in an open body of liquid (e.g. a swimming pool or the sea) is the sum of the atmospheric pressure pushing down on the surface *and* the hydrostatic pressure due to the liquid itself, $p_{\text{total}} = p_{\text{atm}} + h\rho g$.

The **pressure difference** between two points at different depths in the same fluid is

$$\Delta p = \Delta h\, \rho g$$

where Δh is the difference in depth between the two points. This principle underlies the operation of a simple **manometer** (a U-tube containing liquid, used to measure gas pressure): the difference in liquid level between the two arms of the tube gives Δh, from which the pressure difference between the gas being measured and the atmosphere can be found directly.

Ví dụ mẫu · Hydrostatic pressure in fresh water

Find the pressure due to water alone at a depth of $10\,\text{m}$ in a lake (density of water $= 1000\,\text{kg}\,\text{m}^{-3}$), and hence the total pressure at that depth, including the atmosphere.

$$p_{\text{water}} = h\rho g = 10 \times 1000 \times 9.81 = \boxed{9.81 \times 10^4\ \text{Pa}}.$$

$$p_{\text{total}} = p_{\text{atm}} + p_{\text{water}} = 1.01 \times 10^5 + 0.981 \times 10^5 = \boxed{1.99 \times 10^5\ \text{Pa}}\ (\approx 2.0 \times 10^5\ \text{Pa}).$$

Ví dụ mẫu · Hydrostatic pressure in seawater

Find the pressure due to seawater alone (density $1030\,\text{kg}\,\text{m}^{-3}$) at a depth of $25\,\text{m}$, and the total pressure including the atmosphere.

$$p_{\text{water}} = h\rho g = 25 \times 1030 \times 9.81 = 25750 \times 9.81 = \boxed{2.53 \times 10^5\ \text{Pa}}.$$

$$p_{\text{total}} = 2.53 \times 10^5 + 1.01 \times 10^5 = \boxed{3.54 \times 10^5\ \text{Pa}}.$$

GIẢI THÍCH TIẾNG VIỆT

VN **Áp suất** $p = F/A$ (đơn vị Pa). Áp suất trong chất lỏng ở độ sâu h: $p = h\rho g$ (từ trọng lượng cột chất lỏng chia cho diện tích đáy), tác dụng **như nhau theo mọi hướng** và **tăng theo độ sâu**, không phụ thuộc hình dạng bình chứa. Áp suất khí quyển tiêu chuẩn $\approx 1.01 \times 10^5$ Pa. Chênh lệch áp suất giữa hai độ sâu $\Delta p = \Delta h\,\rho g$ – nguyên lý này được dùng trong áp kế chữ U (manometer).

4.5.3 Upthrust and Archimedes' Principle

Pressure in a fluid increases with depth (Section 5.2); this means that when an object is submerged, the fluid pressure pushing **upwards** on the *bottom* surface of the object is greater than the fluid pressure pushing **downwards** on its *top* surface (since the bottom surface is at a greater depth). The resulting net upward force is called **upthrust**.

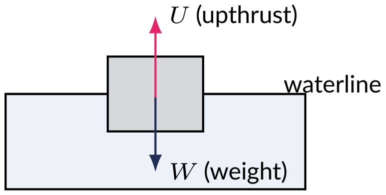
Archimedes' principle: the upthrust on an object that is wholly or partly submerged in a fluid is equal to the **weight of the fluid displaced** by the object:

$$U = \rho_{\text{fluid}} \times V_{\text{displaced}} \times g$$

where ρ_{fluid} is the density of the fluid, $V_{\text{displaced}}$ is the volume of fluid displaced (equal to the volume of the submerged part of the object), and g is the gravitational field strength.

(!) Lỗi thường gặp: Upthrust equals the weight of the fluid **displaced** by the submerged part of the object — *not* the weight, or the total volume, of all the fluid present in the container. A large tank almost full of water and a small bowl with only a little water will each produce exactly the same upthrust on an identical object submerged to the same depth in each, because what matters is only the volume of fluid the object itself pushes out of the way, not how much fluid surrounds it.

Floating and sinking: an object **floats** (in equilibrium, partly or wholly submerged) when the upthrust on it equals its weight, $U = W$ — this happens once it has displaced a volume of fluid whose weight equals its own weight. If, even when *fully* submerged, the maximum possible upthrust (the weight of fluid with volume equal to the whole object) is still less than the object's weight, the object **sinks**, since it can never displace enough fluid to balance its weight.



A block floating with part of its volume submerged below the waterline: the upthrust U (weight of fluid displaced by the submerged part) balances the block's weight W when it floats in equilibrium.

Ví dụ mẫu · Upthrust on a submerged object: does it float or sink?

A solid block of steel (density $7900\,\text{kg m}^{-3}$) has a volume of $1.0 \times 10^{-3}\,\text{m}^3$. It is fully submerged in water (density $1000\,\text{kg m}^{-3}$). Find the upthrust on it and its weight, and hence determine whether it will float or sink if released.

$$U = \rho_{\text{water}} V g = 1000 \times 1.0 \times 10^{-3} \times 9.81 = \boxed{9.81\,\text{N}}.$$

$$W = \rho_{\text{steel}} V g = 7900 \times 1.0 \times 10^{-3} \times 9.81 = \boxed{77.5\,\text{N}}.$$

Since $W\ (77.5\,\text{N}) \gg U\ (9.81\,\text{N})$, even fully submerged the upthrust cannot balance the weight — the block **sinks**.

Ví dụ mẫu · Fraction of an object submerged when floating

A block of ice (density $920\,\text{kg m}^{-3}$) floats in water (density $1000\,\text{kg m}^{-3}$). Find the fraction of the ice's volume that is submerged.

For floating equilibrium, $U = W$:

$$\rho_{\text{water}} V_{\text{sub}}\, g = \rho_{\text{ice}} V_{\text{total}}\, g \;\Rightarrow\; \frac{V_{\text{sub}}}{V_{\text{total}}} = \frac{\rho_{\text{ice}}}{\rho_{\text{water}}} = \frac{920}{1000} = \boxed{0.92\ (92\%)}.$$

Only about 8% of the ice's volume shows above the waterline — the well-known "tip of the iceberg."

Ví dụ mẫu · Floating wooden block: full analysis

A rectangular wooden block, density $600\,\text{kg m}^{-3}$, has dimensions $0.40\,\text{m} \times 0.20\,\text{m} \times 0.10\,\text{m}$, and floats in water ($1000\,\text{kg m}^{-3}$) with the $0.10\,\text{m}$ dimension vertical. Find (a) the block's weight, (b) the fraction of its volume submerged, and (c) the depth of the submerged part.

(a) $V = 0.40 \times 0.20 \times 0.10 = 8.0 \times 10^{-3}\,\text{m}^3$.

$$W = \rho V g = 600 \times 8.0 \times 10^{-3} \times 9.81 = \boxed{47.1\,\text{N}}.$$

(b) At equilibrium, $U = W$, so $\rho_{\text{water}} V_{\text{sub}} g = \rho_{\text{wood}} V g$:

$$\frac{V_{\text{sub}}}{V} = \frac{600}{1000} = \boxed{0.60\ (60\%)}.$$

(c) Depth submerged $= 0.60 \times 0.10\,\text{m} = \boxed{0.060\,\text{m}}$ $(= 6.0\,\text{cm})$.
Check: $V_{\text{sub}} = 0.60 \times 8.0 \times 10^{-3} = 4.8 \times 10^{-3}\,\text{m}^3$; $U = 1000 \times 4.8 \times 10^{-3} \times 9.81 = 47.1\,\text{N} = W$ $\checkmark$.

GIẢI THÍCH TIẾNG VIỆT

VN

Lực đẩy Archimedes: $U = \rho_{\text{chất lỏng}} \times V_{\text{chiếm chỗ}} \times g$ – bằng trọng lượng của phần chất lỏng bị vật **chiếm chỗ**, không phải trọng lượng toàn bộ chất lỏng có trong bình (lỗi rất hay gặp). Nguyên nhân vật lý: áp suất tăng theo độ sâu nên lực đẩy lên ở đáy vật lớn hơn lực đẩy xuống ở đỉnh vật. Vật **nổi** khi $U = W$ (chiếm chỗ đúng bằng trọng lượng của nó); vật **chìm** khi ngay cả lúc ngập hoàn toàn, lực đẩy tối đa vẫn nhỏ hơn trọng lượng.

Practice Bank

Bài 1 · MCQ — Types of force

Which of the following is a **contact** force, not a long-range (field) force?

(A) Gravitational force
(B) Electrostatic force
(C) Tension
(D) Magnetic force

Lời giải chi tiết

(C) Tension. Gravitational, electrostatic and magnetic forces all act at a distance without contact; tension requires a stretched string/rope/cable actually touching the object.

Bài 2 · MCQ — Weight and centre of gravity

The weight of an object may be considered, for the purposes of calculating moments, to act at:

(A) The highest point of the object
(B) The centre of gravity
(C) The point of contact with the ground
(D) Any corner of the object

Lời giải chi tiết

(B) The centre of gravity — the single point at which the entire weight may be considered to act.

Bài 3 · Equilibrium of a particle

A point is acted on by two forces: $9.0\,\text{N}$ along the $+x$-axis and $12\,\text{N}$ along the $+y$-axis. Find the magnitude and direction of the third force needed to keep the point in equilibrium.

Lời giải chi tiết

Resultant of the two given forces: $R = \sqrt{9.0^2 + 12^2} = \sqrt{81 + 144} = \sqrt{225} = 15$ N, at $\theta = \tan^{-1}(12/9.0) = 53.1^\circ$ above the $+x$-axis.
The equilibrant (third force) is $\boxed{15\,\text{N}}$, acting in exactly the opposite direction, i.e. at $180^\circ + 53.1^\circ = 233.1^\circ$ from the $+x$-axis.

Bài 4 · Resolving forces: symmetric case

Three coplanar forces act on a ring, all of magnitude 8.0 N: one at 0°, one at 120°, and the third force F (unknown angle) keeping the ring in equilibrium. Find F and its direction.

Lời giải chi tiết

By symmetry (each force 120° from the next), $F = \boxed{8.0\,\text{N}}$, acting at $\boxed{240^\circ}$ from the $+x$-axis. (Check by components: sum of first two forces $= (8.0 - 4.0,\ 6.93) = (4.0, 6.93)$, magnitude $\sqrt{16 + 48} = \sqrt{64} = 8.0$ N, direction 60° above $+x$; the equilibrant is 8.0 N at $180^\circ + 60^\circ = 240^\circ$.)

Bài 5 · MCQ — Condition for equilibrium of a particle

For a point (particle) to be in equilibrium, which condition must necessarily be true?

(A) The resultant moment about it is zero

(B) The resultant force on it is zero

(C) All the forces acting on it are equal in magnitude

(D) Only two forces can act on it

Lời giải chi tiết

(B). A particle has no physical size, so it cannot experience a net turning effect — the sole condition for its equilibrium is that the vector sum of forces is zero.

Bài 6 · Moment of a force

A force of 15 N is applied perpendicular to a spanner handle, at a perpendicular distance of 0.25 m from the nut. Calculate the moment of the force about the nut.

Lời giải chi tiết

$$M = Fd = 15 \times 0.25 = \boxed{3.75\,\text{N m}}.$$

Bài 7 · Moment of an angled force

A force of 20 N is applied to a door handle 0.80 m from the hinge, directed at 30° to the line from the hinge to the handle. Find the moment of the force about the hinge.

Lời giải chi tiết

Perpendicular distance $= L \sin\theta = 0.80 \times \sin 30^\circ = 0.80 \times 0.50 = 0.40$ m.

$$M = F \times (L \sin\theta) = 20 \times 0.40 = \boxed{8.0\,\text{N m}}.$$

Bài 8 · MCQ — Definition of a couple

A couple consists of:

(A) Two equal, opposite forces acting along the same line

(B) Two equal, opposite, parallel forces whose lines of action do not coincide

(C) Any two forces acting on the same body

(D) A single force applied off-centre

Lời giải chi tiết

(B). A couple is specifically two equal, opposite, parallel forces whose lines of action are offset — producing pure rotation with zero resultant force.

Bài 9 · Torque of a couple

Two forces of 3.0 N act in opposite directions on the rim of a steering wheel of diameter 0.36 m. Calculate the torque produced.

Lời giải chi tiết

$$\text{Torque} = F \times d = 3.0 \times 0.36 = \boxed{1.08\,\text{N m}}.$$

Bài 10 · Seesaw balance

A seesaw is pivoted at its centre. A child of weight 400 N sits 2.0 m from the pivot. Where must a child of weight 500 N sit, on the other side, to balance the seesaw?

Lời giải chi tiết

$$400 \times 2.0 = 500 \times d \;\Rightarrow\; d = \frac{800}{500} = \boxed{1.6\,\text{m}}.$$

Bài 11 · Principle of moments with masses

A light rod is pivoted at its centre. A mass of 200 g hangs 45 cm from the pivot on one side. Find the distance from the pivot at which a mass of 300 g must hang on the other side to balance the rod.

Lời giải chi tiết

Since both weights are mg, the factor g cancels in the moment balance, leaving simply $m_1 d_1 = m_2 d_2$:

$$200 \times 45 = 300 \times d \;\Rightarrow\; d = \frac{9000}{300} = \boxed{30\,\text{cm}}.$$

Bài 12 · Uniform beam on two end supports

A uniform plank, length 4.0 m and weight 200 N, rests on supports at each end, A and B. A load of 100 N is placed 1.0 m from A. Find the reactions R_A and R_B.

Lời giải chi tiết

Total weight: $R_A + R_B = 200 + 100 = 300$ N.
Moments about A: the plank's weight acts at its midpoint, 2.0 m from A; the load is 1.0 m from A.

$$R_B \times 4.0 = 200 \times 2.0 + 100 \times 1.0 = 400 + 100 = 500 \;\Rightarrow\; R_B = \boxed{125\,\text{N}}.$$

$$R_A = 300 - 125 = \boxed{175\,\text{N}}.$$

Check (moments about B): plank's weight is 2.0 m from B; load is 3.0 m from B: $R_A \times 4.0 = 200 \times 2.0 + 100 \times 3.0 = 400 + 300 = 700 \Rightarrow R_A = 175$ N ✓.

Bài 13 · MCQ — Equilibrium of a rigid body

For an extended (rigid) body to be in equilibrium, which condition(s) must be satisfied?

(A) Resultant force zero only

(B) Resultant moment zero only

(C) Resultant force zero AND resultant moment (about any point) zero

(D) All applied forces must be equal in magnitude

Lời giải chi tiết

(C). Unlike a particle, an extended body can rotate, so *both* conditions are required — zero resultant force alone is not sufficient (a couple satisfies it but still causes rotation).

Bài 14 · Hinged beam with an angled cable

A uniform beam of weight 80 N and length 1.2 m is hinged to a wall at one end, held horizontal by a cable attached at the other end, making $40°$ with the beam. Find the tension in the cable.

Lời giải chi tiết

Moment of weight about the hinge: $80 \times 0.60 = 48$ N m (weight acts at the midpoint, 0.60 m horizontally from the hinge).
Moment of tension about the hinge: $T \sin 40° \times 1.2 = 0.771\,T$.
Principle of moments: $0.771\,T = 48 \Rightarrow T = \boxed{62\,\text{N}}$.

Bài 15 · Stability reasoning

Explain why a vehicle with a low centre of gravity and a wide wheelbase is less likely to topple over when cornering than a taller, narrower vehicle.

Lời giải chi tiết

A vehicle topples when the vertical line through its centre of gravity moves outside its base area (the "footprint" of its wheels). A **lower** centre of gravity and a **wider** base both mean the vehicle must tilt through a **larger angle** before that vertical line reaches the edge of the base — so a low, wide vehicle can lean much further (e.g. while cornering) before toppling than a tall, narrow one.

Bài 16 · MCQ — Locating the centre of gravity

The centre of gravity of an irregular lamina can be found experimentally by:

(A) Balancing it on a single point only

(B) Suspending it freely from one point, marking the plumb line, repeating from a second point, and taking the intersection

(C) Weighing it on a balance

(D) Measuring its perimeter

Lời giải chi tiết

(B). This is the standard plumb-line method: the centre of gravity lies on the vertical line below each suspension point, so it must be at the intersection of two such lines.

Bài 17 · Density from mass and dimensions

A metal cube has sides of length $5.0\,\text{cm}$ and mass $675\,\text{g}$. Find its density, in kg m^{-3}.

Lời giải chi tiết

$V = (0.050)^3 = 1.25 \times 10^{-4}\,\text{m}^3$. $\quad m = 0.675\,\text{kg}$.

$$\rho = \frac{m}{V} = \frac{0.675}{1.25 \times 10^{-4}} = \boxed{5400\,\text{kg m}^{-3}}.$$

Bài 18 · MCQ — Comparing densities

Which of the following typically has the **greatest** density?

(A) Air

(B) Ice

(C) Water

(D) Steel

Lời giải chi tiết

(D) Steel ($\approx 7900\,\text{kg m}^{-3}$), far denser than water (1000), ice (≈ 920), or air (≈ 1.2).

Bài 19 · Explaining floating with density

Explain, in terms of density, why ice floats on water.

Lời giải chi tiết

Ice has a lower density ($\approx 920\,\text{kg m}^{-3}$) than liquid water ($1000\,\text{kg m}^{-3}$), so a given mass of ice occupies more volume than the same mass of water. When floating, upthrust equals weight; because $\rho_{\text{ice}}/\rho_{\text{water}} < 1$, the ice only needs to submerge a *fraction* of its volume (in fact 92%, from $\rho_{\text{ice}}/\rho_{\text{water}}$) to displace its own weight of water, so it settles with part of its volume above the surface rather than sinking.

Bài 20 · Density by displacement

A stone of mass $240\,\text{g}$ is lowered into a measuring cylinder; the water level rises from $50\,\text{cm}^3$ to $130\,\text{cm}^3$. Find the density of the stone in kg m^{-3}.

Lời giải chi tiết

$V = 130 - 50 = 80\,\text{cm}^3 = 8.0 \times 10^{-5}\,\text{m}^3$. $m = 0.240\,\text{kg}$.

$$\rho = \frac{0.240}{8.0 \times 10^{-5}} = \boxed{3000\,\text{kg m}^{-3}}.$$

Bài 21 · Pressure from force and area

A crate of weight $240\,\text{N}$ rests on the ground with a base area of $0.60\,\text{m}^2$. Find the pressure it exerts on the ground.

Lời giải chi tiết

$$p = \frac{F}{A} = \frac{240}{0.60} = \boxed{400\,\text{Pa}}.$$

Bài 22 · MCQ — Hydrostatic pressure

The pressure at a given depth in a fluid depends on:

(A) Depth, fluid density and g only — not the shape of the container

(B) The shape and depth of the container

(C) The surface area of the container's base

(D) The total volume of fluid present

Lời giải chi tiết

(A). From $p = h\rho g$, only depth, density and g matter — the shape, base area, or total volume of the container are irrelevant.

Bài 23 · Hydrostatic pressure and total pressure

Find the pressure due to water at a depth of $15\,\text{m}$ (density $1000\,\text{kg m}^{-3}$), and the total pressure at that depth including the atmosphere (1.01×10^5 Pa).

Lời giải chi tiết

$$p_{\text{water}} = h\rho g = 15 \times 1000 \times 9.81 = \boxed{1.47 \times 10^5\,\text{Pa}}.$$

$$p_{\text{total}} = 1.47 \times 10^5 + 1.01 \times 10^5 = \boxed{2.48 \times 10^5\,\text{Pa}}.$$

Bài 24 · Pressure difference (manometer principle)

Two points in a body of water differ in depth by $0.50\,\text{m}$. Find the pressure difference between the two points.

Lời giải chi tiết

$$\Delta p = \Delta h\,\rho g = 0.50 \times 1000 \times 9.81 = \boxed{4.9 \times 10^3\,\text{Pa}}.$$

Bài 25 · Upthrust and float/sink decision

A hollow ball has volume $4.0 \times 10^{-4}\,\text{m}^3$ and mass $0.20\,\text{kg}$. It is fully submerged in water and released. Find the upthrust on it, and determine whether it rises, stays still, or sinks further.

Lời giải chi tiết

$$U = \rho V g = 1000 \times 4.0 \times 10^{-4} \times 9.81 = \boxed{3.92\,\text{N}}.$$

$$W = mg = 0.20 \times 9.81 = \boxed{1.96\,\text{N}}.$$

Since $U\ (3.92\,\text{N}) > W\ (1.96\,\text{N})$, there is a net upward force — the ball **rises** to the surface, where it will float with less than its full volume submerged (just enough to make $U = W$).

Bài 26 · Fraction submerged when floating

A wooden cube of density $700\,\text{kg}\,\text{m}^{-3}$ floats in water. Find the fraction of its volume that is submerged.

Lời giải chi tiết

At equilibrium $U = W$, so $\dfrac{V_\text{sub}}{V} = \dfrac{\rho_\text{wood}}{\rho_\text{water}} = \dfrac{700}{1000} = \boxed{0.70\ (70\%)}$.

Bài 27 · MCQ — Archimedes' principle

The upthrust on an object wholly or partly submerged in a fluid is equal to:

(A) The weight of the object itself

(B) The weight of the fluid displaced by the object

(C) The fluid pressure at the bottom surface of the object

(D) The volume of the object

Lời giải chi tiết

(B). This is Archimedes' principle exactly: $U = \rho_\text{fluid} V_\text{displaced}\, g$.

Bài 28 · Apparent weight loss method for density

An object hangs from a spring balance; its weight in air reads $8.0\,\text{N}$. When fully submerged in water, the reading drops to $5.5\,\text{N}$. Find (a) the upthrust, (b) the volume of the object, and (c) its density.

Lời giải chi tiết

(a) Upthrust = apparent loss of weight $= 8.0 - 5.5 = \boxed{2.5\,\text{N}}$.

(b) $U = \rho_\text{water} V g \Rightarrow V = \dfrac{U}{\rho g} = \dfrac{2.5}{1000 \times 9.81} = \dfrac{2.5}{9810} = \boxed{2.55 \times 10^{-4}\,\text{m}^3}$.

(c) True mass $= \dfrac{W_\text{air}}{g} = \dfrac{8.0}{9.81} = 0.8155\,\text{kg}$. $\quad \rho = \dfrac{m}{V} = \dfrac{0.8155}{2.55 \times 10^{-4}} = \boxed{3.2 \times 10^3\,\text{kg}\,\text{m}^{-3}}$.

Work, Energy, Power & Deformation of Solids

Mục tiêu Unit: Sau chương này, học sinh có thể: tính công thực hiện bởi một lực không đổi và một lực biến thiên (qua diện tích dưới đồ thị lực–độ dời); áp dụng định luật bảo toàn năng lượng cho các bài toán có và không có lực cản (ma sát, lực cản không khí); tính công suất của động cơ, xe cộ khi chuyển động đều và khi tăng tốc; tính hiệu suất của hệ một tầng và hệ nhiều tầng (chuỗi hiệu suất); áp dụng định luật Hooke, tính hằng số lò xo tương đương khi ghép nối tiếp và song song; tính ứng suất, biến dạng, mô-đun Young; đọc và giải thích đồ thị ứng suất–biến dạng, phân biệt vật liệu dẻo, giòn và đàn hồi cao su; tính năng lượng biến dạng đàn hồi (mật độ năng lượng) và năng lượng tiêu tán trong vòng trễ (hysteresis).

5.1 Work, Energy and Power

5.1.1 Work Done by a Constant Force

When a force acts on an object and causes a displacement, the force does work on the object. If the force F is constant and the displacement s makes an angle θ with the direction of the force, the work done is

$$W = Fs\cos\theta$$

Work is a scalar quantity, measured in joules (J), where 1 J $= 1$ N m. Only the component of the force along the direction of motion, $F\cos\theta$, does work. If $\theta = 90^\circ$ (force perpendicular to displacement), no work is done. If the force and displacement are in the same direction, $\theta = 0^\circ$ and $W = Fs$.

GIẢI THÍCH TIẾNG VIỆT

VN

Công thực hiện bởi một lực không đổi được tính bằng $W = Fs\cos\theta$, trong đó θ là góc giữa hướng của lực và hướng của độ dời. Chỉ thành phần lực song song với chuyển động mới sinh công; thành phần vuông góc (như phản lực pháp tuyến) không sinh công.

Ví dụ mẫu · Work done by a force at an angle

A crate is pulled a distance of 12 m across a warehouse floor by a rope tension of 60 N acting at 40° above the horizontal. Calculate the work done by the tension.

Solution. Using $W = Fs\cos\theta$:

$$W = 60 \times 12 \times \cos 40^\circ = 720 \times 0.766 = 5.5 \times 10^2 \text{ J } (\approx 550 \text{ J})$$

5.1.2 Work Done as the Area Under a Force–Displacement Graph

For a force that is not constant (for example, the force needed to stretch a spring, which grows as the extension increases), work done cannot simply be calculated using $Fs\cos\theta$ with a single value of F. Instead:

Work done from a force-displacement graph

The work done by a force is equal to the **area under the force–displacement graph**, between the initial and final positions.

- If the graph is a straight line through the origin (force directly proportional to displacement, as in Hooke's law), the area is a triangle: $W = \frac{1}{2}F_{\text{max}}x$.
- If the force is constant, the area is a rectangle: $W = Fs$.
- If the force varies in a more complicated (non-linear) way, the area must be found by counting squares on the graph or by other numerical methods.

GIẢI THÍCH TIẾNG VIỆT

VN

Đối với một lực biến thiên (không phải hằng số), công thực hiện được xác định bằng diện tích nằm dưới đồ thị lực theo độ dời (hoặc độ giãn). Nếu đồ thị là đường thẳng qua gốc tọa độ (như trong định luật Hooke), diện tích là một tam giác, cho công thức $W = \frac{1}{2}F_{\text{max}}x$.

Ví dụ mẫu · Work done stretching a spring

A spring of spring constant $k = 150\,\text{N m}^{-1}$ is stretched from its natural length to an extension of $0.20\,\text{m}$. Calculate the work done in stretching the spring.

Solution. Since $F = kx$, the force grows linearly with extension from 0 to $F_{\text{max}} = kx = 150 \times 0.20 = 30\,\text{N}$. The force–extension graph is a straight line through the origin, so the work done is the area of the triangle beneath it:

$$W = \frac{1}{2}F_{\text{max}}x = \frac{1}{2}(30)(0.20) = 3.0\,\text{J}$$

This is identical to using $W = \frac{1}{2}kx^2 = \frac{1}{2}(150)(0.20)^2 = 3.0$ J, confirming that the elastic potential energy formula is simply the area under a linear force–extension graph.

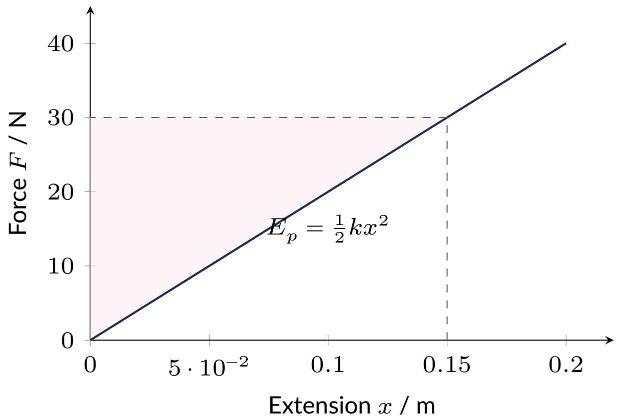


Figure 5.1: Force-extension graph for a spring with $k = 200\,\text{N m}^{-1}$. The shaded triangular area up to $x = 0.15\,\text{m}$ represents the elastic potential energy stored, $E_p = \frac{1}{2}(200)(0.15)^2 = 2.25\,\text{J}$.

5.1.3 Energy Conservation: Kinetic and Gravitational Potential Energy

Kinetic energy:

$$E_k = \tfrac{1}{2}mv^2$$

Change in gravitational potential energy:

$$\Delta E_p = mg\Delta h$$

The principle of conservation of energy states that energy cannot be created or destroyed, only transformed from one form to another. In the absence of resistive forces (friction, air resistance), mechanical energy is conserved:

$$\Delta E_k + \Delta E_p = 0 \quad \text{i.e.} \quad \text{loss in } E_p = \text{gain in } E_k$$

VN **GIẢI THÍCH TIẾNG VIỆT**

Động năng $E_k = \frac{1}{2}mv^2$ và độ biến thiên thế năng trọng trường $\Delta E_p = mg\Delta h$. Khi không có lực cản, cơ năng được bảo toàn: phần thế năng mất đi chuyển hoàn toàn thành động năng thu được. Khi có lực cản (ma sát, lực cản không khí), một phần năng lượng chuyển thành nhiệt và động năng thu được luôn nhỏ hơn thế năng mất đi.

Ví dụ mẫu · Energy conservation on a smooth slope

A skier of mass $60\,\text{kg}$ starts from rest at the top of a smooth (frictionless) slope and descends through a vertical height of $20\,\text{m}$. Calculate the speed of the skier at the bottom.

Solution. Since the slope is smooth, all the loss in gravitational potential energy converts to kinetic energy:

$$mgh = \tfrac{1}{2}mv^2 \implies v = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 20} = \sqrt{392.4} = 19.8\,\text{m}\,\text{s}^{-1}$$

Note that the mass cancels; the result is independent of the skier's mass.

5.1.4 Energy Conservation with Resistive Forces

Real systems are never perfectly frictionless. When resistive forces (friction, air resistance, drag) act, they do negative work on the moving object, removing mechanical energy from the system and converting it into internal energy (heat) and sound. The work-energy relationship becomes:

$$\text{loss in } E_p = \text{gain in } E_k + \text{energy dissipated against resistive forces}$$

Equivalently, the energy dissipated is the work done against the resistive force:

$$W_{\text{dissipated}} = mgh - \tfrac{1}{2}mv^2$$

where v is the *actual* (measured) final speed, which will always be less than the ideal frictionless value $\sqrt{2gh}$.

VN **GIẢI THÍCH TIẾNG VIỆT**

Khi có lực cản, không phải toàn bộ thế năng chuyển thành động năng. Một phần năng lượng bị tiêu tán do công thực hiện chống lại lực ma sát hoặc lực cản không khí, chuyển hóa thành nhiệt. Do đó tốc độ đo được thực tế luôn nhỏ hơn giá trị lý tưởng tính từ $\sqrt{2gh}$.

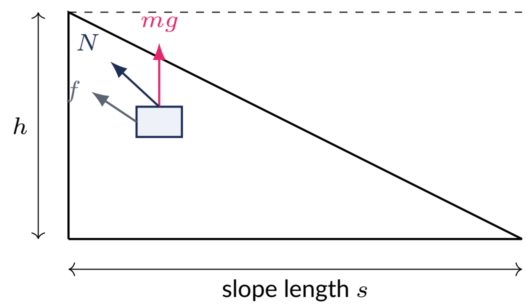


Figure 5.2: A block sliding down a rough slope of height h and slope length s. The friction force f acts up the slope, opposing motion, and does negative work equal to the energy dissipated as heat.

Ví dụ mẫu · Energy conservation with friction on a rough slope

The same skier of mass $60\,\text{kg}$ now descends a rough slope of vertical height $20\,\text{m}$ and slope length $50\,\text{m}$, starting from rest. Due to friction, the measured speed at the bottom is only $18.0\,\text{m}\,\text{s}^{-1}$ (compared with the ideal frictionless value of $19.8\,\text{m}\,\text{s}^{-1}$ found earlier). Calculate (a) the energy dissipated by friction, and (b) the average frictional force acting on the skier.
Solution. (a) Loss in gravitational PE:

$$mgh = 60 \times 9.81 \times 20 = 11772\,\text{J}$$

Actual kinetic energy gained:

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}(60)(18.0)^2 = \tfrac{1}{2}(60)(324) = 9720\,\text{J}$$

Energy dissipated by friction:

$$\Delta E = 11772 - 9720 = 2052\,\text{J} \approx 2.05\,\text{kJ}$$

(b) Since work done against friction $= f \times s$ (average friction force times slope length):

$$f = \frac{2052}{50} = 41.0\,\text{N}$$

5.1.5 Power

Power is the rate at which work is done, or the rate at which energy is transferred:

$$P = \frac{W}{t} = \frac{\Delta E}{t}$$

For an object moving at constant velocity v under a driving force F (parallel to the motion),

$$P = Fv$$

Power is measured in watts (W), where $1\,\text{W} = 1\,\text{J}\,\text{s}^{-1}$.

GIẢI THÍCH TIẾNG VIỆT

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Công suất là tốc độ thực hiện công (hoặc tốc độ truyền năng lượng): $P = W/t = \Delta E/t$. Khi một lực kéo không đổi F tác dụng theo hướng chuyển động với tốc độ v, công suất tức thời là $P = Fv$. Đây là công thức quan trọng cho các bài toán về xe cộ và máy móc chuyển động.

Ví dụ mẫu · Power delivered by a crane winch

A crane winch lifts a load using a cable tension of 500 N, raising the load at a constant speed of 4.0 m s^{-1}. Calculate the power delivered by the winch.
Solution.

$$P = Fv = 500 \times 4.0 = 2000\,\text{W} = 2.0\,\text{kW}$$

Ví dụ mẫu · Vehicle power at constant speed on a level road

A car travels at a constant speed of 25 m s^{-1} along a level road. The total resistive force (rolling friction plus air resistance) acting on the car at this speed is 600 N. Calculate the power output required from the engine.
Solution. At constant speed, the car is in equilibrium, so the driving force from the engine exactly balances the total resistive force: $F_{\text{drive}} = 600$ N. The power output is

$$P = Fv = 600 \times 25 = 15000\,\text{W} = 15\,\text{kW}$$

Ví dụ mẫu · Vehicle power climbing a hill at constant speed

A car of mass 1200 kg climbs a slope at a constant speed of 20 m s^{-1}. The slope is inclined such that $\sin\theta = 0.05$. The total resistive force (friction and air resistance) at this speed is 500 N. Calculate (a) the driving force required, and (b) the power output of the engine.
Solution. (a) At constant speed, the driving force must balance both the resistive force and the component of the car's weight acting down the slope:

$$F_{\text{drive}} = F_{\text{resistive}} + mg\sin\theta = 500 + (1200)(9.81)(0.05) = 500 + 588.6 = 1088.6\,\text{N}$$

(b) Power output:

$$P = Fv = 1088.6 \times 20 = 21772\,\text{W} \approx 21.8\,\text{kW}$$

Ví dụ mẫu · Extra power required to accelerate a vehicle

A car of mass 1000 kg is travelling at 15 m s^{-1} on a level road and accelerating at 2.0 m s^{-2}. At this speed, the total resistive force on the car is 400 N. Calculate (a) the driving force needed, (b) the total power output of the engine at this instant, (c) the power that would be needed simply to overcome resistance at this speed (i.e. to travel at constant 15 m s^{-1}), and (d) the extra power required for the acceleration.
Solution. (a) By Newton's second law, the resultant force needed for the acceleration is ma; the driving force must supply this *in addition to* balancing the resistive force:

$$F_{\text{drive}} = ma + F_{\text{resistive}} = (1000)(2.0) + 400 = 2000 + 400 = 2400\,\text{N}$$

(b) Total instantaneous power:

$$P_{\text{total}} = F_{\text{drive}}v = 2400 \times 15 = 36000\,\text{W} = 36\,\text{kW}$$

(c) Power to overcome resistance alone:

$$P_{\text{resistance}} = F_{\text{resistive}}v = 400 \times 15 = 6000\,\text{W} = 6.0\,\text{kW}$$

(d) Extra power for acceleration:

$$P_{\text{extra}} = P_{\text{total}} - P_{\text{resistance}} = 36000 - 6000 = 30000\,\text{W} = 30\,\text{kW}$$

This confirms that an accelerating vehicle always requires *more* power than Fv evaluated using the resistive force alone — the engine must also supply the power mav associated with increasing the car's kinetic energy.

5.1.6 Efficiency

$$\text{efficiency} = \frac{\text{useful energy (or power) output}}{\text{total energy (or power) input}} \times 100\%$$

No real machine is 100% efficient: some input energy is always wasted, usually as heat due to friction or resistance, in accordance with the principle of conservation of energy.

VN GIẢI THÍCH TIẾNG VIỆT

Hiệu suất được tính bằng tỉ số giữa năng lượng (hoặc công suất) đầu ra hữu ích và năng lượng (hoặc công suất) đầu vào, nhân với 100%. Không có máy nào đạt hiệu suất 100%, vì luôn có một phần năng lượng bị hao phí, thường dưới dạng nhiệt do ma sát.

Ví dụ mẫu · Efficiency of a single-stage motor

An electric motor takes in 2000 J of electrical energy and delivers 1500 J of useful mechanical work. Calculate the efficiency of the motor.

Solution.

$$\text{efficiency} = \frac{1500}{2000} \times 100\% = 75\%$$

Ví dụ mẫu · Two-stage efficiency chain

A hoist system consists of an electric motor (efficiency 85%) connected to a gearbox (efficiency 90%). If the motor receives 2000 W of electrical input power, calculate (a) the mechanical power delivered by the motor to the gearbox, (b) the useful power output of the gearbox, and (c) the overall efficiency of the system.

Solution. (a) Power out of the motor:

$$P_1 = 2000 \times 0.85 = 1700 \text{ W}$$

(b) Power out of the gearbox:

$$P_2 = 1700 \times 0.90 = 1530 \text{ W}$$

(c) Overall efficiency:

$$\text{efficiency} = \frac{1530}{2000} \times 100\% = 76.5\%$$

Note that this equals the *product* of the individual stage efficiencies: $0.85 \times 0.90 = 0.765 = 76.5\%$. When several stages act in sequence, overall efficiency is always found by multiplying (not adding) the individual efficiencies, and it is always lower than the efficiency of the best individual stage.

(!) Lỗi thường gặp: It is tempting to assume that "loss in gravitational PE = gain in KE" always holds. This is only true when no resistive forces act. Whenever friction, air resistance, or any dissipative force is present, some of the energy converts to heat, so the actual kinetic energy gained is always less than the ideal value calculated from mgh. Similarly, in vehicle problems, the power needed to overcome resistance at a given speed ($P = Fv$, using only the resistive force) is *not* the total power output if the vehicle is also accelerating — extra power (mav) must be added, and if the vehicle is also climbing a slope, the term $mgv \sin\theta$ must be added as well.

5.2 Deformation of Solids

5.2.1 Hooke's Law and Elastic Potential Energy

For a spring (or any elastic material) stretched or compressed within its limit of proportionality, the extension is directly proportional to the applied force:

$$F = kx$$

where k is the spring constant (units N m^{-1}), a measure of the stiffness of the spring. The elastic potential energy stored is the area under the (linear) force–extension graph:

$$E_p = \frac{1}{2}kx^2 = \frac{1}{2}Fx$$

VN **GIẢI THÍCH TIẾNG VIỆT**

Định luật Hooke phát biểu rằng độ giãn (hoặc nén) của một lò xo tỉ lệ thuận với lực tác dụng, $F = kx$, miễn là chưa vượt quá giới hạn tỉ lệ. Thế năng đàn hồi tích trữ bằng diện tích dưới đồ thị lực–độ giãn (đường thẳng), $E_p = \frac{1}{2}kx^2$.

Ví dụ mẫu · Elastic potential energy in a stretched spring

A spring of spring constant $k = 200$ N m^{-1} is stretched by 0.15 m. Calculate (a) the force applied, and (b) the elastic potential energy stored.
Solution. (a) $F = kx = 200 \times 0.15 = 30$ N
(b) $E_p = \frac{1}{2}kx^2 = \frac{1}{2}(200)(0.15)^2 = \frac{1}{2}(200)(0.0225) = 2.25$ J

5.2.2 Springs in Series and in Parallel

Combining springs

When springs are combined, they behave like an equivalent single spring of spring constant k_{eff}:

- **Series** (springs joined end to end, sharing the same force, extensions add):

$$\frac{1}{k_{\text{eff}}} = \frac{1}{k_1} + \frac{1}{k_2} + \ldots$$

  The combination is *less stiff* (smaller k_{eff}) than either spring alone — it stretches further for the same load.

- **Parallel** (springs side by side, sharing the same extension, forces add):

$$k_{\text{eff}} = k_1 + k_2 + \ldots$$

  The combination is *stiffer* (larger k_{eff}) than either spring alone.

VN **GIẢI THÍCH TIẾNG VIỆT**

Khi ghép nối tiếp, các lò xo cùng chịu một lực nhưng độ giãn cộng lại; hằng số lò xo tương đương tính theo $1/k_{\text{eff}} = 1/k_1 + 1/k_2$, luôn nhỏ hơn hằng số của mỗi lò xo thành phần. Khi ghép song song, các lò xo cùng chịu một độ giãn nhưng lực cộng lại; $k_{\text{eff}} = k_1 + k_2$, luôn lớn hơn hằng số của mỗi lò xo thành phần.

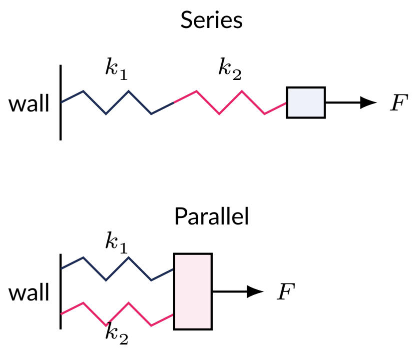

Figure 5.3: Two springs of constants k_1 and k_2 connected in series (top, extensions add) and in parallel (bottom, forces add), each attached to a load.

Ví dụ mẫu · Springs in series and in parallel

Two springs have spring constants $k_1 = 150\,\text{N}\,\text{m}^{-1}$ and $k_2 = 300\,\text{N}\,\text{m}^{-1}$. A load of $6.0\,\text{N}$ is applied. Calculate the extension produced when the springs are combined (a) in series, and (b) in parallel.

Solution. (a) Series.

$$\frac{1}{k_{\text{eff}}} = \frac{1}{150} + \frac{1}{300} = \frac{2}{300} + \frac{1}{300} = \frac{3}{300} = \frac{1}{100} \implies k_{\text{eff}} = 100\,\text{N}\,\text{m}^{-1}$$

$$x = \frac{F}{k_{\text{eff}}} = \frac{6.0}{100} = 0.060\,\text{m} = 6.0\,\text{cm}$$

(b) Parallel.

$$k_{\text{eff}} = k_1 + k_2 = 150 + 300 = 450\,\text{N}\,\text{m}^{-1}$$

$$x = \frac{F}{k_{\text{eff}}} = \frac{6.0}{450} = 0.0133\,\text{m} = 1.33\,\text{cm}$$

As expected, the series combination stretches much further (it is less stiff) than the parallel combination under the same load.

5.2.3 Stress, Strain and the Young Modulus

$$\text{stress, } \sigma = \frac{F}{A} \qquad \text{strain, } \varepsilon = \frac{x}{L}$$

where F is the applied force, A is the cross-sectional area, x is the extension, and L is the original (natural) length. Stress is measured in pascals (Pa), and strain is dimensionless (a ratio). The Young modulus is the ratio of stress to strain, valid only up to the limit of proportionality:

$$E = \frac{\sigma}{\varepsilon} = \frac{F/A}{x/L} = \frac{FL}{Ax}$$

E is a property of the material (independent of the sample's dimensions), measured in pascals.

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Ứng suất $\sigma = F/A$ và biến dạng $\varepsilon = x/L$. Mô-đun Young $E = \sigma/\varepsilon = FL/(Ax)$ là đại lượng đặc trưng cho vật liệu (không phụ thuộc kích thước mẫu vật), chỉ có giá trị không đổi trong vùng biến dạng còn tuân theo định luật Hooke (trước giới hạn tỉ lệ).

Ví dụ mẫu · Young modulus of a wire

A wire of length $L = 2.00$ m and cross-sectional area $A = 0.50$ mm^2 extends by 0.80 mm under a load of 40 N. Calculate the Young modulus of the material and identify the likely material.

Solution. Converting units: $A = 0.50 \times 10^{-6}$ m^2, $x = 0.80 \times 10^{-3}$ m.

$$\text{stress} = \frac{F}{A} = \frac{40}{0.50 \times 10^{-6}} = 8.0 \times 10^{7}\ \text{Pa}$$

$$\text{strain} = \frac{x}{L} = \frac{0.80 \times 10^{-3}}{2.00} = 4.0 \times 10^{-4}$$

$$E = \frac{\sigma}{\varepsilon} = \frac{8.0 \times 10^{7}}{4.0 \times 10^{-4}} = 2.0 \times 10^{11}\ \text{Pa} = 200\,\text{GPa}$$

This value is consistent with steel ($E \approx 200$ GPa).

5.2.4 The Stress–Strain Graph

Key features of a stress-strain graph

- **Limit of proportionality** — the point beyond which stress is no longer proportional to strain (Hooke's law no longer holds); the gradient up to this point equals the Young modulus.
- **Elastic limit** — the point beyond which the material no longer returns to its original length when unloaded; permanent (plastic) deformation begins just beyond this point. It lies very close to the limit of proportionality for most materials.
- **Yield point** — a marked increase in strain for little or no increase in stress; the material begins to deform plastically more readily.
- **Ultimate tensile stress (UTS)** — the maximum stress the material can withstand before its cross-section begins to reduce rapidly (necking).
- **Breaking stress (fracture)** — the stress at which the material actually fractures; for ductile materials this can be lower than the UTS because the true stress at the (now-narrower) neck is higher than the nominal stress plotted.

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Trên đồ thị ứng suất–biến dạng: giới hạn tỉ lệ là điểm ứng suất không còn tỉ lệ với biến dạng; giới hạn đàn hồi là điểm mà vật liệu không còn trở về chiều dài ban đầu sau khi bỏ lực; điểm chảy dẻo là nơi biến dạng tăng mạnh dù ứng suất gần như không đổi; ứng suất kéo cực đại là giá trị ứng suất lớn nhất vật liệu chịu được trước khi bắt đầu thắt lại (necking); ứng suất gãy là ứng suất tại thời điểm vật liệu đứt gãy.

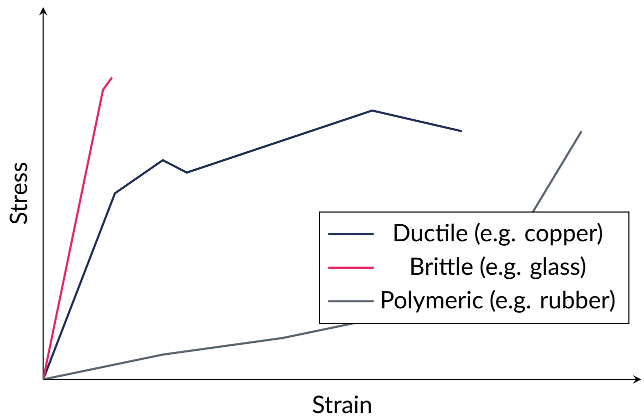

Figure 5.4: Schematic (not to scale) stress-strain curves comparing ductile, brittle and polymeric behaviour. The ductile curve shows a long plastic region before fracture; the brittle curve ends abruptly shortly after the elastic region; the polymeric curve shows very large strain with a non-linear, increasingly steep shape.

Table 5.1: Comparison of ductile, brittle and polymeric (elastomeric) behaviour under stress.

Property	**Ductile** (e.g. copper, gold)	**Brittle** (e.g. glass, cast iron)	**Polymeric** (e.g. rubber)
Plastic region	Large; extensive plastic deformation before fracture	Almost none; fractures shortly after the elastic limit	Not plastic in the usual sense; very large *elastic* strains possible
Elastic limit / yield	Clear yield point, then plastic flow	Elastic limit close to fracture point	No sharp yield; stress-strain curve is markedly non-linear
Fracture strain	Large (tens of %)	Small (fraction of a %)	Very large, but fully recoverable if within elastic range
Loading/unloading curves	Coincide only up to elastic limit	Coincide up to fracture (little plastic flow)	Often do *not* coincide — a hysteresis loop forms

Ví dụ mẫu · Identifying ductile and brittle behaviour

Two wire specimens, X and Y, are stretched to fracture. Specimen X shows a long region of plastic deformation (strain increasing to over 20%) after its elastic limit before finally breaking. Specimen Y fractures suddenly, with almost no visible plastic deformation, shortly after its elastic limit is passed. Identify the type of behaviour shown by each specimen and give a typical example material for each.

Solution. Specimen X shows **ductile** behaviour (large plastic deformation before fracture) — typical of metals such as copper or gold. Specimen Y shows **brittle** behaviour (fractures shortly after the elastic limit, with little or no plastic deformation) — typical of glass or cast iron.

5.2.5 Polymeric Behaviour and Elastic Hysteresis

Rubber-like (polymeric) materials

Rubber and similar polymeric materials can withstand very large elastic strains (often several hundred percent) and show a markedly non-linear stress-strain relationship. When such a material is stretched (loaded) and then released (unloaded), the unloading curve often lies *below* the loading curve on the stress-strain (or force-extension) graph, forming a closed loop known as a **hysteresis loop**.

- The area under the *loading* curve represents the total work done stretching the material.
- The area under the *unloading* curve represents the elastic energy actually recovered (re-

turned as useful work) as the material relaxes.

- The area *enclosed by the loop* (difference between the two) represents the energy that is not recovered — it is dissipated within the material as internal (thermal) energy, causing the material to warm up slightly.

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Vật liệu polymer (như cao su) có thể chịu biến dạng đàn hồi rất lớn và đồ thị ứng suất–biến dạng thường phi tuyến. Khi kéo căng rồi thả ra, đường cong khi thả tải thường nằm dưới đường cong khi kéo tải, tạo thành một vòng trễ (hysteresis loop). Diện tích khép kín của vòng trễ biểu thị năng lượng bị tiêu tán thành nhiệt, không được thu hồi lại dưới dạng công hữu ích.

5.2.6 Strain Energy Density

The elastic energy stored per unit volume of a stretched material (strain energy density) is the area under the stress-strain graph, up to the limit of proportionality:

$$\frac{E_p}{\text{volume}} = \tfrac{1}{2}\sigma\varepsilon$$

The total elastic energy stored is then $E_p = \left(\tfrac{1}{2}\sigma\varepsilon\right) \times (AL)$, which is algebraically identical to $E_p = \tfrac{1}{2}Fx$.

VN **GIẢI THÍCH TIẾNG VIỆT**

Mật độ năng lượng biến dạng đàn hồi (năng lượng trên một đơn vị thể tích) bằng diện tích dưới đồ thị ứng suất–biến dạng, tính bằng $\tfrac{1}{2}\sigma\varepsilon$. Nhân với thể tích của vật liệu ta được tổng năng lượng đàn hồi tích trữ, tương đương với công thức $E_p = \tfrac{1}{2}Fx$.

