AP Physics 1: Algebra-Based

Giáo trình luyện thi chuyên sâu

Song ngữ Anh–Việt

8020 ENGLISH

5A/2 Trần Phú, P.4, Q.5, TP.HCM · luyenthi8020.com

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Lời mở đầu · Foreword

Cuốn sách này thuộc bộ giáo trình luyện thi chương trình quốc tế của **8020 English (8020 Education)** — trung tâm luyện thi trọng điểm **IELTS · SAT · AP · IB · A-level · IGCSE** và săn học bổng du học. Toàn bộ nội dung được biên soạn bởi đội ngũ **Giáo sư · Tiến sĩ · Thạc sĩ** tốt nghiệp các trường top đầu thế giới, bám sát khung chương trình chính thức, trình bày đầy đủ lý thuyết, ví dụ mẫu, hình minh hoạ và hệ thống bài tập có lời giải chi tiết.

This book is part of the 8020 English international exam-prep series: complete English theory with Vietnamese explanations, worked examples, illustrations and fully-solved practice, aligned to the official framework.

Thành tích 8020 English

9.0 IELTS cao nhất	**1570** SAT cao nhất
5/5 AP — điểm tuyệt đối	**90/100** IB Diploma
100% GV $\geq$ 8.0 IELTS / 1500 SAT	**CMU** Tiến sĩ ĐH #1 Hoa Kỳ về CS

> **Đội ngũ tiêu biểu:** Thầy Châu Cát Tường (Academic Head, ThS Western Sydney, top 1% IELTS 8.5) · TS Nguyễn Anh Huy (Carnegie Mellon, cựu kỹ sư Amazon) · TS Nguyễn Phước Trường (Toán, ĐH Klagenfurt) · cùng đội ngũ giảng viên SAT 1500–1600, IELTS 8.0–8.5.

Kết quả học viên — điểm thi thật: IELTS 8.0–9.0 · SAT 1550 / 1570 / 1590 · AP 5/5 (Calculus BC, Microeconomics) · IB 90/100 (Economics) · Duolingo 110 · PTE 90 · TOEFL 120. **Bảng vàng SAT:** Khánh Đăng 1510 · Thảo Nguyên 1520 · Tâm Anh 1530.

"Cuối cùng đã đậu vào trường top như mong muốn." — Phan Thị Thanh Hằng, IELTS Overall 8.5.

Cách dùng sách hiệu quả: mỗi Unit gồm (1) **Lý thuyết trọng tâm** tiếng Anh kèm **giải thích tiếng Việt**; (2) **Ví dụ mẫu** có lời giải; (3) **Hệ thống bài tập** kèm **lời giải chi tiết**. Hãy tự làm bài trước khi xem lời giải để đạt hiệu quả tối đa.

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ĐỘI NGŨ

ĐỘI NGŨ GIÁO VIÊN TẬN TÂM

**Tối thiểu 8.0 IELTS Overall*

Gv. Nguyễn Nhật Nam
IELTS 8.5 Overall

Gv. Nguyễn Khanh
IELTS 8.0 Overall

Gv. Nguyễn Đan Vy
IELTS 8.0 Overall

Giảng viên 8020 — IELTS 8.5 & 8.0 Overall (British Council · IDP · Cambridge)

KẾT QUẢ HỌC VIÊN

THÀNH TÍCH HỌC VIÊN

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Học viên 8020 — IELTS 8.0 Overall (bảng điểm thật)

ĐỘI NGŨ

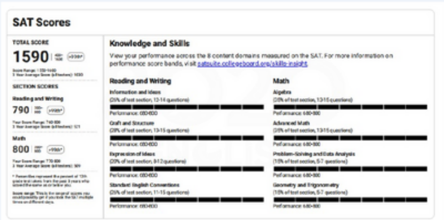
ĐỘI NGŨ GIÁO VIÊN TẬN TÂM

**Tối thiểu 1500 SAT Overall*

Thầy Quang Minh

Đại học Bách Khoa Hà Nội

SAT 1590 | IELTS 8.0



THẾ MẠNH & KINH NGHIỆM

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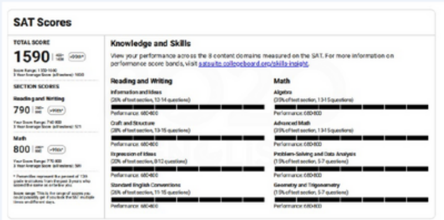
- Dạy nhanh band 1400–1550+
- Chiến thuật làm bài & quản lý thời gian

Cô Phương Vy

Đại học Ngoại thương

SAT 1590 | IELTS 8.0



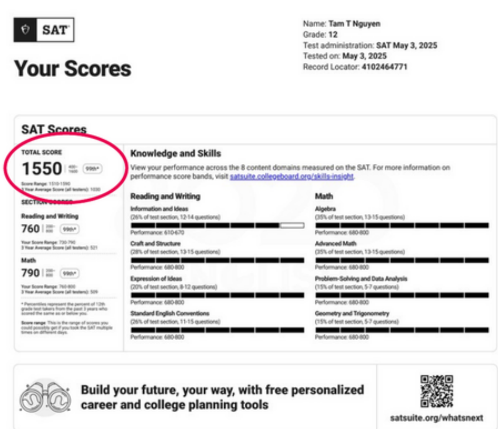
THẾ MẠNH & KINH NGHIỆM

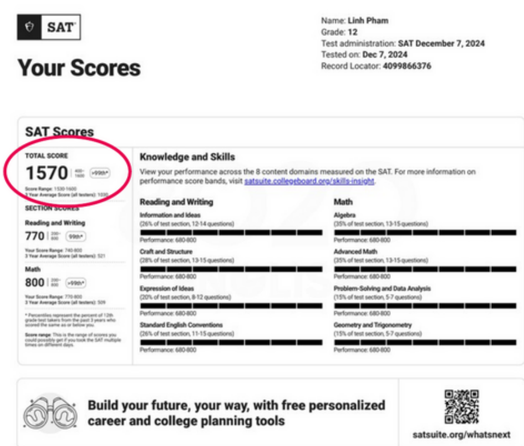
- SAT Verbal Instructor (Springboard, QAS)
- Dạy lớp nhỏ & xây dựng tài liệu riêng
- Chuyên Reading & Writing: Logic – Grammar

Giảng viên 8020 — SAT 1590 (ĐH Bách Khoa Hà Nội & ĐH Ngoại thương)