Ví dụ mẫu · Strain energy density and elastic hysteresis

(a) For the wire in the earlier Young modulus example ($\sigma = 8.0 \times 10^7$ Pa, $\varepsilon = 4.0 \times 10^{-4}$, $A = 0.50 \times 10^{-6}$ m^2, $L = 2.00$ m), calculate the strain energy density and the total elastic energy stored.
(b) A rubber cord is stretched to an extension of 0.20 m. The area under the loading force-extension curve is 0.80 J. When released, the area under the unloading curve is only 0.50 J. Calculate the energy dissipated as heat during the loading-unloading cycle.
Solution. **(a)** Strain energy density:

$$\tfrac{1}{2}\sigma\varepsilon = \tfrac{1}{2}(8.0 \times 10^7)(4.0 \times 10^{-4}) = 1.6 \times 10^4 \text{ J m}^{-3} = 16 \text{ kJ m}^{-3}$$

Volume of wire: $AL = (0.50 \times 10^{-6})(2.00) = 1.0 \times 10^{-6}$ m^3. Total energy stored:

$$E_p = (1.6 \times 10^4) \times (1.0 \times 10^{-6}) = 0.016 \text{ J} = 16 \text{ mJ}$$

(This agrees with $\tfrac{1}{2}Fx = \tfrac{1}{2}(40)(0.80 \times 10^{-3}) = 0.016$ J.)
(b) Energy dissipated = area enclosed by the hysteresis loop = energy supplied during loading − energy recovered during unloading:

$$\Delta E = 0.80 - 0.50 = 0.30 \text{ J}$$

This energy appears as internal (thermal) energy in the rubber; it is not returned as useful mechanical work.

(!) Lỗi thường gặp: When calculating work done or energy stored from a force-extension (or stress-strain) graph, remember that $E_p = \frac{1}{2}Fx$ is only valid for a **straight line through the origin** — that is, within the region where Hooke's law applies. Beyond the limit of proportionality, or for any non-linear relationship (such as rubber), this formula cannot be used; instead, the energy must be found from the actual area under the graph (by counting grid squares or by integration). Similarly, when a loading and unloading curve do not coincide, do not assume all the input energy is recovered — only the area under the *unloading* curve is returned as usable elastic energy.

5.3 Practice

Bài tập luyện tập

Which expression correctly gives the work done by a force F acting at an angle θ to the direction of displacement s?

(A) $W = Fs$

(B) $W = Fs\sin\theta$

(C) $W = Fs\cos\theta$

(D) $W = F/s$

Lời giải chi tiết

Answer: C. Only the component of the force in the direction of motion, $F\cos\theta$, does work, so $W = Fs\cos\theta$.

Bài tập luyện tập

A box is pulled a distance of 8.0 m across a floor by a force of 45 N acting at $30°$ above the horizontal. Calculate the work done by this force.

Lời giải chi tiết

$$W = Fs\cos\theta = 45 \times 8.0 \times \cos 30° = 360 \times 0.866 = 312\,\text{J}\ (\approx 3.1 \times 10^2\ \text{J})$$

Bài tập luyện tập

An object falls from rest through the air, and air resistance acts on it throughout the fall. Which statement is **not** necessarily true?

(A) The object's kinetic energy increases as it falls.

(B) Some of the lost gravitational potential energy is transferred to the surroundings as heat.

(C) All of the lost gravitational potential energy is converted to kinetic energy.

(D) The object's speed is less than it would be in the absence of air resistance.

Lời giải chi tiết

Answer: C. This is false whenever a resistive force acts — some of the gravitational PE lost is dissipated as heat (and sound) rather than converting entirely into kinetic energy.

Bài tập luyện tập

A stone of mass 0.50 kg is dropped from a height of 20 m. Air resistance does 15 J of work against the stone during the fall. Calculate the speed at which the stone lands.

Lời giải chi tiết

Loss in GPE: $mgh = 0.50 \times 9.81 \times 20 = 98.1$ J. Kinetic energy at landing: $E_k = 98.1 - 15 = 83.1$ J.

$$v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2 \times 83.1}{0.50}} = \sqrt{332.4} = 18.2\,\text{m s}^{-1}$$

Bài tập luyện tập

A pendulum bob of mass 0.20 kg is released from a height of 0.45 m above the lowest point of its swing. Air resistance removes 0.30 J of energy from the system by the time the bob reaches the lowest point. Calculate the speed of the bob at the lowest point.

Lời giải chi tiết

Loss in GPE: $mgh = 0.20 \times 9.81 \times 0.45 = 0.883$ J. Kinetic energy at lowest point: $E_k = 0.883 - 0.30 = 0.583$ J.

$$v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2 \times 0.583}{0.20}} = \sqrt{5.83} = 2.41\,\text{m s}^{-1}$$

Bài tập luyện tập

Power may be correctly defined as:

(A) the total energy transferred by a system

(B) the rate at which work is done, or energy is transferred

(C) the force acting per unit area

(D) the total work done by a force over a given displacement

Lời giải chi tiết

Answer: B. Power is the rate of doing work / transferring energy, $P = W/t = \Delta E/t$.

Bài tập luyện tập

A pump lifts 500 kg of water through a height of 12 m in 40 s, at a steady rate. Calculate the useful power output of the pump (ignore any change in kinetic energy of the water).

Lời giải chi tiết

$$P = \frac{mgh}{t} = \frac{500 \times 9.81 \times 12}{40} = \frac{58860}{40} = 1471.5\,\text{W} \approx 1.5\,\text{kW}$$

Bài tập luyện tập

A car experiences a total resistive force of 720 N when travelling at a constant $30\,\text{m s}^{-1}$ along a level road. Calculate the power output of the engine at this speed.

Lời giải chi tiết

$$P = Fv = 720 \times 30 = 21600\,\text{W} = 21.6\,\text{kW}$$

Bài tập luyện tập

A car of mass $1000\,\text{kg}$ travels at a constant speed of $18\,\text{m s}^{-1}$ up a slope for which $\sin\theta = 0.04$. The total resistive force (friction and air resistance) is $350\,\text{N}$. Calculate (a) the driving force required, and (b) the power output of the engine.

Lời giải chi tiết

(a)

$$F_{\text{drive}} = F_{\text{resistive}} + mg\sin\theta = 350 + (1000)(9.81)(0.04) = 350 + 392.4 = 742.4\,\text{N}$$

(b)

$$P = Fv = 742.4 \times 18 = 13363\,\text{W} \approx 13.4\,\text{kW}$$

Bài tập luyện tập

A car of mass $900\,\text{kg}$ is travelling at $12\,\text{m s}^{-1}$ on a level road and accelerating at $1.5\,\text{m s}^{-2}$. The total resistive force at this speed is $300\,\text{N}$. Calculate (a) the driving force, (b) the total power output, (c) the power needed just to overcome resistance at this speed, and (d) the extra power required for the acceleration.

Lời giải chi tiết

(a)

$$F_{\text{drive}} = ma + F_{\text{resistive}} = (900)(1.5) + 300 = 1350 + 300 = 1650\,\text{N}$$

(b)

$$P_{\text{total}} = Fv = 1650 \times 12 = 19800\,\text{W} = 19.8\,\text{kW}$$

(c)

$$P_{\text{resistance}} = 300 \times 12 = 3600\,\text{W} = 3.6\,\text{kW}$$

(d)

$$P_{\text{extra}} = 19800 - 3600 = 16200\,\text{W} = 16.2\,\text{kW}$$

Bài tập luyện tập

Efficiency of a machine is correctly given by:

(A) $\dfrac{\text{total input energy}}{\text{useful output energy}} \times 100\%$

(B) $\dfrac{\text{useful output energy}}{\text{total input energy}} \times 100\%$

(C) useful output energy − total input energy

(D) $\dfrac{\text{energy wasted}}{\text{useful output energy}} \times 100\%$

Lời giải chi tiết

Answer: B. Efficiency is the useful output divided by the total input, expressed as a percentage.

Bài tập luyện tập

An electric motor (efficiency 80%) drives a pulley system (efficiency 75%). The electrical energy supplied to the motor is 500 J. Calculate (a) the mechanical energy delivered by the motor, (b) the useful output energy of the pulley system, and (c) the overall efficiency of the combined system.

Lời giải chi tiết

(a) $500 \times 0.80 = 400$ J **(b)** $400 \times 0.75 = 300$ J **(c)** Overall efficiency $= \dfrac{300}{500} \times 100\% = 60\%$ (consistent with $0.80 \times 0.75 = 0.60$).

Bài tập luyện tập

A machine takes in 8000 J of energy and produces 5000 J of useful work output in 4.0 s. Calculate (a) the efficiency of the machine, (b) the useful power output, and (c) the power wasted.

Lời giải chi tiết

(a) Efficiency $= \dfrac{5000}{8000} \times 100\% = 62.5\%$ **(b)** $P_{\text{useful}} = \dfrac{5000}{4.0} = 1250$ W **(c)** Energy wasted $= 8000 - 5000 = 3000$ J, so $P_{\text{wasted}} = \dfrac{3000}{4.0} = 750$ W

Bài tập luyện tập

A machine receives 9000 J of input energy and produces 6000 J of useful work output in 3.0 s. Calculate (a) the power output, and (b) the efficiency of the machine.

Lời giải chi tiết

(a) $P = \dfrac{6000}{3.0} = 2000$ W **(b)** Efficiency $= \dfrac{6000}{9000} \times 100\% = 66.7\%$

Bài tập luyện tập

On a force-extension graph for a spring obeying Hooke's law, the spring constant k is represented by:

(A) the area under the graph

(B) the gradient of the straight-line region

(C) the intercept on the force axis

(D) the intercept on the extension axis

Lời giải chi tiết

Answer: B. Since $F = kx$, the gradient of the linear force-extension graph equals the spring constant k.

Bài tập luyện tập

A spring stores 4.5 J of elastic potential energy when compressed by 0.30 m. Calculate the spring constant.

Lời giải chi tiết

$$E_p = \tfrac{1}{2}kx^2 \implies k = \frac{2E_p}{x^2} = \frac{2 \times 4.5}{0.30^2} = \frac{9.0}{0.09} = 100\,\text{N m}^{-1}$$

Bài tập luyện tập

A spring of spring constant $250\,\text{N m}^{-1}$ is compressed by $0.12\,\text{m}$. Calculate the elastic potential energy stored.

Lời giải chi tiết

$$E_p = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(250)(0.12)^2 = \tfrac{1}{2}(250)(0.0144) = 1.8\,\text{J}$$

Bài tập luyện tập

Two springs, of spring constants $120\,\text{N m}^{-1}$ and $180\,\text{N m}^{-1}$, are connected **in series**. Calculate (a) the effective spring constant, and (b) the extension produced by a load of $9.0\,\text{N}$.

Lời giải chi tiết

(a)

$$\frac{1}{k_{\text{eff}}} = \frac{1}{120} + \frac{1}{180} = \frac{3}{360} + \frac{2}{360} = \frac{5}{360} = \frac{1}{72} \implies k_{\text{eff}} = 72\,\text{N m}^{-1}$$

(b)

$$x = \frac{F}{k_{\text{eff}}} = \frac{9.0}{72} = 0.125\,\text{m} = 12.5\,\text{cm}$$

Bài tập luyện tập

The same two springs ($120\,\text{N m}^{-1}$ and $180\,\text{N m}^{-1}$) are instead connected **in parallel**. Calculate (a) the effective spring constant, and (b) the extension produced by a load of $9.0\,\text{N}$.

Lời giải chi tiết

(a) $k_{\text{eff}} = 120 + 180 = 300\,\text{N m}^{-1}$ **(b)** $x = \dfrac{9.0}{300} = 0.030\,\text{m} = 3.0\,\text{cm}$

Bài tập luyện tập

Three identical springs, each of spring constant $100\,\text{N m}^{-1}$, are combined so that two of them are connected in parallel, and this parallel combination is then connected in series with the third spring. Calculate the overall effective spring constant.

Lời giải chi tiết

Parallel pair: $k_p = 100 + 100 = 200\,\text{N m}^{-1}$. In series with the third spring ($100\,\text{N m}^{-1}$):

$$\frac{1}{k_{\text{eff}}} = \frac{1}{200} + \frac{1}{100} = \frac{1}{200} + \frac{2}{200} = \frac{3}{200} \implies k_{\text{eff}} = \frac{200}{3} = 66.7\,\text{N m}^{-1}$$

Bài tập luyện tập

A point on a stress-strain graph beyond which a material no longer returns to its original length once the load is removed is called the:

(A) limit of proportionality
(B) elastic limit
(C) ultimate tensile stress
(D) breaking stress

Lời giải chi tiết

Answer: B. Beyond the elastic limit, some permanent (plastic) deformation remains after the load is removed.

Bài tập luyện tập

A wire has a Young modulus of 1.2×10^{11} Pa, a cross-sectional area of 2.0 mm^2, and a natural length of 1.5 m. Calculate the extension produced by a load of 60 N.

Lời giải chi tiết

$$x = \frac{FL}{AE} = \frac{60 \times 1.5}{(2.0 \times 10^{-6})(1.2 \times 10^{11})} = \frac{90}{2.4 \times 10^5} = 3.75 \times 10^{-4}\text{ m} = 0.375\text{ mm}$$

Bài tập luyện tập

A metal wire of length 3.0 m and diameter 0.60 mm extends by 1.2 mm when a load is applied. Given that the Young modulus of the metal is 2.0×10^{11} Pa, calculate the load applied.

Lời giải chi tiết

Radius $r = 0.30$ mm $= 0.30 \times 10^{-3}$ m, so $A = \pi r^2 = \pi(0.30 \times 10^{-3})^2 = 2.83 \times 10^{-7}$ m^2. Strain: $\varepsilon = \dfrac{1.2 \times 10^{-3}}{3.0} = 4.0 \times 10^{-4}$. Stress: $\sigma = E\varepsilon = (2.0 \times 10^{11})(4.0 \times 10^{-4}) = 8.0 \times 10^7$ Pa. Load: $F = \sigma A = (8.0 \times 10^7)(2.83 \times 10^{-7}) = 22.6$ N

Bài tập luyện tập

The diameter of a wire is halved, while the material, length, and applied load remain the same. By what factor does the extension change?

(A) $\times\frac{1}{2}$
(B) $\times 2$
(C) $\times\frac{1}{4}$
(D) $\times 4$

Lời giải chi tiết

Answer: D. Since $x = FL/(AE)$ and $A \propto d^2$, halving the diameter reduces A to one quarter, so x (which is inversely proportional to A) increases by a factor of 4.

Bài tập luyện tập

The diameter of a different wire is instead **doubled**, with the same material, length, and load. State and explain the effect on the extension produced.

Lời giải chi tiết

Since $x = FL/(AE)$ and cross-sectional area $A \propto d^2$, doubling the diameter increases A by a factor of 4. As $x \propto 1/A$ (for fixed F, L, E), the extension is reduced to one quarter of its original value.

Bài tập luyện tập

Stress-strain graphs are obtained for two wire specimens, X and Y. Specimen X shows a long region of plastic deformation (strain increasing well beyond 20%) before fracture. Specimen Y fractures suddenly with almost no plastic deformation, shortly after its elastic limit. Classify the behaviour of each specimen and give one example material for each.

Lời giải chi tiết

Specimen X is **ductile** (e.g. copper, gold) — large plastic deformation occurs before fracture. Specimen Y is **brittle** (e.g. glass, cast iron) — fracture occurs shortly after the elastic limit, with little or no plastic deformation.

Bài tập luyện tập

A rubber band is stretched and then released. Its stress-strain graph shows a large elastic strain, a markedly non-linear shape, and loading/unloading curves that do not coincide, forming a closed loop. (a) What type of behaviour does this describe? (b) What does the fact that the two curves do not coincide indicate? (c) What happens to the energy represented by the area enclosed by the loop?

Lời giải chi tiết

(a) This is typical **polymeric (elastomeric)** behaviour, showing elastic hysteresis. **(b)** The unloading curve lying below the loading curve shows that less energy is returned during unloading than was supplied during loading. **(c)** The area enclosed by the loop represents energy that is not recovered as useful mechanical work — it is dissipated within the material as internal (thermal) energy.

Bài tập luyện tập

During the loading of a rubber cord, the area under the force-extension graph (up to maximum extension) is 1.2 J. When the cord is released, the area under the unloading curve is 0.75 J. Calculate the energy dissipated as heat during the loading-unloading cycle.

Lời giải chi tiết

$$\Delta E = 1.2 - 0.75 = 0.45 \text{ J}$$

Bài tập luyện tập

A wire of cross-sectional area 1.0 mm^2 and length 2.5 m is stretched elastically to a stress of 1.0×10^8 Pa, producing a strain of 5.0×10^{-4}. Calculate (a) the strain energy per unit volume, and (b) the total elastic energy stored in the wire.

Lời giải chi tiết

(a)

$$\tfrac{1}{2}\sigma\varepsilon = \tfrac{1}{2}(1.0\times10^{8})(5.0\times10^{-4}) = 2.5\times10^{4}\ \text{J m}^{-3} = 25\,\text{kJ m}^{-3}$$

(b) Volume $= AL = (1.0\times10^{-6})(2.5) = 2.5\times10^{-6}\ \text{m}^3$.

$$E_p = (2.5\times10^{4})(2.5\times10^{-6}) = 0.0625\ \text{J} = 62.5\,\text{mJ}$$

Bài tập luyện tập

A ball of mass $0.20\,\text{kg}$ is dropped from a height of $5.0\,\text{m}$. Due to air resistance, it lands with a measured speed of $9.0\,\text{m s}^{-1}$ instead of the theoretical (frictionless) value. Calculate (a) the theoretical (ideal) landing speed, and (b) the energy dissipated due to air resistance.

Lời giải chi tiết

(a)

$$v_{\text{ideal}} = \sqrt{2gh} = \sqrt{2\times9.81\times5.0} = \sqrt{98.1} = 9.90\,\text{m s}^{-1}$$

(b) Loss in GPE: $mgh = 0.20\times9.81\times5.0 = 9.81\,\text{J}$. Actual KE: $\tfrac{1}{2}mv^2 = \tfrac{1}{2}(0.20)(9.0)^2 = \tfrac{1}{2}(0.20)(81) = 8.1\,\text{J}$. Energy dissipated: $9.81 - 8.1 = 1.71\,\text{J} \approx 1.7\,\text{J}$

Waves & Superposition

This chapter brings together the two related Cambridge International AS & A Level Physics (9702) topics *Waves* and *Superposition*. We begin with the language used to describe a progressive wave, then build up phase and path difference, wave intensity, the electromagnetic spectrum and polarisation, and the Doppler effect. The second half of the chapter develops the principle of superposition into two-source interference, diffraction (through a single slit and through a diffraction grating), and stationary waves.

> **Mục tiêu Unit:** Sau chương này, học sinh có thể: mô tả sóng truyền bằng biên độ, bước sóng, chu kỳ, tần số và áp dụng phương trình sóng $v = f\lambda$; phân biệt sóng ngang và sóng dọc, và giải thích vì sao hiện tượng phân cực chứng tỏ ánh sáng là sóng ngang; tính độ lệch pha từ hiệu quãng đường và ngược lại; áp dụng định luật nghịch đảo bình phương và mối liên hệ cường độ–biên độ; áp dụng định luật Malus cho ánh sáng qua các tấm phân cực; mô tả các vùng của quang phổ điện từ; giải thích định tính hiệu ứng Doppler; áp dụng nguyên lý chồng chập để phân tích giao thoa hai khe Young, nhiễu xạ qua cách tử nhiễu xạ và qua khe đơn; mô tả sự hình thành sóng dừng trên dây và so sánh sóng dừng với sóng truyền.

6.1 Progressive Waves

A progressive (travelling) wave transfers energy from one place to another without transferring the medium itself. Each particle of the medium oscillates about a fixed equilibrium position, and the disturbance – and the energy it carries – moves outward from the source.

> **Describing a progressive wave**
>
> - **Displacement** x – the distance of a particle from its equilibrium position, at a given instant.
> - **Amplitude** x_0 – the maximum displacement from equilibrium.
> - **Wavelength** λ – the distance between two successive particles that are in phase (e.g. two adjacent peaks).
> - **Period** T – the time for one complete oscillation of a particle, or for the wave pattern to advance by one wavelength.
> - **Frequency** $f = \dfrac{1}{T}$ – the number of complete oscillations per unit time, measured in Hz.
> - **Wave speed** v – the speed at which the wave profile (and the energy) travels.
>
> The three quantities above are linked by the **wave equation**:
>
> $$v = f\lambda$$

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>
> Sóng truyền (progressive wave) mang năng lượng đi xa nguồn nhưng không mang theo vật chất của môi trường – mỗi hạt của môi trường chỉ dao động quanh vị trí cân bằng của nó. Biên độ x_0 là độ lệch lớn nhất so với vị trí cân bằng; bước sóng λ là khoảng cách giữa hai điểm dao

động cùng pha gần nhau nhất; chu kỳ T và tần số $f = 1/T$ mô tả thời gian dao động. Phương trình sóng $v = f\lambda$ liên hệ ba đại lượng: tốc độ, tần số và bước sóng.

6.1.1 Transverse and longitudinal waves

In a **transverse wave** the particles of the medium oscillate *perpendicular* to the direction in which energy is transferred. All electromagnetic waves (radio, light, X-rays, etc.) are transverse, as are waves on a stretched string and water surface ripples.

In a **longitudinal wave** the particles oscillate *parallel* to the direction of energy transfer, producing regions of compression (particles closer together) and rarefaction (particles further apart). Sound waves in air, and the waves produced by pushing a slinky spring back and forth along its length, are longitudinal.

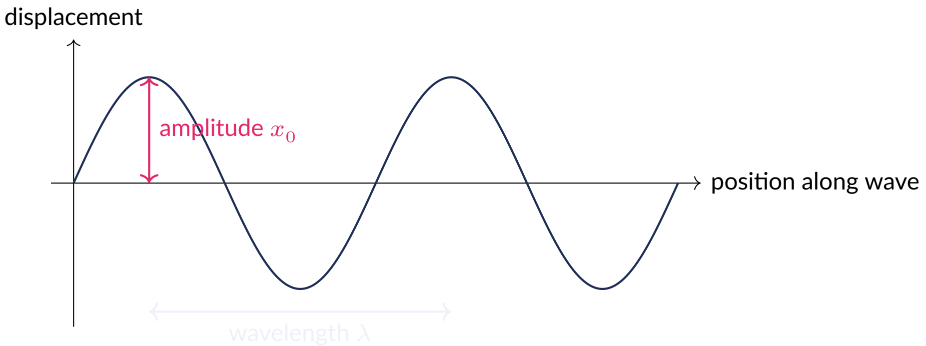

Fig. 6.1 Displacement–position graph for a transverse wave, showing amplitude and wavelength.

Ví dụ mẫu · Wave speed of sound

A loudspeaker emits a sound wave of frequency 500 Hz. The wavelength of the sound in air is measured to be 0.68 m. Calculate the speed of sound in air.
Solution. Using $v = f\lambda$:

$$v = 500 \times 0.68 = 340 \text{ m s}^{-1}$$

This is consistent with the accepted speed of sound in air at room temperature.

6.1.2 Polarisation

Polarisation restricts the oscillations of a transverse wave to a single plane. Only transverse waves can be polarised – a longitudinal wave, whose oscillations are already along the direction of travel, has no perpendicular plane to restrict. The fact that light *can* be polarised (for example, by passing it through a polarising filter, or by reflection from a shiny surface) is therefore direct evidence that light is a transverse wave.

(!) Lỗi thường gặp: Do not confuse polarisation with interference or diffraction. Polarisation is a property that only transverse waves can show; it says nothing about whether two sources are coherent. A common exam error is to claim that sound can be polarised – sound is longitudinal, so it cannot be.

6.2 Phase Difference and Path Difference

Two points on a wave, or two waves arriving at the same point, are described by their **phase difference** – how far out of step their oscillations are. Phase is measured in radians or degrees, with one full cycle corresponding to 2π rad (360°).

When two waves of the same wavelength travel to a point by different routes, the **path difference** (the difference in the distances travelled) determines their phase difference:

$$\text{phase difference} = \frac{\text{path difference}}{\lambda} \times 2\pi \text{ rad} \quad \text{or} \quad \frac{\text{path difference}}{\lambda} \times 360^\circ$$

A path difference of a whole number of wavelengths ($n\lambda$) corresponds to a phase difference of a whole number of cycles – the waves are **in phase**. A path difference of $\left(n + \frac{1}{2}\right)\lambda$ corresponds to a phase difference of an odd number of half-cycles – the waves are in **antiphase**.

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Độ lệch pha (phase difference) cho biết hai dao động lệch nhau bao nhiêu so với một chu kỳ đầy đủ (2π rad hay 360°). Khi hai sóng cùng bước sóng đi theo hai quãng đường khác nhau tới cùng một điểm, hiệu quãng đường (path difference) quyết định độ lệch pha theo công thức: độ lệch pha $=$ (hiệu quãng đường$/\lambda) \times 2\pi$. Hiệu quãng đường bằng số nguyên lần λ nghĩa là hai sóng cùng pha; hiệu quãng đường bằng $(n + \frac{1}{2})\lambda$ nghĩa là hai sóng ngược pha.

Ví dụ mẫu · Phase difference from path difference

Two coherent waves of wavelength 0.48 m arrive at a point P with a path difference of 0.12 m. Find the phase difference between the waves at P, in both radians and degrees.

Solution. The path difference as a fraction of a wavelength is

$$\frac{0.12}{0.48} = 0.25$$

So the phase difference is

$$0.25 \times 360^\circ = 90^\circ \quad \text{or equivalently} \quad 0.25 \times 2\pi = \frac{\pi}{2} \text{ rad}$$

6.3 Intensity of a Wave

The **intensity** I of a wave is the power transmitted per unit area perpendicular to the direction of travel, measured in W m^{-2}. Intensity depends on amplitude in the same way for all waves:

$$I \propto x_0^2$$

For a point source that radiates energy equally in all directions (spherically), the same total power P is spread over the surface of an ever-larger sphere of area $4\pi r^2$ as the wave travels outward, so

$$I = \frac{P}{4\pi r^2} \quad \Rightarrow \quad I \propto \frac{1}{r^2}$$

This is the **inverse square law** for intensity: doubling the distance from a point source reduces the intensity to one quarter.

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Cường độ sóng I tỉ lệ với bình phương biên độ ($I \propto x_0^2$). Đối với một nguồn điểm phát sóng đều theo mọi hướng, năng lượng lan tỏa trên diện tích mặt cầu $4\pi r^2$, nên cường độ giảm theo định luật nghịch đảo bình phương khoảng cách: $I \propto 1/r^2$. Khi khoảng cách tới nguồn tăng gấp đôi, cường độ giảm còn một phần tư.

Ví dụ mẫu · Inverse square law

A small loudspeaker behaves as a point source of sound. At a distance of 2.0 m the intensity of the sound is $8.0\,\mathrm{W\,m^{-2}}$. Calculate the intensity at a distance of 6.0 m.
Solution. Since $I \propto 1/r^2$,

$$I_1 r_1^2 = I_2 r_2^2 \qquad \Rightarrow \qquad I_2 = I_1\left(\frac{r_1}{r_2}\right)^2 = 8.0 \times \left(\frac{2.0}{6.0}\right)^2 = 8.0 \times \frac{1}{9} = 0.89\ \mathrm{W\,m^{-2}}$$

Ví dụ mẫu · Amplitude and intensity

A wave has amplitude 8.0 mm as it leaves a source. After travelling through an absorbing medium its amplitude has fallen to 2.0 mm. What fraction of the original intensity remains?
Solution. Since $I \propto x_0^2$,

$$\frac{I_{\text{final}}}{I_{\text{initial}}} = \left(\frac{2.0}{8.0}\right)^2 = (0.25)^2 = 0.0625$$

So only $6.3\,\%$ of the original intensity remains.

6.4 The Electromagnetic Spectrum

All electromagnetic waves are transverse waves consisting of oscillating electric and magnetic fields, and **all travel at the same speed in a vacuum**, the speed of light:

$$c = 3.00 \times 10^8\ \mathrm{m\,s^{-1}}$$

They differ from one another only in wavelength (and correspondingly frequency, since $c = f\lambda$). Together they form a continuous spectrum, conventionally divided into the regions below.

Region	Approx. wavelength range / m	Typical use
Radio waves	10^{-1} – 10^{4}	Broadcasting, communications
Microwaves	10^{-3} – 10^{-1}	Satellite links, radar, cooking
Infrared	7×10^{-7} – 10^{-3}	Thermal imaging, remote controls
Visible light	4×10^{-7} – 7×10^{-7}	Human vision, optical fibres
Ultraviolet	10^{-9} – 4×10^{-7}	Sterilisation, fluorescent lamps
X-rays	10^{-13} – 10^{-9}	Medical and security imaging
Gamma rays	10^{-16} – 10^{-11}	Cancer treatment, sterilisation

Table 6.1 The electromagnetic spectrum (ranges are approximate order-of-magnitude values; neighbouring regions overlap).

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Toàn bộ sóng điện từ (từ sóng vô tuyến đến tia gamma) đều là sóng ngang gồm điện trường và từ trường dao động vuông góc với nhau, và tất cả đều truyền với cùng tốc độ trong chân không, $c = 3{,}00 \times 10^8$ m s^{-1}. Chúng chỉ khác nhau về bước sóng (và do đó tần số). Bảng quang phổ điện từ liệt kê bảy vùng theo thứ tự bước sóng tăng dần: tia gamma, tia X, tử ngoại, ánh sáng khả kiến, hồng ngoại, vi sóng, sóng vô tuyến.

Ví dụ mẫu · Identifying an EM region

An electromagnetic wave has a wavelength of 1.5×10^{-10} m. Identify the region of the spectrum it belongs to, and calculate its frequency.

Solution. From Table 6.1, a wavelength of 1.5×10^{-10} m ($= 0.15$ nm) lies in the **X-ray** region. Using $c = f\lambda$:

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^{8}}{1.5 \times 10^{-10}} = 2.0 \times 10^{18}\ \text{Hz}$$

6.4.1 Malus's law

When unpolarised light passes through a single polarising filter, only the component of the oscillation parallel to the filter's transmission axis is transmitted, and the transmitted intensity is exactly half of the incident intensity, whatever the orientation of the filter.

If already-polarised light of intensity I_0 is then incident on a second polarising filter whose transmission axis is at angle θ to the plane of polarisation, the transmitted intensity follows **Malus's law**:

$$I = I_0 \cos^2\theta$$

where θ is the angle between the plane of polarisation of the incoming light and the transmission axis of the filter. Note it is $\cos^2\theta$, not $\cos\theta$.

(!) Lỗi thường gặp: Two common errors with Malus's law: (1) forgetting that unpolarised light loses half its intensity on passing through the *first* filter, before Malus's law is even applied; (2) writing $I = I_0\cos\theta$ instead of $I = I_0\cos^2\theta$. Always check whether the incident light is already polarised (apply $\cos^2\theta$ directly) or unpolarised (halve the intensity first, then apply $\cos^2\theta$ to the angle between the two filters).

Ví dụ mẫu · Malus's law – one filter

Polarised light of intensity 8.0 W m^{-2} is incident on a polarising filter whose transmission axis is at 30° to the plane of polarisation of the light. Calculate the transmitted intensity.
Solution.

$$I = I_0\cos^2\theta = 8.0 \times \cos^2(30^\circ) = 8.0 \times 0.75 = 6.0\ \text{W m}^{-2}$$

Ví dụ mẫu · Malus's law – unpolarised light through two filters

Unpolarised light of intensity 12.0 W m^{-2} is incident on two polarising filters in succession. The transmission axes of the two filters are at 40° to each other. Calculate the intensity transmitted through both filters.
Solution. After the first filter, the light is polarised and its intensity is halved:

$$I_1 = \frac{I_0}{2} = \frac{12.0}{2} = 6.0\ \text{W m}^{-2}$$

Applying Malus's law for the second filter, at $\theta = 40^\circ$ to the first:

$$I_2 = I_1\cos^2(40^\circ) = 6.0 \times (0.766)^2 = 6.0 \times 0.587 = 3.5\ \text{W m}^{-2}$$

6.5 The Doppler Effect

When there is relative motion between a wave source and an observer, the observer detects a frequency different from the frequency emitted by the source – this is the **Doppler effect**.

- If the source moves **towards** a stationary observer, successive wavefronts are emitted closer together in the direction of travel: the observed wavelength is shortened and the observed frequency is **higher** than the emitted frequency.
- If the source moves **away** from a stationary observer, successive wavefronts are spread further apart: the observed wavelength is lengthened and the observed frequency is **lower** than the emitted frequency.
- The same reasoning applies (with source and observer roles reversed) if the source is stationary and the observer moves.

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Hiệu ứng Doppler xảy ra khi có chuyển động tương đối giữa nguồn sóng và người quan sát. Khi nguồn tiến lại gần, các mặt sóng bị dồn lại gần nhau hơn theo hướng chuyển động, khiến bước sóng quan sát được ngắn lại và tần số quan sát được cao hơn tần số phát ra. Khi nguồn lùi ra xa, các mặt sóng bị kéo giãn ra, bước sóng quan sát dài hơn và tần số quan sát được thấp hơn.

Ví dụ mẫu · Doppler effect – qualitative

An ambulance approaches a pedestrian standing at the roadside with its siren sounding at a constant emitted frequency, then passes the pedestrian and moves away. Describe and explain how the pitch heard by the pedestrian changes.

Solution. While the ambulance approaches, the source is moving towards the stationary pedestrian, so successive sound wavefronts are compressed in front of it; the pedestrian hears a frequency higher than the frequency emitted by the siren, i.e. a higher pitch. As the ambulance passes and then moves away, the source is now receding, so wavefronts behind it are stretched out; the pedestrian hears a frequency lower than the emitted frequency, i.e. a lower pitch. The pedestrian therefore hears the pitch of the siren drop noticeably as the ambulance passes.

6.6 The Principle of Superposition

When two or more waves meet at a point, the **principle of superposition** states that the resultant displacement at that point is the vector sum of the individual displacements due to each wave.

For a stable, observable interference pattern to form, the sources of the waves must be **coherent**: they must have the same frequency (and wavelength) and a constant phase difference. Two independent lamps, for instance, are not coherent, because the light they emit is produced by huge numbers of unrelated atomic transitions with rapidly and randomly changing phase.

For two coherent waves of the same amplitude arriving at a point with path difference Δ:

- **Constructive interference** (maximum resultant amplitude) occurs when $\Delta = n\lambda$, where $n = 0, 1, 2, \ldots$
- **Destructive interference** (minimum, ideally zero, resultant amplitude) occurs when $\Delta = \left(n + \frac{1}{2}\right)\lambda$

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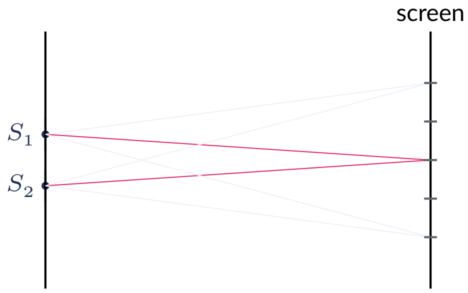
Nguyên lý chồng chập phát biểu rằng khi hai hay nhiều sóng gặp nhau tại một điểm, độ dịch chuyển tổng hợp bằng tổng vectơ của các độ dịch chuyển riêng lẻ. Để có một hình ảnh giao thoa ổn định, hai nguồn sóng phải kết hợp (coherent): cùng tần số, cùng bước sóng và có độ lệch pha không đổi theo thời gian. Giao thoa tăng cường (constructive) xảy ra khi hiệu quãng đường bằng số nguyên lần bước sóng ($n\lambda$); giao thoa triệt tiêu (destructive) xảy ra khi hiệu quãng đường bằng $(n + \frac{1}{2})\lambda$.

6.7 Young's Double-Slit Experiment

Coherent light (commonly obtained by illuminating two narrow, closely-spaced slits with a single laser or with light passed through a single slit first) passing through two slits produces an interference pattern of equally-spaced bright and dark fringes on a distant screen.



Two-slit arrangement (not to scale): bright fringes form where path difference $= n\lambda$; dark fringes where path difference $= (n + \frac{1}{2})\lambda$.

Fig. 6.2 Young's double-slit arrangement.

For slit separation a, screen distance D (with $D \gg a$) and wavelength λ, the fringe (spacing between adjacent bright, or adjacent dark, fringes) spacing x is

$$x = \frac{\lambda D}{a}$$

Ví dụ mẫu · Double-slit fringe spacing

Light of wavelength 600 nm passes through two slits separated by 0.50 mm. The screen is 2.00 m from the slits. Calculate the fringe spacing.

Solution. Using $x = \dfrac{\lambda D}{a}$, with $\lambda = 6.00 \times 10^{-7}$ m, $a = 5.0 \times 10^{-4}$ m, $D = 2.00$ m:

$$x = \frac{6.00 \times 10^{-7} \times 2.00}{5.0 \times 10^{-4}} = \frac{1.2 \times 10^{-6}}{5.0 \times 10^{-4}} = 2.4 \times 10^{-3} \text{ m} = 2.4 \text{ mm}$$

6.8 Diffraction and the Diffraction Grating

Diffraction is the spreading of a wave as it passes through a gap or around an obstacle; it is most pronounced when the gap width is comparable to the wavelength. A **diffraction grating** is a plate ruled with a very large number of closely-spaced, equally-spaced slits, used to split light into sharp, well-defined maxima.

For a grating with slit spacing d, monochromatic light of wavelength λ produces bright maxima (orders $n = 0, 1, 2, \dots$) at angles θ to the incident direction given by

$$d \sin\theta = n\lambda$$

The slit spacing d is related to the number of lines per unit length, N, by $d = \dfrac{1}{N}$.

(!) Lỗi thường gặp: Because $\sin\theta$ cannot exceed 1, the equation $d\sin\theta = n\lambda$ has no solution once $\dfrac{n\lambda}{d} > 1$. This gives a quick way to find the **highest observable order**: find the largest integer n for which $\dfrac{n\lambda}{d} \leq 1$.

Ví dụ mẫu · Diffraction grating angle

Light of wavelength 600 nm is incident normally on a diffraction grating with 500 lines per millimetre. Calculate the angle of the second-order ($n = 2$) maximum.

Solution. The slit spacing is

$$d = \frac{1}{500\ \text{lines mm}^{-1}} = 2.0 \times 10^{-3}\ \text{mm} = 2.0 \times 10^{-6}\ \text{m}$$

Using $d\sin\theta = n\lambda$:

$$\sin\theta = \frac{n\lambda}{d} = \frac{2 \times 6.00 \times 10^{-7}}{2.0 \times 10^{-6}} = 0.60 \qquad \Rightarrow \qquad \theta = 36.9^\circ$$

Ví dụ mẫu · Maximum order of a grating

For the same grating and light as the previous example ($d = 2.0 \times 10^{-6}$ m, $\lambda = 600$ nm), find the highest order of maximum that can be observed.

Solution. The highest possible order satisfies $\dfrac{n\lambda}{d} \leq 1$, so

$$n_{\text{max}} \leq \frac{d}{\lambda} = \frac{2.0 \times 10^{-6}}{6.00 \times 10^{-7}} = 3.33$$

So the largest integer value is $n = 3$. Checking: at $n = 3$, $\sin\theta = \dfrac{3 \times 6.00 \times 10^{-7}}{2.0 \times 10^{-6}} = 0.90$ (valid, $\theta = 64.2^\circ$); at $n = 4$, $\sin\theta = 1.2$, which is impossible. The highest observable order is therefore $n = 3$.

6.8.1 Single-slit diffraction

Passing monochromatic light through a single narrow slit produces a diffraction pattern on a screen consisting of a bright **central maximum**, flanked on either side by much dimmer secondary maxima separated by minima. The central maximum is twice as wide (in terms of angle, or of distance along the screen) as each of the secondary maxima, and is far more intense than any of them.

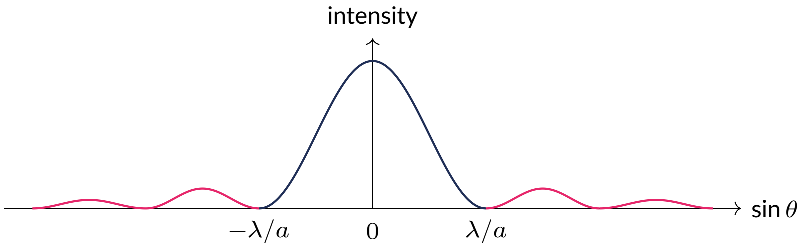


Fig. 6.3 Intensity pattern from a single slit of width a: the central maximum is twice the angular width of, and much more intense than, each side maximum.

> For a single slit of width a, the first minimum (the edge of the central maximum) on either side occurs at angle θ given by
> $$a\sin\theta = \lambda$$

Ví dụ mẫu · Single-slit first minimum

Light of wavelength 600 nm passes through a single slit of width 0.10 mm. A screen is placed 1.50 m from the slit. Find (a) the angle to the first minimum, and (b) the width of the central maximum on the screen.

Solution. (a) Using $a\sin\theta = \lambda$ with $a = 1.0 \times 10^{-4}$ m and $\lambda = 6.0 \times 10^{-7}$ m:

$$\sin\theta = \frac{\lambda}{a} = \frac{6.0 \times 10^{-7}}{1.0 \times 10^{-4}} = 6.0 \times 10^{-3} \qquad \Rightarrow \qquad \theta \approx 0.344^\circ$$

(b) Since θ is small, the distance from the centre of the pattern to the first minimum is $y \approx D\theta$ (with θ in radians, $\theta = 6.0 \times 10^{-3}$ rad):

$$y \approx 1.50 \times 6.0 \times 10^{-3} = 9.0 \times 10^{-3}\ \text{m} = 9.0\ \text{mm}$$

The central maximum extends from $-y$ to $+y$, so its width is

$$2y = 18\ \text{mm}$$

6.9 Stationary (Standing) Waves

A **stationary wave** is formed when two progressive waves of the same frequency, amplitude and wavelength travel through the same region in **opposite directions** and superpose. In practice this is commonly achieved by reflection: for example a wave sent along a string fixed at one end reflects back along itself, or a wave in a tube reflects off a closed or open end.

At certain fixed points, called **nodes**, the two component waves are always in antiphase and cancel completely, so there is no displacement at any time. Midway between adjacent nodes are points called **antinodes**, where the two component waves are always in phase and reinforce, giving the maximum possible amplitude at that point. Unlike a progressive wave, a stationary wave does not transfer energy from one place to another – energy is exchanged locally between adjacent nodes and antinodes but does not travel along the wave as a whole.

Property	Progressive wave	Stationary wave
Energy transfer	Transfers energy away from source	No net energy transfer
Phase along wave	Every point has a different phase (phase varies continuously)	All points between adjacent nodes are in phase; phase jumps by π across each node
Amplitude	Same amplitude for every particle in the wave	Amplitude varies from zero (at nodes) to a maximum (at antinodes)

Table 6.2 Progressive waves compared with stationary waves.

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Sóng dừng hình thành khi hai sóng truyền có cùng tần số, biên độ và bước sóng truyền theo hai hướng ngược nhau (thường do phản xạ) gặp nhau và chồng chập. Tại các nút sóng (node), hai sóng thành phần luôn ngược pha nên triệt tiêu nhau hoàn toàn; tại các bụng sóng (antinode), hai sóng luôn cùng pha nên biên độ đạt cực đại. Khác với sóng truyền, sóng dừng không truyền năng lượng đi xa – năng lượng chỉ trao đổi cục bộ giữa nút và bụng liền kề.

6.9.1 Harmonics on a stretched string

A string of length L fixed rigidly at both ends must have a node at each end. This restricts the possible stationary wave patterns to a discrete set of **harmonics**: the nth harmonic has n antinodes and $(n+1)$ nodes (including the two fixed ends), and satisfies

$$L = n\left(\frac{\lambda}{2}\right), \qquad n = 1, 2, 3, \ldots$$

The first harmonic ($n = 1$) is called the **fundamental**.

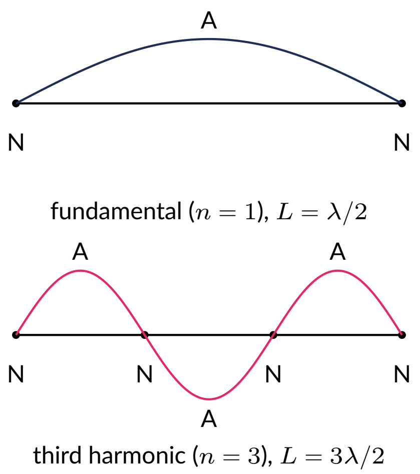


Fig. 6.4 Stationary wave patterns on a string fixed at both ends: the fundamental and the third harmonic.

Ví dụ mẫu · String harmonic (n=3)

A string of length 1.2 m, fixed at both ends, vibrates in its third harmonic (three antinodes visible). Calculate the wavelength of the stationary wave.

Solution. Using $L = n\left(\frac{\lambda}{2}\right)$ with $n = 3$:

$$\lambda = \frac{2L}{n} = \frac{2 \times 1.2}{3} = \frac{2.4}{3} = 0.80 \text{ m}$$

Ví dụ mẫu · String harmonic (n=5)

A different string, of length 0.90 m and fixed at both ends, is set vibrating in its fifth harmonic. The speed of waves on this string is $180\,\text{m}\,\text{s}^{-1}$. Calculate (a) the wavelength and (b) the frequency of vibration.

Solution. (a) Using $L = n\left(\frac{\lambda}{2}\right)$ with $n = 5$:

$$\lambda = \frac{2L}{n} = \frac{2 \times 0.90}{5} = \frac{1.8}{5} = 0.36\ \text{m}$$

(b) Using $v = f\lambda$:

$$f = \frac{v}{\lambda} = \frac{180}{0.36} = 500\ \text{Hz}$$

6.9.2 Investigating stationary sound waves

A stationary sound wave can be set up conveniently in a **resonance tube**: a tuning fork of known frequency is held above the open end of a tube whose length can be varied (for example, by changing the level of water inside it). At certain tube lengths the air column resonates loudly, because a stationary wave has been established with a node at the water surface (or closed end) and an antinode close to the open end. By locating successive positions of resonance as the tube length is changed, and knowing that adjacent resonance lengths correspond to a change in tube length of half a wavelength, the wavelength – and hence, combined with the known frequency of the fork, the speed of sound – can be determined experimentally.

6.10 Practice

Bài tập luyện tập

Which of the following is a longitudinal wave?

(A) Light
(B) Sound in air
(C) Water surface ripples
(D) Radio waves

Lời giải chi tiết

B. In sound, air particles oscillate parallel to the direction the wave travels, producing compressions and rarefactions – this is the defining feature of a longitudinal wave. Light, water ripples and radio waves are all transverse.

Bài tập luyện tập

A water wave has frequency 50 Hz and wavelength 0.24 m. Calculate its speed.

Lời giải chi tiết

$$v = f\lambda = 50 \times 0.24 = 12\ \text{m}\,\text{s}^{-1}$$

Bài tập luyện tập

Which property distinguishes a transverse wave from a longitudinal wave?

(A) The wave speed

(B) The frequency of oscillation

(C) The direction of particle oscillation relative to the direction of energy transfer

(D) The amplitude of oscillation

Lời giải chi tiết

C. In a transverse wave the oscillation is perpendicular to the direction of energy transfer; in a longitudinal wave it is parallel. Speed, frequency and amplitude do not by themselves distinguish the two types.

Bài tập luyện tập

Two coherent point sources S_1 and S_2 emit waves of wavelength 0.040 m. At a point P, $S_1P = 0.62$ m and $S_2P = 0.70$ m. State, with a calculation, whether the interference at P is constructive or destructive.

Lời giải chi tiết

Path difference $= 0.70 - 0.62 = 0.08$ m.

$$\frac{0.08}{0.040} = 2.0$$

The path difference is a whole number (2) of wavelengths, so the interference at P is **constructive**.

Bài tập luyện tập

In a Young's double-slit experiment, the slit separation is halved while the wavelength and the distance to the screen are kept constant. What happens to the fringe spacing?

Lời giải chi tiết

Since $x = \dfrac{\lambda D}{a}$, halving a (with λ and D unchanged) **doubles** the fringe spacing x.

Bài tập luyện tập

Two coherent waves of wavelength 0.36 m arrive at a point with a path difference of 0.27 m. Calculate the phase difference between them, in degrees and in radians.

Lời giải chi tiết

$$\frac{0.27}{0.36} = 0.75$$

$$\text{phase difference} = 0.75 \times 360^\circ = 270^\circ = \frac{3\pi}{2}\ \text{rad}$$

Bài tập luyện tập

A point source produces sound of intensity 2.0×10^{-2} W m^{-2} at a distance of 4.0 m. Calculate the intensity at a distance of 12.0 m, assuming no absorption of energy.

Lời giải chi tiết

$$I_2 = I_1\left(\frac{r_1}{r_2}\right)^2 = 2.0\times10^{-2}\times\left(\frac{4.0}{12.0}\right)^2 = 2.0\times10^{-2}\times\frac{1}{9} = 2.2\times10^{-3}\ \text{W m}^{-2}$$

Bài tập luyện tập

The amplitude of a wave falls to one quarter of its original value as it passes through an absorbing medium. What fraction of the original intensity remains?

Lời giải chi tiết

Since $I \propto x_0^2$,

$$\left(\frac{1}{4}\right)^2 = \frac{1}{16}$$

So $\frac{1}{16}$ $(6.25\,\%)$ of the original intensity remains.

Bài tập luyện tập

Which list places electromagnetic waves in order of *increasing* wavelength?

(A) gamma rays, X-rays, ultraviolet, visible, infrared, microwaves, radio waves

(B) radio waves, microwaves, infrared, visible, ultraviolet, X-rays, gamma rays

(C) visible, ultraviolet, X-rays, gamma rays, radio waves, microwaves, infrared

(D) infrared, visible, ultraviolet, microwaves, radio waves, X-rays, gamma rays

Lời giải chi tiết

A. Gamma rays have the shortest wavelength and radio waves the longest, so listing gamma rays first through to radio waves last gives increasing wavelength.

Bài tập luyện tập

An electromagnetic wave has a wavelength of $2.5\,\text{cm}$. Identify the region of the spectrum it belongs to and calculate its frequency.

Lời giải chi tiết

A wavelength of $2.5\,\text{cm} = 2.5\times10^{-2}$ m lies in the **microwave** region.

$$f = \frac{c}{\lambda} = \frac{3.00\times10^8}{2.5\times10^{-2}} = 1.2\times10^{10}\ \text{Hz}$$

Bài tập luyện tập

Which electromagnetic waves are most strongly absorbed by the ozone layer, and are associated with sunburn and skin damage?

(A) Infrared

(B) Ultraviolet

(C) Microwaves

(D) Radio waves

Lời giải chi tiết

B, ultraviolet. Ozone in the upper atmosphere absorbs much of the Sun's ultraviolet radiation; the UV that reaches the surface is responsible for sunburn and increased skin cancer risk.

Bài tập luyện tập

Polarised light of intensity $15.0\,\mathrm{W\,m^{-2}}$ is incident on a polarising filter whose transmission axis is at 55° to the plane of polarisation of the light. Calculate the transmitted intensity.

Lời giải chi tiết

$$I = I_0 \cos^2\theta = 15.0 \times \cos^2(55^\circ) = 15.0 \times 0.329 = 4.9\ \mathrm{W\,m^{-2}}$$

Bài tập luyện tập

Unpolarised light of intensity $20.0\,\mathrm{W\,m^{-2}}$ passes through two polarising filters whose transmission axes are at 60° to each other. Calculate the transmitted intensity.

Lời giải chi tiết

After the first filter: $I_1 = \dfrac{20.0}{2} = 10.0\ \mathrm{W\,m^{-2}}$. After the second filter:

$$I_2 = I_1 \cos^2(60^\circ) = 10.0 \times 0.25 = 2.5\ \mathrm{W\,m^{-2}}$$

Bài tập luyện tập

Unpolarised light of intensity $40.0\,\mathrm{W\,m^{-2}}$ passes through three polarising filters in succession. The transmission axis of the second filter is at 30° to the first, and the transmission axis of the third filter is at a further 30° to the second. Calculate the intensity transmitted through all three filters.

Lời giải chi tiết

After filter 1: $I_1 = \dfrac{40.0}{2} = 20.0\ \mathrm{W\,m^{-2}}$. After filter 2: $I_2 = 20.0\cos^2(30^\circ) = 20.0 \times 0.75 = 15.0\ \mathrm{W\,m^{-2}}$. After filter 3: $I_3 = 15.0\cos^2(30^\circ) = 15.0 \times 0.75 = 11.3\ \mathrm{W\,m^{-2}}$.

Bài tập luyện tập

Unpolarised light of intensity I_0 passes through two polarising filters. The transmitted intensity is $\frac{3}{8}I_0$. Calculate the angle between the transmission axes of the two filters.

Lời giải chi tiết

$$\frac{I_0}{2}\cos^2\theta = \frac{3}{8}I_0 \quad\Rightarrow\quad \cos^2\theta = \frac{3}{4} \quad\Rightarrow\quad \cos\theta = \frac{\sqrt{3}}{2} = 0.866$$

$$\theta = 30.0^\circ$$

Bài tập luyện tập

Sodium light of wavelength 589 nm illuminates two slits separated by 0.20 mm. The screen is 1.80 m from the slits. Calculate the fringe spacing.

Lời giải chi tiết

$$x = \frac{\lambda D}{a} = \frac{589 \times 10^{-9} \times 1.80}{0.20 \times 10^{-3}} = \frac{1.060 \times 10^{-6}}{2.0 \times 10^{-4}} = 5.30 \times 10^{-3}\text{ m} = 5.30\text{ mm}$$

Bài tập luyện tập

In a double-slit experiment, the slit separation is 0.50 mm and the screen is 2.0 m away. The measured fringe spacing is 2.0 mm. Calculate the wavelength of the light used.

Lời giải chi tiết

$$\lambda = \frac{xa}{D} = \frac{2.0 \times 10^{-3} \times 5.0 \times 10^{-4}}{2.0} = \frac{1.0 \times 10^{-6}}{2.0} = 5.0 \times 10^{-7}\text{ m} = 500\text{ nm}$$

Bài tập luyện tập

In the same apparatus (slit separation 0.50 mm, screen distance 2.0 m), compare the fringe spacing produced by blue light of wavelength 450 nm with that produced by red light of wavelength 650 nm.

Lời giải chi tiết

Blue light: $x = \dfrac{450 \times 10^{-9} \times 2.0}{5.0 \times 10^{-4}} = 1.8 \times 10^{-3}$ m $= 1.8$ mm.

Red light: $x = \dfrac{650 \times 10^{-9} \times 2.0}{5.0 \times 10^{-4}} = 2.6 \times 10^{-3}$ m $= 2.6$ mm.

Red light, having the longer wavelength, produces the wider fringe spacing.

Bài tập luyện tập

Light of wavelength 500 nm is incident normally on a diffraction grating with 600 lines per millimetre. Calculate the angle of the first-order maximum.

Lời giải chi tiết

$$d = \frac{1}{600}\text{ mm} = 1.667 \times 10^{-6}\text{ m}$$

$$\sin\theta = \frac{n\lambda}{d} = \frac{1 \times 500 \times 10^{-9}}{1.667 \times 10^{-6}} = 0.300 \qquad \Rightarrow \qquad \theta = 17.5^\circ$$

Bài tập luyện tập

A diffraction grating has 300 lines per millimetre. Light of wavelength 650 nm is incident on it normally. Find the highest order of maximum that can be observed.

Lời giải chi tiết

$$d = \frac{1}{300}\text{ mm} = 3.333 \times 10^{-6}\text{ m}$$

$$n_{\text{max}} \leq \frac{d}{\lambda} = \frac{3.333 \times 10^{-6}}{650 \times 10^{-9}} = 5.13$$

So the highest integer order is $n = 5$ (checking: at $n = 5$, $\sin\theta = 0.975$, valid; at $n = 6$, $\sin\theta = 1.17$, impossible).

Bài tập luyện tập

A laser of wavelength $633\,\text{nm}$ is incident normally on a diffraction grating. The second-order maximum is observed at 25.0°. Calculate the number of lines per millimetre on the grating.

Lời giải chi tiết

$$d = \frac{n\lambda}{\sin\theta} = \frac{2 \times 633 \times 10^{-9}}{\sin(25.0^\circ)} = \frac{1.266 \times 10^{-6}}{0.4226} = 3.00 \times 10^{-6}\text{ m} = 3.00 \times 10^{-3}\text{ mm}$$

$$N = \frac{1}{d} = \frac{1}{3.00 \times 10^{-3}} \approx 334\text{ lines per mm}$$

Bài tập luyện tập

In single-slit diffraction, how does the width and intensity of the central maximum compare with the secondary (side) maxima?

(A) Same width, same intensity

(B) Twice the width, and much greater intensity

(C) Half the width, and lower intensity

(D) The central maximum has no defined width

Lời giải chi tiết

B. The central maximum is twice the angular width of each secondary maximum, and is far brighter, since most of the diffracted energy is concentrated in it.

Bài tập luyện tập

Light of wavelength $450\,\text{nm}$ passes through a single slit of width $0.030\,\text{mm}$. Calculate the angle to the first minimum.

Lời giải chi tiết

$$\sin\theta = \frac{\lambda}{a} = \frac{450 \times 10^{-9}}{3.0 \times 10^{-5}} = 0.0150 \qquad \Rightarrow \qquad \theta = 0.86^\circ$$

Bài tập luyện tập

In a single-slit diffraction experiment using light of wavelength $600\,\text{nm}$, the first minimum is observed at an angle of 2.0° to the straight-through direction. Calculate the width of the slit.

Lời giải chi tiết

$$a = \frac{\lambda}{\sin\theta} = \frac{600 \times 10^{-9}}{\sin(2.0^\circ)} = \frac{600 \times 10^{-9}}{0.0349} = 1.72 \times 10^{-5}\ \text{m} = 17.2\ \mu\text{m}$$

Bài tập luyện tập

A string of length 1.5 m, fixed at both ends, vibrates in its third harmonic. Calculate the wavelength of the stationary wave, and calculate the frequency of vibration if the wave speed on the string is 250 m s^{-1}.

Lời giải chi tiết

$$\lambda = \frac{2L}{n} = \frac{2 \times 1.5}{3} = 1.0\ \text{m}$$
$$f = \frac{v}{\lambda} = \frac{250}{1.0} = 250\ \text{Hz}$$

Bài tập luyện tập

A string of length 0.80 m, fixed at both ends, vibrates in its fourth harmonic at a frequency of 300 Hz. Calculate the wavelength of the stationary wave and the speed of the wave on the string.

Lời giải chi tiết

$$\lambda = \frac{2L}{n} = \frac{2 \times 0.80}{4} = 0.40\ \text{m}$$
$$v = f\lambda = 300 \times 0.40 = 120\ \text{m s}^{-1}$$

Bài tập luyện tập

A stationary wave of wavelength 0.50 m is set up on a string of length 1.0 m, fixed at both ends. Which harmonic is this, and how many nodes (including the two fixed ends) are present?

Lời giải chi tiết

$$n = \frac{2L}{\lambda} = \frac{2 \times 1.0}{0.50} = 4$$

This is the **fourth harmonic**. The number of nodes is $n + 1 = 5$.

Bài tập luyện tập

In a stationary wave, how does the phase of oscillation compare for all points lying between two adjacent nodes?

(A) They all have completely different, randomly-varying phases

(B) They are all in phase with each other, though their amplitudes differ

(C) They transfer energy continuously from one to the next

(D) They all have exactly the same amplitude

Lời giải chi tiết

B. All points between the same pair of adjacent nodes oscillate in phase with each other (reaching maximum displacement at the same instant); it is only their amplitude that varies, from zero at the nodes to a maximum at the antinode between them.

Bài tập luyện tập

A cyclist rides towards, past, and then away from a stationary observer while continuously ringing a bell of fixed emitted frequency. Describe and explain what the observer hears.

Lời giải chi tiết

As the cyclist approaches, the bell is a moving source approaching a stationary observer, so the wavefronts are compressed and the observer hears a frequency higher than that emitted by the bell (a higher pitch). Once the cyclist has passed and is moving away, the wavefronts are stretched out and the observer hears a frequency lower than that emitted (a lower pitch). The observer therefore perceives a drop in pitch as the cyclist passes.

Bài tập luyện tập

Briefly explain how a resonance tube can be used to demonstrate a stationary sound wave, and how it could be used to help find the wavelength of the sound produced by a tuning fork of known frequency.

Lời giải chi tiết

A tuning fork is held above the open end of a tube whose effective length can be adjusted (for example by changing a water level inside the tube). As the length is changed, at certain lengths the air column resonates strongly with the fork, because a stationary wave has been set up in the tube with a node at the water surface and an antinode near the open end. Locating successive lengths at which resonance occurs shows that adjacent resonance lengths differ by half a wavelength of the sound, allowing the wavelength to be found; combined with the known frequency of the fork, this gives the speed of sound via $v = f\lambda$.

Electricity & D.C. Circuits

Mục tiêu Unit: Chương này giúp học sinh hiểu bản chất dòng điện và hiệu điện thế trong mạch điện một chiều; nắm vững định luật Ohm, điện trở suất, và đặc tuyến dòng điện–hiệu điện thế (I–V) của các linh kiện điển hình: vật dẫn kim loại tuân theo định luật Ohm, đèn sợi đốt, và điốt bán dẫn. Học sinh cũng sẽ hiểu ứng dụng của nhiệt điện trở (thermistor) và quang điện trở (LDR) trong mạch phân áp làm cảm biến, đồng thời vận dụng thành thạo hai định luật Kirchhoff, công thức mạch nối tiếp – song song, điện trở trong của nguồn điện, và mạch phân áp (có tải và không tải) để giải các bài toán mạch điện, kể cả mạch nhiều vòng cần hệ phương trình đồng thời.

This chapter develops the quantitative treatment of direct current (d.c.) electricity: the microscopic picture of current flow, the definitions of potential difference and e.m.f., resistance and resistivity, the characteristic behaviour of common circuit components, and the two laws of Kirchhoff that allow any resistor network – however complex – to be solved.

7.1 Electric Current and Charge

Electric current I is the rate of flow of charge:

$$I = \frac{Q}{t}$$

where Q is the charge (in coulombs, C) passing a point in time t (in seconds, s). Current is measured in amperes (A); 1 A $= 1$ C s^{-1}. Conventional current flows from positive to negative terminal in the external circuit, opposite to the direction of electron drift.

7.1.1 Microscopic model of current

In a conductor of cross-sectional area A, containing n charge carriers per unit volume, each of charge q, moving with average drift velocity v, the current is

$$I = nAvq$$

This follows because in a time t, the carriers travel a distance vt, so all carriers within a cylinder of volume $A(vt)$ pass through the cross-section; the charge is $Q = nAvtq$, giving $I = Q/t = nAvq$.

$I = Q/t$ (macroscopic definition); $I = nAvq$ (microscopic model), where n is number density of charge carriers, A is cross-sectional area, v is mean drift velocity, and q is the charge on each carrier.

VN **GIẢI THÍCH TIẾNG VIỆT**

Dòng điện I là tốc độ dịch chuyển điện tích qua một tiết diện: $I = Q/t$. Ở mức vi mô, dòng điện được sinh ra bởi các hạt mang điện (ví dụ electron tự do trong kim loại) chuyển động với vận tốc trôi trung bình v rất nhỏ (thường cỡ 10^{-4} m/s), theo công thức $I = nAvq$, với n là mật độ hạt mang điện, A là tiết diện dây dẫn, q là điện tích mỗi hạt.

Ví dụ mẫu · Drift velocity in a copper wire

A copper wire carries a current of 2.0 A. The wire has a cross-sectional area of $1.0e-6$ m^2 and the number density of free electrons in copper is $n = 8.5 \times 10^{28}$ m^{-3}. Calculate the mean drift velocity of the electrons.
Solution. Using $I = nAvq$ with $q = 1.6 \times 10^{-19}$ C:

$$v = \frac{I}{nAq} = \frac{2.0}{(8.5 \times 10^{28})(1.0 \times 10^{-6})(1.6 \times 10^{-19})}$$

The denominator is $8.5 \times 10^{28} \times 1.0 \times 10^{-6} = 8.5 \times 10^{22}$, and $8.5 \times 10^{22} \times 1.6 \times 10^{-19} = 1.36 \times 10^4$.

$$v = \frac{2.0}{1.36 \times 10^4} = 1.5 \times 10^{-4} \text{ m s}^{-1}$$

This confirms that although current is established almost instantly throughout a circuit (the electric field propagates near the speed of light), the individual electrons drift extremely slowly.

7.2 Potential Difference, E.M.F. and Power

The **potential difference** (p.d.) between two points is the energy converted from electrical energy to other forms, per unit charge, as charge passes between those points:

$$V = \frac{W}{Q}$$

measured in volts (V); 1 V $= 1$ J C^{-1}.

The **electromotive force** (e.m.f.) ε of a source is the energy converted from other forms (chemical, mechanical, etc.) into electrical energy, per unit charge, driven around the complete circuit by the source. E.m.f. and p.d. have the same unit (the volt) but are conceptually distinct: e.m.f. describes energy *supplied* by a source, while p.d. describes energy *dissipated* or converted in a component.

7.2.1 Electrical power

Since $P = W/t$ and $V = W/Q$, $I = Q/t$, the electrical power delivered to (or dissipated in) a component is

$$P = IV$$

Combining with $V = IR$ (Ohm's law, valid for a resistor) gives two equivalent forms:

$$P = I^2R \qquad \text{and} \qquad P = \frac{V^2}{R}$$

Power dissipated in a resistor: $P = IV = I^2R = \dfrac{V^2}{R}$. Choose whichever form uses the quantities given directly in the problem, to avoid unnecessary intermediate calculation (and rounding error).

GIẢI THÍCH TIẾNG VIỆT

VN

Hiệu điện thế $V = W/Q$ là năng lượng chuyển hóa trên mỗi đơn vị điện tích khi đi qua một linh kiện; suất điện động ε là năng lượng cung cấp cho mỗi đơn vị điện tích bởi nguồn điện (kể cả phần hao phí trên điện trở trong). Công suất điện có thể tính theo ba công thức tương đương $P = IV = I^2R = V^2/R$; nên chọn công thức phù hợp với dữ kiện đã cho để tránh tính toán trung gian không cần thiết.

Ví dụ mẫu · Choosing the correct power formula

For each part below, calculate the power dissipated, choosing the most direct formula.
(a) A lamp is labelled "$12\,\text{V}$, $30\,\text{W}$". Calculate the current it draws and its resistance at rated conditions.
(b) A resistor of resistance $15\,\Omega$ carries a current of $3.0\,\text{A}$. Calculate the power dissipated.
(c) A component of resistance $27\,\Omega$ has a p.d. of $9.0\,\text{V}$ across it. Calculate the power dissipated.
Solution.
(a) Given P and V: $I = P/V = 30/12 = 2.5\,\text{A}$. Then $R = V/I = 12/2.5 = 4.8\,\Omega$.
(b) Given I and R: $P = I^2R = (3.0)^2 \times 15 = 9.0 \times 15 = 135\,\text{W}$.
(c) Given V and R: $P = V^2/R = (9.0)^2/27 = 81/27 = 3.0\,\text{W}$.

7.3 Resistance and Resistivity

The **resistance** of a component is defined as

$$R = \frac{V}{I}$$

measured in ohms (Ω); $1\ \Omega = 1$ V A^{-1}. **Ohm's law** states that the current through a metallic conductor is directly proportional to the p.d. across it, provided the physical conditions (in particular, temperature) remain constant – equivalently, R is constant for such a conductor.

7.3.1 Resistivity

For a uniform wire or sample of length L and cross-sectional area A, the resistance depends on the material through its **resistivity** ρ:

$$R = \frac{\rho L}{A}$$

Resistivity has units of $\Omega\,\text{m}$ (ohm metre) and is a property of the material alone (at a given temperature), independent of the sample's dimensions.

$R = V/I$ (Ohm's law: R constant for a metallic conductor at constant temperature); $R = \rho L/A$, where ρ is the resistivity of the material (unit $\Omega\,\text{m}$).

VN **GIẢI THÍCH TIẾNG VIỆT**
Điện trở $R = V/I$ theo định luật Ohm là hằng số đối với vật dẫn kim loại ở nhiệt độ không đổi. Điện trở suất ρ là đại lượng đặc trưng cho vật liệu (không phụ thuộc kích thước), liên hệ với điện trở qua công thức $R = \rho L/A$: điện trở tỉ lệ thuận với chiều dài L và tỉ lệ nghịch với tiết diện A.

Ví dụ mẫu · Resistance of a copper wire

A copper wire of length $4.0\,\text{m}$ has a cross-sectional area of $2.0e{-}7\,\text{m}^2$. The resistivity of copper is $\rho = 1.7 \times 10^{-8}\ \Omega\,\text{m}$. Calculate the resistance of the wire.
Solution.

$$R = \frac{\rho L}{A} = \frac{(1.7 \times 10^{-8})(4.0)}{2.0 \times 10^{-7}} = \frac{6.8 \times 10^{-8}}{2.0 \times 10^{-7}} = 0.34\,\Omega$$

Ví dụ mẫu · Choosing wire dimensions for a target resistance

A resistor is to be made from nichrome wire ($\rho = 1.10 \times 10^{-6}\ \Omega\,\text{m}$) of length 1.5 m, and must have a resistance of exactly $5.0\,\Omega$. Calculate the required cross-sectional area and the diameter of the wire.

Solution. Rearranging $R = \rho L/A$:

$$A = \frac{\rho L}{R} = \frac{(1.10 \times 10^{-6})(1.5)}{5.0} = \frac{1.65 \times 10^{-6}}{5.0} = 3.3 \times 10^{-7}\ \text{m}^2$$

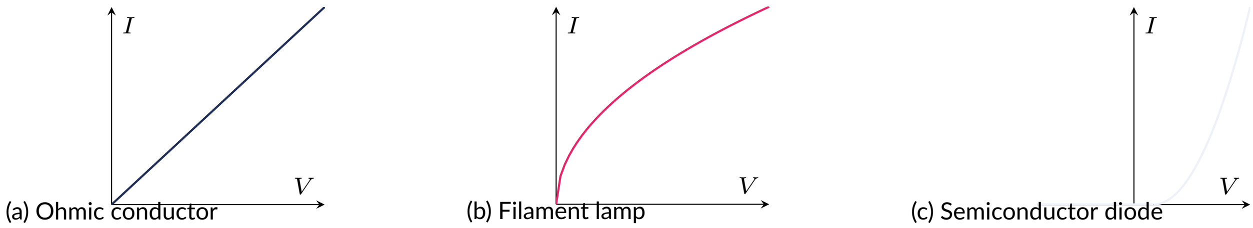
Since $A = \pi r^2$:

$$r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{3.3 \times 10^{-7}}{\pi}} = \sqrt{1.05 \times 10^{-7}} = 3.2 \times 10^{-4}\ \text{m}$$

The diameter is $d = 2r = 6.5 \times 10^{-4}$ m ≈ 0.65 mm.

7.4 Current--Voltage Characteristics

The I–V characteristic of a component is a graph of current against p.d. across it, and reveals how its resistance behaves. Three characteristics are especially important.



(a) Ohmic conductor (b) Filament lamp (c) Semiconductor diode

Current–voltage characteristics. (a) An ohmic (metallic) conductor at constant temperature: a straight line through the origin, since $R = V/I$ is constant. (b) A filament lamp: the curve bends towards the V-axis because current increases less than proportionally as V increases – the filament heats up, and its resistance rises with temperature. (c) A semiconductor diode: negligible current flows until a threshold (forward) voltage is exceeded, after which current rises rapidly; in reverse bias, current remains negligible.

Explaining the shapes of I–V graphs

Ohmic conductor. At constant temperature, the resistance of a metal is constant, so $I \propto V$ – a straight line through the origin, with gradient $1/R$.

Filament lamp. As current increases, the filament dissipates more power ($P = I^2R$) and its temperature rises. The increased thermal vibration of the metal lattice ions scatters the conduction electrons more frequently, increasing resistivity and hence resistance. Consequently, for equal increases in V, the resulting increase in I becomes progressively smaller – the graph curves towards the V-axis.

Semiconductor diode. A diode consists of a p-n junction that allows conventional current to flow easily in one direction (forward bias) but blocks it in the other (reverse bias). In forward bias, negligible current flows until the p.d. reaches a threshold (forward) voltage (typically around 0.6 –0.7 V for silicon), beyond which the junction conducts freely and current rises steeply. In reverse bias, only a negligible leakage current flows.

VN **GIẢI THÍCH TIẾNG VIỆT**

Đặc tuyến I–V của vật dẫn kim loại (tuân theo định luật Ohm) là đường thẳng qua gốc tọa độ vì điện trở không đổi. Đặc tuyến của đèn sợi đốt là đường cong uốn về phía trục V: khi dòng điện tăng, dây tóc nóng lên, điện trở suất và điện trở tăng theo nhiệt độ, khiến dòng điện tăng chậm hơn hiệu điện thế. Đặc tuyến của điốt bán dẫn gần như bằng không cho đến khi đạt điện áp ngưỡng (điện áp thuận), sau đó dòng điện tăng rất nhanh; ở chiều phân cực ngược, dòng điện gần như bằng không.

Ví dụ mẫu · Reading resistance from an I–V graph

The I–V characteristic of an ohmic conductor is a straight line through the origin. At $V = 4.0\,$V, the graph shows $I = 0.80\,$A. Calculate the resistance.
Solution. Since the conductor is ohmic, $R = V/I$ is constant everywhere on the line:

$$R = \frac{4.0}{0.80} = 5.0\,\Omega$$

Ví dụ mẫu · Current in a forward-biased diode circuit

A silicon diode has a constant forward voltage of $0.60\,$V once conducting. It is connected in series with a resistor of $100\,\Omega$ across a $3.0\,$V supply. Calculate the current in the circuit.
Solution. By Kirchhoff's second law, the supply e.m.f. equals the sum of the p.d.s around the loop:

$$\varepsilon = IR + V_{\text{diode}} \implies 3.0 = I(100) + 0.60$$

$$I = \frac{3.0 - 0.60}{100} = \frac{2.40}{100} = 0.024\,\text{A} = 24\,\text{mA}$$

Below the threshold voltage of $0.60\,$V, the diode would conduct negligible current; once forward biased beyond this threshold, the current is limited only by the series resistor.

7.5 Thermistors and Light-Dependent Resistors as Sensors

Some components are deliberately designed so that their resistance responds strongly to a physical condition, making them useful as sensors.

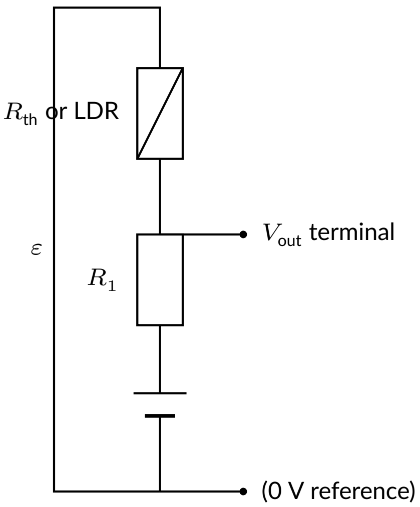
Thermistor (NTC type). A negative temperature coefficient (NTC) thermistor has a resistance that decreases sharply as its temperature increases. As temperature rises, more charge carriers become available in the semiconductor material, and this effect dominates over increased lattice scattering, so resistance falls.

Light-dependent resistor (LDR). An LDR has a resistance that decreases as the intensity of light falling on it increases: more incident light liberates more charge carriers in the semiconductor.

(!) Lỗi thường gặp: An NTC thermistor behaves *oppositely* to a metallic conductor: metals have resistance that *increases* with temperature, while an NTC thermistor's resistance *decreases* with temperature. Do not assume all components behave like ohmic conductors.

7.5.1 Sensor circuits: the potential divider

Both components are commonly used as one arm of a potential divider, so that a changing resistance produces a changing output voltage that can be monitored, amplified, or used to trigger a switch.



A potential-divider sensor circuit: a fixed resistor R_1 in series with a thermistor or LDR across a supply ε. The output V_{out} is taken across one of the two components; as the sensor's resistance changes, so does the fraction of ε appearing across each component.

Ví dụ mẫu · Thermistor potential-divider sensor

A fixed resistor $R_1 = 1.0\,k\Omega$ is connected in series with an NTC thermistor across a 6.0 V supply of negligible internal resistance. The output is taken across R_1. At $20\,^\circ$C, the thermistor's resistance is $3.0\,k\Omega$; when warmed, its resistance falls to $0.50\,k\Omega$. Calculate the output voltage at each temperature.

Solution.

At $20\,^\circ$C: total resistance $= 1.0 + 3.0 = 4.0\,k\Omega$.

$$I = \frac{6.0}{4.0 \times 10^3} = 1.5 \times 10^{-3}\ \text{A} = 1.5\,\text{mA}$$

$$V_{out} = IR_1 = (1.5 \times 10^{-3})(1.0 \times 10^3) = 1.5\,\text{V}$$

When warmed: total resistance $= 1.0 + 0.50 = 1.5\,k\Omega$.

$$I = \frac{6.0}{1.5 \times 10^3} = 4.0 \times 10^{-3}\ \text{A} = 4.0\,\text{mA}$$

$$V_{out} = IR_1 = (4.0 \times 10^{-3})(1.0 \times 10^3) = 4.0\,\text{V}$$

As temperature rises, the thermistor's resistance falls, so a larger fraction of the supply voltage appears across the fixed resistor R_1: the output rises from 1.5 V to 4.0 V. Such a circuit could drive a comparator to switch on a cooling fan once V_{out} exceeds a set threshold.

Ví dụ mẫu · LDR potential-divider sensor

A fixed resistor $R_1 = 4.0\,k\Omega$ is connected in series with an LDR across an 8.0 V supply of negligible internal resistance. The output is taken across the LDR. In darkness the LDR's resistance is $36\,k\Omega$; in bright light it falls to $4.0\,k\Omega$. Calculate the output voltage in each case.

Solution.

In darkness: total resistance $= 4.0 + 36 = 40\,k\Omega$.

$$V_{out} = \varepsilon\left(\frac{R_{LDR}}{R_1 + R_{LDR}}\right) = 8.0 \times \frac{36}{40} = 7.2\,\text{V}$$

In bright light: total resistance $= 4.0 + 4.0 = 8.0\,\text{k}\Omega$.

$$V_{\text{out}} = 8.0 \times \frac{4.0}{8.0} = 4.0\,\text{V}$$

As light intensity increases, the LDR's resistance falls, so the output voltage taken across it falls, from 7.2 V in darkness to 4.0 V in bright light – the opposite trend to that seen when the output is taken across the fixed resistor instead.

VN **GIẢI THÍCH TIẾNG VIỆT**

Nhiệt điện trở loại NTC có điện trở giảm mạnh khi nhiệt độ tăng (ngược với vật dẫn kim loại); quang điện trở (LDR) có điện trở giảm khi cường độ ánh sáng tăng. Cả hai thường được mắc trong mạch phân áp cùng một điện trở cố định: khi điện trở của cảm biến thay đổi, điện áp ra V_{out} lấy trên một trong hai linh kiện cũng thay đổi theo, cho phép mạch điện tử phát hiện sự thay đổi nhiệt độ hoặc ánh sáng.

7.6 Kirchhoff's Laws and Resistor Networks

Kirchhoff's first law states that the sum of currents entering a junction equals the sum of currents leaving it – a consequence of conservation of charge (charge cannot accumulate at a junction in steady conditions).

Kirchhoff's second law states that, around any closed loop in a circuit, the sum of the e.m.f.s equals the sum of the p.d.s (IR drops) – a consequence of conservation of energy (each unit of charge returns to its starting point with no net change in energy).

> Kirchhoff 1: $\sum I_{\text{in}} = \sum I_{\text{out}}$ at any junction. Kirchhoff 2: around any closed loop, $\sum \varepsilon = \sum IR$.

7.6.1 Series and parallel combinations

For resistors in series (same current through each):

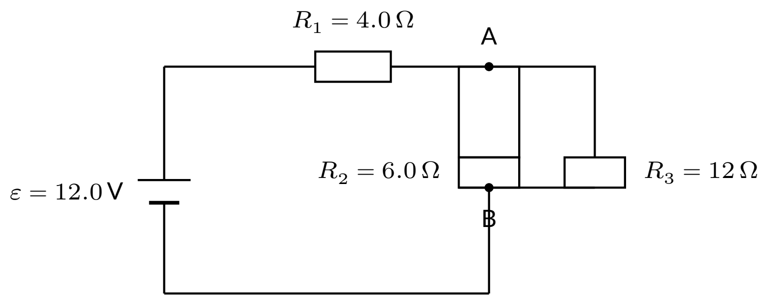
$$R_{\text{total}} = R_1 + R_2 + \cdots$$

For resistors in parallel (same p.d. across each):

$$\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$$

VN **GIẢI THÍCH TIẾNG VIỆT**

Định luật Kirchhoff 1 (bảo toàn điện tích): tổng dòng điện đi vào một nút bằng tổng dòng điện đi ra. Định luật Kirchhoff 2 (bảo toàn năng lượng): quanh một mạch vòng kín, tổng suất điện động bằng tổng độ giảm thế trên các điện trở. Điện trở tương đương: mắc nối tiếp thì cộng trực tiếp $R = R_1 + R_2 + \ldots$; mắc song song thì cộng nghịch đảo $1/R = 1/R_1 + 1/R_2 + \ldots$



$R_1 = 4.0\,\Omega$ in series with the parallel combination of $R_2 = 6.0\,\Omega$ and $R_3 = 12\,\Omega$, driven by an e.m.f. of $12.0\,$V (negligible internal resistance).

Ví dụ mẫu · Series-parallel network

For the circuit above, calculate (a) the resistance of the parallel section, (b) the total resistance, (c) the total current from the source, (d) the p.d. across the parallel section, and (e) the current through each of R_2 and R_3.

Solution.

(a) $\dfrac{1}{R_{23}} = \dfrac{1}{6.0} + \dfrac{1}{12} = \dfrac{2}{12} + \dfrac{1}{12} = \dfrac{3}{12}$, so $R_{23} = \dfrac{12}{3} = 4.0\,\Omega$.

(b) $R_{\text{total}} = R_1 + R_{23} = 4.0 + 4.0 = 8.0\,\Omega$.

(c) $I_{\text{total}} = \dfrac{\varepsilon}{R_{\text{total}}} = \dfrac{12.0}{8.0} = 1.5\,$A.

(d) $V_{23} = I_{\text{total}}R_{23} = 1.5 \times 4.0 = 6.0\,$V.

(e) $I_2 = \dfrac{V_{23}}{R_2} = \dfrac{6.0}{6.0} = 1.0\,$A; $I_3 = \dfrac{V_{23}}{R_3} = \dfrac{6.0}{12} = 0.50\,$A.

Check (Kirchhoff's first law): $I_2 + I_3 = 1.0 + 0.50 = 1.5$ A $= I_{\text{total}}$. ✓

7.7 Internal Resistance of a Source

A real source of e.m.f. (e.g. a battery) has **internal resistance** r, arising from the resistance of the chemicals and electrodes inside it. When a current I is drawn, some of the e.m.f. is used to drive current through r itself, so

$$\varepsilon = I(R + r) = IR + Ir$$

where R is the external (load) resistance. The p.d. across the external circuit – the **terminal p.d.** – is

$$V = \varepsilon - Ir$$

The terminal p.d. equals the e.m.f. only when no current flows (open circuit, $I = 0$).

GIẢI THÍCH TIẾNG VIỆT

VN

Nguồn điện thực luôn có điện trở trong r. Khi có dòng điện I chạy qua, một phần suất điện động bị "hao phí" trên điện trở trong: $\varepsilon = I(R + r)$. Hiệu điện thế ở hai cực của nguồn (điện áp ra ngoài mạch) là $V = \varepsilon - Ir$, luôn nhỏ hơn suất điện động khi có dòng điện chạy qua, và chỉ bằng suất điện động khi mạch hở ($I = 0$).

Ví dụ mẫu · Internal resistance

A cell of e.m.f. $6.0\,$V and internal resistance $0.50\,\Omega$ is connected to an external resistor of resistance $2.5\,\Omega$. Calculate the current in the circuit and the terminal p.d. of the cell.

Solution.

$$I = \frac{\varepsilon}{R + r} = \frac{6.0}{2.5 + 0.50} = \frac{6.0}{3.0} = 2.0\,\text{A}$$
$$V = \varepsilon - Ir = 6.0 - (2.0)(0.50) = 6.0 - 1.0 = 5.0\,\text{V}$$

Ví dụ mẫu · Power delivered versus power wasted

For the cell in the previous example ($\varepsilon = 6.0\,$V, $r = 0.50\,\Omega$, $R = 2.5\,\Omega$, $I = 2.0\,$A), calculate (a) the power dissipated in the internal resistance, (b) the power delivered to the external resistor, and (c) the total power generated by the source. Verify that (a) and (b) sum to (c).

Solution.
(a) $P_{\text{internal}} = I^2 r = (2.0)^2 \times 0.50 = 4.0 \times 0.50 = 2.0\,\text{W}$.
(b) $P_{\text{external}} = I^2 R = (2.0)^2 \times 2.5 = 4.0 \times 2.5 = 10.0\,\text{W}$.
(c) $P_{\text{total}} = I\varepsilon = 2.0 \times 6.0 = 12.0\,\text{W}$.
Check: $P_{\text{internal}} + P_{\text{external}} = 2.0 + 10.0 = 12.0\,\text{W} = P_{\text{total}}$. ✓ This confirms that all the electrical power generated by the source is accounted for: some is usefully delivered to the external circuit, and some is wasted as heat inside the source itself.

7.8 Potential Dividers: Unloaded and Loaded

A potential divider consists of two resistors R_1 and R_2 in series across a supply, with the output taken across R_2 (say). Since the same current flows through both, and $V \propto R$ for a given current,

$$V_{\text{out}} = \varepsilon\left(\frac{R_2}{R_1 + R_2}\right)$$

(!) Lỗi thường gặp: The output of a potential divider is proportional to the *fraction* $R_2/(R_1 + R_2)$, not to R_2 alone. A common error is to write $V_{\text{out}} = \varepsilon R_2$ or to forget to divide by the total resistance of the chain.

7.8.1 Effect of a load

If a load resistor R_L is connected across the output terminals (across R_2), it forms a parallel combination with R_2, reducing the effective resistance of that arm below R_2. This reduces the output voltage below the unloaded value – the divider is said to be **loaded**. The heavier the load (the smaller R_L is compared with R_2), the greater this reduction.

Ví dụ mẫu · Unloaded versus loaded potential divider

A potential divider consists of $R_1 = 3.0\,\text{k}\Omega$ and $R_2 = 6.0\,\text{k}\Omega$ in series across a $9.0\,\text{V}$ supply of negligible internal resistance, with the output taken across R_2.
(a) Calculate the unloaded output voltage.
(b) A load resistor of $6.0\,\text{k}\Omega$ is now connected across R_2. Calculate the new output voltage.
Solution.
(a) $V_{\text{out}} = \varepsilon\left(\dfrac{R_2}{R_1 + R_2}\right) = 9.0 \times \dfrac{6.0}{3.0 + 6.0} = 9.0 \times \dfrac{6.0}{9.0} = 6.0\,\text{V}$.
(b) R_2 and R_L are now in parallel:

$$R_2' = \frac{R_2 R_L}{R_2 + R_L} = \frac{6.0 \times 6.0}{6.0 + 6.0} = \frac{36}{12} = 3.0\,\text{k}\Omega$$

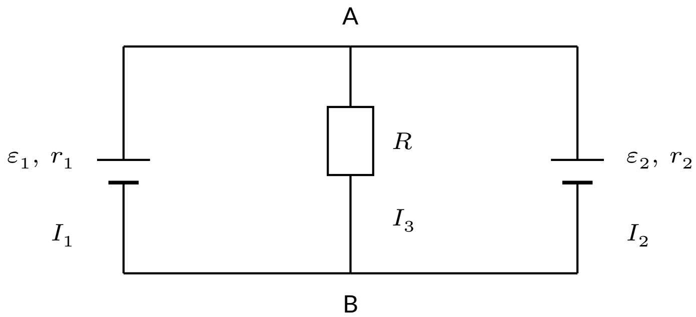
The new total resistance of the chain is $R_1 + R_2' = 3.0 + 3.0 = 6.0\,\text{k}\Omega$, so

$$V_{\text{out}}' = \varepsilon\left(\frac{R_2'}{R_1 + R_2'}\right) = 9.0 \times \frac{3.0}{6.0} = 4.5\,\text{V}$$

Connecting the load reduces the output from $6.0\,\text{V}$ to $4.5\,\text{V}$: the load current, drawn through R_1, increases the p.d. dropped across R_1 and so reduces the p.d. available across the output.

7.9 Multi-Loop Circuits and Simultaneous Equations

Networks containing more than one source, or more loops than can be reduced by simple series/-parallel combination, require both of Kirchhoff's laws applied together: the junction rule gives a relationship between the branch currents, and the loop rule (applied to each independent loop) gives one equation per loop. Together these form a set of simultaneous equations that can be solved for the unknown currents.



Two cells (e.m.f. ε_1, ε_2, internal resistances r_1, r_2) each connected between the same pair of nodes A and B as a shared resistor R. Currents I_1 and I_2 combine at A to give $I_3 = I_1 + I_2$ through R.

Ví dụ mẫu · Multi-loop circuit (Kirchhoff's second law)

In the circuit above, $\varepsilon_1 = 15.0\,\text{V}$, $r_1 = 1.0\,\Omega$, $\varepsilon_2 = 13.0\,\text{V}$, $r_2 = 1.0\,\Omega$, and $R = 3.0\,\Omega$. Currents I_1 and I_2 flow from B to A through the two cells respectively, combining into current I_3 which flows from A to B through R. Find I_1, I_2 and I_3.

Solution. By Kirchhoff's first law at junction A:

$$I_3 = I_1 + I_2 \qquad (1)$$

Applying Kirchhoff's second law to the loop containing ε_1 and R (sum of e.m.f.s = sum of IR drops):

$$\varepsilon_1 = I_1 r_1 + I_3 R \implies 15.0 = 1.0\,I_1 + 3.0\,I_3 \qquad (2)$$

Applying Kirchhoff's second law to the loop containing ε_2 and R:

$$\varepsilon_2 = I_2 r_2 + I_3 R \implies 13.0 = 1.0\,I_2 + 3.0\,I_3 \qquad (3)$$

From (2): $I_1 = 15.0 - 3.0 I_3$. From (3): $I_2 = 13.0 - 3.0 I_3$. Substituting into (1):

$$I_3 = (15.0 - 3.0 I_3) + (13.0 - 3.0 I_3) = 28.0 - 6.0 I_3$$

$$7.0 I_3 = 28.0 \implies I_3 = 4.0\,\text{A}$$

Then $I_1 = 15.0 - 3.0(4.0) = 15.0 - 12.0 = 3.0\,\text{A}$, and $I_2 = 13.0 - 3.0(4.0) = 13.0 - 12.0 = 1.0\,\text{A}$.
Check: $I_1 + I_2 = 3.0 + 1.0 = 4.0\,\text{A} = I_3$. ✓
Further check (p.d. across R must agree from both loops): $V_{AB} = I_3 R = 4.0 \times 3.0 = 12.0\,\text{V}$. Via branch 1: $\varepsilon_1 - I_1 r_1 = 15.0 - 3.0(1.0) = 12.0\,\text{V}$. Via branch 2: $\varepsilon_2 - I_2 r_2 = 13.0 - 1.0(1.0) = 12.0\,\text{V}$. Both agree. ✓

GIẢI THÍCH TIẾNG VIỆT

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Với mạch điện có từ hai vòng trở lên (thường có từ hai nguồn điện trở lên), ta không thể chỉ dùng công thức mắc nối tiếp/song song đơn giản mà phải áp dụng đồng thời cả hai định luật Kirchhoff: định luật 1 tại nút cho một phương trình liên hệ giữa các dòng điện nhánh, định luật 2 áp dụng cho từng mạch vòng độc lập cho thêm các phương trình. Giải hệ phương trình đồng thời này ta tìm được tất cả các dòng điện nhánh, và nên kiểm tra lại bằng cách thay số vào cả hai phương trình vòng.

7.10 Practice Questions

Bài tập luyện tập

Q1. A charge of 6.0 C passes a point in a wire in 4.0 s. Calculate the current.

Lời giải chi tiết

$I = Q/t = 6.0/4.0 = 1.5$ A.

Bài tập luyện tập

Q2. A conductor of cross-sectional area $2.0e{-}6$ m^2 contains charge carriers of number density 8.5×10^{28} m^{-3}, each of charge 1.6×10^{-19} C, moving with mean drift velocity 2.0×10^{-4} m s^{-1}. Calculate the current.

Lời giải chi tiết

$I = nAvq = (8.5 \times 10^{28})(2.0 \times 10^{-6})(2.0 \times 10^{-4})(1.6 \times 10^{-19})$. Step by step: $8.5 \times 10^{28} \times 2.0 \times 10^{-6} = 1.7 \times 10^{23}$; $\times 2.0 \times 10^{-4} = 3.4 \times 10^{19}$; $\times 1.6 \times 10^{-19} = 5.44$. $I \approx 5.4$ A.

Bài tập luyện tập

Q3.

(A) Current is the rate of flow of energy.

(B) Current is the rate of flow of charge.

(C) Current is the rate of flow of potential difference.

(D) Current is the charge stored per unit volume.

Lời giải chi tiết

B. By definition, $I = Q/t$: current is the rate of flow of charge.

Bài tập luyện tập

Q4. A nichrome wire of length 2.0 m has diameter 0.50 mm. The resistivity of nichrome is 1.10×10^{-6} Ω m. Calculate its resistance.

Lời giải chi tiết

Radius $= 0.25$ mm $= 2.5 \times 10^{-4}$ m. $A = \pi r^2 = \pi(2.5 \times 10^{-4})^2 = \pi \times 6.25 \times 10^{-8} = 1.96 \times 10^{-7}$ m^2. $R = \rho L/A = (1.10 \times 10^{-6})(2.0)/(1.96 \times 10^{-7}) = 2.20 \times 10^{-6}/1.96 \times 10^{-7} \approx 11\,\Omega$.

Bài tập luyện tập

Q5. A wire of resistance R has its length doubled and its diameter doubled (same material). Express the new resistance in terms of R.

Lời giải chi tiết

Doubling the diameter quadruples the cross-sectional area ($A \propto d^2$). New resistance:

$$R' = \frac{\rho(2L)}{4A} = \frac{1}{2} \cdot \frac{\rho L}{A} = \frac{R}{2}$$

The new resistance is half the original, $R' = R/2$.

Bài tập luyện tập

Q6. A resistor of resistance $8.0\,\Omega$ is connected across a 12 V supply.

(A) $P = 1.5$ W

(B) $P = 9.0$ W

(C) $P = 18$ W

(D) $P = 144$ W

Lời giải chi tiết

C. $P = V^2/R = 144/8.0 = 18$ W.

Bài tập luyện tập

Q7. A device draws a current of 0.50 A from a 240 V mains supply. Calculate the power it consumes.

Lời giải chi tiết

$P = IV = 0.50 \times 240 = 120$ W.

Bài tập luyện tập

Q8. A resistor of resistance $100\,\Omega$ carries a current of 0.20 A. Calculate the power dissipated.

Lời giải chi tiết

$P = I^2R = (0.20)^2 \times 100 = 0.040 \times 100 = 4.0$ W.

Bài tập luyện tập

Q9. A component has a p.d. of 6.0 V across it and a resistance of $12\,\Omega$. Calculate the power dissipated.

Lời giải chi tiết

$P = V^2/R = (6.0)^2/12 = 36/12 = 3.0$ W.

Bài tập luyện tập

Q10. An electric heater rated at 2.0 kW operates for 3.0 h. Calculate the energy transferred, in kilowatt-hours and in joules.

Lời giải chi tiết

$E = Pt = 2.0$ kW $\times$ 3.0 h $= 6.0$ kWh. Since 1 kWh $= 3.6 \times 10^6$ J: $E = 6.0 \times 3.6 \times 10^6 = 2.16 \times 10^7$ J.

Bài tập luyện tập

Q11. Two resistors of resistance $10\,\Omega$ and $15\,\Omega$ are connected in parallel. Calculate the combined resistance.

Lời giải chi tiết

$\dfrac{1}{R} = \dfrac{1}{10} + \dfrac{1}{15} = \dfrac{3}{30} + \dfrac{2}{30} = \dfrac{5}{30}$, so $R = \dfrac{30}{5} = 6.0\,\Omega$.

Bài tập luyện tập

Q12. For the circuit of $R_1 = 4.0\,\Omega$ in series with $R_2 = 6.0\,\Omega$ parallel $R_3 = 12\,\Omega$ (worked example above, $\varepsilon = 12.0$ V), the current through R_3 was found to be 0.50 A and through R_2 to be 1.0 A. Verify Kirchhoff's first law at the junction where the parallel branches recombine, given the total current is 1.5 A.

Lời giải chi tiết

Current entering the junction from the two branches: $I_2 + I_3 = 1.0 + 0.50 = 1.5$ A, which equals the total current $I_{\text{total}} = 1.5$ A leaving the junction. Kirchhoff's first law is satisfied. ✓

Bài tập luyện tập

Q13. A resistor $R_1 = 2.0\,\Omega$ is in series with the parallel combination of $R_2 = 3.0\,\Omega$ and $R_3 = 6.0\,\Omega$, driven by an e.m.f. of 9.0 V (negligible internal resistance). Find the total current and the current through R_3.

Lời giải chi tiết

$\dfrac{1}{R_{23}} = \dfrac{1}{3.0} + \dfrac{1}{6.0} = \dfrac{2}{6.0} + \dfrac{1}{6.0} = \dfrac{3}{6.0}$, so $R_{23} = 2.0\,\Omega$. $R_{\text{total}} = 2.0 + 2.0 = 4.0\,\Omega$; $I_{\text{total}} = 9.0/4.0 = 2.25$ A. $V_{23} = I_{\text{total}} R_{23} = 2.25 \times 2.0 = 4.5$ V. $I_3 = V_{23}/R_3 = 4.5/6.0 = 0.75$ A (and $I_2 = 4.5/3.0 = 1.5$ A; check $I_2 + I_3 = 2.25$ A $= I_{\text{total}}$ ✓).

Bài tập luyện tập

Q14. A cell of e.m.f. 9.0 V and internal resistance $1.0\,\Omega$ is connected to an external resistor of $8.0\,\Omega$. Calculate the current and the terminal p.d.

Lời giải chi tiết

$I = \varepsilon/(R + r) = 9.0/(8.0 + 1.0) = 9.0/9.0 = 1.0$ A. $V = \varepsilon - Ir = 9.0 - (1.0)(1.0) = 8.0$ V.

Bài tập luyện tập

Q15.

(A) The terminal p.d. always equals the e.m.f.

(B) The terminal p.d. equals the e.m.f. only when the current is zero (open circuit).

(C) The terminal p.d. is always greater than the e.m.f.

(D) The terminal p.d. is independent of the internal resistance.

Lời giải chi tiết

B. Since $V = \varepsilon - Ir$, the terminal p.d. equals ε only when $I = 0$; whenever current flows, energy is dissipated in the internal resistance, so $V < \varepsilon$.

Bài tập luyện tập

Q16. A cell of e.m.f. $1.5\,\mathrm{V}$ and internal resistance $0.50\,\Omega$ is short-circuited (external resistance zero). Calculate the maximum current it can deliver.

Lời giải chi tiết

With $R = 0$: $I = \varepsilon/r = 1.5/0.50 = 3.0\,\mathrm{A}$.

Bài tập luyện tập

Q17. A graph of terminal p.d. against current for a battery is a straight line with a y-intercept of $10\,\mathrm{V}$ and gradient of magnitude $2.0\,\Omega$. Calculate the terminal p.d. when the current is $1.5\,\mathrm{A}$.

Lời giải chi tiết

The intercept gives $\varepsilon = 10\,\mathrm{V}$ and the (negative) gradient gives $-r = -2.0\,\Omega$, since $V = \varepsilon - Ir$. $V = 10 - 2.0 \times 1.5 = 10 - 3.0 = 7.0\,\mathrm{V}$.

Bài tập luyện tập

Q18. A potential divider has $R_1 = 4.0\,\mathrm{k\Omega}$ and $R_2 = 8.0\,\mathrm{k\Omega}$ in series across a $12\,\mathrm{V}$ supply (negligible internal resistance), output taken across R_2. Calculate the unloaded output voltage.

Lời giải chi tiết

$$V_{\text{out}} = 12 \times \frac{8.0}{4.0 + 8.0} = 12 \times \frac{8.0}{12} = 8.0\,\mathrm{V}.$$

Bài tập luyện tập

Q19. For the divider in Q18, a load resistor of $8.0\,\mathrm{k\Omega}$ is now connected across R_2. Calculate the new output voltage.

Lời giải chi tiết

$R_2' = \dfrac{8.0 \times 8.0}{8.0 + 8.0} = \dfrac{64}{16} = 4.0\,\mathrm{k\Omega}$. New total resistance $= R_1 + R_2' = 4.0 + 4.0 = 8.0\,\mathrm{k\Omega}$.

$V'_{\text{out}} = 12 \times \dfrac{4.0}{8.0} = 6.0\,\text{V}$ (down from $8.0\,\text{V}$ unloaded).

Bài tập luyện tập

Q20.

(A) Adding a load resistor across the output of a potential divider always increases the output voltage.

(B) Adding a load resistor across the output of a potential divider has no effect on the output voltage.

(C) Adding a load resistor across the output of a potential divider reduces the output voltage below the unloaded value.

(D) Adding a load resistor across the output of a potential divider makes the output voltage equal to the e.m.f.

Lời giải chi tiết

C. The load resistor forms a parallel combination with the output-side resistor, lowering its effective resistance and hence reducing its share of the total p.d.

Bài tập luyện tập

Q21. A fixed resistor $R_1 = 2.0\,\text{k}\Omega$ is in series with an NTC thermistor across a $5.0\,\text{V}$ supply (negligible internal resistance), output taken across the thermistor. At a cool temperature the thermistor's resistance is $8.0\,\text{k}\Omega$; when warmed it falls to $0.50\,\text{k}\Omega$. Calculate the output voltage at each temperature.