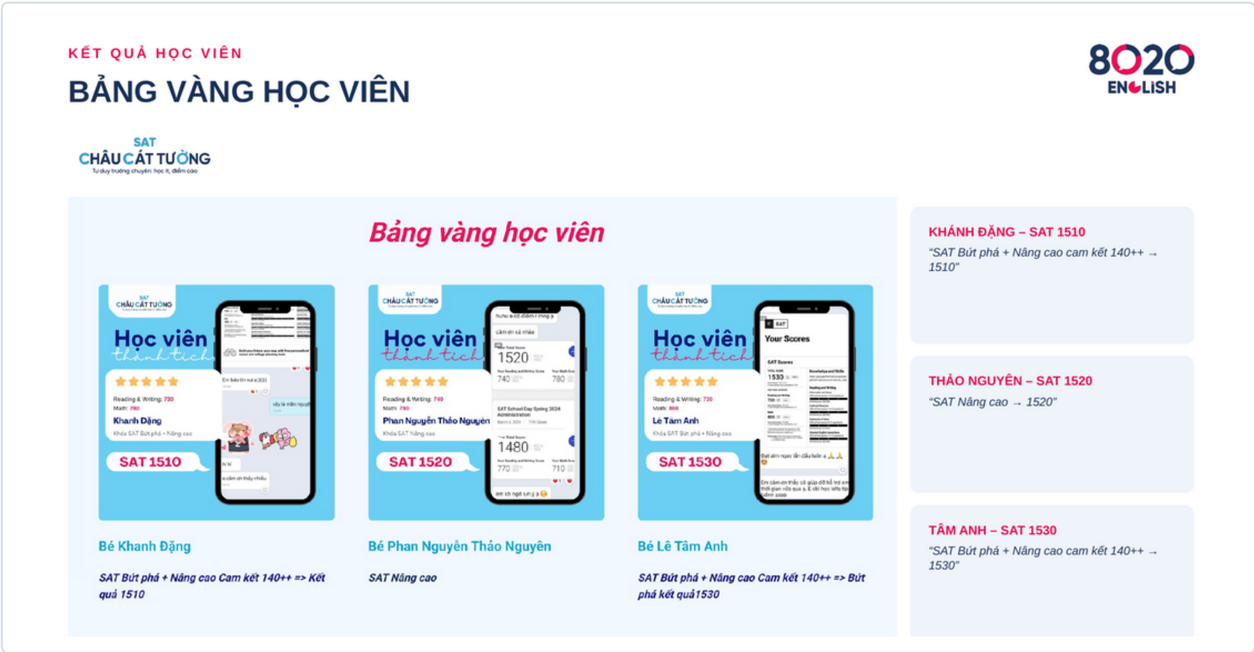
KẾT QUẢ HỌC VIÊN

TUTOR 1-1 – ĐIỂM SỐ ẤN TƯỢNG

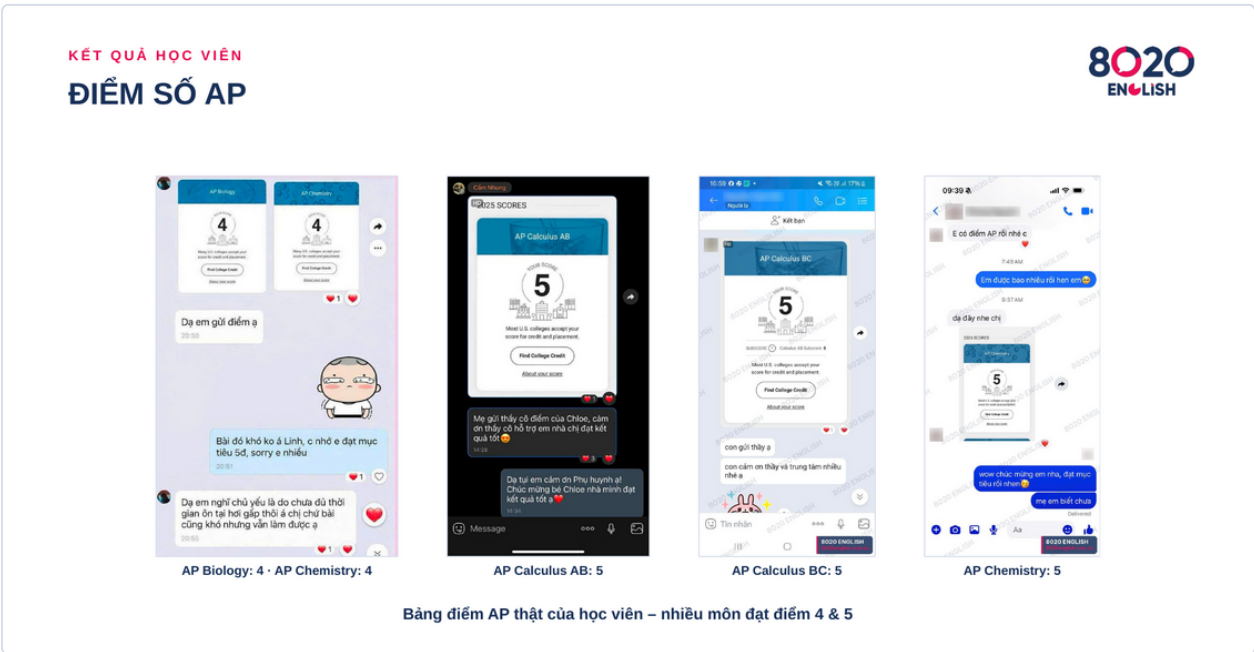




Học viên 8020 — SAT 1550 & 1570 (College Board)



Bảng vàng SAT — Học viên đạt 1510 · 1520 · 1530



Điểm AP thật — Calculus AB/BC 5/5 · Biology & Chemistry 4–5 (College Board)

Mục lục • Contents

Convention used throughout this book: free-fall / gravitational acceleration $g = 9.8\,\mathrm{m/s^2}$ (College Board formula sheet value), taken as positive magnitude; direction is tracked with $+/-$ signs chosen per problem. For incline problems we often use the classic 3–4–5 angle $\theta = 37^\circ$, for which $\sin 37^\circ \approx 0.60$ and $\cos 37^\circ \approx 0.80$.

Kinematics

Mục tiêu Unit: Đại lượng vô hướng/vector; vị trí, độ dịch chuyển, quãng đường; vận tốc trung bình/tức thời, tốc độ; gia tốc và quy tắc dấu; đọc đồ thị x–t, v–t, a–t; bốn phương trình động học với gia tốc không đổi; rơi tự do; chuyển động ném ngang và ném xiên (2D).

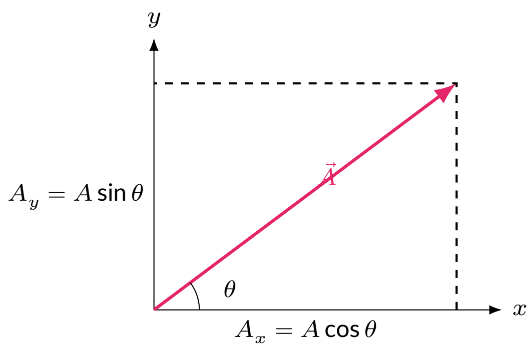
1.1 Scalars, Vectors, and Reference Frames

A **scalar** is a quantity fully described by a magnitude (a number with a unit): mass, time, distance, speed, temperature. A **vector** is a quantity that has both magnitude *and* direction: displacement, velocity, acceleration, force, momentum. Vectors are written with an arrow, $\vec{A}$, or in bold; their magnitude is written $|\vec{A}|$ or simply A.

All motion is described relative to a **reference frame** — a coordinate system with a chosen origin and a chosen positive direction. Every kinematics problem should begin by stating this choice explicitly, because every sign in the problem (of position, velocity, acceleration) depends on it.

Khái niệm cốt lõi

In one dimension, a positive/negative sign carries all the directional information a vector needs: choose "positive" to mean one direction (e.g. right, up, east) and every quantity pointing that way is $+$, everything pointing the opposite way is $-$. In two dimensions, a vector is described either by a magnitude and an angle, or by perpendicular **components**: $A_x = A\cos\theta$, $A_y = A\sin\theta$ (with θ measured from the $+x$ axis), and $A = \sqrt{A_x^2 + A_y^2}$.



Resolving a vector $\vec{A}$ into perpendicular x- and y-components.

VN GIẢI THÍCH TIẾNG VIỆT

Đại lượng **vô hướng** (scalar) chỉ có độ lớn (khối lượng, thời gian, quãng đường, tốc độ). Đại lượng **vector** có cả độ lớn lẫn hướng (độ dịch chuyển, vận tốc, gia tốc, lực). Trong bài toán 1 chiều, dấu $+/-$ thể hiện hướng; trong bài toán 2 chiều, ta phân tích vector thành hai thành phần vuông góc $A_x = A\cos\theta$ và $A_y = A\sin\theta$, rồi xử lý từng phương độc lập.

Ví dụ 1 · Adding perpendicular vectors

A hiker walks 8.0 km east, then 6.0 km north. Find the magnitude and direction of her total displacement.

The two legs are perpendicular, so they form the legs of a right triangle with the resultant as hypotenuse:

$$A = \sqrt{(8.0)^2 + (6.0)^2} = \sqrt{64 + 36} = \sqrt{100} = 10\,\text{km}.$$

Direction: $\theta = \arctan\left(\frac{6.0}{8.0}\right) = \arctan(0.75) = 37^\circ$ north of east.

1.2 Position, Displacement, and Distance

Position x (or y) locates an object relative to the chosen origin. **Displacement** $\Delta x = x_f - x_i$ is the change in position — a vector whose sign shows direction, and whose size can be smaller than the total path length. **Distance** is the total length of the path traveled, always a positive scalar.

> Displacement can be zero (or even point "backward") even when an object travels a long path, because displacement only compares the *start* and *end* points. Distance can never decrease and is always $\geq |\Delta x|$.

Ví dụ 2 · Displacement vs. distance

A particle starts at $x = +2.0$ m, moves to $x = -3.0$ m, then to $x = +1.0$ m. Find its total displacement and total distance traveled.

Displacement: $\Delta x = x_f - x_i = 1.0 - 2.0 = -1.0$ m (magnitude 1.0 m, in the negative direction). Distance: leg 1 $= |(-3.0) - (2.0)| = 5.0$ m; leg 2 $= |1.0 - (-3.0)| = 4.0$ m. Total distance $= 5.0 + 4.0 = 9.0$ m.

> **(!) Lỗi thường gặp:** Distance and $|\Delta x|$ are **only** equal when the object moves in one direction the whole time without reversing. The instant an object turns around, distance $> |\Delta x|$.

1.3 Average and Instantaneous Velocity, Speed

Average velocity is displacement divided by the elapsed time, $\bar{v} = \frac{\Delta x}{\Delta t}$ — a vector, sign matters. **Average speed** is total distance divided by total time, always $\geq |\bar{v}|$. **Instantaneous velocity** is the velocity at one particular moment; graphically, it is the slope of the tangent line to the x–t curve at that instant. **Instantaneous speed** is simply $|v|$.

Ví dụ 3 · Average velocity vs. average speed

A runner covers 400 m east in 100 s, then 100 m west in 25 s. Find her average velocity and average speed for the whole trip.

Net displacement: $\Delta x = 400 - 100 = 300$ m east. Total time: $t = 100 + 25 = 125$ s.

$$\bar{v} = \frac{300}{125} = 2.4\,\text{m/s (east)}.$$

Total distance $= 400 + 100 = 500$ m, so average speed $= \frac{500}{125} = 4.0$ m/s — nearly double the average velocity, because the runner reversed direction.

VN **GIẢI THÍCH TIẾNG VIỆT**

Vận tốc trung bình là độ dịch chuyển chia thời gian ($\bar{v} = \Delta x/\Delta t$) — có thể âm, dương, hoặc bằng 0. **Tốc độ trung bình** là quãng đường chia thời gian — luôn dương và luôn $\geq |\bar{v}|$. Trên đồ thị x–t, vận tốc tức thời chính là **độ dốc của tiếp tuyến** tại thời điểm đó.

1.4 Acceleration

Average acceleration is the rate of change of velocity, $\bar{a} = \dfrac{\Delta v}{\Delta t}$. **Instantaneous acceleration** is the slope of the tangent line on a v–t graph. Acceleration is a vector; its sign is independent of velocity's sign.

Khái niệm cốt lõi

An object **speeds up** when v and a point in the *same* direction (same sign) and **slows down** when they point in *opposite* directions (opposite signs) — regardless of whether the object itself moves in the $+$ or $-$ direction. "Deceleration" is a description of slowing down, *not* a synonym for "negative acceleration."

Ví dụ 4 · Sign of velocity vs. sign of acceleration

A car moving at $v_0 = +20\,\text{m/s}$ (east) experiences a constant acceleration $a = -4.0\,\text{m/s}^2$ (i.e. pointing west). Find its velocity at $t = 3.0\,\text{s}$ and at $t = 6.0\,\text{s}$, and describe the motion in each case.

At $t = 3.0\,\text{s}$: $v = v_0 + at = 20 + (-4.0)(3.0) = +8.0\,\text{m/s}$. Still moving east ($v > 0$), but $a < 0$ opposes v, so the car is **slowing down**.
At $t = 6.0\,\text{s}$: $v = 20 + (-4.0)(6.0) = -4.0\,\text{m/s}$. The car is now moving **west** ($v < 0$), and $a < 0$ points the same way, so it is **speeding up** — in the negative direction.

VN **GIẢI THÍCH TIẾNG VIỆT**

Vật **tăng tốc** khi v và a **cùng dấu**, **giảm tốc** khi v và a **trái dấu** — không phụ thuộc vật đang đi theo chiều dương hay âm. Ví dụ 4 cho thấy cùng một gia tốc âm có thể vừa làm xe *chậm lại* (khi $v > 0$) vừa làm xe *nhanh dần* theo chiều ngược lại (khi $v < 0$) sau khi đổi chiều.