Lời giải chi tiết

Cool: total resistance $= 2.0 + 8.0 = 10.0\,\text{k}\Omega$. $V_{\text{out}} = 5.0 \times \dfrac{8.0}{10.0} = 4.0\,\text{V}$. Warm: total resistance $= 2.0 + 0.50 = 2.50\,\text{k}\Omega$. $V_{\text{out}} = 5.0 \times \dfrac{0.50}{2.50} = 1.0\,\text{V}$. As temperature rises, the thermistor's resistance falls, so the output taken across it falls, from $4.0\,\text{V}$ to $1.0\,\text{V}$.

Bài tập luyện tập

Q22. A fixed resistor $R = 5.0\,\text{k}\Omega$ is in series with an LDR across a $6.0\,\text{V}$ supply (negligible internal resistance), output taken across R. In darkness the LDR's resistance is $95\,\text{k}\Omega$; in bright light it falls to $5.0\,\text{k}\Omega$. Calculate the output voltage in each case.

Lời giải chi tiết

Dark: total resistance $= 5.0 + 95 = 100\,\text{k}\Omega$. $V_{\text{out}} = 6.0 \times \dfrac{5.0}{100} = 0.30\,\text{V}$. Light: total resistance $= 5.0 + 5.0 = 10\,\text{k}\Omega$. $V_{\text{out}} = 6.0 \times \dfrac{5.0}{10} = 3.0\,\text{V}$. The output rises from $0.30\,\text{V}$ in darkness to $3.0\,\text{V}$ in bright light, since the LDR's resistance falls as light intensity increases.

Bài tập luyện tập

Q23.

(A) An NTC thermistor's resistance increases with temperature, like a metal.

(B) An NTC thermistor's resistance decreases as temperature increases.

(C) An NTC thermistor's resistance is independent of temperature.

(D) An NTC thermistor conducts only above a threshold voltage, like a diode.

Lời giải chi tiết

B. "NTC" stands for negative temperature coefficient: resistance falls as temperature rises, the opposite of a metallic conductor.

Bài tập luyện tập

Q24. Which I–V characteristic shows negligible current until a threshold (forward) voltage is reached, followed by a rapid increase in current?

(A) The ohmic conductor

(B) The filament lamp

(C) The semiconductor diode (forward bias)

(D) None of these

Lời giải chi tiết

C. This is the defining feature of a diode's forward-bias characteristic: negligible conduction below the threshold voltage, then a steep rise.

Bài tập luyện tập

Q25. Explain, in terms of the microscopic behaviour of the conductor, why the I–V characteristic of a filament lamp curves towards the V-axis at higher currents.

Lời giải chi tiết

As current increases, the power dissipated ($P = I^2R$) raises the filament's temperature. Increased thermal vibration of the lattice ions scatters the conduction electrons more frequently, increasing the resistivity and hence the resistance of the filament. Since R increases with I, equal increases in V produce progressively smaller increases in I, so the graph bends towards the V-axis.

Bài tập luyện tập

Q26. Two cells drive current into a shared resistor $R = 4.0\,\Omega$ connected between nodes A and B, as in the multi-loop circuit studied above. Cell 1 has $\varepsilon_1 = 16.0\,$V, $r_1 = 2.0\,\Omega$; cell 2 has $\varepsilon_2 = 14.0\,$V, $r_2 = 2.0\,\Omega$. Currents I_1 and I_2 (through the cells) combine into $I_3 = I_1 + I_2$ through R. Find I_1, I_2 and I_3.

Lời giải chi tiết

Loop equations: $16.0 = 2.0I_1 + 4.0I_3$ (1); $14.0 = 2.0I_2 + 4.0I_3$ (2); $I_3 = I_1 + I_2$ (3).
From (1): $I_1 = 8.0 - 2.0I_3$. From (2): $I_2 = 7.0 - 2.0I_3$. Substituting into (3):

$$I_3 = (8.0 - 2.0I_3) + (7.0 - 2.0I_3) = 15.0 - 4.0I_3 \implies 5.0I_3 = 15.0 \implies I_3 = 3.0\,\text{A}$$

$I_1 = 8.0 - 2.0(3.0) = 2.0\,$A; $I_2 = 7.0 - 2.0(3.0) = 1.0\,$A.
Check: $I_1 + I_2 = 2.0 + 1.0 = 3.0$ A $= I_3$. ✓ Check p.d. across R: $I_3R = 3.0 \times 4.0 = 12.0\,$V; via

branch 1: $\varepsilon_1 - I_1 r_1 = 16.0 - 2.0(2.0) = 12.0$ V; via branch 2: $\varepsilon_2 - I_2 r_2 = 14.0 - 1.0(2.0) = 12.0$ V. Both agree. ✓

Bài tập luyện tập

Q27. A power station must deliver 200 kW of power to a distant town through cables of total resistance $0.50\,\Omega$. Compare the power lost as heat in the cables when transmission occurs at (a) 1000 V and (b) 100000 V.

Lời giải chi tiết

(a) $I = P/V = 2.0 \times 10^5/1000 = 200$ A. Power loss $= I^2R = (200)^2 \times 0.50 = 4.0 \times 10^4 \times 0.50 = 2.0e4$ W $= 20$ kW – a tenth of the transmitted power.
(b) $I = P/V = 2.0 \times 10^5/1.0 \times 10^5 = 2.0$ A. Power loss $= I^2R = (2.0)^2 \times 0.50 = 4.0 \times 0.50 = 2.0$ W – negligible compared with the transmitted power.
Transmitting at very high voltage keeps the current (and hence the I^2R loss) small, which is why electricity is distributed over long distances at high voltage.

Bài tập luyện tập

Q28. A silicon diode, with a constant forward voltage of 0.70 V once conducting, is connected in series with a resistor of $330\,\Omega$ across a 6.0 V supply. Calculate the current in the circuit.

Lời giải chi tiết

By Kirchhoff's second law: $\varepsilon = IR + V_{\text{diode}}$, so $6.0 = I(330) + 0.70$.

$$I = \frac{6.0 - 0.70}{330} = \frac{5.30}{330} = 0.0161 \text{ A} \approx 16 \text{ mA}$$

Particle & Nuclear Physics

> **Mục tiêu Unit:** Cấu trúc nguyên tử và hạt nhân (proton, neutron, electron; số proton Z, số nucleon A; đồng vị; kí hiệu hạt nhân); quark và lepton, thành phần quark của hadron (baryon, meson), sự thay đổi quark trong phân rã beta; bốn lực cơ bản (hấp dẫn, điện từ, hạt nhân yếu, hạt nhân mạnh) và các hạt truyền tương tác (gluon, photon, boson W/Z, graviton); phản hạt, sinh cặp và huỷ cặp; phân rã phóng xạ (alpha, beta, gamma) và cân bằng phương trình hạt nhân; khuyết tật khối lượng và năng lượng liên kết ($E = mc^2$), đường cong năng lượng liên kết trên mỗi nucleon, phân hạch & nhiệt hạch; hằng số phân rã, độ phóng xạ (activity), chu kì bán rã, $N = N_0 e^{-\lambda t}$ và $T_{1/2} = \ln 2/\lambda$.

8.1 Atomic and Nuclear Structure

[AS]Every atom consists of a small, dense, positively-charged **nucleus**, surrounded by orbiting **electrons**. The nucleus itself is made up of **protons** and **neutrons**, collectively called **nucleons**.

> **Proton number (atomic number)** Z**:** the number of protons in the nucleus (this also equals the number of electrons in a neutral atom, and determines which chemical element the atom is).
> **Nucleon number (mass number)** A**:** the **total** number of protons and neutrons in the nucleus.
> **Neutron number** $N = A - Z$.

Particle	Relative charge	Relative mass	Location
Proton	$+1$	1	nucleus
Neutron	0	1	nucleus
Electron	-1	$\approx 1/1840$	orbiting the nucleus

Relative charges and masses of the three particles that make up an atom. The proton and neutron have approximately equal mass, roughly 1840 times that of the electron.

8.1.1 Nuclide Notation and Isotopes

A specific nuclide (a specific type of nucleus) is written using the standard notation

$$^{A}_{Z}\mathrm{X}$$

where X is the chemical symbol of the element, A is the nucleon number (top left) and Z is the proton number (bottom left). For example, $^{12}_{6}\mathrm{C}$ is a carbon nucleus with 6 protons and $12 - 6 = 6$ neutrons.

> **Isotopes** are atoms of the **same element** (same proton number Z, and hence the same number of electrons and the same chemical properties) but with **different numbers of neutrons** (and therefore different nucleon numbers A). Isotopes of the same element have identical chemical behaviour but different masses, and often different nuclear stability — some isotopes are stable, others are radioactive.

Ví dụ mẫu · Reading off proton, neutron and electron numbers

For the nuclide ${}^{235}_{92}$U (a well-known isotope of uranium), state the number of protons, neutrons and electrons in a neutral atom.
Proton number $Z = 92$, so there are $\boxed{92 \text{ protons}}$ and (in a neutral atom) $\boxed{92 \text{ electrons}}$.
Neutron number $= A - Z = 235 - 92 = \boxed{143 \text{ neutrons}}$.

8.1.2 Conservation Laws in Nuclear Reactions

In any nuclear reaction or decay, two quantities are always conserved:
(i) Nucleon number A — the total number of nucleons before the reaction equals the total after.
(ii) Proton number Z **(equivalently, charge)** — the total proton number (and total charge) before equals the total after.
These conservation rules are the basis for balancing any nuclear equation, and for identifying an unknown product or reactant.

Ví dụ mẫu · Balancing a nuclear equation using conservation laws

In the reaction ${}^{14}_{7}$N + ${}^{4}_{2}$He → ${}^{17}_{8}$O + ${}^{A}_{Z}$X, identify the unknown particle X.
Conserve A: $14 + 4 = 17 + A \Rightarrow A = 18 - 17 = 1$.
Conserve Z: $7 + 2 = 8 + Z \Rightarrow Z = 9 - 8 = 1$.
A particle with $A = 1$, $Z = 1$ is a **proton**, ${}^{1}_{1}$p (this is, in fact, Rutherford's famous 1919 nitrogen-transmutation reaction — the first artificially induced nuclear reaction).

VN **GIẢI THÍCH TIẾNG VIỆT**
Hạt nhân gồm **proton** (điện tích $+1$) và **neutron** (điện tích 0), gọi chung là **nucleon**; electron (điện tích -1, khối lượng rất nhỏ) quay quanh hạt nhân. **Số proton** Z xác định nguyên tố; **số nucleon** A là tổng proton + neutron. **Đồng vị** là các nguyên tử cùng Z nhưng khác A (khác số neutron) – tính chất hoá học giống nhau nhưng khối lượng và độ bền hạt nhân có thể khác nhau. Trong mọi phản ứng hạt nhân, **số nucleon** A và **số proton/điện tích** Z luôn được **bảo toàn** – đây là cơ sở để cân bằng phương trình hạt nhân.

8.2 Particles: Quarks, Leptons and Hadrons

[AS]Beyond protons, neutrons and electrons, physicists have identified a large family of subatomic particles. These can be organised according to whether or not they are made of smaller constituent particles.

Leptons are **fundamental** particles — they are not made up of anything smaller, as far as is currently known. The **electron** and the (electron) **neutrino** are examples of leptons relevant to this course.
Hadrons are particles built from smaller constituent particles called **quarks**, held together by the strong nuclear force. Hadrons are divided into two families: **baryons** (three quarks) and **mesons** (a quark and an antiquark pair).

8.2.1 Quarks and Their Properties

Six types ("**flavours**") of quark exist, but at this level only the **up (u)** and **down (d)** quarks are required in detail — these are the quarks that make up ordinary matter (protons and neutrons).

Quark	Symbol	Charge (units of e)	Antiparticle (charge)
Up	u	$+\frac{2}{3}$	ū $(-\frac{2}{3})$
Down	d	$-\frac{1}{3}$	d̄ $(+\frac{1}{3})$

Charges of the up and down quarks, and their antiquarks, in units of the elementary charge e.

Baryons are made of **three quarks**: the **proton** = uud, with total charge $\frac{2}{3}+\frac{2}{3}-\frac{1}{3}=+1$; the **neutron** = udd, with total charge $\frac{2}{3}-\frac{1}{3}-\frac{1}{3}=0$. **Antibaryons** are made of three **antiquarks**: the **antiproton** = ūūd̄, total charge $-\frac{2}{3}-\frac{2}{3}+\frac{1}{3}=-1$.
Mesons are made of a **quark-antiquark pair** (e.g. the pion π^+ = ud̄, charge $\frac{2}{3}+\frac{1}{3}=+1$).

Particle	Quark composition	Total charge
Proton (p)	uud	$+1$
Neutron (n)	udd	0
Antiproton (p̄)	ūūd̄	-1
Antineutron (n̄)	ūd̄d̄	0

Quark composition of the proton, neutron and their antiparticles, with the resulting total charge (verify each by summing the quark charges above).

Ví dụ mẫu · Verifying the charge of the antiproton from its quark content

Show that the quark composition ūūd̄ gives the antiproton the correct total charge of $-1e$. Each ū has charge $-\frac{2}{3}e$; each d̄ has charge $+\frac{1}{3}e$.

$$Q=(-\tfrac{2}{3})+(-\tfrac{2}{3})+(+\tfrac{1}{3})=-\tfrac{4}{3}+\tfrac{1}{3}=\boxed{-1\,e}\ \checkmark$$

This matches the antiproton's known charge, confirming it is the antiparticle of the proton (uud, charge $+1e$), with every quark replaced by its corresponding antiquark.

8.2.2 Beta-Minus Decay at the Quark Level

In **beta-minus (β^-) decay**, a **neutron** inside the nucleus transforms into a **proton**. At the quark level, this is a single **down quark changing into an up quark**, emitting an electron and an antineutrino:

$$\mathrm{d}\rightarrow\mathrm{u}+\beta^-+\bar{\nu}_e$$

Since the neutron is udd and the proton is uud, this single quark change (one d becomes a u) is exactly the transformation neutron → proton, consistent with the nuclear-level equation for β^- decay (Section 3).

VN **GIẢI THÍCH TIẾNG VIỆT**

Lepton là hạt cơ bản, không cấu tạo từ hạt nhỏ hơn (ví dụ: electron, neutrino). **Hadron** cấu tạo từ **quark**: **baryon** gồm 3 quark (proton = uud, điện tích $+1$; neutron = udd, điện tích 0), **meson** gồm 1 quark + 1 phản quark. Quark up có điện tích $+2/3\,e$, quark down có điện tích $-1/3\,e$; phản quark có điện tích trái dấu. Phản proton = ūūd̄, điện tích -1. Trong **phân rã beta-trừ**, một quark down biến thành quark up (d $\rightarrow$ u $+\beta^-+\bar{\nu}_e$), tương ứng với việc neutron biến thành proton ở cấp độ hạt nhân.

8.3 Fundamental Forces and Exchange Particles

[AS]All interactions between particles — from the everyday push of a hand on a door to the binding of quarks inside a proton — can be attributed to just **four fundamental forces**.

The four fundamental forces, in decreasing order of relative strength, are: the **strong nuclear force**, the **electromagnetic force**, the **weak nuclear force**, and the **gravitational force**. Each force is understood, in modern particle physics, as arising from the exchange of a particular **exchange particle** (**gauge boson**) between the interacting particles.

Force	Relative strength	Range	Exchange particle
Strong (nuclear)	strongest	very short ($\sim 10^{-15}$ m)	gluon
Electromagnetic	intermediate	infinite (inverse-square)	photon
Weak (nuclear)	weaker still	extremely short ($\sim 10^{-18}$ m)	W^+, W^-, Z^0 bosons
Gravitational	weakest	infinite (inverse-square)	graviton (not yet observed)

The four fundamental forces, ranked qualitatively by relative strength, with their range and associated exchange particle (gauge boson).

Strong nuclear force: acts between **nucleons** (and, more fundamentally, between quarks), holding the nucleus together against the electrostatic repulsion of the protons; its range is only of the order of a femtometre (10^{-15} m, roughly the size of a nucleon), so it has essentially no effect beyond a few nucleon diameters. Its exchange particle is the **gluon**.
Electromagnetic force: acts between all **charged particles**, responsible for the attraction/repulsion of charges, chemical bonding, and the structure of atoms; it has **infinite range**, obeying an inverse-square law, and its exchange particle is the **photon**.
Weak nuclear force: responsible for processes such as **beta decay**; its range is even shorter than the strong force's ($\sim 10^{-18}$ m). Its exchange particles are the **W^+**, **W^-** and **Z^0 bosons**.
Gravitational force: acts between all particles that have **mass**; by far the **weakest** of the four forces (utterly negligible between subatomic particles, but dominant on astronomical scales because mass is always positive and so gravitational effects always add up). It has **infinite range**, obeying an inverse-square law. Its hypothesised exchange particle, the **graviton**, has not yet been directly observed.

(!) Lỗi thường gặp: Do not confuse **range** with **strength**. The strong force is the **strongest** of the four but has an extremely **short range**; the gravitational force is the **weakest** but has **infinite range**. A force being "weak" (in the everyday sense) is not the same as it being classified as the fundamental **weak** force — the weak nuclear force is a specific, named fundamental interaction responsible for processes like beta decay, distinct from (and, in this context, unrelated in name to) gravity simply being numerically small.

VN GIẢI THÍCH TIẾNG VIỆT

Có **bốn lực cơ bản**, xếp theo độ mạnh giảm dần: **lực hạt nhân mạnh** (mạnh nhất, tầm hoạt động rất ngắn $\sim 10^{-15}$m, hạt truyền tương tác là **gluon**) $>$ **lực điện từ** (tầm hoạt động vô hạn, tuân theo định luật bình phương nghịch đảo, hạt truyền tương tác là **photon**) $>$ **lực hạt nhân yếu** (tầm hoạt động cực ngắn $\sim 10^{-18}$m, chịu trách nhiệm cho phân rã beta, hạt truyền tương tác là các boson **W^+, W^-, Z^0**) $>$ **lực hấp dẫn** (yếu nhất nhưng tầm hoạt động vô hạn, hạt truyền tương tác giả thuyết là **graviton**, chưa quan sát được trực tiếp). Cần phân biệt rõ **độ mạnh** và **tầm hoạt động** – đây là hai khái niệm độc lập.

8.3.1 Antiparticles, Pair Production and Annihilation [AS]

Every particle has a corresponding **antiparticle**, with the **same mass** but **opposite charge** (and opposite values of certain other quantum properties). For example, the antiparticle of the electron is the **positron** (e^+), identical in mass to the electron but carrying charge $+1e$ instead of $-1e$.

Pair production: a photon with sufficiently high energy can spontaneously convert into a particle-antiparticle pair (e.g. an electron and a positron), converting electromagnetic energy into mass in accordance with $E = mc^2$ — this can only happen if the photon's energy is at least equal to the combined rest energy of the two particles produced (and typically requires the presence of a nearby nucleus to conserve momentum).
Annihilation is the reverse process: when a particle meets its corresponding antiparticle (e.g. an electron meeting a positron), the pair is converted entirely into energy, typically emitted as **two photons** (two are required, rather than one, so that both energy and momentum can be conserved simultaneously).

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Mỗi hạt đều có một **phản hạt** tương ứng, cùng khối lượng nhưng điện tích trái dấu (ví dụ: positron là phản hạt của electron). **Sinh cặp (pair production)**: một photon có năng lượng đủ lớn có thể biến thành một cặp hạt-phản hạt (ví dụ electron-positron), theo $E = mc^2$. **Huỷ cặp (annihilation)**: khi một hạt gặp phản hạt của nó, cả hai biến hoàn toàn thành năng lượng, thường phát ra dưới dạng **hai photon** (cần hai photon để bảo toàn cả năng lượng và động lượng).

8.4 Radioactive Decay

Certain nuclides are **unstable**, and decay spontaneously, emitting radiation in the process. There are three types of radioactive emission relevant to this course: **alpha** (α), **beta-minus** (β^-), and **gamma** (γ).

Alpha decay: the nucleus emits an **alpha particle** (a helium nucleus, ${}^4_2\text{He}$ — two protons and two neutrons):

$$ {}^A_Z\text{X} \rightarrow {}^{A-4}_{Z-2}\text{Y} + {}^4_2\text{He} $$

Beta-minus decay: a neutron in the nucleus converts to a proton, emitting a fast electron and an (anti)neutrino:

$$ {}^A_Z\text{X} \rightarrow {}^{A}_{Z+1}\text{Y} + {}^{0}_{-1}\beta + \bar{\nu}_e $$

Gamma decay: an excited nucleus (often left in an excited state after an earlier α or β decay) loses energy by emitting a high-energy photon; **neither A nor Z changes**, since no particles are ejected from the nucleus — only energy is released.

Ví dụ mẫu · Alpha decay of radium-226

Write the equation for the alpha decay of ${}^{226}_{88}\text{Ra}$, and verify that both nucleon number and proton number balance.

$$ {}^{226}_{88}\text{Ra} \rightarrow {}^{222}_{86}\text{Rn} + {}^4_2\text{He} $$

Check A: $226 = 222 + 4$ ✓. **Check Z:** $88 = 86 + 2$ ✓.

Ví dụ mẫu · Beta-minus decay of carbon-14

Write the equation for the beta-minus decay of ${}^{14}_{6}$C to nitrogen, and verify conservation of A and Z.

$$ {}^{14}_{6}\text{C} \rightarrow {}^{14}_{7}\text{N} + {}^{0}_{-1}\beta + \bar{\nu}_e $$

Check A: $14 = 14 + 0 + 0$ ✓. **Check** Z: $6 = 7 + (-1) + 0$ ✓. At the quark level, one down quark in a neutron of the carbon-14 nucleus converts to an up quark, turning that neutron into a proton (so Z increases by 1 while A stays fixed), with the electron and antineutrino carrying away the released energy.

Ví dụ mẫu · A decay chain: alpha followed by beta-minus

${}^{238}_{92}$U decays by alpha emission to a daughter nuclide Y, which itself then decays by beta-minus emission to a nuclide Z. Identify Y and Z.

Alpha decay: A decreases by 4, Z decreases by 2: ${}^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Y} + {}^{4}_{2}\text{He}$, so Y has $A = 234$, $Z = 90$ (thorium, Th).

Beta-minus decay of Y: A unchanged, Z increases by 1: ${}^{234}_{90}\text{Th} \rightarrow {}^{234}_{91}\text{Z} + {}^{0}_{-1}\beta + \bar{\nu}_e$, so Z has $A = 234$, $Z = 91$ (protactinium, Pa). $\boxed{\text{Y} = {}^{234}_{90}\text{Th}, \quad \text{Z} = {}^{234}_{91}\text{Pa}}$

(!) Lỗi thường gặp: Gamma emission changes **neither** A **nor** Z — it is easy to mistakenly think some particle must be emitted from the nucleus, but a gamma ray is a photon, carrying energy and momentum but no charge and no nucleon content. Also remember that an emitted β^- particle is written as ${}^{0}_{-1}\beta$ (nucleon number 0, "proton number" -1, since it carries one unit of negative charge) purely so that the conservation equations balance correctly — it is not literally a nucleon.

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Phân rã alpha: hạt nhân phát ra hạt alpha (${}^{4}_{2}$He), A giảm 4, Z giảm 2. **Phân rã beta-trừ**: một neutron biến thành proton, phát ra electron và phản neutrino, A không đổi, Z tăng 1. **Phân rã gamma**: hạt nhân đang ở trạng thái kích thích phát ra photon năng lượng cao, cả A và Z đều **không đổi**.

8.4.1 Mass Defect and Binding Energy

When nucleons combine to form a nucleus, the mass of the resulting nucleus is measurably **less** than the total mass of the separate, unbound nucleons. This missing mass is called the **mass defect**, Δm.

The **binding energy** of a nucleus is the energy that would be required to separate it completely into its individual, unbound nucleons (equivalently, the energy released when the nucleus is assembled from separate nucleons). It is related to the mass defect Δm by Einstein's mass-energy equation:

$$ E = \Delta m\, c^2 $$

where $c = 3.00 \times 10^8\ \text{m s}^{-1}$ is the speed of light in a vacuum. A nucleus is therefore **more stable** the greater its binding energy **per nucleon** (E/A), since more energy per nucleon would be needed to pull it apart.

Ví dụ mẫu · Binding energy from a given mass defect

A certain nucleus has a mass defect of $\Delta m = 3.20 \times 10^{-28}$ kg. Using $c = 3.00 \times 10^{8}$ m s^{-1}, calculate its binding energy, in joules and in MeV (1 MeV $= 1.60 \times 10^{-13}$ J).

$$E = \Delta m\, c^2 = 3.20 \times 10^{-28} \times (3.00 \times 10^{8})^2 = 3.20 \times 10^{-28} \times 9.00 \times 10^{16} = \boxed{2.88 \times 10^{-11}\ \text{J}}.$$

Converting to MeV:

$$E = \frac{2.88 \times 10^{-11}}{1.60 \times 10^{-13}} = \boxed{180\ \text{MeV}}.$$

Ví dụ mẫu · Binding energy from a mass defect given in atomic mass units

A nucleus has a mass defect of $\Delta m = 0.0304$ u, where 1 u $= 1.66 \times 10^{-27}$ kg. Taking $c = 3.00 \times 10^{8}$ m s^{-1}, find the binding energy in joules.
First convert to kilograms:

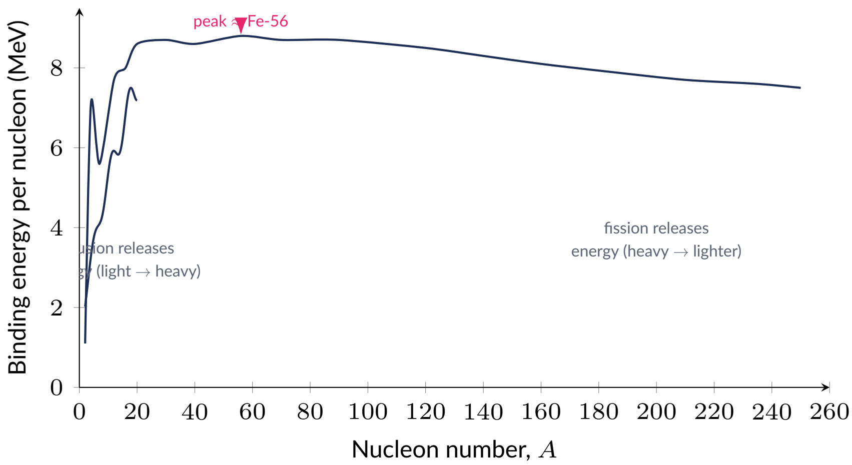
$$\Delta m = 0.0304 \times 1.66 \times 10^{-27} = 5.05 \times 10^{-29}\ \text{kg}.$$

Then:

$$E = \Delta m\, c^2 = 5.05 \times 10^{-29} \times 9.00 \times 10^{16} = \boxed{4.55 \times 10^{-12}\ \text{J}}\ (\approx 28.4\ \text{MeV}).$$

8.4.2 Binding Energy per Nucleon

Comparing the total binding energy of different nuclei is not, by itself, a fair measure of stability, since larger nuclei naturally have more nucleons to bind. Instead, nuclear stability is compared using the **binding energy per nucleon**, E/A.



Binding energy per nucleon against nucleon number A: the curve rises steeply for light nuclei, peaks near iron-56 ($A \approx 56$, the most stable nuclide), then falls slowly for heavier nuclei. Both **fusion** of light nuclei and **fission** of heavy nuclei move the products towards the peak, releasing energy.

Nuclear fission is the splitting of a heavy nucleus into two lighter nuclei (plus typically some neutrons); **nuclear fusion** is the joining of two light nuclei into a single heavier nucleus. Both processes release energy **because** the products lie **closer to the peak** of the binding-energy-per-nucleon curve (near $A \approx 56$) than the reactants did — a higher binding energy per nucleon in the products means the products are more tightly bound, and the difference in total binding energy is released as energy (kinetic energy of the products and/or radiation).

Ví dụ mẫu · Energy released in a fission reaction, given the mass defect

In a particular fission reaction, the total mass of the products is measured to be less than the total mass of the reactants by $\Delta m = 3.30 \times 10^{-28}$ kg. Calculate the energy released, in joules and in MeV.

$$E = \Delta m\, c^2 = 3.30 \times 10^{-28} \times (3.00 \times 10^8)^2 = 3.30 \times 10^{-28} \times 9.00 \times 10^{16} = \boxed{2.97 \times 10^{-11}\ \text{J}}.$$

$$E = \frac{2.97 \times 10^{-11}}{1.60 \times 10^{-13}} = \boxed{186\ \text{MeV}}.$$

(This is consistent with the typical scale of energy released per fission event in a nuclear reactor, of the order of 200 MeV.)

Ví dụ mẫu · Energy released in a fusion reaction, given the mass defect

In a fusion reaction between two light nuclei, the mass decrease is $\Delta m = 4.30 \times 10^{-29}$ kg. Calculate the energy released, in joules and in MeV.

$$E = \Delta m\, c^2 = 4.30 \times 10^{-29} \times 9.00 \times 10^{16} = \boxed{3.87 \times 10^{-12}\ \text{J}}.$$

$$E = \frac{3.87 \times 10^{-12}}{1.60 \times 10^{-13}} = \boxed{24.2\ \text{MeV}}.$$

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Khuyết tật khối lượng Δm: khối lượng hạt nhân nhỏ hơn tổng khối lượng các nucleon riêng lẻ; **năng lượng liên kết** $E = \Delta m\, c^2$ là năng lượng cần để tách hạt nhân thành các nucleon rời rạc. So sánh độ bền hạt nhân dùng **năng lượng liên kết trên mỗi nucleon** E/A – đỉnh đường cong ở khoảng $A \approx 56$ (Fe-56), là hạt nhân bền nhất. Cả **phân hạch** (hạt nhân nặng tách thành hai mảnh nhẹ hơn) và **nhiệt hạch** (hai hạt nhân nhẹ kết hợp thành một hạt nhân nặng hơn) đều giải phóng năng lượng vì sản phẩm nằm gần đỉnh đường cong hơn, tức có năng lượng liên kết/nucleon lớn hơn.

8.4.3 Decay Constant, Half-Life and Activity

[A2 – A Level]Radioactive decay is a **random** process at the level of any individual nucleus, but the behaviour of a large sample of identical nuclei can be described precisely using statistics.

The **decay constant**, λ, is the **probability that any one given nucleus will decay per unit time** (units: s^{-1}, or another time^{-1} unit). It is a fixed property of a particular nuclide, unaffected by temperature, pressure, or chemical environment.
The **activity**, A, of a sample is the number of decays occurring per unit time:

$$A = \lambda N$$

where N is the number of undecayed nuclei present at that instant. Activity is measured in **becquerels** (Bq), where 1 Bq $= 1$ decay per second.

The number of undecayed nuclei remaining after time t follows exponential decay:

$$N = N_0\, e^{-\lambda t}$$

where N_0 is the initial number of undecayed nuclei. Since activity is proportional to N, activity decays with exactly the same exponential form: $A = A_0 e^{-\lambda t}$.
The **half-life**, $T_{1/2}$, is the time taken for the number of undecayed nuclei (or the activity) to fall to **half** its original value:

$$T_{1/2} = \frac{\ln 2}{\lambda}$$

Ví dụ mẫu · Number of nuclei remaining after several half-lives

A radioactive sample has a half-life of 6.0 h and initially contains $N_0 = 8.0 \times 10^{10}$ undecayed nuclei. How many remain after 18 h?
18 h corresponds to $18/6.0 = 3$ half-lives. Each half-life halves the number remaining:

$$N = N_0\left(\frac{1}{2}\right)^3 = 8.0 \times 10^{10} \times \frac{1}{8} = \boxed{1.0 \times 10^{10}}.$$

Ví dụ mẫu · Finding activity from the decay constant and number of nuclei

A sample contains $N = 5.0 \times 10^{12}$ undecayed nuclei of a nuclide with decay constant $\lambda = 1.2 \times 10^{-4}\ \text{s}^{-1}$. Calculate the activity of the sample, in Bq and in MBq.

$$A = \lambda N = 1.2 \times 10^{-4} \times 5.0 \times 10^{12} = \boxed{6.0 \times 10^{8}\ \text{Bq}} = \boxed{600\ \text{MBq}}.$$

Ví dụ mẫu · Finding the decay constant and half-life from activity measurements

The activity of a radioactive source is measured to be $A_0 = 4.80 \times 10^6$ Bq. After $t = 12.0$ days, the activity has fallen to $A = 1.20 \times 10^6$ Bq. Find (a) the decay constant λ, and (b) the half-life, in days.
Since $A = A_0 e^{-\lambda t}$, rearranging and taking natural logarithms of both sides:

$$\frac{A}{A_0} = e^{-\lambda t} \implies \ln\left(\frac{A}{A_0}\right) = -\lambda t \implies \lambda = -\frac{1}{t}\ln\left(\frac{A}{A_0}\right).$$

$$\frac{A}{A_0} = \frac{1.20 \times 10^6}{4.80 \times 10^6} = 0.250, \qquad \ln(0.250) = -1.386.$$

$$\lambda = -\frac{1}{12.0} \times (-1.386) = \boxed{0.1155\ \text{day}^{-1}}\ (\approx 0.116\ \text{day}^{-1}).$$

(b)

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{0.1155} = \boxed{6.00\ \text{days}}.$$

Check: 12.0 days is exactly 2 half-lives of 6.00 days, so the activity should fall to $(1/2)^2 = 1/4$ of its original value — exactly matching the given ratio $1.20/4.80 = 0.250$ ✓.

(!) Lỗi thường gặp: The half-life is a fixed statistical property of a nuclide, unaffected by temperature, pressure, or chemical bonding — it is **not** the time taken for **all** the nuclei in a sample to decay (in principle, a tiny number of undecayed nuclei remain, in theory, forever, since the decay is random and exponential decay never mathematically reaches exactly zero). Each successive half-life always halves **whatever amount currently remains**, not the original starting amount — after 2 half-lives, $1/4$ remains (not $1/2 - 1/2 = 0$), after 3 half-lives, $1/8$ remains, and so on.

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Hằng số phân rã λ là xác suất phân rã của một hạt nhân trong một đơn vị thời gian. **Độ phóng xạ (activity)** $A = \lambda N$ (đơn vị Bq). Số hạt nhân chưa phân rã giảm theo hàm mũ $N = N_0 e^{-\lambda t}$; **chu kì bán rã** $T_{1/2} = \ln 2/\lambda$ là thời gian để số hạt nhân (hoặc độ phóng xạ) giảm còn một nửa. Nếu biết độ phóng xạ ban đầu và sau thời gian t, có thể tìm λ bằng cách lấy logarit tự nhiên: $\ln(A/A_0) = -\lambda t$. Chu kì bán rã là tính chất cố định, không phụ thuộc điều kiện vật lý/hoá học; mỗi chu kì bán rã làm giảm một nửa lượng **hiện có**, không phải lượng ban đầu.

Practice Bank

Bài 1 · Nuclide notation

For the nuclide ${}^{40}_{19}$K, state the number of protons, neutrons and electrons (in a neutral atom).

Lời giải chi tiết

Protons $= Z = \boxed{19}$; electrons (neutral atom) $= \boxed{19}$; neutrons $= A - Z = 40 - 19 = \boxed{21}$.

Bài 2 · MCQ — Isotopes

Two nuclides are isotopes of the same element if they have:

(A) The same A, different Z

(B) The same Z, different A

(C) The same Z and the same A

(D) Different Z and different A, but the same mass

Lời giải chi tiết

(B). Isotopes share the same proton number Z (same element) but differ in neutron number, and hence in nucleon number A.

Bài 3 · Balancing a nuclear reaction

Identify the unknown particle in ${}^{9}_{4}$Be + ${}^{4}_{2}$He → ${}^{12}_{6}$C + ${}^{A}_{Z}$X.

Lời giải chi tiết

Conserve A: $9 + 4 = 13 = 12 + A \;\Rightarrow\; A = 1$. Conserve Z: $4 + 2 = 6 = 6 + Z \;\Rightarrow\; Z = 0$.
A particle with $A = 1$, $Z = 0$ is a **neutron**, ${}^{1}_{0}$n — this is, in fact, the historic reaction (Chadwick, 1932) used to discover the neutron.

Bài 4 · Antiproton quark composition

State the quark composition of the antiproton, and verify it gives the correct total charge.

Lời giải chi tiết

Antiproton $=$ $\bar{u}\bar{u}\bar{d}$: charge $= -\frac{2}{3} - \frac{2}{3} + \frac{1}{3} = \boxed{-1\,e}$, correctly matching the antiproton's known charge (opposite to the proton's $+1e$).

Bài 5 · MCQ — Leptons vs hadrons

Which statement correctly distinguishes leptons from hadrons?

(A) Leptons are made of quarks; hadrons are fundamental

(B) Leptons are fundamental particles; hadrons are made of quarks

(C) Both are fundamental particles

(D) Both are made of exactly three quarks

Lời giải chi tiết

(B). Leptons (e.g. the electron) are fundamental; hadrons (baryons and mesons) are composite particles built from quarks.

Bài 6 · Beta decay of carbon-14, with quark explanation

Write the equation for the beta-minus decay of $^{14}_{6}$C, check conservation of A and Z, and explain the decay at the quark level.

Lời giải chi tiết

$$^{14}_{6}\text{C} \to {}^{14}_{7}\text{N} + {}^{0}_{-1}\beta + \bar{\nu}_e.$$

Check: A: $14 = 14 + 0 + 0$ ✓; Z: $6 = 7 - 1 + 0$ ✓. At the quark level, one down quark in a neutron of the nucleus changes to an up quark (d $\to$ u $+ \beta^- + \bar{\nu}_e$), converting that neutron into a proton, so Z increases by 1 while A (total nucleon count) is unchanged.

Bài 7 · MCQ — Fundamental forces: range

Which fundamental force has the shortest range?

(A) Gravitational

(B) Electromagnetic

(C) Weak nuclear

(D) Strong nuclear

Lời giải chi tiết

(C) Weak nuclear force, with a range of order 10^{-18} m — shorter even than the strong force's range of order 10^{-15} m.

Bài 8 · MCQ — Exchange particles

Which exchange particle is associated with the electromagnetic force?

(A) Gluon
(B) Photon
(C) W boson
(D) Graviton

Lời giải chi tiết

(B) Photon.

Bài 9 · Comparing strength and range of fundamental forces

Explain why the strong nuclear force does not prevent large nuclei (many nucleon diameters across) from being pulled apart by electrostatic repulsion between protons.

Lời giải chi tiết

The strong force, although the strongest of the four fundamental forces at very short range, has an extremely short range (of order 10^{-15} m, roughly one nucleon diameter) — it only acts effectively between **immediately neighbouring** nucleons. The electromagnetic (electrostatic) repulsion between protons, by contrast, has **infinite range** (inverse-square law), so in a large nucleus, protons far apart still repel each other, while the short-range strong force cannot bind equally distant nucleons — this is why very large nuclei tend to be unstable.

Bài 10 · MCQ — Antiparticles

Compared with a particle, its corresponding antiparticle has:

(A) The same mass, opposite charge
(B) Opposite mass, same charge
(C) Half the mass, same charge
(D) The same mass and the same charge

Lời giải chi tiết

(A). Antiparticles share the same mass as their corresponding particle but carry opposite charge (and opposite values of certain other quantum properties).

Bài 11 · Pair production and annihilation

Explain what is meant by pair production and by annihilation, and state why annihilation of an electron-positron pair typically produces two photons rather than one.

Lời giải chi tiết

Pair production: a sufficiently energetic photon converts into a particle-antiparticle pair (e.g. electron + positron), converting electromagnetic energy into mass via $E = mc^2$. **Annihilation:** a particle meeting its antiparticle converts entirely into energy. Two photons (rather than one) are produced because a single photon could not simultaneously conserve both energy *and* momentum in the annihilation process (in the centre-of-mass frame the initial total momentum is zero, so the two photons must be emitted in opposite directions with equal energy).

Bài 12 · Alpha decay balancing

Write the equation for the alpha decay of $^{238}_{92}$U, and check conservation of A and Z.

Lời giải chi tiết

$$^{238}_{92}\text{U} \to ^{234}_{90}\text{Th} + ^{4}_{2}\text{He}.$$

Check: A: $238 = 234 + 4$ ✓; Z: $92 = 90 + 2$ ✓.

Bài 13 · Combined decay chain

$^{232}_{90}$Th decays by alpha emission to Y, which then decays by beta-minus emission to Z. Identify Y and Z.

Lời giải chi tiết

Alpha: A drops by 4, Z drops by 2: Y $= ^{228}_{88}$Ra.
Beta-minus: A unchanged, Z rises by 1: Z $= ^{228}_{89}$Ac.
$\boxed{\text{Y} = ^{228}_{88}\text{Ra}, \quad \text{Z} = ^{228}_{89}\text{Ac}}$

Bài 14 · MCQ — Gamma decay

Gamma decay of an excited nucleus results in:

(A) A decreases by 4, Z decreases by 2

(B) A unchanged, Z increases by 1

(C) Both A and Z unchanged

(D) A unchanged, Z decreases by 1

Lời giải chi tiết

(C). A gamma ray is a photon, carrying no charge and no nucleon content, so neither A nor Z changes.

Bài 15 · Binding energy from mass defect in kg

A nucleus has a mass defect $\Delta m = 2.50 \times 10^{-28}$ kg. Using $c = 3.00 \times 10^{8}$ m s^{-1}, calculate the binding energy in joules and in MeV.

Lời giải chi tiết

$$E = \Delta m\, c^2 = 2.50 \times 10^{-28} \times 9.00 \times 10^{16} = \boxed{2.25 \times 10^{-11}\ \text{J}}.$$

$$E = \frac{2.25 \times 10^{-11}}{1.60 \times 10^{-13}} = \boxed{141\ \text{MeV}}.$$

Bài 16 · Binding energy from mass defect in u

A nucleus has mass defect $\Delta m = 0.0304$ u (1 u $= 1.66 \times 10^{-27}$ kg). Find its binding energy in joules and in MeV, given $c = 3.00 \times 10^{8}$ m s^{-1}.

Lời giải chi tiết

$\Delta m = 0.0304 \times 1.66 \times 10^{-27} = 5.05 \times 10^{-29}$ kg.

$$E = 5.05 \times 10^{-29} \times 9.00 \times 10^{16} = \boxed{4.55 \times 10^{-12}\ \text{J}} = \boxed{28.4\ \text{MeV}}.$$

Bài 17 · Helium-4 binding energy per nucleon

The total binding energy of a helium-4 nucleus is 28.4 MeV. Calculate the binding energy per nucleon, and explain briefly why this makes the alpha particle unusually stable.

Lời giải chi tiết

$$\frac{E}{A} = \frac{28.4}{4} = \boxed{7.1 \text{ MeV per nucleon}}.$$

This value is unusually high for such a light nucleus (comparable to nuclei several times heavier), meaning each nucleon in helium-4 is very tightly bound — this is why the alpha particle is emitted whole in alpha decay rather than nucleons being emitted individually, and why helium-4 is exceptionally stable compared with its light neighbours.

Bài 18 · MCQ — Binding energy curve

The nuclide with the highest binding energy per nucleon (most stable) lies at approximately:

(A) $A \approx 4$ (helium)

(B) $A \approx 56$ (iron)

(C) $A \approx 235$ (uranium)

(D) $A \approx 1$ (hydrogen)

Lời giải chi tiết

(B). The binding-energy-per-nucleon curve peaks near $A \approx 56$ (iron-56).

Bài 19 · Fission energy from given mass defect

In a fission reaction, the mass decrease is $\Delta m = 3.50 \times 10^{-28}$ kg. Calculate the energy released, in joules and MeV.

Lời giải chi tiết

$$E = \Delta m\, c^2 = 3.50 \times 10^{-28} \times 9.00 \times 10^{16} = \boxed{3.15 \times 10^{-11} \text{ J}}.$$

$$E = \frac{3.15 \times 10^{-11}}{1.60 \times 10^{-13}} = \boxed{197 \text{ MeV}}.$$

Bài 20 · Fusion energy from given mass defect

In a fusion reaction, the mass decrease is $\Delta m = 6.00 \times 10^{-29}$ kg. Calculate the energy released, in joules and MeV.

Lời giải chi tiết

$$E = \Delta m\, c^2 = 6.00 \times 10^{-29} \times 9.00 \times 10^{16} = \boxed{5.40 \times 10^{-12} \text{ J}}.$$

$$E = \frac{5.40 \times 10^{-12}}{1.60 \times 10^{-13}} = \boxed{33.8 \text{ MeV}}.$$

Bài 21 · MCQ — Why fission and fusion both release energy

Both nuclear fission of heavy nuclei and nuclear fusion of light nuclei release energy because:

(A) The number of nucleons increases in both processes

(B) The products have a lower binding energy per nucleon than the reactants

(C) The products lie closer to the peak of the binding-energy-per-nucleon curve than the reactants

(D) Mass is created in both processes

Lời giải chi tiết

(C). In both cases, the products are more tightly bound per nucleon (closer to the $A \approx 56$ peak) than the reactants, so total binding energy increases and the corresponding mass difference is released as energy.

Bài 22 · Half-life: fraction remaining after multiple half-lives

A radioactive isotope has a half-life of 5.0 days. What fraction of the original sample remains after 20 days?

Lời giải chi tiết

$20/5.0 = 4$ half-lives.

$$\left(\frac{1}{2}\right)^4 = \boxed{\frac{1}{16}}.$$

Bài 23 · Number of nuclei remaining

A sample initially contains $N_0 = 6.4 \times 10^9$ nuclei of a nuclide with half-life 4.0 h. How many nuclei remain after 16 h?

Lời giải chi tiết

$16/4.0 = 4$ half-lives.

$$N = N_0 \left(\frac{1}{2}\right)^4 = 6.4 \times 10^9 \times \frac{1}{16} = \boxed{4.0 \times 10^8}.$$

Bài 24 · Activity from decay constant and number of nuclei

A sample contains $N = 2.0 \times 10^{13}$ nuclei of a nuclide with decay constant $\lambda = 3.5 \times 10^{-5}\ \text{s}^{-1}$. Find its activity, in Bq.

Lời giải chi tiết

$$A = \lambda N = 3.5 \times 10^{-5} \times 2.0 \times 10^{13} = \boxed{7.0 \times 10^8\ \text{Bq}}.$$

Bài 25 · Decay constant from half-life

A nuclide has a half-life of 8.0 days. Calculate its decay constant λ, in s^{-1}.

Lời giải chi tiết

$T_{1/2} = 8.0$ days $= 8.0 \times 24 \times 3600 = 6.912 \times 10^5$ s.

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{6.912 \times 10^5} = \boxed{1.00 \times 10^{-6}\ \text{s}^{-1}}.$$

Bài 26 · Finding decay constant and half-life from activity data (logarithms)

The activity of a source falls from $A_0 = 8.00 \times 10^5$ Bq to $A = 5.00 \times 10^4$ Bq over $t = 40.0$ minutes. Find (a) the decay constant λ (in min^{-1}), and (b) the half-life, in minutes.

Lời giải chi tiết

$$\ln\left(\frac{A}{A_0}\right) = -\lambda t \ \Rightarrow\ \lambda = -\frac{1}{t}\ln\left(\frac{A}{A_0}\right).$$

$$\frac{A}{A_0} = \frac{5.00 \times 10^4}{8.00 \times 10^5} = 0.0625, \qquad \ln(0.0625) = -2.773.$$

$$\lambda = -\frac{1}{40.0} \times (-2.773) = \boxed{0.0693\ \text{min}^{-1}}.$$

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{0.0693} = \boxed{10.0\ \text{minutes}}.$$

Check: 40.0 minutes $= 4$ half-lives of 10.0 min, so activity should fall to $(1/2)^4 = 1/16 = 0.0625$ of its original value — matching the given ratio exactly ✓.

Bài 27 · MCQ — Properties of half-life

Which statement about half-life is correct?

(A) It is the time for all nuclei in a sample to decay

(B) It depends on the temperature of the sample

(C) It is the time for the number of undecayed nuclei (or activity) to fall to half its current value

(D) It increases as the sample ages

Lời giải chi tiết

(C). Half-life is fixed and independent of physical/chemical conditions; it describes the time for whatever amount currently remains to be halved, not a fixed absolute quantity of decay.

Bài 28 · Combining half-life and initial activity

A source has initial activity $A_0 = 2.40 \times 10^6$ Bq and half-life 15 days. Find its activity after 60 days.

Lời giải chi tiết

$60/15 = 4$ half-lives.

$$A = A_0\left(\frac{1}{2}\right)^4 = 2.40 \times 10^6 \times \frac{1}{16} = \boxed{1.50 \times 10^5\ \text{Bq}}.$$

Bài 29 · Time to reach a given fraction remaining

A nuclide has a half-life of 12 h. How long does it take for the activity to fall to $1/32$ of its initial value?

Lời giải chi tiết

$1/32 = (1/2)^5$, so 5 half-lives are needed:

$$t = 5 \times 12 = \boxed{60\,\text{h}}.$$

Bài 30 · Extended problem: mass defect, binding energy and stability comparison

Nucleus P has mass defect $\Delta m_P = 4.00 \times 10^{-28}$ kg and nucleon number $A_P = 50$. Nucleus Q has mass defect $\Delta m_Q = 3.00 \times 10^{-28}$ kg and nucleon number $A_Q = 30$. Using $c = 3.00 \times 10^8$ m s^{-1} and 1 MeV $= 1.60 \times 10^{-13}$ J, find the binding energy per nucleon (in MeV) for each nucleus, and state which is more stable.

Lời giải chi tiết

Nucleus P: $E_P = \Delta m_P c^2 = 4.00 \times 10^{-28} \times 9.00 \times 10^{16} = 3.60 \times 10^{-11}\,\text{J} = \dfrac{3.60 \times 10^{-11}}{1.60 \times 10^{-13}} = 225$ MeV.

$$\frac{E_P}{A_P} = \frac{225}{50} = \boxed{4.50\ \text{MeV/nucleon}}.$$

Nucleus Q: $E_Q = \Delta m_Q c^2 = 3.00 \times 10^{-28} \times 9.00 \times 10^{16} = 2.70 \times 10^{-11}\,\text{J} = \dfrac{2.70 \times 10^{-11}}{1.60 \times 10^{-13}} = 168.75$ MeV.

$$\frac{E_Q}{A_Q} = \frac{168.75}{30} = \boxed{5.6\ \text{MeV/nucleon}}\ (\approx 5.63).$$

Since E_Q/A_Q $(5.6\,\text{MeV/nucleon}) > E_P/A_P$ $(4.50\,\text{MeV/nucleon})$, nucleus **Q is more stable** — each of its nucleons is, on average, more tightly bound.

Motion in a Circle & Gravitational Fields

[A2 – A Level only, not in the AS syllabus]

Mục tiêu Unit: Học sinh mô tả chuyển động tròn đều bằng tốc độ góc, chu kỳ và tần số; giải thích lực hướng tâm là hợp lực của các lực thực (lực căng, trọng lực, phản lực, ma sát) chứ không phải một loại lực mới; áp dụng cho mặt đường nghiêng, chuyển động tròn trong mặt phẳng đứng và con lắc nón. Học sinh phát biểu định luật hấp dẫn Newton, tính cường độ trường hấp dẫn và thế năng hấp dẫn, suy ra vận tốc thoát và năng lượng toàn phần của vệ tinh, đồng thời áp dụng định luật Kepler thứ ba cho quỹ đạo vệ tinh và hành tinh.

9.1 Angular Speed and Centripetal Acceleration

9.1.1 Describing circular motion

A body moving in a circle of radius r at constant speed v sweeps out equal angles in equal times. The **angular speed** ω is the rate of change of angle:

$$\omega = \frac{v}{r} = \frac{2\pi}{T} = 2\pi f$$

where T is the period (time for one revolution) and f is the frequency (revolutions per second). Angular speed is measured in rad s^{-1}.

$$\omega = \frac{v}{r} = \frac{2\pi}{T} = 2\pi f, \quad v = \omega r$$

VN GIẢI THÍCH TIẾNG VIỆT

Tốc độ góc ω đo bằng radian trên giây, liên hệ với tốc độ dài v qua bán kính r: $v = \omega r$. Chu kỳ T là thời gian đi hết một vòng, tần số $f = 1/T$.

9.1.2 Centripetal acceleration and force

Even though the speed is constant, the *direction* of the velocity is continuously changing, so the body accelerates. This acceleration points toward the centre of the circle and has magnitude

$$a = \frac{v^2}{r} = \omega^2 r$$

By Newton's second law, a resultant (net) force must act toward the centre to produce this acceleration:

$$F = ma = \frac{mv^2}{r} = m\omega^2 r$$

This is called the **centripetal force**. It is essential to understand that centripetal force is *not* a new, separate force that appears in addition to the real forces acting on a body. It is simply the name given to the *resultant* of the real forces (tension, weight, normal contact force, friction, gravitational attraction) that happens to point toward the centre of the circular path.

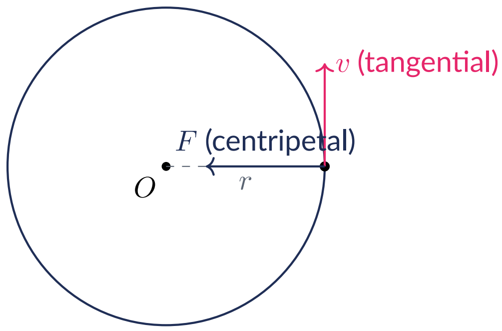

Figure 9.1. Uniform circular motion: the velocity v is always tangential to the path, while the resultant force F (and the acceleration it causes) always points toward the centre O.

Ví dụ mẫu · Centripetal force

A ball of mass $m = 0.50$ kg is whirled on a string in a horizontal circle of radius $r = 0.80$ m at a constant speed $v = 4.0\,\mathrm{m\,s^{-1}}$. Find the tension in the string and the angular speed.
Solution. The tension supplies the entire centripetal force (the string is horizontal, so weight is supported separately in this idealised case):

$$F = \frac{mv^2}{r} = \frac{0.50 \times 4.0^2}{0.80} = \frac{8.0}{0.80} = 10.0\,\mathrm{N}$$

$$\omega = \frac{v}{r} = \frac{4.0}{0.80} = 5.0\,\mathrm{rad\,s^{-1}}$$

(!) Lỗi thường gặp: Never add a separate "centripetal force" arrow to a free-body diagram alongside the real forces. Draw only the real forces (tension, weight, normal force, friction, gravitational pull), then state that *their resultant* equals mv^2/r directed toward the centre. Writing an equation such as $T - mg - F_c = 0$ double-counts the centripetal force and is incorrect.

9.2 Banking of Roads and Tracks

On a flat road, the centripetal force needed to turn a corner is provided entirely by friction between the tyres and the road. Friction has a limit, so at high speed or small radius a flat curve becomes unsafe. Engineers reduce this reliance on friction by **banking** the road or track: tilting the surface so that the normal contact force has a horizontal component directed toward the centre of the curve.

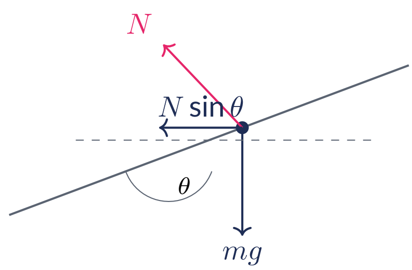

Figure 9.2. Cross-section of a banked road. The normal force N is perpendicular to the road surface. Its horizontal component $N \sin\theta$ supplies the centripetal force; its vertical component $N \cos\theta$ balances the weight mg.

Derivation: ideal banking angle

Resolve the normal force N into vertical and horizontal components. At the design ("ideal") speed, no friction is required, so only N and mg act.
Vertical equilibrium: $N\cos\theta = mg$
Horizontal (centripetal) equation: $N\sin\theta = \dfrac{mv^2}{r}$
Dividing the two equations eliminates both N and m:

$$\tan\theta = \frac{v^2}{rg}$$

This is the angle at which a vehicle travelling at exactly speed v around a curve of radius r needs *no* friction at all to stay on the track. At any other speed, friction must supply the difference.

$\tan\theta = \dfrac{v^2}{rg}$ (ideal banking angle, zero friction required)

VN **GIẢI THÍCH TIẾNG VIỆT**

Mặt đường nghiêng một góc θ so với phương ngang giúp thành phần nằm ngang của phản lực N đóng vai trò lực hướng tâm, giảm bớt yêu cầu về ma sát. Ở tốc độ thiết kế v với bán kính r, góc nghiêng lý tưởng thỏa $\tan\theta = v^2/(rg)$.

Ví dụ mẫu · Banking angle

A curve of radius $r = 150$ m is to be banked so that cars travelling at $v = 30\,\text{m s}^{-1}$ ($108\,\text{km h}^{-1}$) need no friction. Find the banking angle.
Solution.

$$\tan\theta = \frac{v^2}{rg} = \frac{30.0^2}{150\times 9.81} = \frac{900}{1471.5} = 0.6116$$
$$\theta = \tan^{-1}(0.6116) = 31.5^\circ$$

9.3 Vertical Circular Motion

When a body moves in a *vertical* circle (e.g. a bucket of water on a string, a car over a hilltop, a roller-coaster loop), gravity acts on the body at every point but the direction of gravity relative to the centre of the circle changes as the body moves around. The resultant of gravity and the contact/tension force must always equal mv^2/r directed toward the centre.

9.3.1 At the top of the circle

At the top, both the tension (or normal force) T and the weight mg point downward, toward the centre. Both contribute to the centripetal force:

$$T + mg = \frac{mv^2}{r} \quad\Rightarrow\quad T = \frac{mv^2}{r} - mg$$

As v decreases, T decreases. The **minimum speed** at the top occurs when T just reaches zero (the string is on the point of going slack, or a car is on the point of leaving the road surface); gravity alone then supplies the centripetal force:

$$mg = \frac{mv_{\text{min}}^2}{r} \quad\Rightarrow\quad v_{\text{min}} = \sqrt{gr}$$

9.3.2 At the bottom of the circle

At the bottom, the tension T points upward (toward the centre) while the weight mg points downward (away from the centre). The resultant toward the centre is $T - mg$:

$$T - mg = \frac{mv^2}{r} \quad \Rightarrow \quad T = \frac{mv^2}{r} + mg$$

The tension at the bottom is always *greater* than at the top for the same speed, because at the bottom the string tension must both support the weight *and* supply the centripetal force, whereas at the top gravity assists the centripetal requirement.

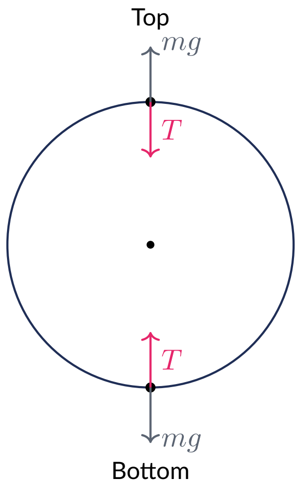

Figure 9.3. At the top, T and mg both point toward the centre: $T + mg = mv^2/r$. At the bottom, T points toward the centre while mg points away from it: $T - mg = mv^2/r$.

VN GIẢI THÍCH TIẾNG VIỆT

Ở đỉnh vòng tròn đứng, lực căng dây và trọng lực cùng hướng vào tâm nên $T + mg = mv^2/r$; tốc độ tối thiểu để dây không bị chùng là $v_{\text{min}} = \sqrt{gr}$. Ở đáy vòng tròn, lực căng hướng vào tâm còn trọng lực hướng ra ngoài nên $T - mg = mv^2/r$, do đó lực căng ở đáy luôn lớn hơn ở đỉnh với cùng một tốc độ.

Ví dụ mẫu · Minimum speed at the top

A small mass on a string moves in a vertical circle of radius $r = 0.50$ m. Find the minimum speed at the top of the circle for the string to remain taut.

Solution.

$$v_{\text{min}} = \sqrt{gr} = \sqrt{9.81 \times 0.50} = \sqrt{4.905} = 2.21\,\text{m s}^{-1}$$

Ví dụ mẫu · Tension at the bottom

A bucket and its contents, total mass $m = 1.5$ kg, are swung in a vertical circle of radius $r = 0.75$ m. At the lowest point the speed is $v = 6.0\,\text{m s}^{-1}$. Find the tension in the rope at that instant.

Solution.

$$T = \frac{mv^2}{r} + mg = \frac{1.5 \times 6.0^2}{0.75} + 1.5 \times 9.81 = \frac{54.0}{0.75} + 14.72 = 72.0 + 14.72 = 86.7\,\text{N}$$

Ví dụ mẫu · Comparing top and bottom

A mass $m = 0.20$ kg moves in a vertical circle of radius $r = 0.60$ m. Assuming the speed is $v = 5.0\,\text{m s}^{-1}$ at both the top and the bottom, find the tension at each point and comment on the difference.

Solution. First, $\dfrac{mv^2}{r} = \dfrac{0.20 \times 5.0^2}{0.60} = \dfrac{5.0}{0.60} = 8.33$ N, and $mg = 0.20 \times 9.81 = 1.96$ N.

At the top: $T_{\text{top}} = \dfrac{mv^2}{r} - mg = 8.33 - 1.96 = 6.37$ N

At the bottom: $T_{\text{bottom}} = \dfrac{mv^2}{r} + mg = 8.33 + 1.96 = 10.3$ N

The difference, $T_{\text{bottom}} - T_{\text{top}} = 2mg = 3.92$ N, exactly equals twice the weight, confirming that gravity assists the centripetal requirement at the top and opposes it at the bottom.

9.4 The Conical Pendulum

A **conical pendulum** consists of a mass on a string, moving in a horizontal circle while the string sweeps out a cone at a constant angle θ to the vertical. Unlike a simple pendulum, the mass does not swing back and forth; it moves at constant speed around a horizontal circle of radius r.

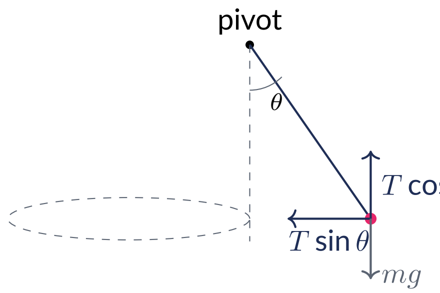

Figure 9.4. Conical pendulum: the string makes a constant angle θ with the vertical. The vertical component of tension $T\cos\theta$ balances gravity; the horizontal component $T\sin\theta$ supplies the centripetal force.

Derivation: conical pendulum

Resolve the tension T into vertical and horizontal components.
Vertical equilibrium: $T\cos\theta = mg$

Horizontal (centripetal) equation: $T\sin\theta = \dfrac{mv^2}{r}$

Dividing gives the same form as the banking equation:

$$\tan\theta = \frac{v^2}{rg}$$

Using $v = \omega r$ and $r = L\sin\theta$ (where L is the string length), the horizontal equation becomes $T\sin\theta = m\omega^2 L\sin\theta$, so $T = m\omega^2 L$. Combining with $T\cos\theta = mg$ gives

$$\cos\theta = \frac{g}{\omega^2 L} \quad\Rightarrow\quad \omega = \sqrt{\frac{g}{L\cos\theta}} \quad\Rightarrow\quad T_{\text{period}} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L\cos\theta}{g}}$$

Conical pendulum: $\tan\theta = \dfrac{v^2}{rg}$, $\quad T_{\text{period}} = 2\pi\sqrt{\dfrac{L\cos\theta}{g}}$

VN **GIẢI THÍCH TIẾNG VIỆT**

Con lắc nón có dây treo dài L hợp góc θ không đổi với phương thẳng đứng khi vật chuyển động tròn đều trên mặt phẳng ngang. Thành phần thẳng đứng của lực căng cân bằng trọng lực, thành phần ngang đóng vai trò lực hướng tâm, dẫn tới $\tan\theta = v^2/(rg)$ và chu kỳ $T = 2\pi\sqrt{L\cos\theta/g}$.

Ví dụ mẫu · Conical pendulum period

A conical pendulum has a string of length $L = 0.80$ m making a constant angle of 25° with the vertical. Find the period of the motion.
Solution.

$$T_{\text{period}} = 2\pi\sqrt{\frac{L\cos\theta}{g}} = 2\pi\sqrt{\frac{0.80\times\cos 25^\circ}{9.81}} = 2\pi\sqrt{\frac{0.80\times 0.9063}{9.81}} = 2\pi\sqrt{0.07393} = 2\pi\times 0.2719 = 1.71\ \text{s}$$

(As a check: the radius is $r = L\sin\theta = 0.80\times\sin 25^\circ = 0.338$ m, giving $\omega = 2\pi/T = 3.68$ rad s^{-1} and $v = \omega r = 1.24$ m s^{-1}, which satisfies $\tan 25^\circ = v^2/(rg)$.)

9.5 Newton's Law of Gravitation

Every point mass attracts every other point mass with a force proportional to the product of their masses and inversely proportional to the square of their separation:

$$F = \frac{Gm_1m_2}{r^2}$$

where $G = 6.67\times 10^{-11}$ N m^2 kg^{-2} is the gravitational constant. The force is always attractive and acts along the line joining the two masses. Throughout this chapter, unless stated otherwise, we take Earth's mass $M_E = 5.97\times 10^{24}$ kg and Earth's radius $R_E = 6.37\times 10^6$ m.

$$F = \frac{Gm_1m_2}{r^2}, \quad G = 6.67\times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2}$$

VN **GIẢI THÍCH TIẾNG VIỆT**

Định luật hấp dẫn Newton: hai chất điểm bất kỳ hút nhau bằng một lực tỉ lệ thuận với tích hai khối lượng và tỉ lệ nghịch với bình phương khoảng cách. Lực này luôn là lực hút, không bao giờ là lực đẩy.

Ví dụ mẫu · Gravitational force between two bodies

Find the gravitational attraction between the Earth ($M_E = 5.97\times 10^{24}$ kg) and the Moon (mass 7.35×10^{22} kg), separated by a distance of 3.84×10^8 m.
Solution.

$$F = \frac{GM_EM_{\text{Moon}}}{r^2} = \frac{6.67\times 10^{-11}\times 5.97\times 10^{24}\times 7.35\times 10^{22}}{(3.84\times 10^8)^2}$$

The numerator is $GM_E\times M_{\text{Moon}} = (3.982\times 10^{14})(7.35\times 10^{22}) = 2.927\times 10^{37}$, and the denominator is 1.475×10^{17}, so

$$F = \frac{2.927\times 10^{37}}{1.475\times 10^{17}} = 1.98\times 10^{20}\ \text{N}$$

9.6 Gravitational Field Strength

The gravitational field strength g at a point is the gravitational force per unit mass placed at that point:

$$g = \frac{F}{m} = \frac{GM}{r^2}$$

where M is the mass creating the field and r is the distance from its centre. As a check, at Earth's surface ($r = R_E$):

$$g = \frac{GM_E}{R_E^2} = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(6.37 \times 10^6)^2} = \frac{3.982 \times 10^{14}}{4.058 \times 10^{13}} = 9.81\,\mathrm{N\,kg^{-1}}$$

which matches the standard surface value, confirming the consistency of G, M_E and R_E used throughout this chapter.

9.6.1 Field strength outside a uniform sphere

Outside a uniform spherical mass (or any spherically symmetric mass distribution), the field behaves as though all the mass were concentrated at the centre, so $g \propto 1/r^2$: doubling the distance from the centre reduces g to one quarter.

9.6.2 Field strength inside a uniform sphere

Inside a uniform sphere of radius R, only the mass enclosed within radius r contributes to the field at that point (the mass outside radius r contributes zero net field, by symmetry). Since the enclosed mass is proportional to r^3 for a uniform density, the field strength there is

$$g \propto r$$

rising linearly from zero at the centre to the full surface value $g_s = GM/R^2$ at $r = R$. The field is therefore continuous at the surface: the linear (inside) and inverse-square (outside) expressions agree exactly at $r = R$.

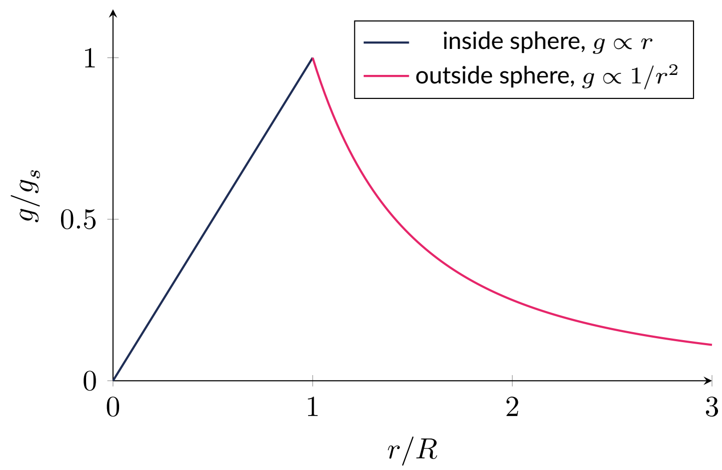

Figure 9.5. Gravitational field strength g (normalised to the surface value g_s) against distance from the centre r (normalised to the sphere's radius R), for a uniform sphere. Inside, g rises linearly from zero; outside, g falls as $1/r^2$. The two curves meet continuously at $r = R$.

GIẢI THÍCH TIẾNG VIỆT

VN

Bên ngoài một khối cầu đồng chất, cường độ trường hấp dẫn tỉ lệ nghịch với bình phương khoảng cách tới tâm ($g \propto 1/r^2$), giống như toàn bộ khối lượng tập trung tại tâm. Bên trong khối cầu, chỉ phần khối lượng nằm trong bán kính r mới đóng góp vào trường tại đó, nên g tăng tuyến tính theo r từ 0 tại tâm đến giá trị bề mặt tại $r = R$.

9.7 Gravitational Potential and Potential Energy

9.7.1 Gravitational potential

The gravitational potential ϕ at a point is the work done per unit mass in bringing a small test mass from infinity to that point:

$$\phi = -\frac{GM}{r}$$

Potential is defined to be zero at infinity and is negative everywhere else, since gravity does positive work on a mass falling inward (energy is released), meaning the mass has lower, i.e. more negative, potential energy than at infinity. The work done moving a mass m between two points of potential ϕ_1 and ϕ_2 is

$$W = m(\phi_2 - \phi_1)$$

$$\phi = -\frac{GM}{r}, \quad W = m(\phi_2 - \phi_1)$$

9.7.2 Potential energy of a two-body system

The gravitational potential energy of a system of two masses M and m separated by distance r is

$$E_p = m\phi = -\frac{GMm}{r}$$

This follows directly from the definition of potential and represents the total work needed to bring the two masses together from infinite separation (negative, since gravity does positive work as they come together).

$E_p = -\dfrac{GMm}{r}$ (zero at infinite separation, negative for all finite r)

GIẢI THÍCH TIẾNG VIỆT

VN

Thế năng hấp dẫn $\phi = -GM/r$ được quy ước bằng 0 ở vô cực và âm ở mọi điểm hữu hạn, vì trọng trường thực hiện công dương khi vật rơi vào gần. Thế năng của hệ hai vật $E_p = -GMm/r$ cũng luôn âm, thể hiện hệ hai vật liên kết với nhau bởi lực hấp dẫn.

Ví dụ mẫu · Work done raising a satellite

Find the work needed to raise a 1000 kg satellite (considering gravitational potential energy only) from Earth's surface to a circular orbit of radius $r = 6.77 \times 10^6$ m (400 km above the surface).

Solution. Potential at the surface:

$$\phi_{\text{surface}} = -\frac{GM_E}{R_E} = -\frac{3.982 \times 10^{14}}{6.37 \times 10^6} = -6.251 \times 10^7 \text{ J kg}^{-1}$$

Potential at the orbit:

$$\phi_{\text{orbit}} = -\frac{GM_E}{r} = -\frac{3.982 \times 10^{14}}{6.77 \times 10^6} = -5.882 \times 10^7 \text{ J kg}^{-1}$$

Work done:

$$W = m(\phi_{\text{orbit}} - \phi_{\text{surface}}) = 1000 \times (-5.882 \times 10^7 - (-6.251 \times 10^7)) = 1000 \times 3.693 \times 10^6 = 3.69 \times 10^9 \text{ J}$$

(This is only the change in gravitational potential energy; launching the satellite into orbit also requires giving it kinetic energy, considered separately in Section 9.10.)

Ví dụ mẫu · Two-body potential energy

Find the gravitational potential energy of the 1000 kg satellite of the previous example while in its circular orbit at $r = 6.77 \times 10^6$ m.
Solution.

$$E_p = -\frac{GM_E m}{r} = -\frac{3.982 \times 10^{14} \times 1000}{6.77 \times 10^6} = -\frac{3.982 \times 10^{17}}{6.77 \times 10^6} = -5.88 \times 10^{10}\ \text{J}$$

9.8 Escape Velocity

The **escape velocity** is the minimum speed a body must be given at a planet's surface so that it can travel to infinity without any further propulsion, arriving at infinity with (at most) zero final speed.

Derivation: escape velocity

At the surface, the body has kinetic energy $\frac{1}{2}mv_{\text{esc}}^2$ and gravitational potential energy $-\frac{GMm}{R}$. At infinity, both the potential energy and (in the limiting case) the kinetic energy are zero. Setting the total mechanical energy equal at both points (energy conservation, with zero final kinetic energy at infinity defining the minimum case):

$$\frac{1}{2}mv_{\text{esc}}^2 - \frac{GMm}{R} = 0$$

$$\frac{1}{2}v_{\text{esc}}^2 = \frac{GM}{R} \quad \Rightarrow \quad v_{\text{esc}} = \sqrt{\frac{2GM}{R}}$$

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}}$$

VN **GIẢI THÍCH TIẾNG VIỆT**

Vận tốc thoát là tốc độ tối thiểu để một vật rời khỏi bề mặt hành tinh và đi tới vô cực mà không cần thêm lực đẩy, tại vô cực vật có vận tốc bằng 0. Ta có được bằng cách cho tổng động năng và thế năng bằng 0: $v_{\text{esc}} = \sqrt{2GM/R}$.

Ví dụ mẫu · Escape velocity from Earth

Calculate the escape velocity from the surface of the Earth.
Solution.

$$v_{\text{esc}} = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{\frac{2 \times 3.982 \times 10^{14}}{6.37 \times 10^6}} = \sqrt{\frac{7.964 \times 10^{14}}{6.37 \times 10^6}} = \sqrt{1.250 \times 10^8} = 1.12 \times 10^4\ \text{m s}^{-1} = 11.2\ \text{km s}^{-1}$$

9.9 Orbital Motion and Kepler's Third Law

For a satellite of mass m in a stable circular orbit of radius r around a mass M, gravity alone supplies the centripetal force:

$$\frac{GMm}{r^2} = \frac{mv^2}{r} = m\omega^2 r$$

From this, $v = \sqrt{GM/r}$, and using $\omega = 2\pi/T$ we obtain **Kepler's third law**:

$$\frac{GMm}{r^2} = m\left(\frac{2\pi}{T}\right)^2 r \quad \Rightarrow \quad T^2 = \frac{4\pi^2}{GM} r^3 \quad \Rightarrow \quad T^2 \propto r^3$$

The period of a satellite's orbit depends only on its orbital radius (and the central mass M) – not on the satellite's own mass.

$$\frac{GMm}{r^2} = \frac{mv^2}{r} = m\omega^2 r, \qquad T^2 = \frac{4\pi^2}{GM} r^3$$

VN **GIẢI THÍCH TIẾNG VIỆT**

Với vệ tinh chuyển động tròn đều, lực hấp dẫn đóng vai trò lực hướng tâm duy nhất, dẫn tới định luật Kepler thứ ba: $T^2 \propto r^3$. Chu kỳ quỹ đạo chỉ phụ thuộc bán kính quỹ đạo và khối lượng thiên thể trung tâm, không phụ thuộc khối lượng vệ tinh.

Ví dụ mẫu · Low Earth orbit

A satellite orbits at a height of 400 km above the Earth's surface (comparable to the International Space Station). Find its orbital speed and period.

Solution. The orbital radius, measured from Earth's centre, is

$$r = R_E + h = 6.37 \times 10^6 + 4.00 \times 10^5 = 6.77 \times 10^6 \text{ m}$$

Orbital speed:

$$v = \sqrt{\frac{GM_E}{r}} = \sqrt{\frac{3.982 \times 10^{14}}{6.77 \times 10^6}} = \sqrt{5.882 \times 10^7} = 7669 \text{ m s}^{-1} \approx 7.67 \text{ km s}^{-1}$$

Period:

$$T = \frac{2\pi r}{v} = \frac{2\pi \times 6.77 \times 10^6}{7669} = 5546 \text{ s} = 92.4 \text{ min}$$

This matches the roughly 90-minute period observed for the ISS.

(!) Lỗi thường gặp: Always measure orbital radius r (and the distance in the gravitation and field-strength equations) from the *centre* of the planet, never from its surface. For a satellite at height h above the surface, use $r = R_E + h$, not $r = h$ or $r = R_E$ alone.

9.10 Total Energy of an Orbiting Satellite

A satellite in a circular orbit possesses both kinetic energy (from its orbital speed) and gravitational potential energy (from its position in the field). Using $v^2 = GM/r$ from the orbit equation:

$$E_k = \frac{1}{2}mv^2 = \frac{GMm}{2r}$$

$$E_p = -\frac{GMm}{r}$$

$$E_{\text{total}} = E_k + E_p = \frac{GMm}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r}$$

The total mechanical energy of a circular orbit is negative, indicating that the satellite is gravitationally bound to the central body; it would require additional energy (bringing the total to zero or above) to escape to infinity. Note that the kinetic energy exactly equals half the magnitude of the potential energy, a general feature of circular gravitational orbits.

$$E_k = \frac{GMm}{2r}, \quad E_p = -\frac{GMm}{r}, \quad E_{\text{total}} = -\frac{GMm}{2r}$$

VN **GIẢI THÍCH TIẾNG VIỆT**

Vệ tinh trên quỹ đạo tròn có động năng $E_k = GMm/2r$ và thế năng $E_p = -GMm/r$, nên tổng năng lượng $E_{\text{total}} = -GMm/2r$ luôn âm, cho thấy vệ tinh bị "giữ" bởi lực hấp dẫn quanh thiên thể trung tâm.

Ví dụ mẫu · Total orbital energy

Find the kinetic energy, potential energy, and total mechanical energy of the 1000 kg satellite orbiting at $r = 6.77 \times 10^6$ m from the earlier example.

Solution.

$$E_k = \frac{GM_E m}{2r} = \frac{3.982 \times 10^{14} \times 1000}{2 \times 6.77 \times 10^6} = \frac{3.982 \times 10^{17}}{1.354 \times 10^7} = 2.94 \times 10^{10}\ \text{J}$$

$$E_p = -\frac{GM_E m}{r} = -5.88 \times 10^{10}\ \text{J} \quad \text{(from the previous section)}$$

$$E_{\text{total}} = E_k + E_p = 2.94 \times 10^{10} - 5.88 \times 10^{10} = -2.94 \times 10^{10}\ \text{J}$$

As expected, $E_{\text{total}} = -GM_E m/(2r)$, and its magnitude equals E_k.

9.11 Practice Questions

Bài tập luyện tập

1. In uniform circular motion, which pair of quantities remains constant in magnitude throughout the motion?

(A) Velocity and acceleration

(B) Speed and angular speed

(C) Momentum and net force

(D) Displacement and velocity

Lời giải chi tiết

Speed v and angular speed ω are both constant in magnitude in uniform circular motion (only their directions, in the case of velocity, change). Velocity and momentum change direction continuously, and there is always a nonzero net (centripetal) force, so it cannot be constant at zero. **Answer: B.**

Bài tập luyện tập

2. A fairground ride rotates at 12 revolutions per minute. Its radius is 8.0 m. Calculate (a) the period, (b) the angular speed, (c) the speed of a rider.

Lời giải chi tiết

(a) $T = \dfrac{60}{12} = 5.0$ s

(b) $\omega = \frac{2\pi}{T} = \frac{2\pi}{5.0} = 1.26\ \text{rad s}^{-1}$
(c) $v = \omega r = 1.26 \times 8.0 = 10.1\ \text{m s}^{-1}$

Bài tập luyện tập

3. A car of mass 900 kg rounds a flat, unbanked curve of radius 60 m at 15 m s^{-1}. Calculate the centripetal force required and the minimum coefficient of friction between the tyres and the road needed to prevent skidding.

Lời giải chi tiết

$$F = \frac{mv^2}{r} = \frac{900 \times 15.0^2}{60} = \frac{900 \times 225}{60} = 3375\ \text{N}$$

Friction supplies this force, up to a maximum of μmg, so

$$\mu_{\text{min}} = \frac{F}{mg} = \frac{3375}{900 \times 9.81} = \frac{3375}{8829} = 0.382$$

Bài tập luyện tập

4. The main purpose of banking a curve is to...

(A) Increase the frictional force available

(B) Allow a component of the normal force to supply the centripetal force, reducing the need for friction

(C) Increase the gravitational force on the vehicle

(D) Decrease the radius of the curve

Lời giải chi tiết

Banking tilts the normal force so that part of it points horizontally, toward the centre of the curve, supplying some or all of the centripetal force without relying on friction. **Answer: B.**

Bài tập luyện tập

5. A highway curve of radius 250 m is to be banked for vehicles travelling at 100 km h^{-1} with no friction required. Find the banking angle.

Lời giải chi tiết

$v = 100\ \text{km h}^{-1} = 27.78\ \text{m s}^{-1}$.

$$\tan\theta = \frac{v^2}{rg} = \frac{27.78^2}{250 \times 9.81} = \frac{771.6}{2452.5} = 0.3146$$

$$\theta = \tan^{-1}(0.3146) = 17.5^\circ$$

Bài tập luyện tập

6. A track is banked at 20° and has radius 100 m. Find the maximum speed at which a vehicle can travel around the curve without any friction being required.

Lời giải chi tiết

$$v = \sqrt{rg\tan\theta} = \sqrt{100 \times 9.81 \times \tan 20^\circ} = \sqrt{981 \times 0.3640} = \sqrt{357.1} = 18.9\,\mathrm{m\,s^{-1}}\ (\approx 68.0\,\mathrm{km\,h^{-1}})$$

Bài tập luyện tập

7. A bucket of water is swung in a vertical circle of radius 0.40 m. Find the minimum speed at the top of the circle needed to keep the water in the bucket.

Lời giải chi tiết

$$v_{\text{min}} = \sqrt{gr} = \sqrt{9.81 \times 0.40} = \sqrt{3.924} = 1.98\,\mathrm{m\,s^{-1}}$$

Bài tập luyện tập

8. The bucket and water of question 7 have total mass 0.30 kg. If the speed at the bottom of the circle (radius still 0.40 m) is $3.0\,\mathrm{m\,s^{-1}}$, find the tension in the handle at that point.

Lời giải chi tiết

$$T = \frac{mv^2}{r} + mg = \frac{0.30 \times 3.0^2}{0.40} + 0.30 \times 9.81 = \frac{2.70}{0.40} + 2.943 = 6.75 + 2.943 = 9.69\,\mathrm{N}$$

Bài tập luyện tập

9. At the top of a vertical circular loop, if a car's speed is greater than the minimum required speed, the normal contact force from the track...

(A) Becomes negative

(B) Remains zero

(C) Is greater than zero and increases with speed

(D) Equals the weight of the car

Lời giải chi tiết

Above the minimum speed, $N = mv^2/r - mg > 0$, and N increases as v increases. **Answer: C.**

Bài tập luyện tập

10. A roller-coaster loop has radius 12 m. (a) Find the minimum speed at the top of the loop. (b) Using energy conservation (height difference $= 2r$), find the minimum speed required at the bottom of the loop for the coaster to just complete it.

Lời giải chi tiết

(a)

$$v_{\text{top}} = \sqrt{gr} = \sqrt{9.81 \times 12} = \sqrt{117.7} = 10.9\,\mathrm{m\,s^{-1}}$$

(b) By energy conservation between bottom and top (height gained $= 2r$):

$$\tfrac{1}{2}v_{\text{bottom}}^2 = \tfrac{1}{2}v_{\text{top}}^2 + g(2r)$$

$$v_{\text{bottom}}^2 = 117.7 + 4 \times 9.81 \times 12 = 117.7 + 470.9 = 588.6$$

$$v_{\text{bottom}} = 24.3\,\text{m s}^{-1}$$

Bài tập luyện tập

11. A conical pendulum has string length 1.2 m, mass 0.25 kg, and the string makes a constant angle of 20° with the vertical. Find (a) the tension in the string, (b) the period of the motion.

Lời giải chi tiết

(a)

$$T = \frac{mg}{\cos\theta} = \frac{0.25 \times 9.81}{\cos 20^\circ} = \frac{2.453}{0.9397} = 2.61\,\text{N}$$

(b)

$$T_{\text{period}} = 2\pi\sqrt{\frac{L\cos\theta}{g}} = 2\pi\sqrt{\frac{1.2 \times 0.9397}{9.81}} = 2\pi\sqrt{0.1149} = 2\pi \times 0.339 = 2.13\,\text{s}$$

Bài tập luyện tập

12. A conical pendulum has a string of length 0.50 m rotating at 1.0 revolution per second. Find the angle the string makes with the vertical.

Lời giải chi tiết

$$\omega = 2\pi f = 2\pi \times 1.0 = 6.283\,\text{rad s}^{-1}$$

From $\cos\theta = \dfrac{g}{\omega^2 L}$:

$$\cos\theta = \frac{9.81}{6.283^2 \times 0.50} = \frac{9.81}{19.74} = 0.4969$$

$$\theta = \cos^{-1}(0.4969) = 60.2^\circ$$

Bài tập luyện tập

13. Two ships, each of mass 5.0×10^7 kg, are 100 m apart. Calculate the gravitational force of attraction between them.

Lời giải chi tiết

$$F = \frac{Gm_1m_2}{r^2} = \frac{6.67 \times 10^{-11} \times (5.0 \times 10^7)^2}{100^2} = \frac{6.67 \times 10^{-11} \times 2.5 \times 10^{15}}{1.0 \times 10^4} = \frac{1.6675 \times 10^5}{1.0 \times 10^4} = 16.7\,\text{N}$$

Bài tập luyện tập

14. At what height above Earth's surface does the gravitational field strength fall to half its surface value?

Lời giải chi tiết

Since $g \propto 1/r^2$, we need $r = R_E\sqrt{2}$ for g to halve.

$$r = 6.37 \times 10^6 \times \sqrt{2} = 9.007 \times 10^6\ \text{m}$$

$$h = r - R_E = 9.007 \times 10^6 - 6.37 \times 10^6 = 2.64 \times 10^6 \text{ m} \approx 2640\,\text{km}$$

Bài tập luyện tập

15. Inside a uniform sphere, the gravitational field strength g varies with distance r from the centre according to...

(A) $g \propto r$

(B) $g \propto 1/r$

(C) $g \propto 1/r^2$

(D) g is constant, independent of r

Lời giải chi tiết

Only the mass enclosed within radius r contributes, and this enclosed mass is proportional to r^3 for a uniform density; combined with the $1/r^2$ factor from the field law, $g \propto r^3/r^2 = r$. **Answer: A.**

Bài tập luyện tập

16. Find the work done raising a 50 kg mass from Earth's surface to a distance of $2R_E$ from Earth's centre.

Lời giải chi tiết

$$\phi_{\text{surface}} = -\frac{GM_E}{R_E} = -6.251 \times 10^7 \text{ J kg}^{-1}$$

$$\phi_{2R_E} = -\frac{GM_E}{2R_E} = -3.126 \times 10^7 \text{ J kg}^{-1}$$

$$W = m(\phi_{2R_E} - \phi_{\text{surface}}) = 50 \times (-3.126 \times 10^7 - (-6.251 \times 10^7)) = 50 \times 3.126 \times 10^7 = 1.56 \times 10^9 \text{ J}$$

Bài tập luyện tập

17. The Moon has mass 7.35×10^{22} kg and radius 1.74×10^6 m. Calculate the escape velocity from its surface.

Lời giải chi tiết

$$v_{\text{esc}} = \sqrt{\frac{2GM_{\text{Moon}}}{R_{\text{Moon}}}} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 7.35 \times 10^{22}}{1.74 \times 10^6}} = \sqrt{\frac{9.805 \times 10^{12}}{1.74 \times 10^6}} = \sqrt{5.635 \times 10^6} = 2.37e3\,\text{m s}^{-1} \approx 2$$

Bài tập luyện tập

18. A projectile is launched from a planet's surface at exactly the escape velocity. Which statement is correct?

(A) It returns to the surface after reaching maximum height

(B) It settles into a circular orbit at that speed

(C) Its total mechanical energy is exactly zero, and it reaches infinity with zero final speed

(D) Its total mechanical energy is positive, and it reaches infinity with nonzero final speed

Lời giải chi tiết

Escape velocity is defined by setting total mechanical energy (KE + PE) to exactly zero, so the projectile reaches infinity with (in the limit) zero remaining speed. **Answer: C.**

Bài tập luyện tập

19. Calculate the gravitational potential energy of the Earth-Moon system, given $M_E = 5.97 \times 10^{24}$ kg, $M_{\text{Moon}} = 7.35 \times 10^{22}$ kg, separation 3.84×10^8 m.

Lời giải chi tiết

$$E_p = -\frac{GM_E M_{\text{Moon}}}{r} = -\frac{2.927 \times 10^{37}}{3.84 \times 10^8} = -7.62 \times 10^{28} \text{ J}$$

Bài tập luyện tập

20. Two satellites orbit Earth: satellite 1 at radius $r_1 = 7.0 \times 10^6$ m, satellite 2 at radius $r_2 = 2.8 \times 10^7$ m. Find the ratio of their periods T_2/T_1, and calculate T_1 explicitly.

Lời giải chi tiết

Since $r_2/r_1 = 4.0$, Kepler's third law gives

$$\frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2} = 4.0^{1.5} = 8.0$$

$$T_1 = 2\pi\sqrt{\frac{r_1^3}{GM_E}} = 2\pi\sqrt{\frac{(7.0 \times 10^6)^3}{3.982 \times 10^{14}}} = 2\pi\sqrt{\frac{3.43 \times 10^{20}}{3.982 \times 10^{14}}} = 2\pi\sqrt{8.614 \times 10^5} = 2\pi \times 928 = 5832\,\text{s} \approx 97.2\,\text{min}$$

(and hence $T_2 = 8 \times 5832 = 46656\,\text{s} \approx 13.0\,\text{h}$.)

Bài tập luyện tập

21. Explain why there is only one possible orbital radius for a geostationary satellite, and calculate that radius (and the corresponding height above Earth's surface) given $T = 24\,\text{h}$.

Lời giải chi tiết

A geostationary satellite must have a period equal to Earth's rotation period ($T = 24\,\text{h}$) and must orbit in the equatorial plane so that it stays above the same point on the ground. Since Kepler's third law fixes a unique relationship between T and r for orbits around a given mass M_E, specifying $T = 24\,\text{h}$ determines r uniquely – there is exactly one radius that gives this period.

$$r^3 = \frac{GM_E T^2}{4\pi^2} = \frac{3.982 \times 10^{14} \times (86400)^2}{4\pi^2} = \frac{3.982 \times 10^{14} \times 7.465 \times 10^9}{39.48} = \frac{2.973 \times 10^{24}}{39.48} = 7.53 \times 10^{22}\,\text{m}^3$$

$$r = (7.53 \times 10^{22})^{1/3} = 4.22 \times 10^7 \text{ m}$$

Height above the surface:

$$h = r - R_E = 4.22 \times 10^7 - 6.37 \times 10^6 \approx 3.59 \times 10^4 \text{ km}$$

This unique altitude is used for communications satellites because a receiving dish on the ground can be pointed permanently at a fixed spot in the sky, with no need to track the satellite's motion.

Bài tập luyện tập

22. A 2000 kg satellite orbits Earth at $r = 8.0 \times 10^6$ m. Find its kinetic energy, potential energy, and total mechanical energy.

Lời giải chi tiết

$$E_k = \frac{GM_E m}{2r} = \frac{3.982 \times 10^{14} \times 2000}{2 \times 8.0 \times 10^6} = \frac{7.964 \times 10^{17}}{1.6 \times 10^7} = 4.98e10\,\text{J}$$

$$E_p = -\frac{GM_E m}{r} = -\frac{7.964 \times 10^{17}}{8.0 \times 10^6} = -9.96e10\,\text{J}$$

$$E_{\text{total}} = E_k + E_p = 4.98 \times 10^{10} - 9.96 \times 10^{10} = -4.98e10\,\text{J}$$

Bài tập luyện tập

23. If the orbital radius of a satellite is doubled while it remains in a circular orbit, the magnitude of its total mechanical energy...

(A) Doubles
(B) Halves
(C) Stays the same
(D) Quadruples

Lời giải chi tiết

$E_{\text{total}} = -GMm/(2r)$, so the magnitude is inversely proportional to r. Doubling r halves the magnitude of E_{total}. **Answer: B.**

Bài tập luyện tập

24. A satellite orbits just above the Moon's surface, at radius equal to the Moon's radius $R_{\text{Moon}} = 1.74 \times 10^6$ m ($M_{\text{Moon}} = 7.35 \times 10^{22}$ kg). Find its orbital speed and period.

Lời giải chi tiết

$$v = \sqrt{\frac{GM_{\text{Moon}}}{R_{\text{Moon}}}} = \sqrt{\frac{4.902 \times 10^{12}}{1.74 \times 10^6}} = \sqrt{2.818 \times 10^6} = 1679\,\text{m s}^{-1} \approx 1.68\,\text{km s}^{-1}$$

$$T = \frac{2\pi R_{\text{Moon}}}{v} = \frac{2\pi \times 1.74 \times 10^6}{1679} = 6512\,\text{s} \approx 108.5\,\text{min}$$

Bài tập luyện tập

25. According to Newton's law of gravitation, the force between two point masses is inversely proportional to...

(A) Their separation, r
(B) The square of their separation, r^2
(C) The cube of their separation, r^3
(D) The square root of their separation

Lời giải chi tiết

$F = Gm_1m_2/r^2$, so the force is inversely proportional to r^2. **Answer: B.**

Bài tập luyện tập

26. A curve of radius $80\,\text{m}$ is banked at $25°$. (a) Find the design speed, at which no friction is required. (b) If a car actually travels at $25\,\text{m s}^{-1}$, state – with a reason – which way friction must act.

Lời giải chi tiết

(a)

$$v = \sqrt{rg\tan\theta} = \sqrt{80 \times 9.81 \times \tan 25°} = \sqrt{784.8 \times 0.4663} = \sqrt{365.9} = 19.1\,\text{m s}^{-1}$$

(b) $25\,\text{m s}^{-1} > 19.1\,\text{m s}^{-1}$, so at this higher speed the car requires *more* centripetal force than the normal force alone can supply at this angle; the car tends to slide up and outward across the banked surface. Friction must therefore act down the slope, with a component directed toward the centre of the curve, to supply the additional centripetal force needed.

Bài tập luyện tập

27. For a mass on a string moving in a vertical circle, at the instant the string is horizontal (mass at the side of the circle), the centripetal force is provided by...

(A) Tension alone, since gravity acts tangentially at this point

(B) Gravity alone

(C) The horizontal component of gravity only

(D) Neither tension nor gravity, since velocity is constant

Lời giải chi tiết

When the string is horizontal, the string tension is horizontal (along the radius, toward the centre) while gravity acts vertically, i.e. entirely tangential to the circle at that point, contributing nothing to the centripetal direction. So $T = mv^2/r$ at that instant. **Answer: A.**

Bài tập luyện tập

28. Mars orbits the Sun at a mean distance of 1.524 times Earth's orbital radius. Given Earth's orbital period is 1.00 year, use Kepler's third law to find the orbital period of Mars.

Lời giải chi tiết

$$\frac{T_{\text{Mars}}}{T_{\text{Earth}}} = \left(\frac{r_{\text{Mars}}}{r_{\text{Earth}}}\right)^{3/2} = 1.524^{1.5}$$

$$1.524^{1.5} = 1.524 \times \sqrt{1.524} = 1.524 \times 1.2345 = 1.881$$

$$T_{\text{Mars}} = 1.00 \times 1.881 = 1.88\ \text{years}$$

Thermal Physics, Ideal Gases & Oscillations

[A2 – A Level only, not in the AS syllabus]

Mục tiêu Unit: **Phần I – Nhiệt học & khí lý tưởng:** thang nhiệt độ Kelvin; nhiệt dung riêng $Q = mc\Delta\theta$ và nhiệt hoá riêng $Q = mL$ (tại nhiệt độ không đổi); định luật I nhiệt động lực học $\Delta U = q + w$; công của khí ở áp suất không đổi $W = p\Delta V$; quá trình đẳng nhiệt và đoạn nhiệt; phương trình khí lý tưởng $pV = nRT = NkT$; các giả thiết của thuyết động học phân tử khí; động năng trung bình phân tử $\langle E_k\rangle = \frac{3}{2}kT$. **Phần II – Dao động:** phương trình dao động điều hoà $a = -\omega^2 x$ và các nghiệm x, v; độ lệch pha giữa hai dao động; năng lượng dao động điều hoà (tổng, động năng, thế năng); con lắc đơn và con lắc lò xo (ngang & treo thẳng đứng); các loại tắt dần (nhẹ, tới hạn, mạnh) và hiện tượng cộng hưởng.

10.1 Heat, Temperature and Specific Heat Capacity

[A2 – A Level]Thermal physics is concerned with the transfer of energy as **heat**, and with the way that transferred energy changes the temperature or physical state of a substance.

Two bodies are in **thermal equilibrium** when they are at the same temperature: there is then no net flow of thermal energy between them. Temperature is measured on the **thermodynamic (Kelvin) scale**, related to the Celsius scale by

$$T/\text{K} = \theta/^{\circ}\text{C} + 273$$

A change of 1 K is identical in size to a change of $1\,^{\circ}$C – only the zero points of the two scales differ (absolute zero, 0 K, is $-273\,^{\circ}$C).

(!) Lỗi thường gặp: Every gas-law equation and every kinetic-theory equation in this chapter requires temperature in **kelvin**, never in $^{\circ}$C. Forgetting to convert (or using θ in $^{\circ}$Cdirectly in $pV = nRT$) is one of the most common numerical errors at this level – always convert first, then substitute.

10.1.1 Heat Capacity and Specific Heat Capacity

The word **"specific"** in physics always means **"per unit mass"**. The **heat capacity** C of an object is the energy needed to raise *that particular object's* temperature by 1 K (units J K^{-1}); it depends on the mass and material of the specific object. The **specific heat capacity** c of a *material* is the energy needed to raise the temperature of 1 **kg** of that material by 1 K (units J kg^{-1} K^{-1}) – a property of the substance itself, independent of how much of it you have. For a mass m undergoing a temperature change $\Delta\theta$ with no change of state:

$$Q = mc\Delta\theta$$

Ví dụ mẫu · Time to heat water using specific heat capacity

A 1500 W electric heater is used to heat 2.0 kg of water from $20\,^\circ$C to $80\,^\circ$C. Taking the specific heat capacity of water as $c = 4200$ J kg^{-1} K^{-1}, and assuming no heat losses, calculate the time taken.

$$Q = mc\Delta\theta = 2.0 \times 4200 \times (80 - 20) = 2.0 \times 4200 \times 60 = \boxed{504\,000\,\text{J}}.$$

Since power $P = Q/t$:

$$t = \frac{Q}{P} = \frac{504\,000}{1500} = \boxed{336\,\text{s}}\ (= 5.6\,\text{minutes}).$$

10.1.2 Change of State and Specific Latent Heat

When a substance changes state (melts, boils, freezes or condenses), energy is transferred **without any change in temperature** – the energy instead changes the arrangement of, and bonding between, the molecules. The **specific latent heat** L of a substance is the energy required to change the state of 1 **kg** of it, at constant temperature (units J kg^{-1}):

$$Q = mL$$

Specific latent heat of fusion L_f applies to melting/freezing; **specific latent heat of vaporisation** L_v applies to boiling/condensing. For any given substance, L_v is **always much larger** than L_f: melting only needs to break the relatively small number of bonds that hold molecules in a fixed, ordered lattice (the molecules remain close together, just free to move past one another), whereas boiling must completely separate the molecules against **all** the intermolecular attractive forces, pulling them far apart into a gas.

Ví dụ mẫu · Time to boil a mass of water using specific latent heat

A 1200 W kettle element boils away 50 g of water once the water has already reached $100\,^\circ$C. Taking the specific latent heat of vaporisation of water as $L_v = 2.26 \times 10^6$ J kg^{-1}, find the time taken to boil this mass away completely (assume all electrical energy goes into vaporisation).

$$Q = mL_v = 0.050 \times 2.26 \times 10^6 = \boxed{1.13 \times 10^5\,\text{J}}.$$

$$t = \frac{Q}{P} = \frac{1.13 \times 10^5}{1200} = \boxed{94\,\text{s}}.$$

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Hai vật ở **cân bằng nhiệt** khi cùng nhiệt độ. Thang Kelvin: $T/\text{K} = \theta/^\circ\text{C} + 273$ – luôn đổi sang Kelvin trước khi thay vào công thức. **"Specific"** nghĩa là **"trên một đơn vị khối lượng"**. **Nhiệt dung riêng** c: $Q = mc\Delta\theta$ (không đổi trạng thái). **Nhiệt hoá riêng** L: $Q = mL$ (đổi trạng thái ở nhiệt độ không đổi); nhiệt hoá hơi L_v luôn lớn hơn nhiệt nóng chảy L_f vì hoá hơi phải phá vỡ **toàn bộ** lực liên kết phân tử để tách các phân tử ra xa nhau, trong khi nóng chảy chỉ cần phá vỡ cấu trúc trật tự của chất rắn (các phân tử vẫn ở gần nhau).

10.2 The First Law of Thermodynamics

[A2 – A Level]The first law of thermodynamics is simply a statement of **conservation of energy** applied to a system that can both receive heat and have work done on (or by) it.

The **first law of thermodynamics** states that the increase in internal energy ΔU of a system equals the heat q supplied **to** the system plus the work w done **on** the system:

$$\Delta U = q + w$$

Sign convention: q is positive when heat is supplied **to** the system (and negative if heat leaves the system); w is positive when work is done **on** the system by its surroundings (and negative when the system does work **on** the surroundings, i.e. when the system expands and pushes its surroundings back).