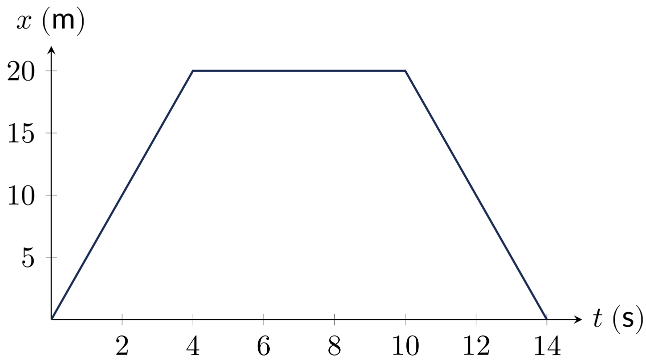
1.5 Interpreting Motion Graphs

Three graphs summarize 1D motion: position vs. time (x–t), velocity vs. time (v–t), and acceleration vs. time (a–t). Reading them correctly is a core AP Physics 1 skill.

On an x–t graph: the *slope* equals velocity. A horizontal segment means the object is at rest. A steeper slope means greater speed; a change in slope sign means the object reversed direction.

On a v–t graph: the *slope* equals acceleration; the *signed area* between the curve and the t-axis equals displacement (area above the axis is positive displacement, area below is negative). A horizontal segment means constant velocity (zero acceleration).

On an a–t graph: the *signed area* between the curve and the t-axis equals the change in velocity, Δv.



x–t graph: rises with slope $5.0\,\text{m/s}$ (0–4 s), flat (4–10 s, at rest), falls with slope $-5.0\,\text{m/s}$ (10–14 s).

Ví dụ 5 · Reading a v–t graph with a sign change

A v–t graph is a straight line from $v = +15\,\text{m/s}$ at $t = 0$ to $v = -5.0\,\text{m/s}$ at $t = 4.0\,\text{s}$. Find (a) the acceleration, (b) the time at which $v = 0$, and (c) the total displacement over the $4.0\,\text{s}$.

(a) $a = \dfrac{\Delta v}{\Delta t} = \dfrac{-5.0 - 15}{4.0 - 0} = \dfrac{-20}{4.0} = -5.0\,\text{m/s}^2$.

(b) $v = 15 + (-5.0)t = 0 \Rightarrow t = 3.0\,\text{s}$.

(c) From 0–$3.0\,\text{s}$ the graph is a triangle above the axis: area $= \frac{1}{2}(3.0)(15) = +22.5\,\text{m}$. From 3.0–$4.0\,\text{s}$ it is a triangle below the axis: area $= \frac{1}{2}(1.0)(5.0) = 2.5\,\text{m}$, counted as $-2.5\,\text{m}$. Total displacement $= 22.5 - 2.5 = +20.0\,\text{m}$ (total *distance* would be $22.5 + 2.5 = 25.0\,\text{m}$).

Quantity	Symbol	SI unit
Position / displacement	$x,\ \Delta x$	m
Velocity / speed	v	m/s
Acceleration	a	m/s^2
Time	t	s

Table 1.1: *
Core kinematic quantities and their SI units.

1.6 Constant-Acceleration Kinematics Equations

When acceleration is constant, velocity changes linearly with time and position changes quadratically. Four equations connect displacement Δx, initial velocity v_0, final velocity v, acceleration a, and time t.

1.6.1 Deriving the Equations

From the definition of acceleration, $a = \dfrac{v - v_0}{t}$, rearranged:

$$v = v_0 + at. \tag{1}$$

Because acceleration is constant, velocity increases linearly, so the *average* velocity over the interval is simply the mean of the initial and final values:

$$\bar{v} = \tfrac{1}{2}(v_0 + v) \;\Rightarrow\; \Delta x = \tfrac{1}{2}(v_0 + v)t. \tag{2}$$

Substituting equation (1) into (2) eliminates v:

$$\Delta x = \tfrac{1}{2}(v_0 + v_0 + at)t = v_0 t + \tfrac{1}{2}at^2. \tag{3}$$

Solving (1) for $t = \dfrac{v - v_0}{a}$ and substituting into (2) eliminates t:

$$v^2 = v_0^2 + 2a\,\Delta x. \tag{4}$$

$$v = v_0 + at, \qquad \Delta x = v_0 t + \tfrac{1}{2}at^2, \qquad \Delta x = \tfrac{1}{2}(v_0 + v)t, \qquad v^2 = v_0^2 + 2a\,\Delta x.$$

Each equation involves only four of the five variables $(\Delta x, v_0, v, a, t)$. Identify the three quantities you know and the one you want, then pick the equation that omits the fifth (unwanted) variable.

VN **GIẢI THÍCH TIẾNG VIỆT**

Bốn phương trình động học chỉ đúng khi gia tốc **không đổi**. Mẹo chọn công thức: liệt kê 5 đại lượng $(\Delta x, v_0, v, a, t)$, gạch chân 3 đại lượng đã biết và 1 đại lượng cần tìm, rồi chọn phương trình **không chứa** đại lượng còn lại (chưa biết và không cần tìm).

Ví dụ 6 · Takeoff distance

An airplane starts from rest and must reach a takeoff speed of $80\,\text{m/s}$ after accelerating uniformly down a $1000\,\text{m}$ runway. Find the required acceleration and the time it takes.

Known: $v_0 = 0$, $v = 80\,\text{m/s}$, $\Delta x = 1000\,\text{m}$; unknown a (equation 4, no t):

$$v^2 = v_0^2 + 2a\,\Delta x \;\Rightarrow\; a = \frac{v^2 - v_0^2}{2\Delta x} = \frac{80^2}{2(1000)} = \frac{6400}{2000} = 3.2\,\text{m/s}^2.$$

Then from equation (1): $t = \dfrac{v - v_0}{a} = \dfrac{80}{3.2} = 25\,\text{s}$.

Ví dụ 7 · Braking distance

A car traveling at $30\,\text{m/s}$ brakes to a stop over $75\,\text{m}$. Find the (constant) deceleration and the time to stop.

$v^2 = v_0^2 + 2a\Delta x \Rightarrow 0 = (30)^2 + 2a(75) \Rightarrow a = \dfrac{-900}{150} = -6.0\,\text{m/s}^2$.

$t = \dfrac{v - v_0}{a} = \dfrac{0 - 30}{-6.0} = 5.0\,\text{s}$.

(!) Lỗi thường gặp: Always define a positive direction *first*, then assign signs to v_0, v, and a consistently for the whole problem. A common error is plugging in the ”size” of a deceleration as positive when the chosen positive direction requires it to be negative.

1.7 Free Fall

Free fall is one-dimensional motion under gravity alone, with constant acceleration of magnitude $g = 9.8\,\text{m/s}^2$, always directed downward. Choosing up as positive gives $a = -g$ for every object in free fall, whether it is rising, falling, or momentarily at rest at the top of its path.

At the top of a vertical throw, $v = 0$ for an instant, but $a = -g \neq 0$ throughout the entire flight — velocity is momentarily zero, acceleration never is. By symmetry, the time to rise equals the time to fall back to the same height, and the speed on the way down (at any given height) equals the speed on the way up at that same height.

Ví dụ 8 · Ball thrown straight up

A ball is thrown straight up with initial speed $v_0 = 19.6\,\text{m/s}$. Find (a) the time to reach maximum height, (b) the maximum height, and (c) the total time until it returns to the launch point.

(a) At maximum height $v = 0$: $t_{up} = \dfrac{v_0}{g} = \dfrac{19.6}{9.8} = 2.0\,\text{s}$.

(b) $h_{\text{max}} = \dfrac{v_0^2}{2g} = \dfrac{(19.6)^2}{2(9.8)} = \dfrac{384.16}{19.6} = 19.6\,\text{m}$.

(c) By symmetry, total flight time $= 2t_{up} = 4.0\,\text{s}$.

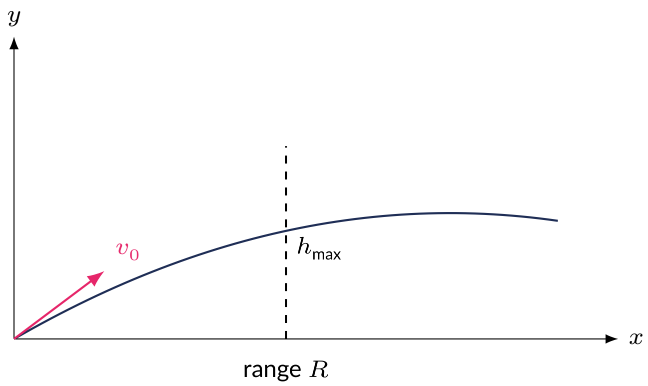
Ví dụ 9 · Dropped stone from a bridge

A stone is thrown straight down from a bridge at $5.0\,\text{m/s}$ and strikes the water $2.0\,\text{s}$ later. Find the height of the bridge above the water and the stone's impact speed.

Taking down as positive: $h = v_0 t + \frac{1}{2}gt^2 = (5.0)(2.0) + \frac{1}{2}(9.8)(2.0)^2 = 10 + 19.6 = 29.6\,\text{m}$.
$v = v_0 + gt = 5.0 + (9.8)(2.0) = 24.6\,\text{m/s}$.

1.8 Projectile Motion (Two Dimensions)

Projectile motion combines horizontal and vertical motion that are completely **independent** of one another (ignoring air resistance): horizontally $a_x = 0$ so v_x is constant; vertically $a_y = -g$, exactly like free fall. Time t is the one variable that links the two directions.



Projectile launched at angle θ above horizontal traces a parabola; v_x stays constant while v_y changes at rate $-g$.

VN GIẢI THÍCH TIẾNG VIỆT

Chuyển động ngang và dọc của vật ném xiên **độc lập** với nhau: theo phương ngang v_x **không đổi** (không có lực nào theo phương ngang khi bỏ qua sức cản không khí); theo phương dọc vật rơi tự do với $a_y = -g$. Thời gian bay t là biến chung nối hai chuyển động — thường giải phương trình **dọc trước** để tìm t, rồi thay vào phương trình **ngang**.

1.8.1 Horizontal Launch

An object launched horizontally has $v_{y0} = 0$; it falls exactly like a dropped object while moving sideways at constant $v_x = v_0$.