10.2.1 Work Done by a Gas at Constant Pressure

When a gas expands or is compressed at **constant pressure** p, the work done **by** the gas on its surroundings is

$$W = p\,\Delta V$$

where ΔV is the change in volume. If the gas **expands** ($\Delta V > 0$), the gas does positive work *on* the surroundings, which is equivalent to *negative* work being done *on* the gas ($w = -p\Delta V$ in the first-law sign convention above). If the gas is **compressed** ($\Delta V < 0$), the surroundings do positive work on the gas ($w = +p|\Delta V|$).

Ví dụ mẫu · Work done by an expanding gas, applied to the first law

A gas is held at a constant pressure of $p = 2.0 \times 10^5$ Pa in a cylinder fitted with a piston. The gas expands from a volume of $V_1 = 4.0 \times 10^{-4}$ m^3 to $V_2 = 6.0 \times 10^{-4}$ m^3, and in doing so its internal energy increases by $\Delta U = 60$ J. Calculate (a) the work done **by** the gas on its surroundings, and (b) the heat supplied to the gas.

(a)

$$\Delta V = V_2 - V_1 = 6.0 \times 10^{-4} - 4.0 \times 10^{-4} = 2.0 \times 10^{-4}\ \text{m}^3.$$

$$W_{\text{by gas}} = p\,\Delta V = 2.0 \times 10^5 \times 2.0 \times 10^{-4} = \boxed{40\ \text{J}}.$$

(b) Since the gas expands, it does 40 J of work *on* the surroundings, so the work done *on* the gas is $w = -40$ J. Using the first law $\Delta U = q + w$:

$$60 = q + (-40) \implies q = 60 + 40 = \boxed{100\ \text{J}}.$$

So 100 J of heat must have been supplied to the gas: part of this heat increased the internal energy (60 J) and the rest was used doing work as the gas expanded (40 J).

10.2.2 Isothermal and Adiabatic Changes

An **isothermal** change is one carried out at **constant temperature**: it typically occurs slowly enough that heat can flow into or out of the gas to keep its temperature fixed as its pressure and volume change. An **adiabatic** change is one in which **no heat enters or leaves** the system ($q = 0$ in the first law); this typically happens when a change occurs **rapidly**, too fast for significant heat transfer to take place, so any change in internal energy comes entirely from work done on or by the gas.

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Định luật I nhiệt động lực học: $\Delta U = q + w$, trong đó q là nhiệt truyền **vào** hệ (dương) và w là công thực hiện **lên** hệ (dương). Khi khí giãn nở ở áp suất không đổi, công do khí thực hiện lên môi trường là $W = p\Delta V$ (tương đương $w = -p\Delta V$ trong định luật I). Quá trình **đẳng nhiệt**: nhiệt độ không đổi, thường xảy ra chậm và có trao đổi nhiệt với môi trường. Quá trình **đoạn nhiệt**: không có trao đổi nhiệt ($q = 0$), thường xảy ra nhanh.

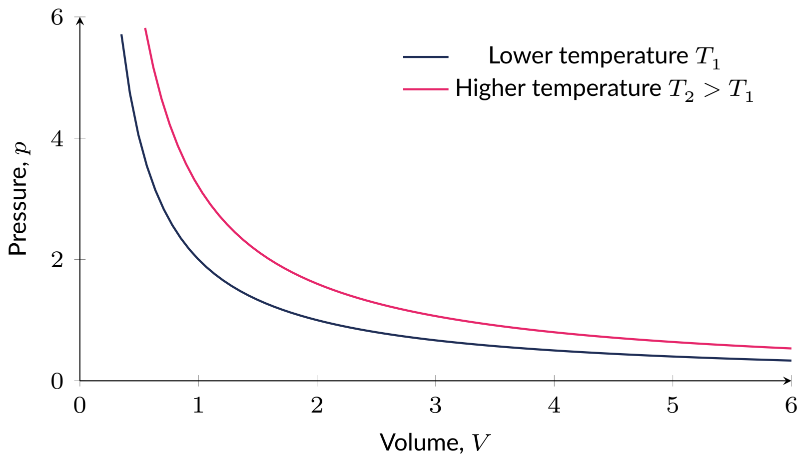
10.3 The Ideal Gas Law

[A2 – A Level]An **ideal gas** is a theoretical model of a gas that obeys the gas laws exactly at all pressures, volumes and temperatures; real gases approximate this behaviour well at low pressure and high temperature (i.e. far from the conditions under which they would liquefy).

The **equation of state for an ideal gas** may be written in two equivalent forms:

$$pV = nRT \qquad \text{or} \qquad pV = NkT$$

where p is pressure (Pa), V is volume ($\mathrm{m^3}$), T is thermodynamic temperature (K), n is the number of moles, N is the number of molecules, $R = 8.31\,\mathrm{J\,mol^{-1}\,K^{-1}}$ is the molar gas constant, and $k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$ is the Boltzmann constant. Since one mole contains the Avogadro number $N_A = 6.02 \times 10^{23}$ of molecules, $N = nN_A$ and $R = N_A k$.



Isothermal (p–V) curves for a fixed mass of ideal gas: each curve satisfies pV = constant at its own fixed temperature. The higher curve (further from the origin) corresponds to the **higher** temperature, since $pV = nRT$ means pV must increase if T increases.

Ví dụ mẫu · Combined gas law: finding a new volume

A fixed mass of gas occupies $V_1 = 2.0\,\mathrm{dm^3}$ at pressure $p_1 = 1.0 \times 10^5$ Pa and temperature $T_1 = 300$ K. It is compressed and heated so that its pressure becomes $p_2 = 1.5 \times 10^5$ Pa and its temperature becomes $T_2 = 350$ K. Calculate the new volume V_2.

For a fixed mass of gas, n and R are constant, so $\dfrac{p_1V_1}{T_1} = \dfrac{p_2V_2}{T_2}$. Rearranging:

$$V_2 = \frac{p_1V_1T_2}{T_1p_2} = \frac{(1.0 \times 10^5)(2.0)(350)}{(300)(1.5 \times 10^5)}.$$

Numerator: $1.0 \times 10^5 \times 2.0 \times 350 = 7.0 \times 10^7$. Denominator: $300 \times 1.5 \times 10^5 = 4.5 \times 10^7$.

$$V_2 = \frac{7.0 \times 10^7}{4.5 \times 10^7} = \boxed{1.56\,\text{dm}^3}.$$

Ví dụ mẫu · Finding the number of moles from the ideal gas law

A cylinder contains gas at a pressure of $p = 3.00 \times 10^5$ Pa, in a volume of $V = 8.00 \times 10^{-3}\,\text{m}^3$, at a temperature of $T = 290$ K. Calculate the number of moles of gas present.

From $pV = nRT$:

$$n = \frac{pV}{RT} = \frac{(3.00 \times 10^5)(8.00 \times 10^{-3})}{(8.31)(290)} = \frac{2400}{2409.9} = \boxed{0.996\,\text{mol}}\ (\approx 1.00\,\text{mol}).$$

Ví dụ mẫu · Finding a new temperature from the ideal gas law at constant volume

A fixed mass of gas is sealed in a rigid container of constant volume. Initially $p_1 = 1.00 \times 10^5$ Pa at $T_1 = 290$ K. The gas is heated until the pressure becomes $p_2 = 1.45 \times 10^5$ Pa. Find the new temperature T_2.

At constant volume, $\dfrac{p_1}{T_1} = \dfrac{p_2}{T_2}$, so

$$T_2 = T_1 \times \frac{p_2}{p_1} = 290 \times \frac{1.45 \times 10^5}{1.00 \times 10^5} = 290 \times 1.45 = \boxed{420.5\,\text{K}}\ (\approx 420\,\text{K}).$$

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Phương trình khí lý tưởng: $pV = nRT$ (theo số mol n, $R = 8.31\,\text{J mol}^{-1}\,\text{K}^{-1}$) hoặc $pV = NkT$ (theo số phân tử N, $k = 1.38 \times 10^{-23}\,\text{J K}^{-1}$). Với khối lượng khí không đổi, dùng định luật khí kết hợp $\dfrac{p_1V_1}{T_1} = \dfrac{p_2V_2}{T_2}$ để tìm đại lượng chưa biết. Đường đẳng nhiệt trên đồ thị p–V ($pV =$ hằng số): đường càng xa gốc toạ độ ứng với nhiệt độ càng cao.

10.4 Kinetic Theory of Gases

[A2 – A Level]The ideal gas law can be derived from a simple microscopic model of a gas as a very large number of tiny particles in ceaseless random motion – the **kinetic theory of gases**.

The kinetic theory of gases rests on the following assumptions:

- the gas consists of a very large number of **point molecules** (of negligible volume compared with the volume of the container), in constant **random motion**;
- **intermolecular forces** are negligible **except** during collisions;
- all collisions (between molecules, and between molecules and the container walls) are **perfectly elastic** (no loss of kinetic energy overall);
- the **time of a collision** is negligible compared with the time between successive collisions;
- there is a **large enough number** of molecules that a statistical (average) treatment is valid;
- the total **volume of the molecules themselves** is negligible compared with the volume occupied by the gas.

A key result of kinetic theory (derivable from the above assumptions, combined with $pV = NkT$) is that the **average translational kinetic energy** of a gas molecule depends only on the thermodynamic temperature:

$$\langle E_k \rangle = \tfrac{3}{2}kT$$

This shows directly that temperature is a measure of the average kinetic energy of the random motion of the particles making up a substance – it is **not** a measure of the total internal energy of a sample (which also depends on how many molecules are present).

Ví dụ mẫu · Average molecular kinetic energy at a given temperature

Calculate the average translational kinetic energy of a gas molecule at $T = 300$ K, taking $k = 1.38 \times 10^{-23}$ J K^{-1}.

$$\langle E_k \rangle = \tfrac{3}{2}kT = 1.5 \times 1.38 \times 10^{-23} \times 300 = \boxed{6.21 \times 10^{-21}\ \text{J}}.$$

Ví dụ mẫu · Number of molecules in a gas sample

A gas occupies $V = 1.0\,\text{dm}^3 = 1.0 \times 10^{-3}\,\text{m}^3$ at pressure $p = 2.0 \times 10^5$ Pa and temperature $T = 300$ K. Using $pV = NkT$, calculate the number of molecules N present.

$$N = \frac{pV}{kT} = \frac{(2.0 \times 10^5)(1.0 \times 10^{-3})}{(1.38 \times 10^{-23})(300)} = \frac{2.0 \times 10^2}{4.14 \times 10^{-21}} = \boxed{4.83 \times 10^{22}\ \text{molecules}}\ (\approx 4.8 \times 10^{22}).$$

(!) Lỗi thường gặp: Temperature must be in **kelvin** in $\langle E_k \rangle = \frac{3}{2}kT$, $pV = nRT$ and $pV = NkT$ – there is no exception. Also, remember that the assumptions of kinetic theory (elastic collisions, negligible molecular volume and negligible intermolecular forces except during collisions) are precisely the conditions under which a **real** gas behaves most like an **ideal** gas: low pressure (molecules far apart, so their own volume and mutual forces matter less) and high temperature (well above the boiling point, so the gas is far from the conditions where it would liquefy).

VN **GIẢI THÍCH TIẾNG VIỆT**

Thuyết động học phân tử khí dựa trên các giả thiết: phân tử coi như chất điểm chuyển động hỗn loạn không ngừng; lực tương tác phân tử không đáng kể trừ lúc va chạm; va chạm hoàn toàn đàn hồi; thời gian va chạm rất ngắn so với thời gian giữa hai va chạm; số phân tử đủ lớn để xử lý thống kê; thể tích phân tử không đáng kể so với thể tích bình chứa. Kết quả quan trọng: động năng trung bình của một phân tử $\langle E_k \rangle = \frac{3}{2}kT$ chỉ phụ thuộc nhiệt độ tuyệt đối – đây chính là ý nghĩa vật lý của nhiệt độ.

10.5 Oscillations

[A2 – A Level]The second half of this chapter turns to **simple harmonic motion (SHM)** – a special type of oscillation that underlies the behaviour of pendulums, springs, vibrating molecules and countless other periodic systems.

10.5.1 The Defining Equation of SHM

A body performs **simple harmonic motion** if its acceleration is always **directly proportional** to its displacement from a fixed (equilibrium) point, and always directed **towards** that point (i.e. acceleration and displacement are always in opposite directions):

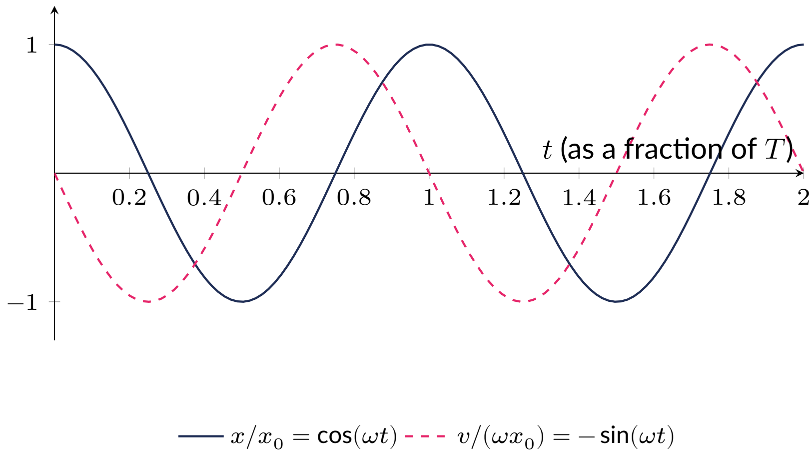
$$a = -\omega^2 x$$

where ω is the **angular frequency** (rad s^{-1}), related to the period T and frequency f by $\omega = 2\pi f = \dfrac{2\pi}{T}$.

If the body starts at maximum displacement x_0 (the **amplitude**) at $t = 0$, the solutions of the SHM equation are:

$$x = x_0 \cos(\omega t), \qquad v = -\omega x_0 \sin(\omega t), \qquad v = \pm\omega\sqrt{x_0^2 - x^2}$$

The **maximum speed** occurs at $x = 0$ (passing through equilibrium): $v_0 = \omega x_0$. The **maximum acceleration** occurs at $x = \pm x_0$ (the extremes of the motion): $a_0 = \omega^2 x_0$.



Displacement–time and velocity–time graphs for SHM starting at maximum positive displacement. The velocity graph is a quarter-cycle **ahead** of the displacement graph – velocity **leads** displacement by $90°$ (or $\pi/2$ rad), reaching its own maximum a quarter-period before displacement next does.

Ví dụ mẫu · Angular frequency, maximum speed and maximum acceleration

An oscillator has frequency $f = 2.0$ Hz and amplitude $x_0 = 5.0$ cm $= 0.050$ m. Calculate (a) the angular frequency ω, (b) the maximum speed v_0, (c) the maximum acceleration a_0.

(a)

$$\omega = 2\pi f = 2\pi \times 2.0 = \boxed{12.57 \text{ rad s}^{-1}} \ (\approx 12.6).$$

(b)

$$v_0 = \omega x_0 = 12.57 \times 0.050 = \boxed{0.628 \text{ m s}^{-1}}.$$

(c)

$$a_0 = \omega^2 x_0 = (12.57)^2 \times 0.050 = 158.0 \times 0.050 = \boxed{7.90 \text{ m s}^{-2}}.$$

Ví dụ mẫu · Speed at a given displacement

For the same oscillator as above ($\omega = 12.57$ rad s^{-1}, $x_0 = 0.050$ m), find the speed when the displacement is $x = 3.0$ cm $= 0.030$ m.

$$v = \omega\sqrt{x_0^2 - x^2} = 12.57\sqrt{(0.050)^2 - (0.030)^2} = 12.57\sqrt{0.0025 - 0.0009} = 12.57\sqrt{0.0016}.$$

$$v = 12.57 \times 0.040 = \boxed{0.503\ \text{m s}^{-1}}.$$

10.5.2 Phase Difference Between Two Oscillators

Two bodies performing SHM at the **same frequency** may reach corresponding points in their cycles (e.g. maximum displacement) at different times; this time offset, expressed as an angle, is the **phase difference** $\Delta\phi$ (in radians, or sometimes degrees). A full cycle corresponds to a phase difference of 2π rad $(360°)$; a phase difference of π rad $(180°)$ means the two oscillators are exactly **out of phase** (one at maximum positive displacement while the other is at maximum negative displacement).

Ví dụ mẫu · Phase difference between two pendulums

Two identical pendulums, A and B, both oscillate with period $T = 2.4$ s. Pendulum A reaches its maximum positive displacement at $t = 0$; pendulum B reaches its maximum positive displacement 0.60 s later. Calculate the phase difference between the two pendulums, in radians.

A full cycle ($T = 2.4$ s) corresponds to a phase angle of 2π rad. The time delay of 0.60 s is a fraction $\dfrac{0.60}{2.4} = 0.25$ of a full cycle, so

$$\Delta\phi = 0.25 \times 2\pi = \boxed{\frac{\pi}{2}\ \text{rad}}\ (= 1.57\ \text{rad},\ \text{i.e. } 90°).$$

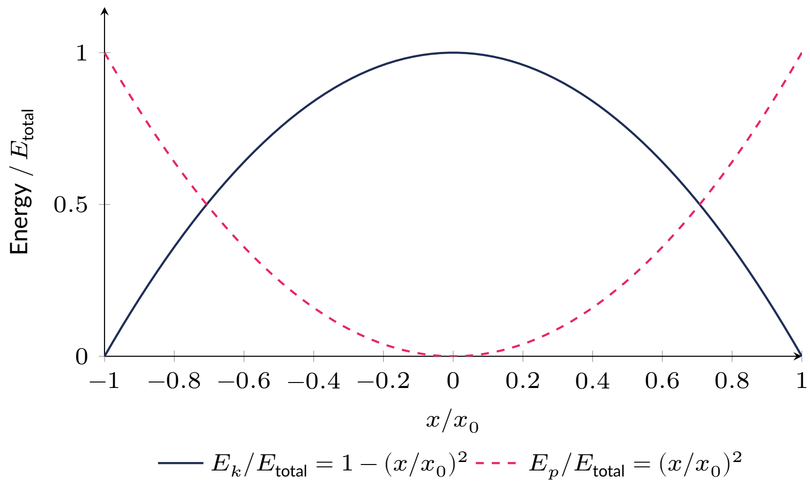
10.5.3 Energy in SHM

Provided there is no resistive (damping) force, the **total energy** of an oscillator undergoing SHM remains constant, continuously exchanged between kinetic and potential forms:

$$E_{\text{total}} = \tfrac{1}{2}m\omega^2 x_0^2 \qquad \text{(constant)}$$

$$E_k = \tfrac{1}{2}m\omega^2(x_0^2 - x^2), \qquad E_p = \tfrac{1}{2}m\omega^2 x^2$$

At any instant, $E_k + E_p = E_{\text{total}}$: kinetic energy is maximum (equal to E_{total}) at $x = 0$, and zero at $x = \pm x_0$; potential energy is zero at $x = 0$ and maximum (equal to E_{total}) at $x = \pm x_0$.



Kinetic and potential energy against fractional displacement x/x_0 for an SHM oscillator. The two curves always sum to the constant total energy (horizontal line at 1), swapping dominance as the oscillator passes through equilibrium (E_k maximum) or reaches its extremes (E_p maximum).

Ví dụ mẫu · Total energy of an oscillator

A mass $m = 0.20$ kg oscillates with angular frequency $\omega = 12.57\,\text{rad s}^{-1}$ and amplitude $x_0 = 5.0$ cm $= 0.050$ m. Calculate its total energy.

$$E_{\text{total}} = \tfrac{1}{2}m\omega^2 x_0^2 = 0.5 \times 0.20 \times (12.57)^2 \times (0.050)^2.$$

$(12.57)^2 = 158.0$; $(0.050)^2 = 2.5 \times 10^{-3}$.

$$E_{\text{total}} = 0.5 \times 0.20 \times 158.0 \times 2.5 \times 10^{-3} = \boxed{3.95 \times 10^{-2}\,\text{J}}\ (= 39.5\,\text{mJ}).$$

Ví dụ mẫu · Using energy conservation directly to find kinetic energy at a given displacement

For the oscillator above ($m = 0.20$ kg, $\omega = 12.57\,\text{rad s}^{-1}$, $x_0 = 0.050$ m, $E_{\text{total}} = 39.5$ mJ), find the kinetic energy when $x = 0.030$ m, using (i) the energy formula directly, and (ii) the speed found from $v = \omega\sqrt{x_0^2 - x^2}$, to confirm the two methods agree.

(i) Direct energy method:

$$E_k = \tfrac{1}{2}m\omega^2(x_0^2 - x^2) = 0.5 \times 0.20 \times 158.0 \times [(0.050)^2 - (0.030)^2].$$

$(0.050)^2 - (0.030)^2 = 2.5 \times 10^{-3} - 0.9 \times 10^{-3} = 1.6 \times 10^{-3}\,\text{m}^2$.

$$E_k = 0.5 \times 0.20 \times 158.0 \times 1.6 \times 10^{-3} = \boxed{2.53 \times 10^{-2}\,\text{J}}\ (= 25.3\,\text{mJ}).$$

(ii) Via speed: from the earlier worked example, $v = 0.503\,\text{m s}^{-1}$ at $x = 0.030$ m. Then

$$E_k = \tfrac{1}{2}mv^2 = 0.5 \times 0.20 \times (0.503)^2 = 0.5 \times 0.20 \times 0.253 = \boxed{2.53 \times 10^{-2}\,\text{J}}\ (= 25.3\,\text{mJ}).$$

Both methods agree exactly ✓, confirming that the energy expressions and the SHM kinematic equations are consistent.

GIẢI THÍCH TIẾNG VIỆT

VN Phương trình dao động điều hoà: $a = -\omega^2 x$; nghiệm $x = x_0\cos(\omega t)$, $v = -\omega x_0\sin(\omega t)$,

$v = \pm\omega\sqrt{x_0^2 - x^2}$. Tốc độ cực đại $v_0 = \omega x_0$ tại $x = 0$; gia tốc cực đại $a_0 = \omega^2 x_0$ tại $x = \pm x_0$. Vận tốc **sớm pha một phần tư chu kì** so với li độ. **Độ lệch pha** giữa hai dao động cùng tần số được tính theo tỉ lệ thời gian trễ trên chu kì, nhân với 2π rad. Năng lượng toàn phần $E_{\text{total}} = \frac{1}{2}m\omega^2 x_0^2$ không đổi, luôn bằng $E_k + E_p$; có thể tính E_k tại một li độ bất kỳ bằng công thức năng lượng trực tiếp hoặc bằng $\frac{1}{2}mv^2$ sau khi tìm v – hai cách phải cho cùng kết quả.

10.6 Simple Harmonic Oscillators: Pendulum and Spring

[A2 – A Level]Two classic mechanical systems that (to a good approximation) undergo SHM are the simple pendulum and the mass-spring system.

The period of a **simple pendulum**, of length L, swinging with small amplitude, is

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where g is the local gravitational field strength. The period of a mass m oscillating on a spring of spring constant k (whether horizontal or vertical) is

$$T = 2\pi\sqrt{\frac{m}{k}}$$

In both cases, notice that the period does **not** depend on the amplitude of the oscillation – this is a defining feature of SHM.

Ví dụ mẫu · Period of a simple pendulum

A simple pendulum has length $L = 0.90$ m. Taking $g = 9.81\,\text{m s}^{-2}$, calculate its period.

$$T = 2\pi\sqrt{\frac{0.90}{9.81}} = 2\pi\sqrt{0.09175} = 2\pi \times 0.3029 = \boxed{1.90\text{ s}}.$$

Ví dụ mẫu · Period of a horizontal mass-spring system

A mass of $m = 0.40$ kg is attached to a spring of spring constant $k = 25\,\text{N m}^{-1}$ and set oscillating on a frictionless horizontal surface. Calculate the period.

$$T = 2\pi\sqrt{\frac{0.40}{25}} = 2\pi\sqrt{0.0160} = 2\pi \times 0.1265 = \boxed{0.795\text{ s}}.$$

Ví dụ mẫu · Vertical mass-spring oscillation

A spring of spring constant $k = 40\,\text{N m}^{-1}$ hangs vertically from a fixed support. A mass of $m = 0.50$ kg is attached to its lower end and allowed to hang at rest, then is pulled down a further 3.0 cm and released, so that it oscillates vertically with SHM about its new equilibrium position. (a) Calculate the extra extension of the spring, beyond its natural length, needed to support the mass in equilibrium (take $g = 9.81\,\text{m s}^{-2}$). (b) Calculate the period of the vertical oscillation. (c) Calculate the maximum speed of the mass during the oscillation.

(a) At equilibrium the spring's tension supports the weight: $ke = mg$, so

$$e = \frac{mg}{k} = \frac{0.50 \times 9.81}{40} = \frac{4.905}{40} = \boxed{0.1226\,\text{m}}\ (\approx 12.3\,\text{cm}).$$

(This extension simply shifts the equilibrium position; the mass then oscillates **about this new equilibrium** with SHM, exactly as for a horizontal spring, because the restoring force for any *further* displacement from this equilibrium is still $F = -kx$.)
(b) The period of vertical SHM on a spring uses the same formula as the horizontal case:

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.50}{40}} = 2\pi\sqrt{0.0125} = 2\pi \times 0.1118 = \boxed{0.702\text{ s}}.$$

(c) Angular frequency: $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.702} = 8.95\,\text{rad s}^{-1}$ (equivalently $\omega = \sqrt{k/m} = \sqrt{40/0.50} = \sqrt{80} = 8.94\,\text{rad s}^{-1}$). With amplitude $x_0 = 0.030\,\text{m}$:

$$v_0 = \omega x_0 = 8.94 \times 0.030 = \boxed{0.268\text{ m s}^{-1}}.$$

(!) Lỗi thường gặp: The period of an SHM oscillator is completely **independent of amplitude** – doubling the amplitude of a pendulum's swing or a spring's oscillation does **not** change its period. However, the maximum speed $v_0 = \omega x_0$ and maximum acceleration $a_0 = \omega^2 x_0$ **do** depend on amplitude, since a larger swing must cover more distance (and change direction with the same period), so it must reach a higher maximum speed and a larger maximum acceleration.

VN GIẢI THÍCH TIẾNG VIỆT

Chu kì con lắc đơn: $T = 2\pi\sqrt{L/g}$. Chu kì con lắc lò xo (ngang hoặc treo thẳng đứng): $T = 2\pi\sqrt{m/k}$ – không phụ thuộc biên độ. Với lò xo treo thẳng đứng, độ giãn thêm ở vị trí cân bằng $e = mg/k$ chỉ dịch chuyển vị trí cân bằng; vật vẫn dao động điều hoà quanh vị trí cân bằng mới với cùng công thức chu kì. **Lưu ý:** chu kì không phụ thuộc biên độ, nhưng tốc độ cực đại $v_0 = \omega x_0$ và gia tốc cực đại $a_0 = \omega^2 x_0$ thì có phụ thuộc biên độ.

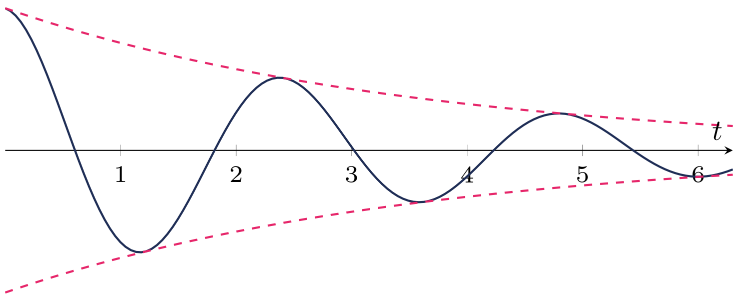
10.7 Damping and Resonance

[A2 – A Level]Real oscillating systems always experience some resistive force (friction, air resistance, etc.) that removes energy from the system – this is called **damping**.

Damping is the progressive loss of energy (and hence amplitude) from an oscillating system due to a resistive force acting on it. Three regimes are distinguished by how strong the damping is:

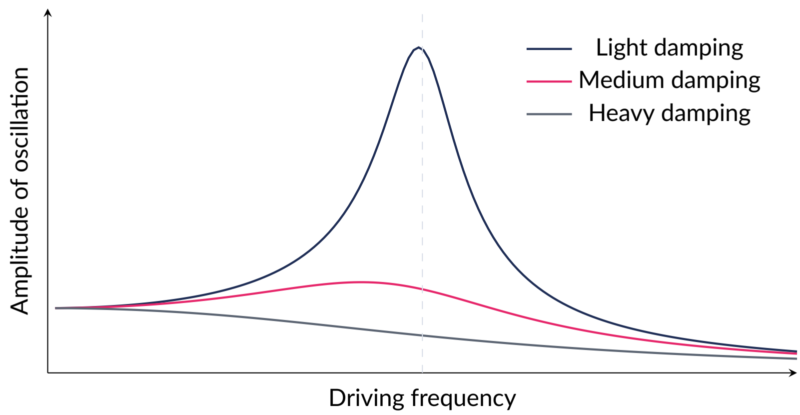
Damping type	Behaviour
Light damping	Amplitude decays gradually (approximately exponentially) over **many** complete oscillations; the system continues to oscillate at close to its natural frequency for a long time before coming to rest.
Critical damping	The system returns to equilibrium in the **shortest possible time**, **without** overshooting or oscillating at all.
Heavy damping (overdamping)	The system returns to equilibrium **without oscillating**, but takes **longer** than the critically damped case to reach equilibrium.

Comparison of the three damping regimes for a displaced oscillator released from rest.



Displacement–time graph for a **lightly damped** oscillator: the oscillation continues at close to the natural frequency, but its amplitude decays within an exponential "envelope" (dashed curves), gradually dying away over many cycles.

Resonance occurs when a system is driven by an external periodic force whose **driving frequency equals the system's own natural frequency of free oscillation**: at this point, energy is transferred into the system most efficiently, and the amplitude of the driven oscillation reaches a **maximum**. The degree of damping present strongly affects the resonance peak: **light damping** produces a **tall, sharp** resonance peak (amplitude rises very high, and only over a narrow range of driving frequencies close to the natural frequency); **heavy damping** produces a **shorter, broader** peak (a smaller maximum amplitude, spread over a wider range of driving frequencies), and also shifts the peak slightly to a lower frequency.



Amplitude of a driven oscillator against driving frequency, for three degrees of damping. All three curves peak close to the natural frequency f_0, but the peak becomes **shorter and broader** (and shifts slightly to lower frequency) as damping increases.

(!) Lỗi thường gặp: Resonance is defined by matching the **driving** frequency to the system's own **natural** frequency – it has nothing to do with matching amplitudes or matching two different systems' natural frequencies to each other. Also, do not confuse "heavy damping" with "critical damping": critical damping is specifically the **fastest non-oscillatory** return to equilibrium, whereas heavy (over-) damping returns to equilibrium **without oscillating but more slowly** than the critical case.

VN GIẢI THÍCH TIẾNG VIỆT

Tắt dần nhẹ: biên độ giảm chậm, hệ vẫn dao động qua nhiều chu kì trước khi dừng hẳn. **Tắt dần tới hạn:** hệ trở về vị trí cân bằng **nhanh nhất** mà **không dao động**. **Tắt dần mạnh:** hệ trở về cân bằng không dao động nhưng **chậm hơn** tắt dần tới hạn. **Cộng hưởng** xảy ra khi tần số của lực cưỡng bức bằng tần số dao động riêng của hệ, khi đó biên độ dao động đạt **cực đại**. Tắt dần **nhẹ** cho đỉnh cộng hưởng **cao và nhọn**; tắt dần **mạnh** cho đỉnh **thấp và tù** hơn (và hơi lệch về tần số thấp hơn). Ứng dụng thực tế: hệ thống giảm xóc ô tô thường dùng tắt dần mạnh để tránh xóc nảy liên tục sau va chạm với mặt đường gồ ghề, dù phải hi sinh tốc độ trở về vị trí cân bằng.

Practice Bank

Bài 1 · MCQ — Kelvin scale

A temperature of $27\,^\circ$C corresponds to:

(A) 27 K
(B) 246 K
(C) 300 K
(D) 373 K

Lời giải chi tiết

(C). $T = \theta + 273 = 27 + 273 = 300$ K.

Bài 2 · Specific heat capacity: heating a metal block

A 0.50 kg block of metal, specific heat capacity $c = 450$ J kg^{-1} K^{-1}, is heated from $18\,^\circ$C to $98\,^\circ$C. Calculate the heat energy required.

Lời giải chi tiết

$$Q = mc\Delta\theta = 0.50 \times 450 \times (98 - 18) = 0.50 \times 450 \times 80 = \boxed{1.80 \times 10^4\ \text{J}}.$$

Bài 3 · Constant volume: pressure and temperature

A fixed mass of gas at constant volume has its temperature increased from 300 K to 600 K. What happens to its pressure?

Lời giải chi tiết

At constant volume, $p \propto T$, so doubling T (from 300 K to 600 K) **doubles the pressure**.

Bài 4 · First law: heat and work

An ideal gas absorbs $500\,\text{J}$ of heat and does $150\,\text{J}$ of work on its surroundings as it expands. Calculate the change in internal energy of the gas.

Lời giải chi tiết

Heat supplied to gas $q = +500\,\text{J}$. Work done *by* the gas on the surroundings is $150\,\text{J}$, so work done *on* the gas is $w = -150\,\text{J}$.

$$\Delta U = q + w = 500 + (-150) = \boxed{+350\,\text{J}}.$$

Bài 5 · Latent heat: melting ice with an electric heater

A $2.0\,\text{kW}$ heater is used to melt $1.2\,\text{kg}$ of ice already at $0\,^\circ\text{C}$. Taking the specific latent heat of fusion of ice as $L_f = 3.34 \times 10^5\,\text{J kg}^{-1}$, find the time taken (assume all heat goes into melting).

Lời giải chi tiết

$$Q = mL_f = 1.2 \times 3.34 \times 10^5 = 4.008 \times 10^5\,\text{J}.$$

$$t = \frac{Q}{P} = \frac{4.008 \times 10^5}{2000} = \boxed{200\,\text{s}}.$$

Bài 6 · RMS speed of helium at a given temperature

Estimate the root-mean-square speed of a helium atom (mass $m = 6.65 \times 10^{-27}\,\text{kg}$) at $T = 300\,\text{K}$, using $\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$ and $k = 1.38 \times 10^{-23}\,\text{J K}^{-1}$.

Lời giải chi tiết

$$\langle c^2\rangle = \frac{3kT}{m} = \frac{3 \times 1.38 \times 10^{-23} \times 300}{6.65 \times 10^{-27}} = \frac{1.242 \times 10^{-20}}{6.65 \times 10^{-27}} = 1.868 \times 10^6\,\text{m}^2\text{s}^{-2}.$$

$$c_{\text{rms}} = \sqrt{1.868 \times 10^6} = \boxed{1370\,\text{m s}^{-1}}\ (\approx 1.37 \times 10^3\,\text{m s}^{-1}).$$

Bài 7 · MCQ — Latent heat of vaporisation vs fusion

The specific latent heat of vaporisation of a substance is always greater than its specific latent heat of fusion because:

(A) Vaporisation always happens at a higher temperature

(B) Vaporisation must completely separate molecules against all intermolecular forces, whereas fusion only breaks the ordered solid structure

(C) Fusion releases energy while vaporisation absorbs it

(D) Vaporisation involves a larger mass of substance

Lời giải chi tiết

(B). Melting only needs to disrupt the fixed, ordered arrangement of a solid (molecules stay close together); boiling must pull molecules completely apart against all the attractive forces between them.

Bài 8 · Work done by an expanding gas, and heat supplied

A gas at constant pressure $p = 1.5 \times 10^5$ Pa expands from $V_1 = 2.0 \times 10^{-3}$ m^3 to $V_2 = 2.8 \times 10^{-3}$ m^3. If the internal energy of the gas increases by 85 J during this expansion, find the heat supplied to the gas.

Lời giải chi tiết

$$\Delta V = 2.8 \times 10^{-3} - 2.0 \times 10^{-3} = 8.0 \times 10^{-4}\ \text{m}^3.$$

$$W_{\text{by gas}} = p\Delta V = 1.5 \times 10^5 \times 8.0 \times 10^{-4} = 120\ \text{J}.$$

Work done *on* gas $w = -120$ J. Using $\Delta U = q + w$:

$$85 = q + (-120) \ \Rightarrow\ q = \boxed{205\ \text{J}}.$$

Bài 9 · MCQ — Isothermal vs adiabatic

A change in which no heat enters or leaves a gas is called:

(A) Isothermal
(B) Isobaric
(C) Adiabatic
(D) Isochoric

Lời giải chi tiết

(C) Adiabatic. (Isothermal means constant temperature; isobaric means constant pressure; isochoric means constant volume.)

Bài 10 · Ideal gas law: finding volume

A gas sample contains $n = 2.50$ mol at pressure $p = 2.00 \times 10^5$ Pa and temperature $T = 310$ K. Using $R = 8.31$ J mol^{-1} K^{-1}, find its volume.

Lời giải chi tiết

$$V = \frac{nRT}{p} = \frac{2.50 \times 8.31 \times 310}{2.00 \times 10^5} = \frac{6440.25}{2.00 \times 10^5} = \boxed{3.22 \times 10^{-2}\ \text{m}^3}.$$

Bài 11 · Combined gas law

A gas occupies 3.0 dm^3 at $p_1 = 1.2 \times 10^5$ Pa and $T_1 = 280$ K. It is compressed to 2.0 dm^3 and heated to $T_2 = 350$ K. Find the new pressure p_2.

Lời giải chi tiết

$$p_2 = \frac{p_1 V_1 T_2}{T_1 V_2} = \frac{(1.2 \times 10^5)(3.0)(350)}{(280)(2.0)} = \frac{1.26 \times 10^8}{560} = \boxed{2.25 \times 10^5\ \text{Pa}}.$$

Bài 12 · MCQ — Kinetic theory assumption

Which of the following is **not** an assumption of the kinetic theory of gases?

(A) Collisions between molecules are perfectly elastic

(B) Intermolecular forces are negligible except during collisions

(C) Molecules have significant volume compared with the container

(D) The time of collision is negligible compared with the time between collisions

Lời giải chi tiết

(C). Kinetic theory assumes the molecules' own volume is **negligible** compared with the container volume — the opposite of this statement.

Bài 13 · Average molecular kinetic energy

Calculate the average translational kinetic energy of a gas molecule at $T = 400$ K.

Lời giải chi tiết

$$\langle E_k \rangle = \tfrac{3}{2}kT = 1.5 \times 1.38 \times 10^{-23} \times 400 = \boxed{8.28 \times 10^{-21}\ \text{J}}.$$

Bài 14 · Finding N from p, V, T

A container holds gas at $p = 1.5 \times 10^5$ Pa, $V = 2.0 \times 10^{-3}$ m^3, $T = 320$ K. Find the number of molecules present.

Lời giải chi tiết

$$N = \frac{pV}{kT} = \frac{(1.5 \times 10^5)(2.0 \times 10^{-3})}{(1.38 \times 10^{-23})(320)} = \frac{300}{4.416 \times 10^{-21}} = \boxed{6.79 \times 10^{22}}.$$

Bài 15 · Finding number of moles from p, V, T

A gas sample has $p = 1.80 \times 10^5$ Pa, $V = 5.00 \times 10^{-3}$ m^3, $T = 250$ K. Find the number of moles.

Lời giải chi tiết

$$n = \frac{pV}{RT} = \frac{(1.80 \times 10^5)(5.00 \times 10^{-3})}{8.31 \times 250} = \frac{900}{2077.5} = \boxed{0.433\ \text{mol}}.$$

Bài 16 · MCQ — Meaning of temperature

Physically, thermodynamic temperature is a measure of:

(A) Total internal energy of a sample

(B) Average kinetic energy per molecule

(C) Total kinetic energy of all molecules in a sample

(D) The volume occupied by a gas

Lời giải chi tiết

(B). $\langle E_k \rangle = \frac{3}{2}kT$ shows that (thermodynamic) temperature measures average kinetic energy *per molecule*, not total energy (which also depends on the number of molecules present).

Bài 17 · SHM: maximum speed from amplitude and period

An oscillator has amplitude 4.0 cm and period $T = 2.0$ s. Calculate its maximum speed.

Lời giải chi tiết

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2.0} = 3.14\,\text{rad s}^{-1}.$$

$$v_0 = \omega x_0 = 3.14 \times 0.040 = \boxed{0.126\ \text{m s}^{-1}}.$$

Bài 18 · Displacement at which Ek = Ep

For an SHM oscillator with amplitude x_0, find the displacement x (in terms of x_0) at which kinetic energy equals potential energy.

Lời giải chi tiết

$E_k = E_p \Rightarrow \frac{1}{2}m\omega^2(x_0^2 - x^2) = \frac{1}{2}m\omega^2 x^2 \Rightarrow x_0^2 - x^2 = x^2 \Rightarrow x_0^2 = 2x^2.$

$$x = \boxed{\frac{x_0}{\sqrt{2}}}\ (\approx 0.707\,x_0).$$

Bài 19 · Pendulum period on the Moon

A simple pendulum has period $T = 1.90$ s on Earth ($g = 9.81\ \text{m s}^{-2}$). Find its period on the Moon, where $g_{\text{moon}} = 1.6\ \text{N kg}^{-1}$.

Lời giải chi tiết

Since $T \propto 1/\sqrt{g}$ for fixed L:

$$T_{\text{moon}} = T_{\text{Earth}}\sqrt{\frac{g_{\text{Earth}}}{g_{\text{moon}}}} = 1.90 \times \sqrt{\frac{9.81}{1.6}} = 1.90 \times \sqrt{6.13} = 1.90 \times 2.476 = \boxed{4.70\ \text{s}}.$$

Bài 20 · MCQ — Car suspension damping

A car's suspension system is deliberately designed with heavy damping. This is because heavy damping:

(A) Maximises the amplitude of bouncing after a bump

(B) Makes the car oscillate for as long as possible

(C) Brings the car body back to its normal position without repeated bouncing over bumps

(D) Increases the natural frequency of the suspension

Lời giải chi tiết

(C). Heavy damping ensures that after hitting a bump, the car body returns to its equilibrium position without oscillating repeatedly (which would be uncomfortable and reduce control), even though this is slightly slower than critical damping would be.

Bài 21 · Phase difference from time delay

Two identical oscillators have period $T = 4.0\,\text{s}$. Oscillator B reaches maximum positive displacement $1.0\,\text{s}$ after oscillator A. Find the phase difference between them, in radians.

Lời giải chi tiết

Fraction of a cycle: $1.0/4.0 = 0.25$.

$$\Delta\phi = 0.25 \times 2\pi = \boxed{\tfrac{\pi}{2}\ \text{rad}}\ (1.57\,\text{rad}).$$

Bài 22 · Energy method: finding amplitude from total energy

An SHM oscillator of mass $m = 0.30\,\text{kg}$ and angular frequency $\omega = 10.0\,\text{rad}\,\text{s}^{-1}$ has total energy $E_{\text{total}} = 6.0 \times 10^{-2}\,\text{J}$. Find its amplitude x_0.

Lời giải chi tiết

$$E_{\text{total}} = \tfrac{1}{2}m\omega^2 x_0^2 \;\Rightarrow\; x_0^2 = \frac{2E_{\text{total}}}{m\omega^2} = \frac{2 \times 6.0 \times 10^{-2}}{0.30 \times (10.0)^2} = \frac{0.12}{30} = 4.0 \times 10^{-3}\,\text{m}^2.$$

$$x_0 = \sqrt{4.0 \times 10^{-3}} = \boxed{0.0632\ \text{m}}\ (\approx 6.3\,\text{cm}).$$

Bài 23 · Vertical spring: extension at equilibrium

A mass of $0.80\,\text{kg}$ hangs at rest from a vertical spring of spring constant $k = 50\,\text{N}\,\text{m}^{-1}$. Taking $g = 9.81\,\text{m}\,\text{s}^{-2}$, find the extension of the spring at equilibrium.

Lời giải chi tiết

$$e = \frac{mg}{k} = \frac{0.80 \times 9.81}{50} = \frac{7.848}{50} = \boxed{0.157\,\text{m}}\ (\approx 15.7\,\text{cm}).$$

Bài 24 · Vertical spring: period

For the mass and spring in the previous question ($m = 0.80\,\text{kg}$, $k = 50\,\text{N}\,\text{m}^{-1}$), find the period of vertical oscillation if the mass is displaced slightly from equilibrium and released.

Lời giải chi tiết

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.80}{50}} = 2\pi\sqrt{0.0160} = 2\pi \times 0.1265 = \boxed{0.795\ \text{s}}.$$

Bài 25 · MCQ — Effect of amplitude on period

Doubling the amplitude of a mass-spring SHM oscillator (while keeping m and k fixed) causes the period to:

(A) Double

(B) Halve

(C) Increase by a factor of $\sqrt{2}$

(D) Stay the same

Lời giải chi tiết

(D). The period of SHM, $T = 2\pi\sqrt{m/k}$, does not depend on amplitude at all.

Bài 26 · Damping type identification

A displaced oscillator is released and returns to its equilibrium position in the shortest possible time, without ever crossing back through equilibrium or oscillating. Identify the type of damping.

Lời giải chi tiết

This describes **critical damping** – the fastest possible return to equilibrium without any oscillation.

Bài 27 · Resonance peak comparison

Two identical oscillators are driven by an external periodic force at a range of frequencies near their natural frequency. Oscillator A is lightly damped; oscillator B is heavily damped. Describe two differences between their resonance curves.

Lời giải chi tiết

(1) Oscillator A (light damping) reaches a much **greater maximum amplitude** at resonance than oscillator B (heavy damping). (2) Oscillator A's resonance peak is much **narrower/sharper** (large amplitude only very close to the natural frequency), while oscillator B's peak is **broader/flatter**, spread over a wider range of driving frequencies (and B's peak is also shifted slightly towards a lower frequency than A's).

Bài 28 · Energy method: finding mass from energy, frequency and amplitude

An SHM oscillator has amplitude $x_0 = 0.080$ m, frequency $f = 1.5$ Hz, and total energy $E_{\text{total}} = 0.90$ J. Find its mass m.

Lời giải chi tiết

$$\omega = 2\pi f = 2\pi \times 1.5 = 9.42\,\text{rad s}^{-1}, \qquad \omega^2 = 88.8\,\text{s}^{-2}.$$

$$E_{\text{total}} = \tfrac{1}{2}m\omega^2 x_0^2 \;\Rightarrow\; m = \frac{2E_{\text{total}}}{\omega^2 x_0^2} = \frac{2 \times 0.90}{88.8 \times (0.080)^2} = \frac{1.80}{88.8 \times 6.4 \times 10^{-3}} = \frac{1.80}{0.568} = \boxed{3.17\text{ kg}}.$$

Bài 29 · Combined SHM problem: speed and kinetic energy

A mass of 0.25 kg oscillates with SHM, amplitude 6.0 cm and period 1.2 s. Find (a) the maximum speed, (b) the kinetic energy at $x = 4.0$ cm, using energy conservation.

Lời giải chi tiết

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{1.2} = 5.236\,\text{rad s}^{-1}, \qquad \omega^2 = 27.4\,\text{s}^{-2}.$$

(a)

$$v_0 = \omega x_0 = 5.236 \times 0.060 = \boxed{0.314\text{ m s}^{-1}}.$$

(b)

$$E_k = \tfrac{1}{2}m\omega^2(x_0^2 - x^2) = 0.5 \times 0.25 \times 27.4 \times \left[(0.060)^2 - (0.040)^2\right].$$

$(0.060)^2 - (0.040)^2 = 3.6 \times 10^{-3} - 1.6 \times 10^{-3} = 2.0 \times 10^{-3}\,\text{m}^2$.

$$E_k = 0.5 \times 0.25 \times 27.4 \times 2.0 \times 10^{-3} = \boxed{6.85 \times 10^{-3}\,\text{J}}\ (\approx 6.9\,\text{mJ}).$$

Bài 30 · Latent heat and specific heat combined

A 0.30 kg block of ice at $0\,^\circ$C is to be completely melted and then the resulting water heated to $40\,^\circ$C. Using $L_f = 3.34 \times 10^5$ J kg^{-1} and $c_{\text{water}} = 4200$ J kg^{-1} K^{-1}, calculate the total energy required.

Lời giải chi tiết

Melting:

$$Q_1 = mL_f = 0.30 \times 3.34 \times 10^5 = 1.002 \times 10^5\,\text{J}.$$

Heating the water from $0\,^\circ$C to $40\,^\circ$C:

$$Q_2 = mc\Delta\theta = 0.30 \times 4200 \times 40 = 5.04 \times 10^4\,\text{J}.$$

Total:

$$Q = Q_1 + Q_2 = 1.002 \times 10^5 + 0.504 \times 10^5 = \boxed{1.506 \times 10^5\,\text{J}}\ (\approx 151\,\text{kJ}).$$

Bài 31 · MCQ — First law sign convention

A gas is compressed, and its surroundings do 80 J of work on it, while the gas releases 30 J of heat to its surroundings. The change in internal energy of the gas is:

(A) $+50$ J

(B) -50 J

(C) $+110$ J

(D) -110 J

Lời giải chi tiết

(A). Work done *on* the gas $w = +80$ J; heat leaving the gas means $q = -30$ J.

$$\Delta U = q + w = -30 + 80 = \boxed{+50\,\text{J}}.$$

Bài 32 · Extended problem: pendulum period, energy and gas law combined

A simple pendulum of length $L = 1.20$ m swings with small amplitude on Earth ($g = 9.81$ m s^{-2}). (a) Find its period. (b) A sealed gas syringe of fixed volume $V = 4.00 \times 10^{-4}$ m^3 contains $n = 0.0160$ mol of gas at the same temperature as the room, $T = 293$ K. Using $R = 8.31$ J mol^{-1} K^{-1}, find the pressure of the gas.

Lời giải chi tiết

(a)

$$T_{\text{pendulum}} = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{1.20}{9.81}} = 2\pi\sqrt{0.1223} = 2\pi \times 0.3497 = \boxed{2.20\,\text{s}}.$$

(b)

$$p = \frac{nRT}{V} = \frac{0.0160 \times 8.31 \times 293}{4.00 \times 10^{-4}} = \frac{38.96}{4.00 \times 10^{-4}} = \boxed{9.74 \times 10^{4}\ \text{Pa}}\ (\approx 0.96\,\text{atm}).$$

Electric Fields, Capacitance, Magnetic Fields & Alternating Current

[A2 – A Level only, not in the AS syllabus]

Mục tiêu Unit: Học sinh nắm vững khái niệm điện trường (cường độ điện trường, điện thế, nguyên lý chồng chất của hai điện tích điểm, thế năng tương tác và công dịch chuyển điện tích); tụ điện (điện dung, năng lượng tích trữ, ghép nối tiếp/song song, tụ phẳng, nạp và xả qua điện trở); từ trường và cảm ứng điện từ (lực từ lên dòng điện và điện tích chuyển động, chuyển động tròn, hiệu ứng Hall, định luật Faraday và Lenz, máy phát điện xoay chiều, máy biến áp và hiệu suất); và dòng điện xoay chiều (giá trị hiệu dụng, công suất trung bình, chỉnh lưu nửa chu kỳ và toàn chu kỳ) theo đúng chuẩn đầu ra của Cambridge International AS & A Level Physics 9702.