Ví dụ 10 · Horizontal launch from a height

A stone is thrown horizontally at $8.0\,\text{m/s}$ from a height of $19.6\,\text{m}$. Find the time of flight, the horizontal range, and the impact speed.

Vertical (falls from rest): $h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(19.6)}{9.8}} = \sqrt{4.0} = 2.0\,\text{s}$.
Horizontal: $R = v_0 t = (8.0)(2.0) = 16\,\text{m}$.
At impact: $v_y = gt = (9.8)(2.0) = 19.6\,\text{m/s}$, $v_x = 8.0\,\text{m/s}$ (unchanged).
Impact speed: $v = \sqrt{v_x^2 + v_y^2} = \sqrt{8.0^2 + 19.6^2} = \sqrt{64 + 384.16} = \sqrt{448.16} = 21.2\,\text{m/s}$.

1.8.2 Launch at an Angle (Level Ground)

For a launch angle θ above horizontal, resolve v_0 into components $v_{x0} = v_0 \cos\theta$ and $v_{y0} = v_0 \sin\theta$ first.

Ví dụ 11 · Launch at an angle, complementary-angle check

A ball is launched at $v_0 = 24.5\,\text{m/s}$, 37° above horizontal, from level ground. Find the flight time, maximum height, and range. ($\sin 37^\circ = 0.60$, $\cos 37^\circ = 0.80$.)

$v_{x0} = 24.5(0.80) = 19.6\,\text{m/s}$, $\quad v_{y0} = 24.5(0.60) = 14.7\,\text{m/s}$.
Time up: $t_{up} = \frac{v_{y0}}{g} = \frac{14.7}{9.8} = 1.5\,\text{s}$; total flight time $t = 2t_{up} = 3.0\,\text{s}$.
Max height: $h_{\text{max}} = \frac{v_{y0}^2}{2g} = \frac{(14.7)^2}{19.6} = \frac{216.09}{19.6} = 11.0\,\text{m}$.
Range: $R = v_{x0}\,t = (19.6)(3.0) = 58.8\,\text{m}$.

Complementary launch angles (θ and $90^\circ - \theta$) give the *same range* on level ground for the same launch speed, because $R = \frac{v_0^2 \sin 2\theta}{g}$ and $\sin(2\theta) = \sin\left(2(90^\circ - \theta)\right)$. The range is maximum at $\theta = 45^\circ$.

1.8.3 Launch at an Angle from a Height

When the projectile lands at a different height than it launched, the vertical displacement is nonzero and t must be found from the quadratic $\Delta y = v_{y0}t - \frac{1}{2}gt^2$.

Ví dụ 12 · Projectile launched from a wall, landing below

A ball is launched from the top of a $20\,\text{m}$ wall at $25\,\text{m/s}$, 37° above horizontal, over flat ground beyond the wall's base. Find the total flight time and the horizontal range.

$v_{x0} = 25(0.80) = 20\,\text{m/s}$, $v_{y0} = 25(0.60) = 15\,\text{m/s}$. Taking up as positive, the ball's net vertical displacement is $\Delta y = -20\,\text{m}$:

$$-20 = 15t - 4.9t^2 \;\Rightarrow\; 4.9t^2 - 15t - 20 = 0.$$

Quadratic formula: $t = \frac{15 \pm \sqrt{15^2 + 4(4.9)(20)}}{2(4.9)} = \frac{15 \pm \sqrt{225 + 392}}{9.8} = \frac{15 \pm \sqrt{617}}{9.8} = \frac{15 \pm 24.84}{9.8}$.
Taking the positive root: $t = \frac{39.84}{9.8} = 4.07\,\text{s}$.
Range: $R = v_{x0}t = (20)(4.07) = 81.3\,\text{m}$.

(!) Lỗi thường gặp: Never reuse a level-ground formula (like $t = 2v_{y0}/g$ or $R = v_0^2 \sin 2\theta/g$) when the launch and landing heights differ. Go back to the vertical displacement equation $\Delta y = v_{y0}t - \frac{1}{2}gt^2$ and solve for t directly.

1.9 Relative Motion

Every velocity discussed so far has been measured relative to the ground. But motion is often described more naturally relative to another moving object: the velocity of a passenger walking inside a moving train, of a boat moving through a river current, or of one car as observed from another car on the highway. This section develops the vector-addition rule that connects velocities measured in different reference frames.

1.9.1 Relative Velocity in One Dimension

When two objects move along the same straight line, the velocity of one relative to the other is found by combining their velocities relative to the ground.

For any two objects (or reference frames) A and B, relative velocities add like vectors:

$$\vec{v}_{A/\text{ground}} = \vec{v}_{A/B} + \vec{v}_{B/\text{ground}}.$$

Equivalently, the velocity of A as seen by an observer riding along with B is

$$\vec{v}_{A/B} = \vec{v}_{A/\text{ground}} - \vec{v}_{B/\text{ground}}.$$

The inner subscripts must "chain" and cancel (B cancels between the two terms on the right, leaving A/ground). In one dimension this is a signed subtraction along the line of motion; in two dimensions the same rule applies component by component, or graphically by placing the vectors tip-to-tail.

When two objects move *toward* each other, their relative speed — the *closing speed* — is the *sum* of their individual ground speeds. When one object chases another moving in the *same* direction, their relative speed is the *difference* of the two ground speeds.

GIẢI THÍCH TIẾNG VIỆT

VN

Vận tốc tương đối cộng theo quy tắc vector: $\vec{v}_{A/\text{đất}} = \vec{v}_{A/B} + \vec{v}_{B/\text{đất}}$, suy ra vận tốc của A so với B là $\vec{v}_{A/B} = \vec{v}_{A/\text{đất}} - \vec{v}_{B/\text{đất}}$. Khi hai vật chuyển động **ngược chiều** (tiến lại gần nhau), tốc độ tương đối (tốc độ khép lại khoảng cách) bằng **tổng** hai tốc độ; khi một vật đuổi theo vật kia theo **cùng chiều**, tốc độ tương đối bằng **hiệu** hai tốc độ.

Ví dụ 13 · Closing speed and meeting time

Two towns are connected by a straight 300 km highway. A car leaves Town A heading toward Town B at a constant 60 km/h; at the same instant a truck leaves Town B heading toward Town A at a constant 40 km/h. Taking the direction from A to B as positive, find (a) the relative velocity of the car with respect to the truck, and (b) the time and location at which they meet.

(a) $v_{car} = +60$ km/h; $v_{truck} = -40$ km/h (opposite direction). Relative velocity of the car with respect to the truck:

$$v_{car/truck} = v_{car} - v_{truck} = 60 - (-40) = +100 \text{ km/h}.$$

This is the rate at which the 300 km gap between them closes — the closing speed is 100 km/h.

(b) Time to close the gap: $t = \dfrac{300}{100} = 3.0$ h.

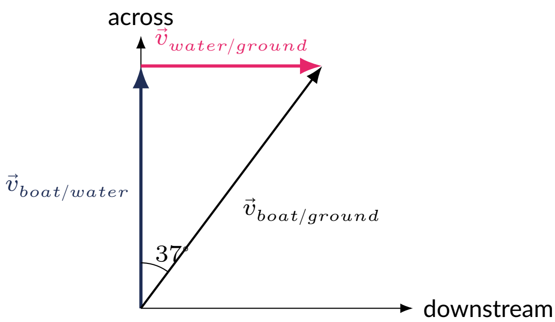
Location, measured from Town A: $x = v_{car}\,t = (60)(3.0) = 180$ km. Check from Town B: the truck has traveled $(40)(3.0) = 120$ km, and $180+120 = 300$ km — consistent. They meet 180 km from Town A.

1.9.2 Relative Velocity in Two Dimensions: Crossing a Current

A boat crossing a river, or a plane flying through wind, illustrates the same addition rule in two dimensions. The pilot controls the object's velocity *relative to the medium* (water or air); the medium itself moves relative to the ground (current or wind). The actual velocity over the ground is the vector sum:

$$\vec{v}_{boat/ground} = \vec{v}_{boat/water} + \vec{v}_{water/ground}.$$

Two natural headings are worth comparing. Aiming the boat **straight across** (perpendicular to the current) gives the *minimum crossing time*, because the entire boat speed contributes to the across-river component of velocity — but the current carries the boat downstream of the point directly opposite its start. Aiming the boat partly **upstream**, so that the upstream component of its velocity exactly cancels the current, produces *zero drift* (the boat's path over the ground is a straight line straight across) — but the across-river component of velocity is reduced, so the crossing takes *longer*.



Tip-to-tail vector addition: the boat's velocity relative to the water (4.0 m/s, straight across) plus the water's velocity relative to the ground (3.0 m/s, downstream) gives the resultant velocity relative to the ground (5.0 m/s, 37° off the straight-across direction).

Ví dụ 14 · River crossing: minimum time vs. zero drift

A river is 80 m wide and flows at 3.0 m/s. A boat can move at 4.0 m/s relative to the water. (a) If the boat is aimed straight across, find the resultant velocity relative to the ground, the crossing time, and the downstream drift. (b) If instead the pilot wants to land directly opposite the starting point (zero drift), find the required heading and the new crossing time.

(a) Components (across $= y$, downstream $= x$): $v_x = 3.0$ m/s (current only), $v_y = 4.0$ m/s (boat only, since it is aimed straight across).

$$v_{boat/ground} = \sqrt{v_x^2 + v_y^2} = \sqrt{3.0^2 + 4.0^2} = \sqrt{9+16} = \sqrt{25} = 5.0\,\text{m/s},$$

at $\theta = \arctan\left(\dfrac{3.0}{4.0}\right) = 37^\circ$ downstream of the straight-across direction.

Crossing time depends only on the across-river component: $t = \dfrac{80}{4.0} = 20$ s.
Downstream drift: $x = v_{current}\,t = (3.0)(20) = 60$ m.