11.1 Electric Fields

11.1.1 Field strength

An electric field is a region of space in which a charge experiences a force. The electric field strength E at a point is defined as the force per unit positive charge acting at that point:

$$E = \frac{F}{Q}$$

where F is the force (in newtons) on a small positive test charge Q (in coulombs) placed at that point. E is a vector quantity with units $\mathrm{N\,C^{-1}}$, equivalent to $\mathrm{V\,m^{-1}}$.

Electric field strength: $E = F/Q$.

Field of an isolated point charge: $E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$, directed radially outward from a positive charge (or radially inward toward a negative charge).

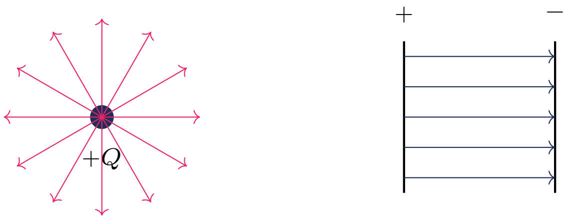
Electric potential of a point charge: $V = \dfrac{Q}{4\pi\varepsilon_0 r}$ (a scalar, taking $V = 0$ at infinity).

Uniform field between parallel plates: $E = \dfrac{V}{d}$, where V is the potential difference between the plates and d is their separation.

$\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F\,m^{-1}}$ is the permittivity of free space, and $\dfrac{1}{4\pi\varepsilon_0} = 8.99 \times 10^{9}\ \mathrm{N\,m^2\,C^{-2}}$.

VN **GIẢI THÍCH TIẾNG VIỆT**

Cường độ điện trường E tại một điểm bằng lực tác dụng lên một đơn vị điện tích dương đặt tại điểm đó. Xung quanh một điện tích điểm, điện trường giảm theo bình phương khoảng cách (tỉ lệ nghịch với r^2), còn điện thế giảm tỉ lệ nghịch với r. Giữa hai bản phẳng song song tích điện trái dấu, điện trường là đều và bằng $E = V/d$.



radial field of a point charge   uniform field between plates, $E = V/d$

Figure 1. Left: the field of an isolated point charge is radial and its magnitude falls as $1/r^2$. Right: between oppositely charged parallel plates, the field is uniform in magnitude and direction, with $E = V/d$.

Ví dụ mẫu · Điểm điện trường và bản phẳng

(a) Two parallel plates are separated by $d = 5.0$ mm with a potential difference $V = 200$ V across them. Find the electric field strength between the plates.
Solution. $E = V/d = 200/(5.0 \times 10^{-3}) = 4.0 \times 10^4$ V m^{-1}.
(b) A point charge $Q = 2.0$ nC is fixed in space. Find the electric field strength and the potential at a point $r = 0.30$ m from the charge.
Solution.

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{8.99 \times 10^9 \times 2.0 \times 10^{-9}}{(0.30)^2} \approx 2.0 \times 10^2 \text{ N C}^{-1}$$

$$V = \frac{Q}{4\pi\varepsilon_0 r} = \frac{8.99 \times 10^9 \times 2.0 \times 10^{-9}}{0.30} \approx 60 \text{ V}$$

11.1.2 Superposition: fields and potentials due to two point charges

When more than one charge is present, the resultant field at a point is found by **vector addition** of the individual field vectors (each contributing $Q_i/4\pi\varepsilon_0 r_i^2$ in its own direction), while the resultant potential is found by **scalar addition** of the individual potentials (each contributing $Q_i/4\pi\varepsilon_0 r_i$, with sign taken into account).

Nguyên lý chồng chất

Field superposition is a vector sum – direction matters. Potential superposition is a scalar sum – only magnitude and sign matter, which makes potential calculations for multiple charges considerably simpler than field calculations.

A useful application is finding the **null point**: the point (or points) on the line joining two charges where the resultant field is zero. For two charges of the *same* sign, this point lies *between* them (since the two field contributions point in opposite directions there). For two charges of *opposite* sign, the fields between them point the same way and never cancel; the null point instead lies *outside* the pair, on the side of the smaller-magnitude charge.

Ví dụ mẫu · Điểm triệt tiêu điện trường

Charges $Q_1 = +4.0$ nC and $Q_2 = +1.0$ nC are fixed 0.60 m apart. Find the position of the null point on the line joining them.
Solution. Let x be the distance from Q_1 to the null point, so $(0.60 - x)$ is the distance from Q_2. Since both charges are positive, the null point lies between them, where the two fields (pointing away from each charge) are equal in magnitude and opposite in direction:

$$\frac{Q_1}{x^2} = \frac{Q_2}{(0.60 - x)^2} \implies \frac{4.0}{x^2} = \frac{1.0}{(0.60 - x)^2} \implies \frac{2}{x} = \frac{1}{0.60 - x}$$

$$2(0.60 - x) = x \implies 1.20 = 3x \implies x = 0.40 \text{ m}$$

The null point is 0.40 m from the 4.0 nC charge (and 0.20 m from the 1.0 nC charge) – closer to the smaller charge, as expected. *Check:* $E_1 = 8.99 \times 10^9 \times 4.0 \times 10^{-9}/0.16 \approx 225$ N C^{-1} and $E_2 = 8.99 \times 10^9 \times 1.0 \times 10^{-9}/0.04 \approx 225$ N C^{-1} – equal, confirming the result.

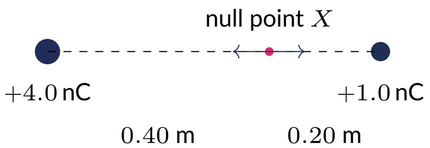


Figure 2. The null point between two like charges lies closer to the smaller charge; the two field contributions there are equal and opposite.

11.1.3 Potential energy and work done in electric fields

The electric potential energy of a system of two point charges Q and q separated by distance r is

$$E_p = \frac{Qq}{4\pi\varepsilon_0 r}$$

This is positive for like charges (energy must be supplied to push them together against repulsion) and negative for unlike charges (energy is released as they are drawn together). $E_p \to 0$ as $r \to \infty$.

When a charge Q moves between two points at different potentials V_1 and V_2, the work done on (or by) the charge is

$$W = Q(V_1 - V_2) = Q\,\Delta V$$

Potential energy of two point charges: $E_p = \dfrac{Qq}{4\pi\varepsilon_0 r}$.
Work done moving charge Q through a potential difference ΔV: $W = Q\,\Delta V$.

Ví dụ mẫu · Thế năng và công dịch chuyển điện tích

(a) Charges $Q_1 = +3.0$ nC and $Q_2 = -2.0$ nC are separated by $r = 0.50$ m. Find the potential energy of the system.
Solution.

$$E_p = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r} = \frac{8.99 \times 10^9 \times (3.0 \times 10^{-9}) \times (-2.0 \times 10^{-9})}{0.50} \approx -1.08 \times 10^{-7} \text{ J}$$

The negative sign shows the pair is bound: work would have to be done to separate them to infinity.
(b) A fixed point charge $Q = +5.0$ nC produces potentials of $V_A \approx 225$ V at point A ($r_A = 0.20$ m) and $V_B \approx 89.9$ V at point B ($r_B = 0.50$ m) (both found from $V = Q/4\pi\varepsilon_0 r$, as in Section 11.1). A test charge $q = +2.0$ nC is moved from B to A. Find the work done.
Solution.

$$W = q(V_A - V_B) = 2.0 \times 10^{-9} \times (225 - 89.9) \approx 2.70 \times 10^{-7} \text{ J}$$

Since q is positive and is being moved to a point of higher potential (closer to the repelling charge Q), positive work must be done *against* the field – consistent with the positive result.

11.1.4 Comparing electric fields and gravitational fields

Both electric and gravitational fields are inverse-square fields with strikingly similar mathematics, but they differ in one crucial respect: gravitational force is always attractive (mass has only one "sign"), while electric force can be attractive or repulsive, since charge can be positive or negative.

Property	Electric field	Gravitational field
Source	Charge Q	Mass M
Force between two sources	$F = \dfrac{Qq}{4\pi\varepsilon_0 r^2}$	$F = \dfrac{GMm}{r^2}$
Field strength	$E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$	$g = \dfrac{GM}{r^2}$
Potential	$V = \dfrac{Q}{4\pi\varepsilon_0 r}$	$\phi = -\dfrac{GM}{r}$
Constant	$\dfrac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9$ N m^2 C^{-2}	$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}
Nature of force	Attractive *or* repulsive	Always attractive
Both fields	$\propto 1/r^2$ (inverse square)	$\propto 1/r^2$ (inverse square)

Figure 3. Electric fields and gravitational fields obey mathematically identical inverse-square laws, but gravitational force has only one sign (always attractive) while electric force can be attractive or repulsive depending on the signs of the charges.

(!) Lỗi thường gặp: Do not assume gravitational potential ϕ is always positive by analogy with electric potential of a positive charge. Because mass is always "positive" but the gravitational force is always attractive, gravitational potential is defined as *negative* everywhere ($\phi = -GM/r$), rising toward zero at infinity. Electric potential, by contrast, can be positive (near positive charge) or negative (near negative charge).

11.2 Capacitance

11.2.1 Capacitance and energy stored

The capacitance of a capacitor is the charge stored per unit potential difference across it:

$$C = \frac{Q}{V}$$

with units of farads (F = C V^{-1}). As a capacitor charges, the p.d. across it rises from 0 to V; the work done in charging it equals the area under a graph of Q against V (a straight line through the origin, so the area is a triangle of base V and height Q):

$$\text{Energy stored} = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \frac{Q^2}{2C}$$

$C = Q/V$. Energy stored: $W = \frac{1}{2}QV = \frac{1}{2}CV^2 = \dfrac{Q^2}{2C}$ (the area under the Q–V graph, a right triangle).

VN **GIẢI THÍCH TIẾNG VIỆT**

Điện dung $C = Q/V$ cho biết khả năng tích điện của tụ ứng với mỗi đơn vị hiệu điện thế. Năng lượng tích trữ trong tụ điện chính là diện tích tam giác dưới đồ thị Q theo V, bằng $\frac{1}{2}QV$, khác với công thức $W = QV$ (công thức này chỉ đúng cho nguồn có hiệu điện thế không đổi, không đúng cho tụ điện vì V tăng dần từ 0 trong quá trình nạp).

Ví dụ mẫu · Điện tích và năng lượng của tụ

A capacitor $C = 2.0\,\mu$F is charged to $V = 50\,$V. Find the charge stored and the energy stored.
Solution.

$$Q = CV = 2.0 \times 10^{-6} \times 50 = 1.0 \times 10^{-4}\text{ C} = 100\ \mu\text{C}$$

$$W = \tfrac{1}{2}QV = \tfrac{1}{2} \times 1.0 \times 10^{-4} \times 50 = 2.5 \times 10^{-3}\text{ J} = 2.5\text{ mJ}$$

11.2.2 Capacitors in series and parallel

Capacitor combination rules are the *opposite* pattern to resistor combination rules:

Series: $\dfrac{1}{C_{\text{total}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \cdots$ (same charge Q on each capacitor; p.d.s add).
Parallel: $C_{\text{total}} = C_1 + C_2 + \cdots$ (same p.d. across each capacitor; charges add).

Ví dụ mẫu · Ghép tụ điện

Capacitors of $4.0\,\mu$F and $12.0\,\mu$F are connected (a) in series and (b) in parallel. Find the combined capacitance in each case.
Solution. (a) Series: $\dfrac{1}{C} = \dfrac{1}{4.0} + \dfrac{1}{12.0} = \dfrac{3}{12} + \dfrac{1}{12} = \dfrac{4}{12}$, so $C = 3.0\ \mu$F.
(b) Parallel: $C = 4.0 + 12.0 = 16.0\ \mu$F.

11.2.3 The parallel-plate capacitor

For a parallel-plate capacitor with plate area A and separation d in vacuum (or air),

$$C = \frac{\varepsilon_0 A}{d}$$

Inserting an insulating dielectric between the plates increases the capacitance, but a full quantitative treatment of relative permittivity is beyond this section; here we use the vacuum/air formula only.

Ví dụ mẫu · Tụ phẳng

A parallel-plate capacitor has circular plates of radius $r = 5.0\,$cm separated by $d = 2.0\,$mm in air. Find its capacitance.
Solution.

$$A = \pi r^2 = \pi \times (0.050)^2 = 7.85 \times 10^{-3}\text{ m}^2$$

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 7.85 \times 10^{-3}}{2.0 \times 10^{-3}} \approx 3.5 \times 10^{-11}\text{ F} = 35\text{ pF}$$

11.2.4 Charging and discharging capacitors

When a charged capacitor discharges through a fixed resistor R, the charge decays exponentially:

$$Q = Q_0 e^{-t/RC}, \qquad \tau = RC$$

where τ (the time constant) is the time for the charge to fall to $1/e \approx 37\%$ of its initial value. Current and voltage decay with the same exponential form.

When a capacitor charges through a resistor from a source of e.m.f. (or steady p.d.) ε, the charge rises according to

$$Q = Q_0\left(1 - e^{-t/RC}\right), \qquad Q_0 = C\varepsilon$$

reaching 63% of its final value after one time constant.

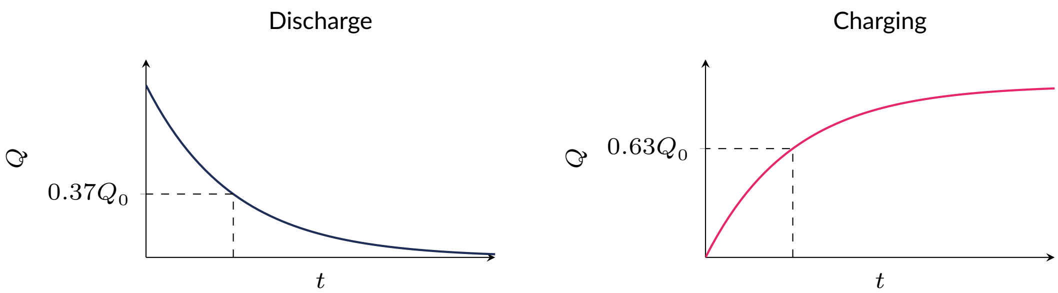


Figure 4. Left: exponential discharge, $Q = Q_0e^{-t/RC}$, falling to 37% after one time constant. Right: exponential charging, $Q = Q_0(1 - e^{-t/RC})$, rising to 63% of the final charge after one time constant.

Ví dụ mẫu · Nạp và xả tụ điện

(a) A capacitor $C = 100\,\mu$F discharges through $R = 2.0\,\text{k}\Omega$. Find the time constant.
Solution. $\tau = RC = 2.0 \times 10^3 \times 100 \times 10^{-6} = 0.20$ s.
(b) A capacitor $C = 470\,\mu$F charges through $R = 1.0\,\text{k}\Omega$ from a 6.0 V supply. Find the charge on the capacitor after $t = 0.5$ s.
Solution. $\tau = RC = 1.0 \times 10^3 \times 470 \times 10^{-6} = 0.47$ s. $Q_0 = C\varepsilon = 470 \times 10^{-6} \times 6.0 = 2.82 \times 10^{-3}$ C.

$$Q = Q_0\left(1 - e^{-t/\tau}\right) = 2.82\times10^{-3}\times\left(1 - e^{-0.5/0.47}\right) = 2.82\times10^{-3}\times(1-0.345) \approx 1.85\times10^{-3}\text{ C} = 1.85\text{ mC}$$

11.3 Magnetic Fields and Electromagnetic Induction

11.3.1 Force on a current-carrying conductor and on a moving charge

A current-carrying conductor in a magnetic field experiences a force given by

$$F = BIL\sin\theta$$

where B is the magnetic flux density, I the current, L the length of conductor in the field, and θ the angle between the conductor and the field. The direction is given by Fleming's left-hand rule.

A single moving charge experiences an analogous force:

$$F = BQv\sin\theta$$

where v is the speed of the charge and θ the angle between its velocity and B. When $\theta = 90^\circ$ (velocity perpendicular to a uniform field), this force is always perpendicular to v, so it provides centripetal force for uniform circular motion:

$$BQv = \frac{mv^2}{r} \implies r = \frac{mv}{BQ}$$

Force on current: $F = BIL\sin\theta$. Force on moving charge: $F = BQv\sin\theta$. Radius of circular path in a uniform field (velocity perpendicular to B): $r = \dfrac{mv}{BQ}$.

Ví dụ mẫu · Lực từ

(a) A straight wire of length $L = 0.20$ m carries a current $I = 3.0$ A perpendicular to a field $B = 0.50$ T. Find the force on the wire.
Solution. $F = BIL = 0.50 \times 3.0 \times 0.20 = 0.30$ N.

(b) An electron travels at $v = 2.0 \times 10^6$ m s^{-1} perpendicular to a field $B = 0.20$ T. Find the force on it and the radius of its circular path.
Solution.

$$F = Bqv = 0.20 \times 1.6 \times 10^{-19} \times 2.0 \times 10^6 = 6.4 \times 10^{-14}\ \text{N}$$

$$r = \frac{mv}{Bq} = \frac{9.11 \times 10^{-31} \times 2.0 \times 10^6}{0.20 \times 1.6 \times 10^{-19}} \approx 5.7 \times 10^{-5}\ \text{m}$$

(!) Lỗi thường gặp: $F = BIL\sin\theta$ and $F = BQv\sin\theta$ both use the angle between the current (or velocity) and the field direction, *not* the angle between the force and anything. When the conductor or charge moves *parallel* to B ($\theta = 0$), the magnetic force is zero, even though B, I and v are all nonzero.

11.3.2 The Hall effect

When a current-carrying conducting slab is placed in a magnetic field perpendicular to the current, the moving charge carriers are deflected sideways by the magnetic force, building up charge on one face of the slab until an electric field develops that balances the magnetic force – this transverse p.d. is the **Hall voltage**, V_H.

Consider a slab of thickness t (measured along the direction of B) carrying current I, with charge carrier number density n and carrier charge q. The drift velocity is $v = I/(nqA_\perp)$ where $A_\perp = t \times w$ (w = slab width in the direction of V_H). Balancing forces, $qE_H = qvB$, so $E_H = vB$ and $V_H = E_H w = vBw$. Substituting for v:

$$V_H = \frac{BI}{ntq}$$

Hall voltage: $V_H = \dfrac{BI}{ntq}$, where n is the number density of charge carriers, t is the slab thickness measured along B, and q is the charge per carrier. Because n is very large in metals, V_H is typically only a few microvolts; semiconductors (much smaller n) give a far more measurable Hall voltage for the same current and field, which is why Hall probes use semiconducting slabs.

VN **GIẢI THÍCH TIẾNG VIỆT**

Hiệu ứng Hall xảy ra khi một tấm dẫn điện mang dòng điện được đặt trong từ trường vuông góc với dòng điện: lực từ làm lệch hạt tải điện sang một cạnh của tấm, tạo ra hiệu điện thế ngang $V_H = BI/(ntq)$. Vì mật độ hạt tải n trong kim loại rất lớn, V_H trong kim loại rất nhỏ; do đó đầu dò Hall thực tế thường dùng chất bán dẫn có n nhỏ hơn nhiều để thu được tín hiệu đo được rõ ràng hơn.

Ví dụ mẫu · Hiệu ứng Hall

A metal slab of thickness $t = 1.0$ mm carries a current $I = 5.0$ A in a magnetic field $B = 0.60$ T perpendicular to the slab's thickness. The metal has a given charge carrier density $n = 1.0 \times 10^{29}$ m^{-3} and each carrier has charge $q = 1.6 \times 10^{-19}$ C. Find the Hall voltage.
Solution.

$$V_H = \frac{BI}{ntq} = \frac{0.60 \times 5.0}{(1.0 \times 10^{29}) \times (1.0 \times 10^{-3}) \times (1.6 \times 10^{-19})} = \frac{3.0}{1.6 \times 10^{7}} \approx 1.9 \times 10^{-7}\ \text{V} \approx 0.19\ \mu\text{V}$$

This tiny voltage illustrates why metals make poor Hall probes – the large n suppresses V_H.

11.3.3 Electromagnetic induction: Faraday's and Lenz's laws

Magnetic flux is $\Phi = BA$ (for B perpendicular to area A); for a coil of N turns, the *flux linkage* is $N\Phi$. Faraday's law states that the induced e.m.f. equals the rate of change of flux linkage:

$$\varepsilon = -\frac{d(N\Phi)}{dt}$$

Lenz's law (the origin of the minus sign) states that the induced e.m.f. always acts in a direction that opposes the change producing it.

A conductor of length L moving with velocity v perpendicular to a field B generates a *motional e.m.f.*: $\varepsilon = BLv$.

> Faraday's law: $\varepsilon = -\dfrac{d(N\Phi)}{dt}$. Lenz's law: induced e.m.f./current opposes the change that creates it. Motional e.m.f.: $\varepsilon = BLv$.

Ví dụ mẫu · Cảm ứng điện từ

(a) A conducting rod of length $L = 0.50\,\text{m}$ moves at $v = 4.0\,\text{m s}^{-1}$ perpendicular to a field $B = 0.25\,\text{T}$. Find the e.m.f. generated.
Solution. $\varepsilon = BLv = 0.25 \times 0.50 \times 4.0 = 0.50\,\text{V}$.
(b) A coil of $N = 200$ turns, cross-sectional area $A = 0.015\ \text{m}^2$, sits in a field $B = 0.40\,\text{T}$ perpendicular to its plane. It is withdrawn from the field, uniformly, in $0.20\,\text{s}$. Find the average e.m.f. induced.
Solution.

$$\varepsilon = \frac{N\Delta(BA)}{\Delta t} = \frac{200 \times 0.40 \times 0.015}{0.20} = \frac{1.2}{0.20} = 6.0\,\text{V}$$

(!) Lỗi thường gặp: Flux linkage $N\Phi = NBA\cos\theta$ can change because B changes, because A changes, *or* because the angle θ between the coil and the field changes (e.g., rotation) – any one of these (or a combination) induces an e.m.f. Do not assume induction requires a changing field strength alone.

11.3.4 The rotating-coil (a.c.) generator

A coil of N turns and area A rotating at constant angular velocity ω in a uniform field B has flux linkage $N\Phi = NBA\cos(\omega t)$ (taking $t = 0$ when the coil plane is parallel to B). Differentiating,

$$\varepsilon = -\frac{d(N\Phi)}{dt} = BAN\omega\sin(\omega t)$$

so the e.m.f. varies sinusoidally with peak value

$$\varepsilon_0 = BAN\omega$$

This is the principle of the simple a.c. generator (dynamo).

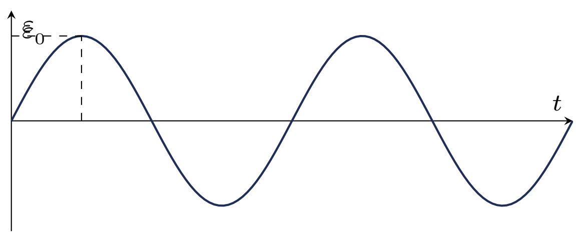


Figure 5. E.m.f. generated by a coil rotating at constant angular velocity in a uniform field: $\varepsilon = \varepsilon_0\sin(\omega t)$, with peak value $\varepsilon_0 = BAN\omega$.

Ví dụ mẫu · Máy phát điện cuộn dây quay

A coil of $N = 100$ turns and area $A = 5.0 \times 10^{-3}$ m^2 rotates at frequency $f = 50$ Hz in a uniform field $B = 0.10$ T. Find (a) the peak e.m.f. generated and (b) the instantaneous e.m.f. when the coil has turned through 60° from the position of zero flux linkage.
Solution. $\omega = 2\pi f = 2\pi \times 50 \approx 314$ rad s^{-1}.

$$\varepsilon_0 = BAN\omega = 0.10 \times 5.0 \times 10^{-3} \times 100 \times 314 \approx 15.7 \text{ V}$$

$$\varepsilon = \varepsilon_0 \sin(\omega t) = 15.7 \times \sin 60^\circ = 15.7 \times 0.866 \approx 13.6 \text{ V}$$

11.4 Alternating Current

11.4.1 RMS values and average power

An alternating current or voltage oscillates sinusoidally, e.g. $I = I_0 \sin(\omega t)$. The root-mean-square (r.m.s.) value is the value of the equivalent direct current/voltage that would dissipate the same average power in a resistor:

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}}, \qquad V_{\text{rms}} = \frac{V_0}{\sqrt{2}}, \qquad P_{\text{avg}} = \tfrac{1}{2} I_0 V_0 = I_{\text{rms}} V_{\text{rms}}$$

$I_{\text{rms}} = I_0/\sqrt{2}$, $V_{\text{rms}} = V_0/\sqrt{2}$, $P_{\text{avg}} = \tfrac{1}{2} I_0 V_0 = I_{\text{rms}} V_{\text{rms}}$.

Ví dụ mẫu · Giá trị hiệu dụng

Domestic mains supply is quoted as $V_{\text{rms}} = 230$ V, and the peak current drawn by an appliance is $I_0 = 5.0$ A. Find the peak voltage, the r.m.s. current, and the average power delivered.
Solution.

$$V_0 = V_{\text{rms}}\sqrt{2} = 230 \times 1.414 \approx 325 \text{ V}$$

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{5.0}{1.414} \approx 3.54 \text{ A}$$

$$P_{\text{avg}} = \tfrac{1}{2} I_0 V_0 = \tfrac{1}{2} \times 5.0 \times 325 \approx 813 \text{ W}$$

11.4.2 Transformers and efficiency

An ideal transformer transfers electrical power from a primary coil to a secondary coil via a changing magnetic flux in a shared iron core. The turns ratio determines the voltage ratio:

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

and for an ideal (100% efficient) transformer, input power equals output power: $I_p V_p = I_s V_s$. Real transformers lose some power (mainly to resistive heating and core losses), so

$$\text{efficiency} = \frac{P_{\text{output}}}{P_{\text{input}}} \times 100\%$$

$V_s/V_p = N_s/N_p$. Ideal transformer: $I_p V_p = I_s V_s$. Efficiency $= \dfrac{P_{\text{out}}}{P_{\text{in}}} \times 100\%$.

Ví dụ mẫu · Máy biến áp và hiệu suất

(a) A step-down transformer has $N_p = 2000$ turns and $N_s = 100$ turns, with $230\,\text{V}$ applied to the primary. Find the secondary voltage.
Solution. $V_s = V_p \times \dfrac{N_s}{N_p} = 230 \times \dfrac{100}{2000} = 11.5\,\text{V}$.
(b) A different transformer draws an input power $P_{\text{in}} = 460\,\text{W}$ (from $V_p = 230\,\text{V}$, $I_p = 2.0\,\text{A}$) and delivers an output of $V_s = 46\,\text{V}$, $I_s = 9.0\,\text{A}$. Find its efficiency.
Solution. $P_{\text{out}} = V_s I_s = 46 \times 9.0 = 414\,\text{W}$.

$$\text{efficiency} = \frac{414}{460} \times 100\% = 90.0\%$$

(The remaining $46\,\text{W}$ is lost, mainly as heat in the windings and core.)

11.4.3 Rectification

Alternating current can be converted to (varying) direct current by **rectification**, using diodes, which conduct in one direction only.

Half-wave rectification uses a single diode, which conducts during one half of each cycle and blocks the other half; the output is a series of one-directional pulses with a "flat zero" gap between them, occurring at the original supply frequency.
Full-wave rectification uses four diodes arranged in a bridge; current is routed through the load in the same direction during both halves of the input cycle, so every half-cycle (positive or negative) produces an output pulse – giving a smoother output at twice the pulse frequency of half-wave rectification, with no dead gaps.

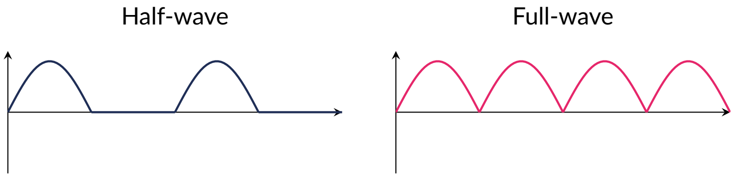


Figure 6. Half-wave rectification (single diode) blocks alternate half-cycles, leaving gaps; full-wave rectification (diode bridge) inverts the negative half-cycles instead of blocking them, giving continuous one-directional pulses.

GIẢI THÍCH TIẾNG VIỆT
VN
Chỉnh lưu nửa chu kỳ dùng một điốt duy nhất, chỉ cho dòng điện đi qua trong một nửa chu kỳ và chặn nửa còn lại, tạo ra các xung có khoảng trống xen giữa. Chỉnh lưu toàn chu kỳ dùng bốn điốt mắc theo kiểu cầu, tận dụng cả hai nửa chu kỳ nên dạng sóng đầu ra liên tục hơn, không có khoảng trống.

11.5 Practice Bank

Bài tập luyện tập

1. Which statement about the electric field of an isolated positive point charge is correct?

(A) The field strength is constant everywhere.

(B) The field strength is proportional to $1/r$.

(C) The field strength is proportional to $1/r^2$ and points radially outward.

(D) The field strength is proportional to $1/r^2$ and points radially inward.

Lời giải chi tiết

Answer: C. $E = Q/4\pi\varepsilon_0 r^2$ is inverse-square, and for a positive charge the field points away from the charge (radially outward).

Bài tập luyện tập

2. Parallel plates are separated by $d = 8.0$ mm with a p.d. $V = 160$ V across them. Find the electric field strength between the plates, and the force on an electron ($q = 1.6 \times 10^{-19}$ C) placed between them.

Lời giải chi tiết

$E = V/d = 160/(8.0 \times 10^{-3}) = 2.0 \times 10^4$ V m^{-1}. $F = qE = 1.6 \times 10^{-19} \times 2.0 \times 10^4 = 3.2 \times 10^{-15}$ N.

Bài tập luyện tập

3. A point charge $Q = +6.0$ nC is fixed in space. Find the electric field strength and the potential at $r = 0.40$ m.

Lời giải chi tiết

$E = \dfrac{8.99 \times 10^9 \times 6.0 \times 10^{-9}}{(0.40)^2} \approx 3.37 \times 10^2$ N C^{-1}. $V = \dfrac{8.99 \times 10^9 \times 6.0 \times 10^{-9}}{0.40} \approx 135$ V.

Bài tập luyện tập

4. Charges $Q_1 = +2.0$ nC and $Q_2 = -8.0$ nC are fixed 0.30 m apart. Find the position of the null point (zero resultant field) on the line joining the charges.

Lời giải chi tiết

Since the charges are opposite in sign, the fields between them point the same way and never cancel; the null point lies outside the pair, beyond the smaller-magnitude charge (Q_1). Let a be the distance from Q_1 to the null point (on the side away from Q_2):

$$\frac{Q_1}{a^2} = \frac{|Q_2|}{(a+0.30)^2} \implies \frac{2.0}{a^2} = \frac{8.0}{(a+0.30)^2} \implies \frac{a+0.30}{a} = 2.0 \implies a = 0.30\text{ m}$$

The null point is 0.30 m from Q_1, on the side opposite to Q_2 (i.e. 0.60 m from Q_2).

Bài tập luyện tập

5. Find the potential energy of a system of two point charges $Q_1 = +4.0$ nC and $Q_2 = +2.0$ nC separated by $r = 0.25$ m.

Lời giải chi tiết

$$E_p = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r} = \frac{8.99 \times 10^9 \times 4.0 \times 10^{-9} \times 2.0 \times 10^{-9}}{0.25} \approx 2.9 \times 10^{-7}\text{ J}$$

Positive, since both charges have the same sign (repulsive interaction).

Bài tập luyện tập

6. A charge $q = +3.0\,\text{nC}$ moves from a point at potential $500\,\text{V}$ to a point at potential $100\,\text{V}$. Find the work done by the electric field on the charge.

Lời giải chi tiết

$$W = q\,\Delta V = 3.0 \times 10^{-9} \times (500 - 100) = 1.2 \times 10^{-6}\ \text{J} = 1.2\ \mu\text{J}$$

Positive, since the field does positive work moving a positive charge to lower potential.

Bài tập luyện tập

7. Which of the following is a correct comparison between electric and gravitational fields?

(A) Both are always attractive.

(B) Both obey an inverse-square law, but gravity is always attractive while electric force may be attractive or repulsive.

(C) Electric potential is always negative, like gravitational potential.

(D) The constant G plays the same numerical role as $1/4\pi\varepsilon_0$.

Lời giải chi tiết

Answer: B. Both fields obey $1/r^2$ laws, but gravitational force (from mass, one sign) is always attractive, whereas electric force (from charge, two signs) can be attractive or repulsive.

Bài tập luyện tập

8. A capacitor $C = 5.0\,\mu\text{F}$ is charged to $V = 12\,\text{V}$. Find the charge stored and the energy stored.

Lời giải chi tiết

$Q = CV = 5.0 \times 10^{-6} \times 12 = 6.0 \times 10^{-5}$ C $= 60\ \mu$C. $W = \frac{1}{2}QV = \frac{1}{2} \times 6.0 \times 10^{-5} \times 12 = 3.6 \times 10^{-4}$ J $= 0.36$ mJ.

Bài tập luyện tập

9. Capacitors of $6.0\,\mu\text{F}$ and $3.0\,\mu\text{F}$ are connected (a) in series, (b) in parallel. Find the total capacitance in each case.

Lời giải chi tiết

(a) $\dfrac{1}{C} = \dfrac{1}{6.0} + \dfrac{1}{3.0} = \dfrac{1}{6} + \dfrac{2}{6} = \dfrac{3}{6}$, so $C = 2.0\ \mu$F. (b) $C = 6.0 + 3.0 = 9.0\ \mu$F.

Bài tập luyện tập

10. A $2.0\,\mu\text{F}$ capacitor is connected in series with a parallel combination of two $4.0\,\mu\text{F}$ capacitors, across a $12\,\text{V}$ supply. Find (a) the total capacitance, (b) the total charge stored, (c) the p.d. across the $2.0\,\mu\text{F}$ capacitor.

Lời giải chi tiết

The two $4.0\,\mu$F capacitors in parallel give $8.0\,\mu$F. (a) $\frac{1}{C} = \frac{1}{2.0} + \frac{1}{8.0} = \frac{4}{8} + \frac{1}{8} = \frac{5}{8}$, so $C = 1.6\,\mu$F. (b) $Q = CV = 1.6 \times 10^{-6} \times 12 = 1.92 \times 10^{-5}$ C $= 19.2\ \mu$C (same charge flows through the series branch). (c) $V_{2\mu F} = Q/C = 19.2\ \mu\text{C}/2.0\ \mu\text{F} = 9.6$ V. *Check:* remaining 2.4 V across the $8.0\ \mu$F combination gives $Q = 8.0 \times 2.4 = 19.2\ \mu$C, consistent.

Bài tập luyện tập

11. A capacitor $C = 20\,\mu$F, initially charged to $Q_0 = 200\,\mu$C, discharges through $R = 5.0\,\text{k}\Omega$. Find the time constant and the charge remaining after $t = 0.30\,$s.

Lời giải chi tiết

$\tau = RC = 5.0 \times 10^3 \times 20 \times 10^{-6} = 0.10$ s.

$$Q = Q_0 e^{-t/\tau} = 200 \times e^{-0.30/0.10} = 200 \times e^{-3} \approx 200 \times 0.0498 \approx 10.0\ \mu\text{C}$$

Bài tập luyện tập

12. A capacitor $C = 50\,\mu$F charges through $R = 2.0\,\text{k}\Omega$ from a $9.0\,$V supply. Find the charge on the capacitor after one time constant.

Lời giải chi tiết

$\tau = RC = 2.0 \times 10^3 \times 50 \times 10^{-6} = 0.10\,$s. $\quad Q_0 = CV = 50 \times 10^{-6} \times 9.0 = 4.5 \times 10^{-4}$ C $= 450\,\mu$C.

$$Q = Q_0(1 - e^{-1}) = 450 \times (1 - 0.368) \approx 284\ \mu\text{C}$$

Bài tập luyện tập

13. A parallel-plate capacitor of capacitance $C = 100\,$pF has plate area $A = 0.020$ m^2. Find the plate separation d (air between plates).

Lời giải chi tiết

$$d = \frac{\varepsilon_0 A}{C} = \frac{8.85 \times 10^{-12} \times 0.020}{100 \times 10^{-12}} \approx 1.8 \times 10^{-3}\ \text{m} = 1.8\ \text{mm}$$

Bài tập luyện tập

14. On a graph of charge Q against p.d. V for a capacitor being charged, the energy stored is best represented by:

(A) The gradient of the line.

(B) The y-intercept.

(C) The area under the line (a triangle of area $\frac{1}{2}QV$).

(D) The area of a rectangle of sides Q and V (i.e. QV).

Lời giải chi tiết

Answer: C. The Q–V graph is a straight line through the origin (since $Q = CV$); the energy stored equals the triangular area under it, $\frac{1}{2}QV$, not the full rectangle QV.

Bài tập luyện tập

15. A straight wire of length $L = 0.15$ m carries current $I = 4.0$ A perpendicular to a field $B = 0.30$ T. Find the force on the wire.

Lời giải chi tiết

$F = BIL = 0.30 \times 4.0 \times 0.15 = 0.18$ N.

Bài tập luyện tập

16. A wire of length $L = 0.25$ m carrying $I = 2.0$ A makes an angle of $30°$ with a field $B = 0.60$ T. Find the force on the wire.

Lời giải chi tiết

$$F = BIL\sin\theta = 0.60 \times 2.0 \times 0.25 \times \sin 30° = 0.30 \times 0.5 = 0.15 \text{ N}$$

Bài tập luyện tập

17. A proton travels at $v = 1.5 \times 10^6$ m s^{-1} perpendicular to a field $B = 0.80$ T. Find the magnetic force on it and the radius of its circular path.

Lời giải chi tiết

$F = Bqv = 0.80 \times 1.6 \times 10^{-19} \times 1.5 \times 10^6 = 1.92 \times 10^{-13}$ N.

$$r = \frac{mv}{Bq} = \frac{1.67 \times 10^{-27} \times 1.5 \times 10^6}{0.80 \times 1.6 \times 10^{-19}} \approx 1.96 \times 10^{-2} \text{ m} \approx 2.0 \text{ cm}$$

Bài tập luyện tập

18. Fleming's left-hand rule is used to find the direction of:

(A) The induced e.m.f. in a generator.

(B) The force on a current-carrying conductor or moving charge in a magnetic field.

(C) The direction of an induced current opposing a changing flux.

(D) The direction of the magnetic field around a straight wire.

Lời giải chi tiết

Answer: B. Fleming's left-hand rule gives the direction of the force (motor effect) on a current or moving charge in a magnetic field.

Bài tập luyện tập

19. A metal slab of thickness $t = 0.50\,\text{mm}$ carries current $I = 3.0\,\text{A}$ in a field $B = 0.45\,\text{T}$ perpendicular to its thickness. Given $n = 8.5 \times 10^{28}\ \text{m}^{-3}$ and $q = 1.6 \times 10^{-19}$ C, find the Hall voltage.

Lời giải chi tiết

$$V_H = \frac{BI}{ntq} = \frac{0.45 \times 3.0}{(8.5 \times 10^{28}) \times (0.50 \times 10^{-3}) \times (1.6 \times 10^{-19})} = \frac{1.35}{6.8 \times 10^{6}} \approx 2.0{\times}10^{-7}\,\text{V} \approx 0.20\,\mu\text{V}$$

Bài tập luyện tập

20. Compared with a metal, a Hall probe made from a semiconductor (with much lower charge-carrier density n) will, for the same B, I, and t:

(A) Produce a smaller Hall voltage.

(B) Produce a larger Hall voltage.

(C) Produce exactly the same Hall voltage.

(D) Produce zero Hall voltage.

Lời giải chi tiết

Answer: B. Since $V_H = BI/ntq$, a smaller n (as in a semiconductor) gives a larger Hall voltage for the same B, I, t and q – this is why semiconductors are used in practical Hall probes.

Bài tập luyện tập

21. A conducting rod of length $L = 0.80\,\text{m}$ moves at $v = 6.0\,\text{m}\,\text{s}^{-1}$ perpendicular to a field $B = 0.15\,\text{T}$. Find the motional e.m.f. generated.

Lời giải chi tiết

$\varepsilon = BLv = 0.15 \times 0.80 \times 6.0 = 0.72$ V.

Bài tập luyện tập

22. A coil of $N = 150$ turns and area $0.010\ \text{m}^2$ is in a field that changes uniformly from $0.50\,\text{T}$ to $0.20\,\text{T}$ in $0.15\,\text{s}$. Find the average induced e.m.f.

Lời giải chi tiết

$\Delta\Phi = \Delta B \times A = (0.50 - 0.20) \times 0.010 = 3.0 \times 10^{-3}$ Wb.

$$\varepsilon = \frac{N\,\Delta\Phi}{\Delta t} = \frac{150 \times 3.0 \times 10^{-3}}{0.15} = 3.0\,\text{V}$$

Bài tập luyện tập

23. A bar magnet is pushed, north pole first, into a coil connected to a sensitive ammeter. By Lenz's law, the induced current in the coil:

(A) Flows so as to create a magnetic field that attracts the approaching magnet, speeding it up.

(B) Flows so as to create a magnetic field that opposes the magnet's motion, i.e. repels the approaching north pole.

(C) Is always zero, since the magnet is not touching the coil.

(D) Flows in a direction independent of the magnet's motion.

Lời giải chi tiết

Answer: B. Lenz's law requires the induced current to oppose the change producing it, so the coil's near face becomes an effective north pole, repelling the approaching magnet (consistent with conservation of energy – work must be done to push the magnet in).

Bài tập luyện tập

24. A generator coil of $N = 150$ turns and area $A = 0.0080$ m^2 rotates at $f = 60$ Hz in a field $B = 0.25$ T. Find (a) the peak e.m.f., (b) the instantaneous e.m.f. at a rotation angle of $45°$ from the position of zero flux linkage.

Lời giải chi tiết

$\omega = 2\pi \times 60 \approx 377$ rad s^{-1}.

$$\varepsilon_0 = BAN\omega = 0.25 \times 0.0080 \times 150 \times 377 \approx 113 \text{ V}$$

$$\varepsilon = \varepsilon_0 \sin 45° = 113 \times 0.707 \approx 80.0 \text{ V}$$

Bài tập luyện tập

25. A transformer has $N_p = 800$ turns and $N_s = 40$ turns, with 240 V applied to the primary. Find the secondary voltage.

Lời giải chi tiết

$$V_s = V_p \times \frac{N_s}{N_p} = 240 \times \frac{40}{800} = 12 \text{ V}$$

Bài tập luyện tập

26. A transformer has an input power of 500 W and an output power of 425 W. Find its efficiency and the power lost.

Lời giải chi tiết

$$\text{efficiency} = \frac{425}{500} \times 100\% = 85.0\%$$

Power lost $= 500 - 425 = 75$ W.

Bài tập luyện tập

27. An a.c. supply has peak voltage $V_0 = 170\,\text{V}$ and peak current $I_0 = 6.0\,\text{A}$. Find the r.m.s. voltage, the r.m.s. current, and the average power delivered.

Lời giải chi tiết

$V_{\text{rms}} = 170/\sqrt{2} \approx 120$ V. $\quad I_{\text{rms}} = 6.0/\sqrt{2} \approx 4.24$ A.

$$P_{\text{avg}} = \tfrac{1}{2}V_0 I_0 = \tfrac{1}{2} \times 170 \times 6.0 \approx 510 \text{ W}$$

Check: $V_{\text{rms}} I_{\text{rms}} = 120 \times 4.24 \approx 510$ W, consistent.

Bài tập luyện tập

28. Which arrangement of diodes is needed to achieve full-wave rectification of an a.c. supply?

(A) A single diode in series with the load.

(B) Two diodes in parallel, both facing the same direction.

(C) Four diodes arranged in a bridge configuration.

(D) No diodes; a capacitor alone is sufficient.

Lời giải chi tiết

Answer: C. A bridge of four diodes routes current through the load in the same direction during both halves of the a.c. cycle, giving full-wave rectification; a single diode (A) gives only half-wave rectification.

Bài tập luyện tập

29. A $3.0\,\mu$F capacitor is connected in series with a $6.0\,\mu$F capacitor across an 18 V supply, then this combination is disconnected and allowed to discharge through $R = 4.0\,\text{k}\Omega$. Find (a) the combined capacitance, (b) the initial charge, (c) the time constant, (d) the charge remaining after $t = 20$ ms.

Lời giải chi tiết

(a) $\dfrac{1}{C} = \dfrac{1}{3.0} + \dfrac{1}{6.0} = \dfrac{2}{6} + \dfrac{1}{6} = \dfrac{3}{6}$, so $C = 2.0\,\mu$F. (b) $Q_0 = CV = 2.0 \times 10^{-6} \times 18 = 3.6 \times 10^{-5}$ C $= 36\,\mu$C. (c) $\tau = RC = 4.0 \times 10^3 \times 2.0 \times 10^{-6} = 8.0 \times 10^{-3}$ s $= 8.0$ ms. (d) $t/\tau = 20/8.0 = 2.5$. $Q = Q_0 e^{-2.5} = 36 \times 0.0821 \approx 3.0\,\mu$C.

Bài tập luyện tập

30. Find the period of revolution of an electron moving perpendicular to a uniform magnetic field $B = 0.60$ T.

Lời giải chi tiết

The period of circular motion is $T = 2\pi m/(Bq)$, independent of speed:

$$T = \frac{2\pi \times 9.11 \times 10^{-31}}{0.60 \times 1.6 \times 10^{-19}} = \frac{5.72 \times 10^{-30}}{9.6 \times 10^{-20}} \approx 6.0 \times 10^{-11} \text{ s}$$

Quantum Physics, Medical Physics & Astronomy/Cosmology

[A2 – A Level only, not in the AS syllabus]

Mục tiêu Unit: Chương cuối cùng của giáo trình gồm ba mảng kiến thức A2 độc lập nhưng cùng dựa trên ý tưởng năng lượng được lượng tử hoá: vật lý lượng tử (photon, hiệu ứng quang điện, lưỡng tính sóng-hạt, mức năng lượng nguyên tử và quang phổ), vật lý y học (siêu âm và tia X trong chẩn đoán hình ảnh), và thiên văn/vũ trụ học (định luật Wien, định luật Stefan-Boltzmann, định luật Hubble và dịch chuyển đỏ). Trọng tâm là thành thạo các công thức $E = hf = hc/\lambda$, $\lambda = h/p$, và khả năng kết hợp nhiều công thức trong một bài toán nhiều bước, ví dụ như tính khoảng cách một ngôi sao từ nhiệt độ, bán kính và cường độ sáng quan sát được.

12.1 Photons and the Photoelectric Effect

12.1.1 The Photon Model of Light

Electromagnetic radiation can behave as a stream of discrete energy packets called **photons**. Each photon of frequency f (or wavelength λ) carries a fixed quantum of energy:

$$E = hf = \frac{hc}{\lambda}$$

where $h = 6.63 \times 10^{-34}$ J s is the Planck constant and $c = 3.00 \times 10^{8}$ m s^{-1} is the speed of light in a vacuum.

Photon energy

A photon's energy depends only on its frequency (or wavelength), never on the intensity of the beam. Intensity is related to the *number* of photons arriving per second, not the energy of each individual photon.

VN GIẢI THÍCH TIẾNG VIỆT

Ánh sáng (và mọi bức xạ điện từ) có thể coi là dòng các ”gói năng lượng” rời rạc gọi là photon. Năng lượng mỗi photon là $E = hf = hc/\lambda$, chỉ phụ thuộc tần số/bước sóng, hoàn toàn không phụ thuộc cường độ chùm sáng. Cường độ sáng lớn hơn nghĩa là có nhiều photon hơn mỗi giây, chứ không phải mỗi photon mang nhiều năng lượng hơn.

12.1.2 The Photoelectric Effect and the Photoelectric Equation

When light of sufficiently high frequency strikes a metal surface, electrons (**photoelectrons**) are emitted instantly. In the photon model, a single photon transfers *all* of its energy to a *single* electron. Part of this energy, the **work function** ϕ, is used to free the electron from the metal surface; any remaining energy becomes the electron's kinetic energy. The fastest electrons (those released with no further energy loss) obey the **photoelectric equation**:

$$hf = \phi + KE_{\text{max}} \qquad \Rightarrow \qquad KE_{\text{max}} = hf - \phi$$

Emission occurs only if $hf \geq \phi$. The minimum frequency at which emission just occurs is the **threshold frequency**:

$$f_0 = \frac{\phi}{h}$$

Below f_0, no photoelectrons are emitted no matter how intense the light is, because no single photon carries enough energy to free an electron.

Photoelectric equation

$KE_{\text{max}} = hf - \phi$ describes a straight-line relationship between KE_{max} and f: gradient $= h$, vertical intercept $= -\phi$, horizontal intercept $= f_0 = \phi/h$.

GIẢI THÍCH TIẾNG VIỆT

VN Hiệu ứng quang điện: một photon truyền toàn bộ năng lượng cho một electron duy nhất. Một phần năng lượng (ϕ – công thoát) dùng để giải phóng electron khỏi bề mặt kim loại, phần còn lại trở thành động năng. Phương trình quang điện $KE_{\text{max}} = hf - \phi$ cho thấy KE_{max} chỉ phụ thuộc tần số ánh sáng, không phụ thuộc cường độ. Tần số ngưỡng $f_0 = \phi/h$ là tần số tối thiểu để có phát xạ electron.

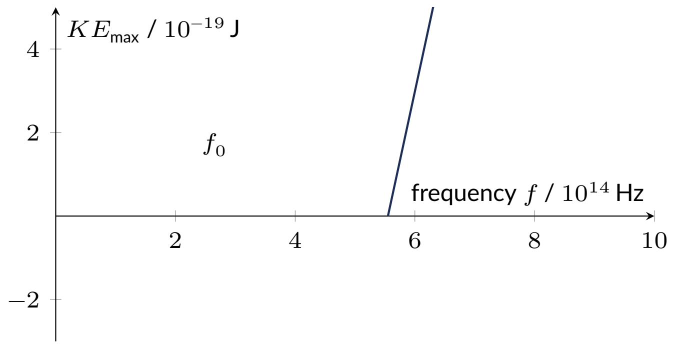
Ví dụ mẫu · năng lượng photon và động năng cực đại

A metal surface has work function $\phi = 2.3\,$eV. Light of frequency $f = 7.5 \times 10^{14}$ Hz is incident on the surface. Find (a) the photon energy, (b) the maximum kinetic energy of the emitted photoelectrons, and (c) the threshold frequency.

Solution. (a) $E = hf = (6.63 \times 10^{-34})(7.5 \times 10^{14}) = 4.97 \times 10^{-19}$ J $= 3.11\,$eV.

(b) $KE_{\text{max}} = E - \phi = 3.11 - 2.3 = 0.81\,$eV (equivalently 1.30×10^{-19} J).

(c) $f_0 = \phi/h = (2.3 \times 1.60 \times 10^{-19})/(6.63 \times 10^{-34}) = 5.55 \times 10^{14}$ Hz.



Graph of KE_{max} against f. The gradient equals the Planck constant h; the horizontal intercept gives the threshold frequency $f_0 = \phi/h$.

12.1.3 Stopping Potential

The maximum kinetic energy of the photoelectrons can be measured by applying a reverse (decelerating) voltage between the emitting surface and a collector. The **stopping potential** V_s is the minimum reverse voltage needed to reduce the photocurrent to exactly zero – it is just large enough to stop even the fastest photoelectrons. Since the work done against the field equals the electron's initial kinetic energy,

$$KE_{\text{max}} = eV_s$$

where $e = 1.60 \times 10^{-19}$ C.

Ví dụ mẫu · điện thế hãm

A metal has work function $\phi = 2.00\,\text{eV}$. Light of frequency $f = 8.00 \times 10^{14}$ Hz is incident on it. Find the stopping potential.
Solution. Photon energy: $hf = (6.63 \times 10^{-34})(8.00 \times 10^{14}) = 5.304 \times 10^{-19}\,\text{J} = 3.32\,\text{eV}$.
$KE_{\text{max}} = 3.32 - 2.00 = 1.32\,\text{eV} = 2.10 \times 10^{-19}\,\text{J}$.
$V_s = KE_{\text{max}}/e = (2.10 \times 10^{-19})/(1.60 \times 10^{-19}) = 1.32\,\text{V}$.
(Numerically, when KE_{max} is expressed in eV, V_s in volts has the same numeric value – a useful shortcut.)

VN **GIẢI THÍCH TIẾNG VIỆT**
Điện thế hãm V_s là điện áp ngược nhỏ nhất đủ để chặn đứng ngay cả các electron quang điện nhanh nhất, khi đó $KE_{\text{max}} = eV_s$. Đây là cách thực nghiệm để đo KE_{max} mà không cần đo trực tiếp vận tốc electron.

(!) Lỗi thường gặp: A common mistake is to assume that increasing the *intensity* of light increases KE_{max} (and hence V_s). It does not: intensity only increases the *number* of photons per second, so it increases the photocurrent (rate of electron emission), never the maximum kinetic energy, which depends solely on frequency.

12.1.4 Evidence for the Photon Model

The photoelectric effect provides strong evidence that light behaves as particles (photons) rather than purely as a continuous wave. Classical wave theory predicts that the energy of a wave depends on its *amplitude* (intensity), spread continuously across the wavefront; it cannot explain the following observations.

Observations that classical wave theory cannot explain:

- Photoelectrons are emitted **instantaneously** (with no measurable time delay) as soon as light of sufficient frequency strikes the surface, even at very low intensity – a wave would need time to accumulate enough energy at one point.
- There is a sharp **threshold frequency** f_0 below which no electrons are emitted at all, regardless of how intense the light is.
- KE_{max} of the emitted electrons is completely **independent of intensity** and depends only on the frequency of the incident light.
- The **photocurrent** (the rate of electron emission) is proportional to the intensity of the light, provided the frequency is above the threshold value.

All four observations are explained naturally by the photon model: each photon carries a fixed quantum hf, transferred entirely to one electron, so emission is instantaneous and depends on individual photon energy (frequency), while intensity (photon number per second) governs only the rate of emission.

12.2 Wave-Particle Duality and Atomic Energy Levels

12.2.1 The De Broglie Wavelength

Just as light (usually a wave) can behave as particles, matter particles (usually treated as particles) can exhibit wave-like behaviour. The **de Broglie wavelength** of a particle of momentum $p = mv$ is

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Electron diffraction, in which a beam of electrons produces diffraction rings when passed through a thin crystalline film (analogous to X-ray diffraction), is direct experimental evidence for the wave nature of matter.

De Broglie wavelength

Every moving particle has an associated wavelength $\lambda = h/p$. Wave effects (diffraction, interference) are only observable when λ is comparable to the size of the obstacle or aperture involved.

GIẢI THÍCH TIẾNG VIỆT

VN Không chỉ ánh sáng có tính hạt, các hạt vật chất (như electron) cũng có tính sóng, với bước sóng de Broglie $\lambda = h/p = h/(mv)$. Hiện tượng nhiễu xạ electron qua màng tinh thể mỏng là bằng chứng thực nghiệm trực tiếp cho tính sóng của vật chất.

Ví dụ mẫu · bước sóng de Broglie của electron

An electron is accelerated from rest through a potential difference of 200 V. Find its de Broglie wavelength. (Electron mass $m_e = 9.11 \times 10^{-31}$ kg.)
Solution. Kinetic energy gained: $KE = eV = (1.60 \times 10^{-19})(200) = 3.20 \times 10^{-17}$ J.
Speed: $v = \sqrt{2KE/m_e} = \sqrt{(2 \times 3.20 \times 10^{-17})/(9.11 \times 10^{-31})} = 8.38 \times 10^6$ m s^{-1}.
Momentum: $p = m_e v = (9.11 \times 10^{-31})(8.38 \times 10^6) = 7.64 \times 10^{-24}$ kg m s^{-1}.
$\lambda = h/p = (6.63 \times 10^{-34})/(7.64 \times 10^{-24}) = 8.68 \times 10^{-11}$ m $= 86.8$ pm.
This wavelength is comparable to atomic spacings (~0.1 nm), which is why electrons diffract from crystal lattices.

Ví dụ mẫu · bước sóng de Broglie của một vật thể vĩ mô

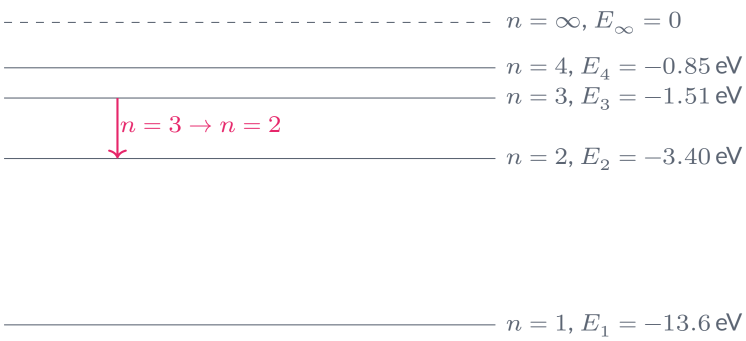
A ball of mass 0.150 kg is thrown at 30.0 m s^{-1}. Find its de Broglie wavelength and comment on the result.
Solution. Momentum: $p = mv = (0.150)(30.0) = 4.50$ kg m s^{-1}.
$\lambda = h/p = (6.63 \times 10^{-34})/(4.50) = 1.47 \times 10^{-34}$ m.
This wavelength is about 10^{24} times smaller than the diameter of an atom ($\sim 10^{-10}$ m). No aperture or obstacle in the everyday world is remotely close to this size, so no diffraction or other wave effect can ever be observed for macroscopic objects – their wave nature, while present in principle, is utterly unobservable.

12.2.2 Atomic Energy Levels and Line Spectra

Electrons in an atom can only occupy discrete, quantised **energy levels** $E_1, E_2, E_3, \ldots$ (all negative, taking a free electron at rest as zero energy). When an electron makes a transition between two levels, a photon is emitted or absorbed with energy exactly equal to the difference between the levels:

$$hf = E_2 - E_1$$

Because only certain discrete energy differences are possible, atoms emit or absorb light only at certain discrete frequencies (wavelengths), producing a **line spectrum** characteristic of the element.



Hydrogen energy-level diagram (not to scale). The transition $n = 3 \to n = 2$ emits the H$_\alpha$ line of the Balmer series.

Ví dụ mẫu · vạch quang phổ Balmer H$_\alpha$

In hydrogen, $E_3 = -1.51$ eV and $E_2 = -3.40$ eV. Find the energy and wavelength of the photon emitted in the transition $n = 3 \to n = 2$.
Solution. $\Delta E = E_3 - E_2 = -1.51 - (-3.40) = 1.89$ eV $= 3.02 \times 10^{-19}$ J.

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.63 \times 10^{-34})(3.00 \times 10^8)}{3.02 \times 10^{-19}} = 6.58 \times 10^{-7}\,\text{m} = 658\,\text{nm}.$$

This is close to the accepted H$_\alpha$ wavelength of 656.3 nm, the small difference arising from rounding the energy-level values.

Ví dụ mẫu · vạch Lyman-alpha

Find the energy and wavelength of the photon emitted when a hydrogen electron falls from $n = 2$ ($E_2 = -3.40$ eV) to $n = 1$ ($E_1 = -13.6$ eV).
Solution. $\Delta E = E_2 - E_1 = -3.40 - (-13.6) = 10.2$ eV $= 1.63 \times 10^{-18}$ J.

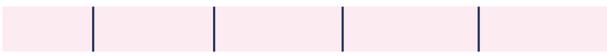
$$\lambda = \frac{hc}{\Delta E} = \frac{(6.63 \times 10^{-34})(3.00 \times 10^8)}{1.63 \times 10^{-18}} = 1.22 \times 10^{-7}\,\text{m} = 122\,\text{nm}.$$

This is close to the accepted Lyman-α wavelength of 121.6 nm (in the ultraviolet).

12.2.3 Emission and Absorption Spectra

A hot, low-pressure gas emits light only at the discrete wavelengths corresponding to downward electron transitions, producing an **emission spectrum**: a series of bright lines on an otherwise dark background. If instead white light (a continuous spectrum) passes through a cooler gas, atoms in the gas absorb photons of exactly the wavelengths needed to drive electrons to higher levels, removing those wavelengths from the transmitted light. This produces an **absorption spectrum**: dark lines at those same wavelengths, superimposed on an otherwise continuous bright background.


Emission spectrum: bright lines on a dark background


Absorption spectrum: dark lines on a continuous bright background

Schematic comparison of emission and absorption line spectra for the same element – the lines occur at identical wavelengths, since both correspond to the same energy-level differences.

Spectrum type	Appearance	Origin
Emission	Bright coloured lines on a dark background	Electrons fall from a higher to a lower energy level, emitting photons of energy $hf = E_{\text{high}} - E_{\text{low}}$
Absorption	Dark lines on a continuous bright background	Electrons absorb photons of energy $hf = E_{\text{high}} - E_{\text{low}}$, jumping to a higher level; those wavelengths are removed from the transmitted light

GIẢI THÍCH TIẾNG VIỆT

VN

Quang phổ vạch phát xạ là các vạch sáng trên nền tối, sinh ra khi electron nhảy từ mức năng lượng cao xuống mức thấp và phát ra photon. Quang phổ vạch hấp thụ là các vạch tối trên nền sáng liên tục, sinh ra khi electron hấp thụ đúng photon có năng lượng bằng hiệu hai mức để nhảy lên mức cao hơn. Hai loại quang phổ của cùng một nguyên tố có các vạch nằm ở đúng cùng bước sóng.

12.3 Medical Physics: Ultrasound

12.3.1 Generating and Detecting Ultrasound

Diagnostic ultrasound uses a **piezoelectric crystal** both to generate and to detect high-frequency sound pulses (typically 1–15 MHz). When an alternating voltage is applied across the crystal, it vibrates mechanically at the same frequency, emitting an ultrasound pulse into the body (the converse piezoelectric effect). The same crystal then acts as the receiver: mechanical vibrations from a returning echo deform the crystal, generating a small alternating voltage across it (the piezoelectric effect) which is amplified and processed to build an image.

GIẢI THÍCH TIẾNG VIỆT

VN

Tinh thể áp điện vừa đóng vai trò máy phát vừa đóng vai trò máy thu siêu âm. Khi có điện áp xoay chiều đặt vào, tinh thể rung động cơ học và phát ra xung siêu âm (hiệu ứng áp điện nghịch). Khi xung phản xạ (tiếng vọng) quay lại làm tinh thể rung động, nó tạo ra điện áp xoay chiều nhỏ (hiệu ứng áp điện thuận), được khuếch đại để tạo ảnh.

12.3.2 Acoustic Impedance and Reflection at Boundaries

Each tissue has a characteristic **acoustic impedance**

$$Z = \rho c$$

where ρ is the density of the tissue and c is the speed of sound in it. When an ultrasound pulse meets a boundary between two tissues of impedances Z_1 and Z_2, part of the beam is reflected. The fraction of *intensity* reflected is

$$\frac{I_r}{I_0} = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2$$

A larger impedance mismatch produces a stronger echo – this is exactly what allows ultrasound to distinguish between different tissue boundaries (e.g. muscle/bone, or a tumour within soft tissue).

Ví dụ mẫu · phản xạ tại ranh giới mô-xương

At a tissue-bone boundary, $Z_1 = 1.63 \times 10^6\,\text{kg m}^{-2}\,\text{s}^{-1}$ (soft tissue) and $Z_2 = 7.75 \times 10^6\,\text{kg m}^{-2}\,\text{s}^{-1}$ (bone). Find the percentage of the incident intensity reflected.

Solution.

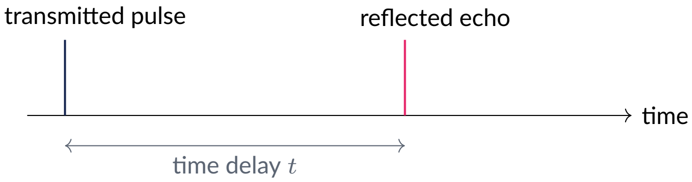
$$\frac{I_r}{I_0} = \left(\frac{7.75 - 1.63}{7.75 + 1.63}\right)^2 = \left(\frac{6.12}{9.38}\right)^2 = (0.652)^2 = 0.426$$

So about 42.6% of the incident intensity is reflected at this boundary (and about 57.4% is transmitted onward).

12.3.3 Pulse-Echo Timing and Depth

A short ultrasound pulse is sent into the body; part of it reflects from each tissue boundary and returns to the transducer as an echo. If the time delay between the transmitted pulse and the returning echo is t, and the speed of the ultrasound in the tissue is c, then since the pulse must travel to the boundary *and back*, the depth of the boundary is

$$d = \frac{ct}{2}$$



Pulse-echo timing: the transducer emits a short pulse and detects the echo reflected from a boundary after a delay t. Because the pulse travels to the boundary and back, depth $d = ct/2$.

Ví dụ mẫu · xác định độ sâu bằng thời gian vọng

An ultrasound pulse is sent into tissue in which the speed of sound is $c = 1.5 \times 10^3$ m s^{-1}. The echo from a boundary returns after a time delay of $40\,\mu$s. Find the depth of the boundary.

Solution.

$$d = \frac{ct}{2} = \frac{(1.5 \times 10^3)(40 \times 10^{-6})}{2} = \frac{0.060}{2} = 0.030\,\text{m} = 3.0\,\text{cm}$$

GIẢI THÍCH TIẾNG VIỆT

VN

Trở kháng âm học $Z = \rho c$ quyết định tỉ lệ phản xạ tại ranh giới hai mô. Kỹ thuật đo thời gian xung-vọng (pulse-echo) cho phép xác định độ sâu của ranh giới: $d = ct/2$, với hệ số 2 vì xung phải đi tới ranh giới rồi quay trở lại.

12.4 Medical Physics: X-ray Imaging

12.4.1 Attenuation of X-rays

As a beam of X-rays passes through matter, its intensity decreases exponentially with the thickness x traversed:

$$I = I_0 e^{-\mu x}$$

where μ is the **linear attenuation coefficient** of the material (units: per metre or per centimetre), which depends on the material and on the X-ray photon energy.

Ví dụ mẫu · suy giảm cường độ tia X

X-rays pass through 5.0 cm of soft tissue for which $\mu = 0.60$ cm^{-1}. Find the fraction of the incident intensity transmitted.

Solution.

$$\frac{I}{I_0} = e^{-\mu x} = e^{-(0.60)(5.0)} = e^{-3.0} = 0.050$$

So 5.0% of the incident intensity is transmitted.

12.4.2 Finding the Attenuation Coefficient from Data

Rearranging the exponential law by taking natural logarithms gives

$$\mu = -\frac{1}{x}\ln\left(\frac{I}{I_0}\right)$$

which allows μ to be found directly from measured input and output intensities.

Ví dụ mẫu · xác định hệ số suy giảm từ số liệu

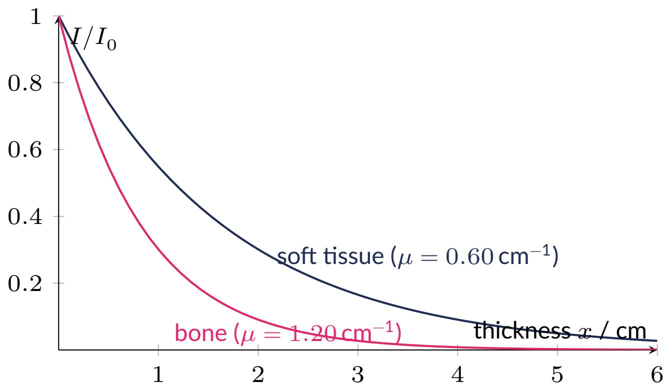
An X-ray beam passes through 4.0 cm of a tissue sample and emerges with 25% of its original intensity. Find the linear attenuation coefficient of the tissue.

Solution.

$$\mu = -\frac{1}{x}\ln\left(\frac{I}{I_0}\right) = -\frac{1}{4.0}\ln(0.25) = -\frac{(-1.386)}{4.0} = 0.347\,\text{cm}^{-1} \approx 0.35\,\text{cm}^{-1}$$

12.4.3 Image Contrast: Why Bone Appears White

Bone has a much higher density and a much larger attenuation coefficient μ than soft tissue, so it absorbs (attenuates) far more of the incident X-ray beam. Consequently, in the region of the image behind bone, very little radiation reaches the photographic film or detector, so that region receives little exposure and appears **light/white**. Behind soft tissue, where μ is much smaller, more radiation is transmitted and reaches the film, exposing it more and producing a **darker** region. This difference in attenuation between tissue types is precisely what creates useful contrast in an X-ray image.



Transmitted intensity fraction against thickness for soft tissue and bone. Bone's much larger μ means far less radiation reaches the film behind it – the basis of X-ray image contrast.

GIẢI THÍCH TIẾNG VIỆT

VN

Xương có hệ số suy giảm μ lớn hơn nhiều so với mô mềm, nên hấp thụ nhiều tia X hơn. Vùng phim phía sau xương nhận ít bức xạ hơn nên có màu sáng/trắng; vùng phía sau mô mềm nhận nhiều bức xạ hơn nên có màu tối hơn. Sự khác biệt suy giảm này chính là nguồn gốc độ tương phản của ảnh X-quang.

12.5 Astrophysics and Cosmology

12.5.1 Black-Body Radiation and Wien's Law

A hot object emits a continuous spectrum of electromagnetic radiation approximating that of a **black body**. The wavelength at which the emitted intensity is greatest, λ_{max}, is related to the object's absolute temperature T by **Wien's displacement law**:

$$\lambda_{\text{max}}T = \text{constant} = 2.90 \times 10^{-3}\,\text{m K}$$

Hotter objects emit their peak intensity at shorter wavelengths.

Ví dụ mẫu · định luật Wien

(a) The Sun's surface temperature is about $5800\,\text{K}$. Find λ_{max}.
(b) A star has $\lambda_{\text{max}} = 290\,\text{nm}$. Find its surface temperature.

Solution. (a) $\lambda_{\text{max}} = \dfrac{2.90 \times 10^{-3}}{5800} = 5.00 \times 10^{-7}\,\text{m} = 500\,\text{nm}$ (blue-green light, consistent with the Sun appearing white/yellow).
(b) $T = \dfrac{2.90 \times 10^{-3}}{290 \times 10^{-9}} = 1.00 \times 10^{4}\,\text{K}$ – a much hotter, blue-white star.

12.5.2 The Stefan-Boltzmann Law and Luminosity

A black body of absolute temperature T and surface area A radiates total power (luminosity)

$$L = \sigma A T^4 = 4\pi r^2 \sigma T^4 \quad \text{(for a sphere of radius } r\text{)}$$

where $\sigma = 5.67 \times 10^{-8}\,\text{W m}^{-2}\,\text{K}^{-4}$ is the Stefan-Boltzmann constant.

Ví dụ mẫu · độ trưng của Mặt Trời

The Sun has radius $r = 6.96 \times 10^{8}\,\text{m}$ and surface temperature $T = 5800\,\text{K}$. Find its luminosity.
Solution. $r^2 = 4.84 \times 10^{17}\,\text{m}^2$; $T^4 = (5800)^4 = 1.13 \times 10^{15}\,\text{K}^4$.

$$L = 4\pi r^2 \sigma T^4 = 4\pi(4.84 \times 10^{17})(5.67 \times 10^{-8})(1.13 \times 10^{15}) = 3.91 \times 10^{26}\,\text{W}$$

(close to the accepted solar luminosity of $3.85 \times 10^{26}\,\text{W}$).

VN GIẢI THÍCH TIẾNG VIỆT

Định luật Wien $\lambda_{\text{max}}T$ = hằng số cho biết vật càng nóng thì bước sóng đỉnh phát xạ càng ngắn. Định luật Stefan-Boltzmann $L = 4\pi r^2 \sigma T^4$ cho biết tổng công suất bức xạ của một vật cầu, phụ thuộc rất mạnh vào nhiệt độ (luỹ thừa bậc 4).

12.5.3 Radiation Flux and Distance

The **radiant flux** (intensity received) F at distance d from a source of luminosity L follows an inverse-square law, since the power spreads out over the surface of an ever-larger sphere:

$$F = \frac{L}{4\pi d^2}$$

Combining this with the Stefan-Boltzmann law allows the distance to a star to be found from its measured temperature, radius, and observed flux.

Ví dụ mẫu · kết hợp Stefan-Boltzmann và định luật nghịch đảo bình phương

A star has surface temperature $T = 6000\,\text{K}$ and radius $r = 2.00\,R_\odot$, where $R_\odot = 6.96 \times 10^8\,\text{m}$. Its observed flux at Earth is $F = 2.50 \times 10^{-11}\,\text{W}\,\text{m}^{-2}$. Find its distance from Earth.
Solution. Radius: $r = 2.00 \times 6.96 \times 10^8 = 1.392 \times 10^9\,\text{m}$, so $r^2 = 1.94 \times 10^{18}\,\text{m}^2$.
$T^4 = (6000)^4 = 1.296 \times 10^{15}\,\text{K}^4$.
Luminosity: $L = 4\pi r^2 \sigma T^4 = 4\pi(1.94 \times 10^{18})(5.67 \times 10^{-8})(1.296 \times 10^{15}) = 1.79 \times 10^{27}\,\text{W}$.
From $F = L/(4\pi d^2)$:

$$d = \sqrt{\frac{L}{4\pi F}} = \sqrt{\frac{1.79 \times 10^{27}}{4\pi(2.50 \times 10^{-11})}} = \sqrt{5.70 \times 10^{36}} = 2.39 \times 10^{18}\,\text{m}$$

Using 1 light-year $\approx 9.47 \times 10^{15}\,\text{m}$, this is about 252 light-years.

12.5.4 Hubble's Law and Redshift

Light from distant galaxies is observed to be shifted to longer wavelengths (**redshift**), because the galaxies are receding from us as the universe expands. For a source moving away at speed $v \ll c$, the redshift is

$$z = \frac{\Delta\lambda}{\lambda} \approx \frac{v}{c}$$

Hubble's law states that the recession speed of a distant galaxy is directly proportional to its distance d:

$$v = H_0 d$$

where H_0 is the Hubble constant, commonly quoted in $\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$.

Ví dụ mẫu · định luật Hubble

A galaxy is receding at $v = 3500\,\text{km}\,\text{s}^{-1}$. Taking $H_0 = 70\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$, find its distance.
Solution.

$$d = \frac{v}{H_0} = \frac{3500}{70} = 50.0\,\text{Mpc}$$

Ví dụ mẫu · dịch chuyển đỏ của một chuẩn tinh

The Lyman-α line, normally at $121.6\,\text{nm}$, is observed in a distant quasar's spectrum at $128.0\,\text{nm}$. Find the redshift z and the recession velocity.
Solution. $\Delta\lambda = 128.0 - 121.6 = 6.4\,\text{nm}$.

$$z = \frac{\Delta\lambda}{\lambda} = \frac{6.4}{121.6} = 0.0526$$

$$v = zc = (0.0526)(3.00 \times 10^5\,\text{km}\,\text{s}^{-1}) = 1.58 \times 10^4\,\text{km}\,\text{s}^{-1}$$

Ví dụ mẫu · ước lượng tuổi vũ trụ

Estimate the age of the universe, taking $H_0 = 70\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$ and treating the universe as having expanded at a constant rate since the Big Bang, so that age $\approx 1/H_0$. (Take $1\,\text{Mpc} = 3.09 \times 10^{22}\,\text{m}$.)

Solution. Convert H_0 to SI units:

$$H_0 = \frac{70 \times 10^3\ \text{m s}^{-1}}{3.09 \times 10^{22}\ \text{m}} = 2.27 \times 10^{-18}\ \text{s}^{-1}$$

$$\text{age} \approx \frac{1}{H_0} = \frac{1}{2.27 \times 10^{-18}} = 4.41 \times 10^{17}\ \text{s}$$

Converting to years (1 year $= 3.156 \times 10^7$ s):

$$\text{age} \approx \frac{4.41 \times 10^{17}}{3.156 \times 10^7} = 1.40 \times 10^{10}\ \text{years}$$

This is close to the currently accepted age of the universe, about 13.8 billion years.

VN GIẢI THÍCH TIẾNG VIỆT

Dịch chuyển đỏ $z = \Delta\lambda/\lambda \approx v/c$ cho biết vận tốc lùi xa của thiên hà. Định luật Hubble $v = H_0 d$ liên hệ vận tốc lùi xa với khoảng cách, và cho phép ước lượng tuổi vũ trụ bằng $1/H_0$ nếu coi tốc độ giãn nở không đổi theo thời gian.

(!) Lỗi thường gặp: The Hubble constant H_0 is usually quoted in mixed units, km s^{-1} Mpc^{-1}. When using $v = H_0 d$, keep v in km s^{-1} and d in Mpc (they will cancel correctly), or convert everything to consistent SI units (m, s) throughout – do not mix km with Mpc without converting.

12.6 Practice Questions

Bài tập luyện tập

1. A metal has threshold frequency $f_0 = 5.0 \times 10^{14}$ Hz. Which of the following frequencies will **not** cause photoelectric emission?

(A) 4.0×10^{14} Hz

(B) 5.5×10^{14} Hz

(C) 6.0×10^{14} Hz

(D) 7.0×10^{14} Hz

Lời giải chi tiết

Emission requires $f \geq f_0 = 5.0 \times 10^{14}$ Hz. Only 4.0×10^{14} Hz is below this. **Answer: A.**

Bài tập luyện tập

2. A sodium surface (work function $\phi = 2.3$ eV) is illuminated with light of wavelength 380 nm. Find KE_{max} and the stopping potential.

Lời giải chi tiết

$f = c/\lambda = (3.00 \times 10^8)/(380 \times 10^{-9}) = 7.89 \times 10^{14}$ Hz.
$hf = (6.63 \times 10^{-34})(7.89 \times 10^{14}) = 5.24 \times 10^{-19}$ J $= 3.27$ eV.
$KE_{\text{max}} = 3.27 - 2.3 = 0.97$ eV; stopping potential $V_s = 0.97$ V.

Bài tập luyện tập

3. In the photoelectric effect, increasing the intensity of light (above the threshold frequency) increases:

(A) KE_{max} of the photoelectrons

(B) the photocurrent (rate of electron emission)

(C) the threshold frequency

(D) the work function

Lời giải chi tiết

Intensity increases the number of photons arriving per second, which increases the rate of photoelectron emission, not their maximum kinetic energy. **Answer: B.**

Bài tập luyện tập

4. Light of frequency 9.0×10^{14} Hz produces a stopping potential of 2.00 V on a metal surface. Find the work function of the metal, in eV.

Lời giải chi tiết

$hf = (6.63 \times 10^{-34})(9.0 \times 10^{14}) = 5.97 \times 10^{-19}$ J $= 3.73$ eV.
$KE_{\text{max}} = eV_s = 2.00$ eV.
$\phi = hf - KE_{\text{max}} = 3.73 - 2.00 = 1.73$ eV.

Bài tập luyện tập

5. A metal has work function $\phi = 4.5 \times 10^{-19}$ J. Find its threshold wavelength.

Lời giải chi tiết

$f_0 = \phi/h = (4.5 \times 10^{-19})/(6.63 \times 10^{-34}) = 6.79 \times 10^{14}$ Hz.
$\lambda_0 = c/f_0 = (3.00 \times 10^8)/(6.79 \times 10^{14}) = 4.42 \times 10^{-7}$ m $= 442$ nm.

Bài tập luyện tập

6. Find the de Broglie wavelength of a neutron ($m = 1.67 \times 10^{-27}$ kg) moving at 2.0×10^3 m s^{-1}.

Lời giải chi tiết

$p = mv = (1.67 \times 10^{-27})(2.0 \times 10^3) = 3.34 \times 10^{-24}$ kg m s^{-1}.
$\lambda = h/p = (6.63 \times 10^{-34})/(3.34 \times 10^{-24}) = 1.99 \times 10^{-10}$ m $= 0.199$ nm.

Bài tập luyện tập

7. An electron has kinetic energy 100 eV. Find its de Broglie wavelength.

Lời giải chi tiết

$KE = 100 \times 1.60 \times 10^{-19} = 1.60 \times 10^{-17}$ J.
$v = \sqrt{2KE/m_e} = \sqrt{(2 \times 1.60 \times 10^{-17})/(9.11 \times 10^{-31})} = 5.93 \times 10^6$ m s^{-1}.
$p = m_e v = (9.11 \times 10^{-31})(5.93 \times 10^6) = 5.40 \times 10^{-24}$ kg m s^{-1}.

$\lambda = h/p = (6.63 \times 10^{-34})/(5.40 \times 10^{-24}) = 1.23 \times 10^{-10}$ m $= 0.123$ nm.

Bài tập luyện tập

8. Explain why the wave nature of a thrown ball (mass 0.150 kg, speed 30 m s^{-1}) is never observed in everyday life.

Lời giải chi tiết

Its de Broglie wavelength is $\lambda = h/p = (6.63 \times 10^{-34})/(4.50) = 1.47 \times 10^{-34}$ m, about 10^{24} times smaller than an atom. No object or slit in the everyday world is anywhere near this size, so diffraction or interference of the ball's matter wave can never be observed.

Bài tập luyện tập

9. In hydrogen, $E_4 = -0.85$ eV and $E_1 = -13.6$ eV. Find the energy and wavelength of the photon emitted in the transition $n = 4 \to n = 1$.

Lời giải chi tiết

$\Delta E = E_4 - E_1 = -0.85 - (-13.6) = 12.75$ eV $= 2.04 \times 10^{-18}$ J.
$\lambda = hc/\Delta E = (6.63 \times 10^{-34})(3.00 \times 10^{8})/(2.04 \times 10^{-18}) = 9.75 \times 10^{-8}$ m $= 97.5$ nm (ultraviolet, Lyman series).

Bài tập luyện tập

10. A hydrogen spectral line has wavelength 486.1 nm. Find the photon energy in eV, and identify which hydrogen transition ($n = 2 \leftrightarrow n = 4$) it corresponds to.

Lời giải chi tiết

$E = hc/\lambda = (6.63 \times 10^{-34})(3.00 \times 10^{8})/(486.1 \times 10^{-9}) = 4.09 \times 10^{-19}$ J $= 2.56$ eV.
This matches $E_4 - E_2 = -0.85 - (-3.40) = 2.55$ eV, the H$_\beta$ line of the Balmer series.

Bài tập luyện tập

11. A spectrum shows dark lines superimposed on a continuous bright background. This is:

(A) an emission spectrum
(B) an absorption spectrum
(C) a black-body spectrum only
(D) a line spectrum with no physical cause

Lời giải chi tiết

Dark lines on a continuous bright background occur when specific wavelengths are absorbed by a cooler gas. **Answer: B.**

Bài tập luyện tập

12. A hydrogen atom's electron moves from $n = 2$ ($E_2 = -3.40$ eV) to $n = 4$ ($E_4 = -0.85$ eV). Is this emission or absorption? Find the photon energy and wavelength involved.

Lời giải chi tiết

The electron moves to a higher energy level, so a photon must be **absorbed**.
$\Delta E = E_4 - E_2 = -0.85 - (-3.40) = 2.55$ eV $= 4.08 \times 10^{-19}$ J.
$\lambda = hc/\Delta E = (6.63 \times 10^{-34})(3.00 \times 10^8)/(4.08 \times 10^{-19}) = 4.88 \times 10^{-7}$ m $= 488$ nm.

Bài tập luyện tập

13. At a muscle-fat boundary, $Z_1 = 1.50 \times 10^6$ kg m^{-2} s^{-1} and $Z_2 = 1.70 \times 10^6$ kg m^{-2} s^{-1}. Find the percentage of intensity reflected.

Lời giải chi tiết

$$\frac{I_r}{I_0} = \left(\frac{1.70 - 1.50}{1.70 + 1.50}\right)^2 = \left(\frac{0.20}{3.20}\right)^2 = (0.0625)^2 = 0.0039$$

About 0.39% of the intensity is reflected (most is transmitted).

Bài tập luyện tập

14. An ultrasound pulse in tissue ($c = 1.5 \times 10^3$ m s^{-1}) returns as an echo after $60\ \mu$s. Find the depth of the reflecting boundary.

Lời giải chi tiết

$$d = \frac{ct}{2} = \frac{(1.5 \times 10^3)(60 \times 10^{-6})}{2} = \frac{0.090}{2} = 0.045\ \text{m} = 4.5\ \text{cm}$$

Bài tập luyện tập

15. A boundary lies at a depth of 8.0 cm in tissue where $c = 1.5 \times 10^3$ m s^{-1}. Find the expected time delay of the echo.

Lời giải chi tiết

$$t = \frac{2d}{c} = \frac{2(0.080)}{1.5 \times 10^3} = 1.07 \times 10^{-4}\ \text{s} = 107\ \mu\text{s}$$

Bài tập luyện tập

16. Explain how a single piezoelectric crystal can act as both the transmitter and receiver of ultrasound pulses.

Lời giải chi tiết

An applied alternating voltage causes the crystal to vibrate mechanically at the same frequency (converse piezoelectric effect), emitting an ultrasound pulse. When a returning echo causes the crystal to vibrate mechanically, this deformation generates a small alternating voltage across the crystal (piezoelectric effect), which is detected and processed as the received signal.

Bài tập luyện tập

17. Soft tissue has $\mu = 0.60\,\text{cm}^{-1}$. What thickness reduces the transmitted X-ray intensity to 10% of its initial value?

Lời giải chi tiết

$$x = -\frac{1}{\mu}\ln\left(\frac{I}{I_0}\right) = -\frac{1}{0.60}\ln(0.10) = \frac{2.303}{0.60} = 3.84\,\text{cm}$$

Bài tập luyện tập

18. X-rays pass through 3.0 cm of a tissue sample and emerge with 40% of the original intensity. Find the linear attenuation coefficient.

Lời giải chi tiết

$$\mu = -\frac{1}{x}\ln\left(\frac{I}{I_0}\right) = -\frac{1}{3.0}\ln(0.40) = \frac{0.916}{3.0} = 0.31\,\text{cm}^{-1}$$

Bài tập luyện tập

19. Explain why bone appears white and soft tissue appears darker on an X-ray image.

Lời giải chi tiết

Bone has a much larger attenuation coefficient μ than soft tissue, so it absorbs far more of the X-ray beam. Little radiation reaches the film behind bone, so that region is under-exposed and appears light/white. Soft tissue transmits more radiation, exposing the film more and appearing darker.

Bài tập luyện tập

20. A tissue has $\mu = 0.23\,\text{cm}^{-1}$. Find the thickness required to reduce the X-ray intensity to half its initial value.

Lời giải chi tiết

$$x_{1/2} = \frac{\ln 2}{\mu} = \frac{0.693}{0.23} = 3.0\,\text{cm}$$

Bài tập luyện tập

21. A star has $\lambda_{\text{max}} = 350\,\text{nm}$. Find its surface temperature.

Lời giải chi tiết

$$T = \frac{2.90 \times 10^{-3}}{350 \times 10^{-9}} = 8.29 \times 10^{3}\,\text{K}$$

Bài tập luyện tập

22. A star has radius $r = 5.0 \times 10^8$ m and surface temperature $T = 4000$ K. Find its luminosity.

Lời giải chi tiết

$r^2 = 2.50 \times 10^{17}$ m^2; $T^4 = (4000)^4 = 2.56 \times 10^{14}$ K^4.

$$L = 4\pi r^2 \sigma T^4 = 4\pi(2.50 \times 10^{17})(5.67 \times 10^{-8})(2.56 \times 10^{14}) = 4.56 \times 10^{25}\text{ W}$$

Bài tập luyện tập

23. A star has luminosity $L = 3.85 \times 10^{26}$ W and its observed flux at Earth is $F = 1.00 \times 10^{-12}$ W m^{-2}. Find its distance in metres and in light-years (1 ly $= 9.47 \times 10^{15}$ m).

Lời giải chi tiết

$$d = \sqrt{\frac{L}{4\pi F}} = \sqrt{\frac{3.85 \times 10^{26}}{4\pi(1.00 \times 10^{-12})}} = \sqrt{3.06 \times 10^{37}} = 5.53 \times 10^{18}\text{ m}$$

In light-years: $d = (5.53 \times 10^{18})/(9.47 \times 10^{15}) = 585$ ly.

Bài tập luyện tập

24. Which of the following is **not** evidence for the Big Bang / an expanding universe?

(A) Cosmic microwave background radiation

(B) Redshift of light from distant galaxies

(C) Hubble's law ($v \propto d$) for galaxies

(D) The Moon's orbital period around the Earth

Lời giải chi tiết

The Moon's orbital period is unrelated to cosmology and is not evidence for the Big Bang. **Answer: D.**

Bài tập luyện tập

25. Star P has surface temperature 1.5 times that of star Q, and both have the same radius. Find the ratio L_P/L_Q.

Lời giải chi tiết

Since $L \propto T^4$ for the same radius,

$$\frac{L_P}{L_Q} = \left(\frac{T_P}{T_Q}\right)^4 = (1.5)^4 = 5.06$$

Bài tập luyện tập

26. A spectral line normally at 656.3 nm is observed at 662.0 nm in a galaxy's spectrum. Find the redshift z and the recession velocity.

Lời giải chi tiết

$\Delta\lambda = 662.0 - 656.3 = 5.7\,\text{nm}$.

$$z = \frac{5.7}{656.3} = 8.69 \times 10^{-3}$$

$$v = zc = (8.69 \times 10^{-3})(3.00 \times 10^{5}\,\text{km s}^{-1}) = 2.61 \times 10^{3}\,\text{km s}^{-1}$$

Bài tập luyện tập

27. A galaxy is at a distance of $200\,\text{Mpc}$. Taking $H_0 = 70\,\text{km s}^{-1}\,\text{Mpc}^{-1}$, find its recession velocity.

Lời giải chi tiết

$$v = H_0 d = (70)(200) = 1.4 \times 10^{4}\,\text{km s}^{-1}$$

Bài tập luyện tập

28. A galaxy's spectrum shows a redshift of $z = 0.020$. Taking $H_0 = 70\,\text{km s}^{-1}\,\text{Mpc}^{-1}$, find its distance.

Lời giải chi tiết

$v = zc = (0.020)(3.00 \times 10^{5}) = 6000\,\text{km s}^{-1}$.

$$d = \frac{v}{H_0} = \frac{6000}{70} = 85.7\,\text{Mpc}$$

Bài tập luyện tập

29. If measurements gave a Hubble constant of $H_0 = 65\,\text{km s}^{-1}\,\text{Mpc}^{-1}$ (take $1\,\text{Mpc} = 3.09 \times 10^{22}\,\text{m}$), estimate the resulting age of the universe.

Lời giải chi tiết

$$H_0 = \frac{65 \times 10^{3}}{3.09 \times 10^{22}} = 2.10 \times 10^{-18}\,\text{s}^{-1}$$

$$\text{age} \approx \frac{1}{H_0} = 4.75 \times 10^{17}\,\text{s} = \frac{4.75 \times 10^{17}}{3.156 \times 10^{7}} = 1.51 \times 10^{10}\,\text{years}$$

(A smaller assumed H_0 gives a larger estimated age.)

Bài tập luyện tập

30. In the photoelectric effect, which quantity depends only on the frequency of the incident light and not on its intensity?

(A) The photocurrent

(B) The number of photoelectrons emitted per second

(C) The maximum kinetic energy of the photoelectrons

(D) The total power absorbed by the metal

Lời giải chi tiết

$KE_{\text{max}} = hf - \phi$ depends only on frequency; intensity affects only the rate of photon arrival and hence the photocurrent. **Answer: C.**

Ôn tập nhanh:

- Luôn kiểm tra đơn vị và thứ nguyên của kết quả trước khi kết luận.
- Bộ công thức SUVAT chỉ áp dụng cho chuyển động có gia tốc không đổi.
- Bảo toàn động lượng là một phương trình vectơ, phải xét đúng dấu và hướng của vận tốc.
- Trong các định luật chất khí, nhiệt độ luôn phải đổi sang kelvin ($T = t + 273$).
- Chu kỳ dao động điều hoà (SHM) không phụ thuộc vào biên độ dao động.
- Hai công thức lượng tử quan trọng nhất: năng lượng photon $E = hf$ và bước sóng de Broglie $\lambda = h/p$.
- Định luật Hubble $v = H_0 d$ và độ dịch chuyển đỏ $z = \Delta\lambda/\lambda$ liên hệ vận tốc lùi xa của thiên hà với khoảng cách của nó.
- Cường độ tia X truyền qua vật chất giảm theo hàm mũ $I = I_0 e^{-\mu x}$, còn cường độ bức xạ ánh sáng giảm theo bình phương khoảng cách $F = L/(4\pi d^2)$.

Đồng hành cùng 8020 English

Cảm ơn bạn đã học cùng bộ giáo trình quốc tế 8020. **Điểm thật · Thầy thật · Kết quả thật** — nhiều học viên chỉ cần 1–2 khoá để đạt kết quả mong muốn.

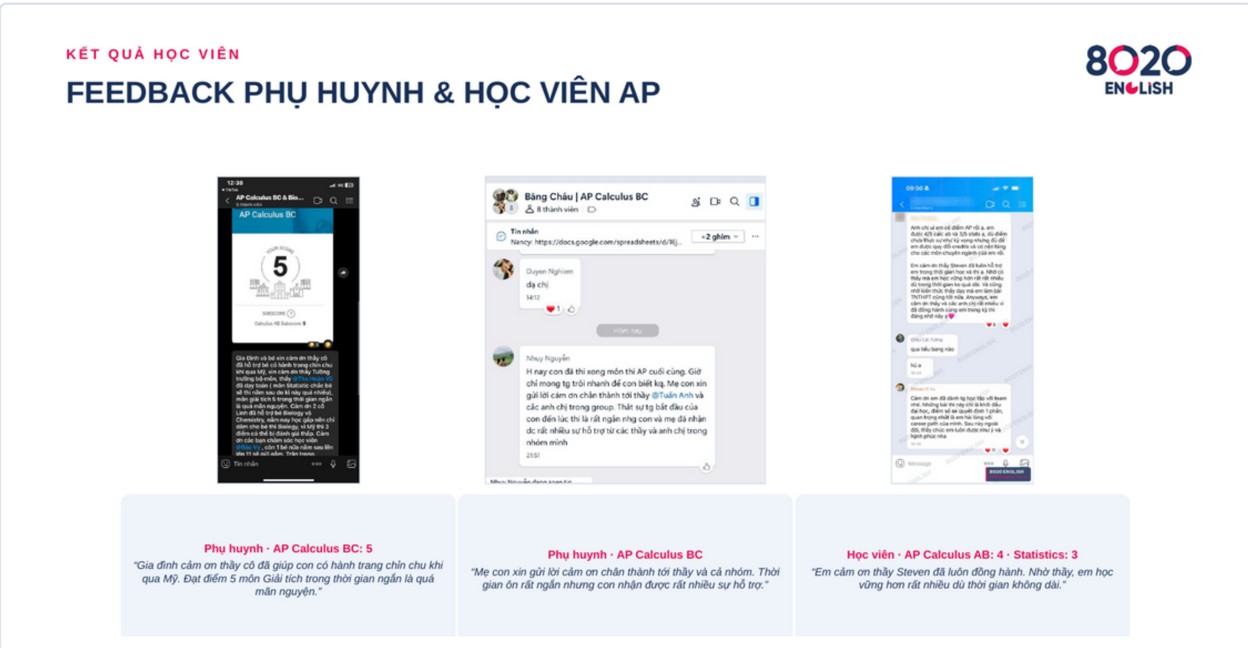
Cảm nhận học viên & phụ huynh

> *"Bạn đã cải thiện được tư duy Writing — kỹ năng từng là nỗi sợ lớn nhất — và cuối cùng đã đậu vào trường top như mong muốn. Thái độ học tập cực kỳ nghiêm túc; ngay cả khi nằm viện vẫn cố gắng học đều đặn."*
>
> — Phan Thị Thanh Hằng · IELTS Overall 8.5

> *"Cần tất cả kỹ năng trên 7.5 để xét vào trường top 1 Anh Quốc về Y học (chương trình UKFPO của NHS) — và đã đạt mục tiêu sau khi học Writing 1–1."*
>
> — Trần Hoàng Bảo Ngọc · IELTS Overall 8.0 (Writing 6.0 → 7.5)

> *"Gia đình và bé xin cảm ơn thầy cô đã giúp con có hành trang chỉn chu khi qua Mỹ. Đạt điểm 5 môn Giải tích trong thời gian ngắn là quá mãn nguyện."*
>
> — Phụ huynh · AP Calculus BC 5/5

> *"Em cảm ơn thầy đã luôn đồng hành. Nhờ thầy, em học vững hơn rất nhiều dù thời gian ôn không dài."*
>
> — Học viên · AP Calculus AB 4



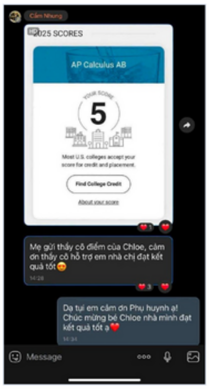
Feedback phụ huynh & học viên AP — nhiều môn đạt điểm 4 & 5

Điểm thi thật của học viên

KẾT QUẢ HỌC VIÊN

ĐIỂM SỐ AP

AP Biology: 4 · AP Chemistry: 4



AP Calculus AB: 5

AP Calculus BC: 5

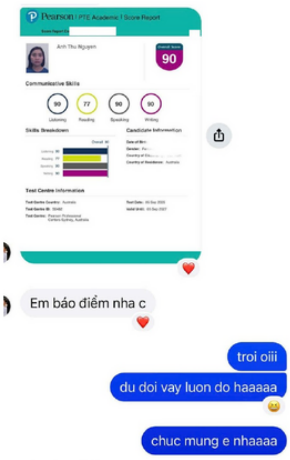
AP Chemistry: 5

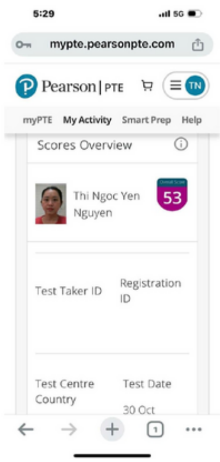
Bảng điểm AP thật của học viên – nhiều môn đạt điểm 4 & 5

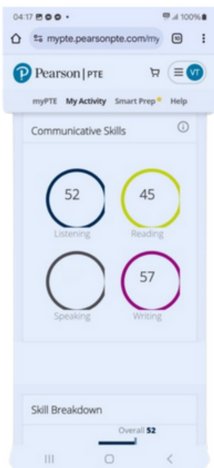
Bảng điểm AP thật — Calculus AB/BC 5/5 · Biology & Chemistry 4–5 (College Board)

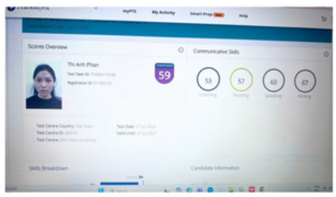
KẾT QUẢ HỌC VIÊN

ĐIỂM SỐ PTE







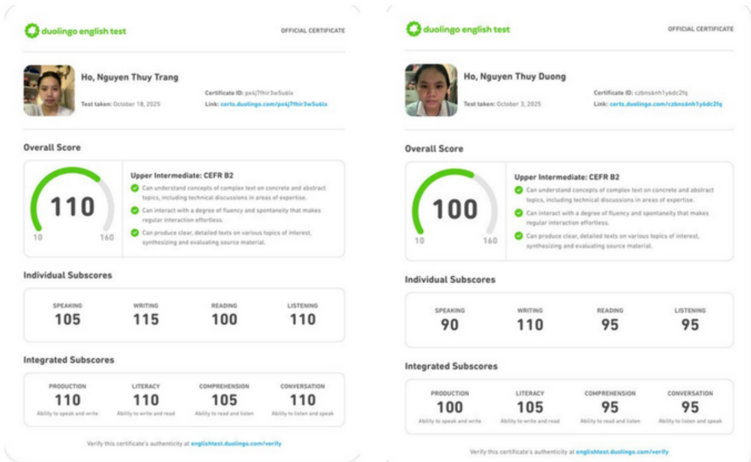


Bảng điểm PTE thật của học viên

Học viên 8020 — PTE Academic 90

KẾT QUẢ HỌC VIÊN

ĐIỂM SỐ DUOLINGO



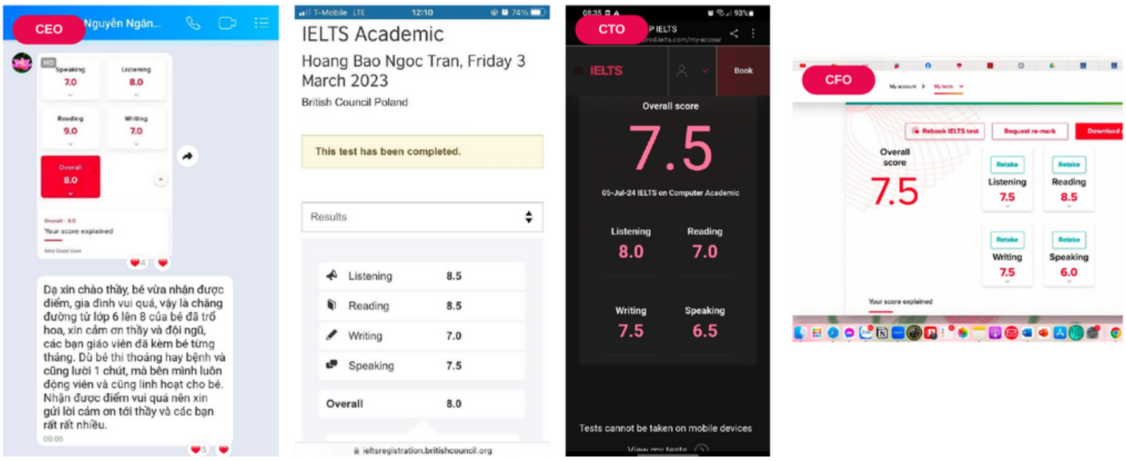
Duolingo English Test – 110 & 100

Học viên 8020 — Duolingo English Test 110 & 100

KẾT QUẢ HỌC VIÊN

THÀNH TÍCH HỌC VIÊN

Bảng điểm thi thật – chất lượng được giám sát nghiêm ngặt. Một số học viên chỉ cần 1–2 khóa để đạt kết quả mong muốn.



Học viên 8020 — IELTS 8.0 / 7.5 / 8.5 (báo điểm thật)