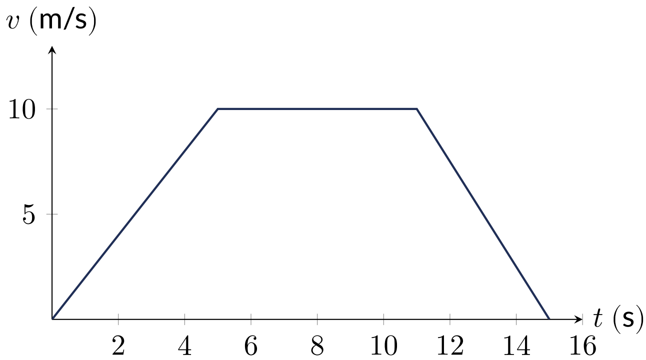
(b) To land directly opposite the start, the boat's upstream velocity component must cancel the current: $4.0\sin\phi = 3.0 \Rightarrow \sin\phi = 0.75 \Rightarrow \phi = 48.6^\circ$ upstream of straight-across. The remaining across-river component is

$$v'_y = 4.0\cos\phi = \sqrt{4.0^2 - 3.0^2} = \sqrt{16-9} = \sqrt{7} = 2.65\,\text{m/s}.$$

New crossing time: $t' = \dfrac{80}{2.65} = 30.2\,\text{s}$ — longer than the $20\,\text{s}$ minimum-time crossing, even though the boat now lands exactly opposite its starting point instead of $60\,\text{m}$ downstream.

1.9.3 Reading Multi-Segment and Curved Graphs

Many real trips involve several distinct phases of motion, each with its own constant acceleration and its own slope on a v–t graph. The tools already introduced — slope for acceleration, signed area for displacement — apply to each segment separately; the results are then combined.



v–t graph with three segments: speeds up from 0 to $10\,\text{m/s}$ (0–5 s, slope $+2.0\,\text{m/s}^2$), cruises at $10\,\text{m/s}$ (5–11 s), then decelerates to rest (11–15 s, slope $-2.5\,\text{m/s}^2$).

Ví dụ 15 · Three-segment trip: displacement, distance, and both kinds of "average"

A car's motion is described by the v–t graph above: it speeds up uniformly from rest to $10\,\text{m/s}$ over the first $5.0\,\text{s}$, cruises at $10\,\text{m/s}$ for the next $6.0\,\text{s}$, then decelerates uniformly to rest over the final $4.0\,\text{s}$. Find the total displacement, the total distance, the average velocity, and the average speed for the whole $15\,\text{s}$ trip.

Break the area under the graph into three pieces:
Triangle 1 (speeding up): $\frac{1}{2}(5.0)(10) = 25\,\text{m}$.
Rectangle (cruising): $(10)(6.0) = 60\,\text{m}$.
Triangle 3 (slowing down): $\frac{1}{2}(4.0)(10) = 20\,\text{m}$.
All three areas lie above the t-axis (the car never reverses direction), so they are all added: total displacement $= 25 + 60 + 20 = 105\,\text{m}$. Because velocity is never negative here, no direction reversal occurs, so total distance $= 105\,\text{m}$ as well.
Average velocity: $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{105}{15} = 7.0\,\text{m/s}$. Average speed $= \dfrac{\text{distance}}{\Delta t} = \dfrac{105}{15} = 7.0\,\text{m/s}$ — the two are equal here specifically *because* the car never backtracks (contrast this with Ví dụ 5, where a sign change made distance exceed $|\Delta x|$).

(!) Lỗi thường gặp: On a v–t graph, area *below* the t-axis represents negative velocity, so it must be **subtracted** (counted as negative) when computing total *displacement* — never added as if it were extra positive area. That same below-axis region still represents real motion covered, however, so when computing total *distance*, every piece of area (above or below the axis) is added as a **positive** magnitude. Mixing up these two rules is one of the most common errors on the AP exam.

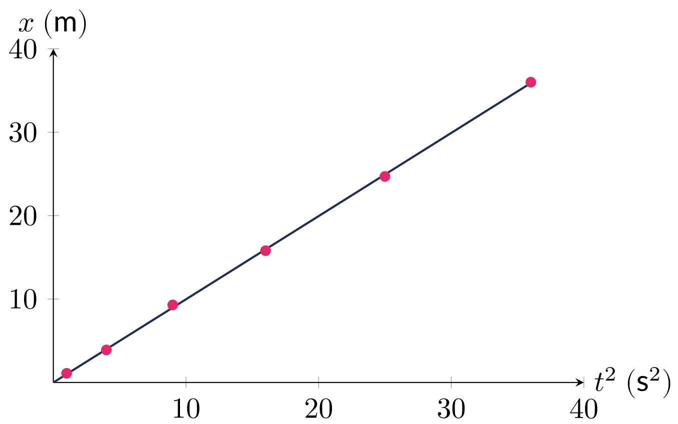
1.9.4 Linearizing Data: Extracting Acceleration from Position–Time Data

Real experimental data are never perfectly clean — every measured (t, x) pair carries some small scatter from timing and measurement error. For an object starting from rest with constant acceleration, $x = \frac{1}{2}at^2$, so a graph of x versus t is a **parabola**. A parabola's slope keeps changing from point to point, so a cannot be read off directly as "the slope" of an x–t graph, and judging a curve's changing steepness by eye — especially through noisy data — is unreliable. AP Physics 1 tests a standard fix for exactly this problem: re-plot the same data against a different horizontal axis so that the relationship becomes a straight line.

If $x = \frac{1}{2}at^2$, define a new variable $u \equiv t^2$. Then $x = \left(\frac{1}{2}a\right) u$ is a **linear** function of u with zero intercept. So a graph of x (vertical axis) versus t^2 (horizontal axis) — *not* x versus t — is a **straight line through the origin**, and its slope equals $\frac{1}{2}a$:

$$\text{slope} = \frac{\Delta x}{\Delta(t^2)} = \frac{1}{2}a \implies a = 2 \times \text{slope}.$$

This process — plotting one variable against a power of another until the graph becomes a straight line — is called **linearizing** the data. A straight line can be fit through scattered data points (by eye, or by a least-squares "best-fit" line) far more reliably than a curve's changing steepness can be judged by eye, so plotting x vs. t^2 is the standard, more reliable way to extract a constant acceleration from real position–time data.



Plotting x vs. t^2 (instead of x vs. t) turns the curved, constant-acceleration relationship into a straight line through the origin; the slope of the best-fit line through the scattered data points equals $\frac{1}{2}a$.

GIẢI THÍCH TIẾNG VIỆT

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Với vật xuất phát từ trạng thái nghỉ và có gia tốc không đổi, $x = \frac{1}{2}at^2$ là hàm bậc hai theo t, nên đồ thị x–t là một đường **cong** (parabol) có độ dốc thay đổi liên tục — rất khó đọc chính xác gia tốc bằng mắt, đặc biệt khi số liệu thực tế có nhiễu (sai số đo đạc). Nếu đổi biến trục hoành thành t^2, phương trình trở thành $x = \left(\frac{1}{2}a\right) t^2$ — một hàm **tuyến tính** theo t^2 với hệ số chặn bằng 0: đồ thị x theo t^2 là một đường **thẳng** đi qua gốc tọa độ, có độ dốc $\frac{1}{2}a$. Kỹ thuật **"tuyến tính hóa"** (linearization) này cho phép vẽ một đường thẳng khớp tốt nhất (best-fit line) qua các điểm số liệu rải rác, rồi suy ra gia tốc $a = 2 \times$ độ dốc — đáng tin cậy hơn nhiều so với việc ước lượng độ cong của đồ thị x–t bằng mắt thường.

Ví dụ 16 · Extracting acceleration from position–time data

A cart starts from rest and rolls down an inclined track. A student measures its position x at six instants of time t, recording the (slightly noisy, real-world) data below.

t (s)	t^2 (s^2)	x (m)
1.0	1.0	1.1
2.0	4.0	3.9
3.0	9.0	9.3
4.0	16.0	15.8
5.0	25.0	24.7
6.0	36.0	36.0

Use the linearized plot of x vs. t^2 to find the cart's acceleration, then check the result against one of the recorded data points.

Step 1 — linearize. Since the cart starts from rest, $x = \frac{1}{2}at^2$ predicts x is proportional to t^2, so t^2 (computed in the middle column above) is the correct horizontal axis for a straight-line plot — not t itself.

Step 2 — find the slope. A best-fit line through all six plotted points (as in the figure above) lies very close to the first and last recorded points, so these two are used as representative points to estimate its slope:

$$\text{slope} = \frac{\Delta x}{\Delta(t^2)} = \frac{36.0 - 1.1}{36.0 - 1.0} = \frac{34.9}{35.0} = 0.997\,\text{m/s}^2.$$

Step 3 — extract the acceleration. Since slope $= \frac{1}{2}a$:

$$a = 2 \times \text{slope} = 2(0.997) = 1.99\,\text{m/s}^2 \approx 2.0\,\text{m/s}^2.$$

Step 4 — sanity check. At $t = 4.0$ s ($t^2 = 16.0\,\text{s}^2$), the fitted line predicts $x = (\text{slope})(t^2) = (0.997)(16.0) = 15.95$ m, which matches the recorded value of 15.8 m to within about 1% — consistent with the small scatter visible in the other rows, and confirming that $a \approx 2.0\,\text{m/s}^2$ is a reliable value extracted from the data.

1.10 Practice Problems

Bài tập luyện tập

A displacement vector has components $\Delta x = 9.0$ m (east) and $\Delta y = 12.0$ m (north). Find its magnitude.

(A) 10.5 m
(B) 12.0 m
(C) 15.0 m
(D) 21.0 m

Lời giải chi tiết

$A = \sqrt{(9.0)^2 + (12.0)^2} = \sqrt{81 + 144} = \sqrt{225} = 15.0$ m. **(C)**.

Bài tập luyện tập

Vector $\vec{A} = 5.0\,\text{m}$ pointing east; vector $\vec{B} = 5.0\,\text{m}$ pointing north. Find the magnitude and direction of the resultant $\vec{A} + \vec{B}$.

Lời giải chi tiết

Since $\vec{A} \perp \vec{B}$: magnitude $= \sqrt{5.0^2 + 5.0^2} = \sqrt{50} = \boxed{7.07\,\text{m}}$, direction $= \arctan(5.0/5.0) = 45°$ north of east.

Bài tập luyện tập

An object moves from $x = -4.0\,\text{m}$ to $x = +6.0\,\text{m}$, then back to $x = +2.0\,\text{m}$. Find its total displacement and its total distance traveled.

Lời giải chi tiết

Displacement: $\Delta x = 2.0 - (-4.0) = 6.0\,\text{m}$.
Distance: $|6.0 - (-4.0)| + |2.0 - 6.0| = 10.0 + 4.0 = \boxed{14\,\text{m}}$.

Bài tập luyện tập

A cyclist rides $5.0\,\text{km}$ east in $0.50\,\text{h}$, rests for $0.25\,\text{h}$, then rides $3.0\,\text{km}$ west in $0.25\,\text{h}$. Find her average velocity and average speed for the entire trip.

Lời giải chi tiết

Net displacement $= 5.0 - 3.0 = 2.0\,\text{km}$ east; total time $= 0.50 + 0.25 + 0.25 = 1.0\,\text{h}$.
Average velocity $= \dfrac{2.0}{1.0} = \boxed{2.0\,\text{km/h east}}$.
Total distance $= 5.0 + 3.0 = 8.0\,\text{km}$; average speed $= \dfrac{8.0}{1.0} = \boxed{8.0\,\text{km/h}}$.

Bài tập luyện tập

An object moves in the $-x$ direction and is slowing down. What are the signs of its velocity and acceleration?

(A) $v > 0,\ a > 0$

(B) $v < 0,\ a < 0$

(C) $v < 0,\ a > 0$

(D) $v > 0,\ a < 0$

Lời giải chi tiết

Moving in $-x$ means $v < 0$. Slowing down requires a to oppose v, so $a > 0$. **(C)**.

Bài tập luyện tập

An x–t graph is a straight line from $(0\,\text{s}, 0\,\text{m})$ to $(4.0\,\text{s}, 20\,\text{m})$, then flat until $10\,\text{s}$, then a straight line down to $(14\,\text{s}, 0\,\text{m})$. Find the velocity in each segment and the total displacement over the $14\,\text{s}$.

Lời giải chi tiết

Segment 1: $v = \dfrac{20}{4.0} = 5.0\,\text{m/s}$. Segment 2 (flat): $v = 0$. Segment 3: $v = \dfrac{0-20}{14-10} = -5.0\,\text{m/s}$. Total displacement $= x_f - x_i = 0 - 0 = \boxed{0\,\text{m}}$ (total distance traveled is $20 + 0 + 20 = 40$ m).

Bài tập luyện tập

A v–t graph rises linearly from 0 to $12\,\text{m/s}$ over $3.0\,\text{s}$, stays constant at $12\,\text{m/s}$ for $5.0\,\text{s}$, then falls linearly to 0 over the next $2.0\,\text{s}$. Find the total displacement.

(A) $60\,\text{m}$ **(C)** $90\,\text{m}$

(B) $78\,\text{m}$ **(D)** $102\,\text{m}$

Lời giải chi tiết

Area $= \frac{1}{2}(3.0)(12) + (12)(5.0) + \frac{1}{2}(2.0)(12) = 18 + 60 + 12 = 90\,\text{m}$. **(C)**.

Bài tập luyện tập

A cyclist accelerates uniformly from $4.0\,\text{m/s}$ to $12\,\text{m/s}$ over $8.0\,\text{s}$. Find her acceleration and the distance she covers.

Lời giải chi tiết

$a = \dfrac{12 - 4.0}{8.0} = \boxed{1.0\,\text{m/s}^2}$. $\Delta x = \frac{1}{2}(v_0 + v)t = \frac{1}{2}(16)(8.0) = \boxed{64\,\text{m}}$.

Bài tập luyện tập

A car decelerates uniformly from $25\,\text{m/s}$ to rest in $5.0\,\text{s}$. Find the distance it travels while braking.

(A) $31.3\,\text{m}$ **(C)** $62.5\,\text{m}$

(B) $50.0\,\text{m}$ **(D)** $125\,\text{m}$

Lời giải chi tiết

$\Delta x = \frac{1}{2}(v_0 + v)t = \frac{1}{2}(25 + 0)(5.0) = 62.5\,\text{m}$. **(C)**.

Bài tập luyện tập

A bullet enters a wood block at $400\,\text{m/s}$ and comes to rest after penetrating $0.20\,\text{m}$. Assuming constant deceleration, find its magnitude.

Lời giải chi tiết

$v^2 = v_0^2 + 2a\Delta x \Rightarrow 0 = (400)^2 + 2a(0.20) \Rightarrow a = \dfrac{-160000}{0.40} = \boxed{-4.0 \times 10^5\,\text{m/s}^2}$ (magnitude 4.0×10^5 m/s^2).

Bài tập luyện tập

A rocket sled starts from rest and accelerates at $4.0\,\text{m/s}^2$ for 5.0 s; a braking chute then decelerates it at $8.0\,\text{m/s}^2$ until it stops. Find the total distance traveled.

Lời giải chi tiết

Phase 1: $v = at = (4.0)(5.0) = 20$ m/s; $\Delta x_1 = \frac{1}{2}at^2 = \frac{1}{2}(4.0)(5.0)^2 = 50$ m.

Phase 2: $\Delta x_2 = \dfrac{v^2}{2a} = \dfrac{(20)^2}{2(8.0)} = 25$ m.

Total $= 50 + 25 = \boxed{75\,\text{m}}$.

Bài tập luyện tập

A ball is dropped from rest and falls for 3.0 s. Find the distance it falls.

(A) 14.7 m
(B) 29.4 m
(C) 44.1 m
(D) 88.2 m

Lời giải chi tiết

$h = \frac{1}{2}gt^2 = \frac{1}{2}(9.8)(3.0)^2 = \frac{1}{2}(9.8)(9.0) = 44.1$ m. **(C)**.

Bài tập luyện tập

A ball is thrown straight up and returns to its launch point after a total flight time of 5.0 s. Find its initial speed and maximum height.

Lời giải chi tiết

By symmetry $t_{up} = 2.5$ s: $v_0 = gt_{up} = (9.8)(2.5) = \boxed{24.5\,\text{m/s}}$.

$h_{\text{max}} = \dfrac{v_0^2}{2g} = \dfrac{(24.5)^2}{19.6} = \dfrac{600.25}{19.6} = \boxed{30.6\,\text{m}}$.

Bài tập luyện tập

An object is launched horizontally at 10 m/s from a 20 m cliff. Find the horizontal range (to 3 significant figures).

(A) 10.0 m
(B) 15.6 m
(C) 20.2 m
(D) 25.0 m

Lời giải chi tiết

$t = \sqrt{2h/g} = \sqrt{40/9.8} = \sqrt{4.08} = 2.02$ s. $R = v_0 t = (10)(2.02) = 20.2$ m. **(C)**.

Bài tập luyện tập

Using the same launch speed $v_0 = 24.5$ m/s as the worked example above, a ball is instead launched at 53° above horizontal (level ground). Compare its range to the 37° case.

(A) Smaller
(B) The same
(C) Larger
(D) Cannot be determined without g

Lời giải chi tiết

$v_{x0} = 24.5(0.60) = 14.7$, $v_{y0} = 24.5(0.80) = 19.6$. $t = 2v_{y0}/g = 2(19.6)/9.8 = 4.0\,\text{s}$. $R = v_{x0}t = (14.7)(4.0) = 58.8\,\text{m}$ — identical to the 37° range found earlier, confirming that complementary angles give equal range. **(B).**

Bài tập luyện tập

A cannonball is fired from the top of a $20\,\text{m}$ wall at $25\,\text{m/s}$, 37° above horizontal, and lands on flat ground beyond the wall. Find its total flight time and range. ($\sin 37^\circ = 0.60$, $\cos 37^\circ = 0.80$.)

Lời giải chi tiết

This is the projectile-from-a-height case: $v_{x0} = 20\,\text{m/s}$, $v_{y0} = 15\,\text{m/s}$, $\Delta y = -20\,\text{m}$.

$$4.9t^2 - 15t - 20 = 0 \Rightarrow t = \frac{15 + \sqrt{225 + 392}}{9.8} = \frac{15 + 24.84}{9.8} = \boxed{4.07\,\text{s}}.$$

$$R = v_{x0}t = (20)(4.07) = \boxed{81.3\,\text{m}}.$$

Bài tập luyện tập

For a projectile in flight (no air resistance), which quantity remains constant throughout?

(A) v_y
(B) v_x
(C) Speed
(D) Height

Lời giải chi tiết

Horizontal acceleration is zero, so v_x never changes. **(B).**

Bài tập luyện tập

A v–t graph decreases linearly from $15\,\text{m/s}$ to $-5.0\,\text{m/s}$ over $4.0\,\text{s}$. Find the acceleration.

(A) $-2.5\,\text{m/s}^2$
(B) $-5.0\,\text{m/s}^2$
(C) $-10\,\text{m/s}^2$
(D) $-20\,\text{m/s}^2$

Lời giải chi tiết

$$a = \frac{-5.0 - 15}{4.0} = \frac{-20}{4.0} = -5.0\,\text{m/s}^2.$$ **(B).**

Bài tập luyện tập

For the v–t graph of the previous problem, find (a) the time at which $v = 0$ and (b) the total displacement over the $4.0\,\text{s}$.

Lời giải chi tiết

(a) $v(t) = 15 - 5.0t = 0 \Rightarrow t = 3.0\,\text{s}$.
(b) Area 0–$3.0\,\text{s}$ (positive): $\frac{1}{2}(3.0)(15) = +22.5\,\text{m}$. Area 3.0–$4.0\,\text{s}$ (negative): $\frac{1}{2}(1.0)(5.0) = -2.5\,\text{m}$.
Total displacement $= 22.5 - 2.5 = \boxed{20.0\,\text{m}}$ (total distance $= 25.0\,\text{m}$).

Bài tập luyện tập

Car A starts from rest at $x = 0$ and accelerates uniformly at $4.0\,\text{m/s}^2$. At the same instant, Car B passes the same point moving at a constant $20\,\text{m/s}$ in the same direction. Find when and where Car A catches up to Car B, and find Car A's speed at that moment.

Lời giải chi tiết

$x_A(t) = \frac{1}{2}(4.0)t^2 = 2.0t^2$; $x_B(t) = 20t$. Setting equal: $2.0t^2 = 20t \Rightarrow t(2.0t - 20) = 0 \Rightarrow t = 10\,\text{s}$.
Position: $x = 20(10) = \boxed{200\,\text{m}}$. Speed of A at that instant: $v_A = at = (4.0)(10) = \boxed{40\,\text{m/s}}$ — twice Car B's speed, since A is still accelerating past B at the meeting point.

Bài tập luyện tập

A projectile launched at angle θ above horizontal lands at the same height from which it was launched. Compared to the time needed to reach maximum height, the total flight time is:

(A) The same
(B) Half
(C) Double
(D) Impossible to determine

Lời giải chi tiết

The parabolic path is symmetric about the peak, so the descent takes exactly as long as the ascent: total time $= 2t_{up}$. **(C)**.

Bài tập luyện tập

For any motion in which an object reverses direction at least once, how does average speed compare to the magnitude of average velocity?

(A) Always strictly less than
(B) Always exactly equal to
(C) Always greater than or equal to
(D) No general relationship exists

Lời giải chi tiết

Distance $\geq |\Delta x|$ always, with equality only when the object never reverses direction. Dividing both sides by the same time interval preserves the inequality: average speed $\geq |\bar{v}|$. **(C)**.

Bài tập luyện tập

An a–t graph is constant at $3.0\,\text{m/s}^2$ from $t = 0$ to $4.0\,\text{s}$, then drops to 0 from $4.0\,\text{s}$ to $6.0\,\text{s}$. If $v(0) = 2.0\,\text{m/s}$, find $v(4.0\,\text{s})$ and $v(6.0\,\text{s})$.

Lời giải chi tiết

Δv from 0 to $4.0\,\text{s}$ = area $= (3.0)(4.0) = 12\,\text{m/s}$, so $v(4.0\,\text{s}) = 2.0 + 12 = \boxed{14.0\,\text{m/s}}$.
From 4.0 to $6.0\,\text{s}$, $a = 0$, so velocity is unchanged: $v(6.0\,\text{s}) = \boxed{14.0\,\text{m/s}}$.

Bài tập luyện tập

A ball is kicked from ground level and lands $24\,\text{m}$ away after $2.0\,\text{s}$ of flight on level ground. Find its initial speed and launch angle.

Lời giải chi tiết

$v_{x0} = \frac{R}{t} = \frac{24}{2.0} = 12\,\text{m/s}$. By symmetry $t_{up} = 1.0\,\text{s}$, so $v_{y0} = gt_{up} = (9.8)(1.0) = 9.8\,\text{m/s}$.
$v_0 = \sqrt{v_{x0}^2 + v_{y0}^2} = \sqrt{144 + 96.04} = \sqrt{240.04} = \boxed{15.5\,\text{m/s}}$.
$\theta = \arctan\left(\frac{9.8}{12}\right) = \arctan(0.817) = \boxed{39.3°}$.

Bài tập luyện tập

Which of the following is a vector quantity?

(A) Distance
(B) Speed
(C) Displacement
(D) Time

Lời giải chi tiết

Displacement has both magnitude and direction. **(C)**.

Bài tập luyện tập

A swimmer swims $50\,\text{m}$ north in a pool in $40\,\text{s}$, then immediately swims back $50\,\text{m}$ south to the start in $45\,\text{s}$. Find her average velocity and average speed for the round trip.

Lời giải chi tiết

Net displacement $= 0$ (returns to start), so average velocity $= \boxed{0}$.
Total distance $= 50 + 50 = 100\,\text{m}$; total time $= 40 + 45 = 85\,\text{s}$. Average speed $= \frac{100}{85} = \boxed{1.18\,\text{m/s}}$.

Bài tập luyện tập

A stone is thrown horizontally from a $45\,\text{m}$ cliff with initial speed $15\,\text{m/s}$. Find (a) the time to hit the ground and (b) the horizontal distance traveled.

Lời giải chi tiết

(a) $t = \sqrt{2h/g} = \sqrt{2(45)/9.8} = \sqrt{9.184} = \boxed{3.03\,\text{s}}$.
(b) $R = v_0 t = (15)(3.03) = \boxed{45.4\,\text{m}}$.

Bài tập luyện tập

Two identical balls are launched from the same height with the same speed: ball A straight up, ball B straight down. Compare their speeds the instant each hits the ground below.

(A) A is faster

(B) B is faster

(C) They are equal

(D) Cannot be determined without the height

Lời giải chi tiết

Ball A first rises and returns to the launch height with the *same* speed it started with (by energy/symmetry), after which both balls fall the same additional height with the same downward speed at launch. Their impact speeds are therefore equal. **(C)**.

Bài tập luyện tập

A particle's position is given by $x(t) = 3.0 + 4.0t - 1.0t^2$ (SI units), valid for constant acceleration. Find its initial velocity, its acceleration, and the time at which it momentarily stops.

Lời giải chi tiết

Comparing to $x = x_0 + v_0 t + \frac{1}{2}at^2$: $v_0 = \boxed{4.0\,\mathrm{m/s}}$, and $\frac{1}{2}a = -1.0 \Rightarrow a = \boxed{-2.0\,\mathrm{m/s^2}}$.

Momentarily stops when $v = v_0 + at = 0$: $t = \dfrac{-4.0}{-2.0} = \boxed{2.0\,\mathrm{s}}$.

Bài tập luyện tập

Two trains approach each other on parallel straight tracks. Train P moves east at $20\,\mathrm{m/s}$ and train Q moves west at $30\,\mathrm{m/s}$; they start $500\,\mathrm{m}$ apart at the same instant. How long until they meet?

(A) $5.0\,\mathrm{s}$

(B) $10\,\mathrm{s}$

(C) $12.5\,\mathrm{s}$

(D) $25\,\mathrm{s}$

Lời giải chi tiết

Taking east as positive, $v_P = +20\,\mathrm{m/s}$ and $v_Q = -30\,\mathrm{m/s}$. The relative velocity of P with respect to Q is $v_{P/Q} = v_P - v_Q = 20 - (-30) = +50\,\mathrm{m/s}$, which is their closing speed. Time to close the $500\,\mathrm{m}$ gap: $t = \dfrac{500}{50} = 10\,\mathrm{s}$. **(B)**.

Bài tập luyện tập

A cyclist starts from position $x = 0$ and moves at a constant $8.0\,\mathrm{m/s}$. At that same instant, a jogger moving at a constant $3.0\,\mathrm{m/s}$ in the same direction is already $100\,\mathrm{m}$ ahead of the cyclist. Using relative velocity, find how long it takes the cyclist to catch the jogger, and how far the cyclist has traveled by then.

Lời giải chi tiết

Relative velocity of the cyclist with respect to the jogger: $v_{cyclist/jogger} = v_{cyclist} - v_{jogger} = 8.0 - 3.0 = +5.0\,\text{m/s}$ — the rate at which the cyclist closes the $100\,\text{m}$ gap. Time to close the gap: $t = \dfrac{100}{5.0} = 20\,\text{s}$.
Cyclist's distance traveled: $x_{cyclist} = v_{cyclist}\,t = (8.0)(20) = \boxed{160\,\text{m}}$.
Check: the jogger travels $(3.0)(20) = 60\,\text{m}$ from her position, i.e. $100 + 60 = 160\,\text{m}$ from the cyclist's start — consistent.

Bài tập luyện tập

A boat heads straight across a river at $5.0\,\text{m/s}$ relative to the water, while the current flows at $12\,\text{m/s}$ parallel to the banks. Find the boat's resultant speed relative to the ground.

(A) $7.0\,\text{m/s}$
(B) $12.5\,\text{m/s}$
(C) $13\,\text{m/s}$
(D) $17\,\text{m/s}$

Lời giải chi tiết

The two velocity components are perpendicular, so they add as a right triangle: $v = \sqrt{5.0^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13\,\text{m/s}$. **(C)**.

Bài tập luyện tập

A river $80\,\text{m}$ wide flows at $6.0\,\text{m/s}$. A boat heads straight across at $8.0\,\text{m/s}$ relative to the water. Find (a) the resultant velocity of the boat relative to the ground (magnitude and direction), (b) the time to cross the river, and (c) how far downstream the boat lands from the point directly opposite its start.

Lời giải chi tiết

(a) $v = \sqrt{8.0^2 + 6.0^2} = \sqrt{64 + 36} = \sqrt{100} = \boxed{10\,\text{m/s}}$, at $\theta = \arctan\left(\dfrac{6.0}{8.0}\right) = \boxed{37^\circ}$ downstream of the straight-across direction.
(b) Crossing time depends only on the across-river component (the current contributes none): $t = \dfrac{80}{8.0} = \boxed{10\,\text{s}}$.
(c) Downstream drift: $x = v_{current}\,t = (6.0)(10) = \boxed{60\,\text{m}}$.

Bài tập luyện tập

Two runners start together from the same point at $t = 0$ and run along the same straight path for a total time T. Runner A accelerates uniformly from rest for the *entire* interval T. Runner B moves at a *constant* speed equal to runner A's average speed over the entire interval T. Without plugging in specific numbers, explain which runner is ahead at the halfway *time* $t = T/2$ (not halfway in distance), and why.

Lời giải chi tiết

Runner B is ahead at the halfway-time mark. Let a be runner A's constant acceleration, starting from rest, so A's velocity at the end of the trip is $v_A(T) = aT$. Because A's acceleration is

constant, A's average velocity over the whole interval is the mean of the initial and final velocities, $\bar{v}_A = \frac{1}{2}(0 + aT) = \frac{1}{2}aT$ — and this is exactly runner B's constant speed, by the problem's definition.
During the *first* half of the time interval, runner A's velocity climbs linearly from 0 up toward $\frac{1}{2}aT$, reaching that value only at the instant $t = T/2$ itself. Runner B, meanwhile, moves at the full value $\frac{1}{2}aT$ from the very start. So for every instant between $t = 0$ and $t = T/2$ (except the single instant $t = T/2$, where their speeds momentarily match), B is moving strictly faster than A — B wins the instantaneous-speed comparison throughout the first half, so B pulls ahead in distance.
This can be confirmed algebraically: $x_A(T/2) = \frac{1}{2}a(T/2)^2 = \dfrac{aT^2}{8}$, while $x_B(T/2) = \left(\frac{1}{2}aT\right)\left(\dfrac{T}{2}\right) = \dfrac{aT^2}{4}$. Since $\dfrac{aT^2}{4} = 2 \times \dfrac{aT^2}{8}$, runner B has covered *twice* the distance runner A has at the halfway-time mark.
(For completeness: during the second half of the interval, A's velocity keeps climbing past $\frac{1}{2}aT$, so A becomes the faster runner and closes the gap; by symmetry the two runners are together again exactly at $t = T$, since both cover the same total distance $\frac{1}{2}aT^2$ over the full interval.)

Bài tập luyện tập

A ball is dropped from rest and falls freely. Compare the distance it falls during the 3rd second of its fall (from $t = 2.0\,\mathrm{s}$ to $t = 3.0\,\mathrm{s}$) to the distance it falls during the 1st second of its fall (from $t = 0$ to $t = 1.0\,\mathrm{s}$).

(A) Ratio of 3
(B) Ratio of 5
(C) Ratio of 7
(D) Ratio of 9

Lời giải chi tiết

Distance fallen from rest up to time t: $y = \frac{1}{2}gt^2$.
1st second: $y(1.0) - y(0) = \frac{1}{2}(9.8)(1.0)^2 - 0 = 4.9\,\mathrm{m}$.
3rd second: $y(3.0) - y(2.0) = \frac{1}{2}(9.8)[(3.0)^2 - (2.0)^2] = \frac{1}{2}(9.8)(9.0 - 4.0) = \frac{1}{2}(9.8)(5.0) = 24.5\,\mathrm{m}$.
Ratio $= \dfrac{24.5}{4.9} = 5$. **(B)**.

Bài tập luyện tập

A train starts from rest and accelerates uniformly at $2.0\,\mathrm{m/s^2}$ for $6.0\,\mathrm{s}$, then travels at that constant velocity for $10\,\mathrm{s}$, and finally brakes to a stop with a uniform deceleration of $3.0\,\mathrm{m/s^2}$. Find (a) the total time for the trip, (b) the total distance traveled, and (c) the average velocity for the entire trip.

Lời giải chi tiết

Phase 1 (speeding up): $v_1 = at_1 = (2.0)(6.0) = 12\,\mathrm{m/s}$; $\Delta x_1 = \frac{1}{2}(0 + 12)(6.0) = 36\,\mathrm{m}$.
Phase 2 (cruising): $\Delta x_2 = (12)(10) = 120\,\mathrm{m}$.
Phase 3 (braking): $t_3 = \dfrac{v_1}{a_3} = \dfrac{12}{3.0} = 4.0\,\mathrm{s}$; $\Delta x_3 = \frac{1}{2}(12 + 0)(4.0) = 24\,\mathrm{m}$.
(a) Total time $= 6.0 + 10 + 4.0 = \boxed{20\,\mathrm{s}}$.
(b) Total distance $= 36 + 120 + 24 = \boxed{180\,\mathrm{m}}$.

(c) Average velocity $= \dfrac{180}{20} = \boxed{9.0\,\text{m/s}}$ (equal to average speed here, since the train never reverses direction).

Bài tập luyện tập

A v–t graph shows an object moving at a constant $+6.0\,\text{m/s}$ for $4.0\,\text{s}$, then abruptly reversing to a constant $-4.0\,\text{m/s}$ for the next $3.0\,\text{s}$. Find the total distance traveled.

(A) $12\,\text{m}$
(B) $24\,\text{m}$
(C) $30\,\text{m}$
(D) $36\,\text{m}$

Lời giải chi tiết

Displacement of segment 1 (above the axis): $+(6.0)(4.0) = +24\,\text{m}$. Displacement of segment 2 (below the axis): $(-4.0)(3.0) = -12\,\text{m}$. Net displacement $= 24 - 12 = 12\,\text{m}$ — but *distance* treats every interval as a positive contribution: distance $= |24| + |-12| = 24 + 12 = 36\,\text{m}$. **(D)**.

Bài tập luyện tập

A soccer ball is kicked from level ground at $v_0 = 20\,\text{m/s}$, 45° above horizontal — the angle that gives maximum range. Using $\sin 45^\circ = \cos 45^\circ = 0.707$, find the total flight time, the maximum height, and the range.

Lời giải chi tiết

$v_{x0} = v_{y0} = 20(0.707) = 14.14\,\text{m/s}$.

Time up: $t_{up} = \dfrac{14.14}{9.8} = 1.44\,\text{s}$; total flight time $t = 2t_{up} = \boxed{2.89\,\text{s}}$ (unrounded $t = 2.886\,\text{s}$, used below for consistency).

Max height: $h_{\text{max}} = \dfrac{v_{y0}^2}{2g} = \dfrac{(14.14)^2}{19.6} = \dfrac{199.9}{19.6} = \boxed{10.2\,\text{m}}$.

Range: $R = v_{x0}\,t = (14.14)(2.886) = \boxed{40.8\,\text{m}}$.

Bài tập luyện tập

At the instant a ball thrown straight up reaches the top of its path, which statement correctly describes its velocity and acceleration?

(A) $v = 0$ and $a = 0$
(B) $v = 0$ and $a = -g$
(C) $v = -g$ and $a = 0$
(D) $v = -g$ and $a = -g$

Lời giải chi tiết

Velocity is momentarily zero at the peak, but gravity never switches off — the ball is still in free fall with $a = -g$ throughout the entire flight, including at the top. **(B)**.

Bài tập luyện tập

An airplane's velocity relative to the air is $150\,\text{m/s}$ due north. A crosswind blows at $40\,\text{m/s}$ due east. Find the plane's resultant velocity relative to the ground: its speed and its direction (measured from due north).

Lời giải chi tiết

$\vec{v}_{plane/ground} = \vec{v}_{plane/air} + \vec{v}_{air/ground}$. The two components are perpendicular (north and east):

$$v = \sqrt{150^2 + 40^2} = \sqrt{22500 + 1600} = \sqrt{24100} = \boxed{155\,\text{m/s}}.$$

Direction: $\theta = \arctan\left(\frac{40}{150}\right) = \arctan(0.267) = \boxed{14.9^\circ}$ east of north.

Bài tập luyện tập

A boat that can move at a fixed speed relative to the water is crossing a river with a current. Compared to heading straight across (perpendicular to the banks), if the pilot instead angles the boat partly upstream to compensate for the current and land directly opposite the starting point, the crossing time will be:

(A) Shorter
(B) Longer
(C) Exactly the same
(D) Cannot be determined without more information

Lời giải chi tiết

Angling upstream diverts part of the boat's fixed speed into canceling the current, leaving a smaller component of velocity actually directed across the river. Since crossing time depends on that across-river component ($t = \text{width}/v_{\text{across}}$), a smaller across-river speed means a **longer** crossing time — exactly as seen when comparing the two headings in this chapter's river-crossing example. **(B)**.

Bài tập luyện tập

A vector has components $A_x = -6.0\,\text{m}$ and $A_y = +8.0\,\text{m}$. Find its magnitude.

(A) $2.0\,\text{m}$
(B) $10\,\text{m}$
(C) $14\,\text{m}$
(D) $48\,\text{m}$

Lời giải chi tiết

$A = \sqrt{(-6.0)^2 + (8.0)^2} = \sqrt{36 + 64} = \sqrt{100} = 10\,\text{m}$. (Magnitude depends only on the squares of the components, so the sign of A_x does not change it — the vector itself points into the second quadrant, at $180^\circ - \arctan(8.0/6.0) = 180^\circ - 53.1^\circ = 126.9^\circ$ measured counterclockwise from $+x$.) **(B)**.

Bài tập luyện tập

A drone starts at $x = +5.0\,\text{m}$, flies to $x = -3.0\,\text{m}$, then to $x = +9.0\,\text{m}$, and finally to $x = +1.0\,\text{m}$. Find its total displacement and its total distance traveled.

Lời giải chi tiết

Displacement: $\Delta x = x_f - x_i = 1.0 - 5.0 = \boxed{-4.0\,\text{m}}$ (magnitude 4.0 m, negative direction). Distance: leg 1 $= |(-3.0) - 5.0| = 8.0$ m; leg 2 $= |9.0 - (-3.0)| = 12.0$ m; leg 3 $= |1.0 - 9.0| = 8.0$ m. Total distance $= 8.0 + 12.0 + 8.0 = \boxed{28\,\text{m}}$.

Bài tập luyện tập

A car travels the first half of a straight-line trip (by *distance*, not time) at a constant 20 m/s, then the second half of the trip at a constant 60 m/s. Find the car's average speed for the entire trip.

Lời giải chi tiết

Let d be the (equal) distance of each half, so the total distance is $2d$. Time for the first half: $t_1 = d/20$. Time for the second half: $t_2 = d/60$. Total time:

$$t = t_1 + t_2 = \frac{d}{20} + \frac{d}{60} = \frac{3d + d}{60} = \frac{4d}{60} = \frac{d}{15}.$$

Average speed $= \dfrac{\text{total distance}}{\text{total time}} = \dfrac{2d}{d/15} = 2d \times \dfrac{15}{d} = \boxed{30\,\text{m/s}}$.

Notice this is *not* the simple average of 20 and 60 m/s (which would be 40 m/s): because the car spends more *time* at the slower speed (covering the same distance takes longer), the true average speed is pulled below the midpoint, toward the slower speed.

Bài tập luyện tập

An a–t graph rises linearly from 0 to $6.0\,\text{m/s}^2$ over $t = 0$ to $t = 4.0$ s, then instantly drops to 0. If $v(0) = 2.0$ m/s, find $v(4.0\,\text{s})$.

(A) 8.0 m/s

(B) 12 m/s

(C) 14 m/s

(D) 24 m/s

Lời giải chi tiết

$\Delta v =$ area under the a–t graph $= \frac{1}{2}(4.0)(6.0) = 12$ m/s (a triangle, not a rectangle, because a itself is changing). $v(4.0\,\text{s}) = v(0) + \Delta v = 2.0 + 12 = 14$ m/s. **(C)**.

Bài tập luyện tập

A skateboarder moving at 5.0 m/s starts up a ramp and decelerates uniformly at $1.0\,\text{m/s}^2$. (a) How far up the ramp does she travel before momentarily coming to rest? (b) How long does this take? (c) If the same constant acceleration continues unchanged after she momentarily stops (so she begins rolling back down), find her velocity at $t = 7.0$ s, measured from when she started up the ramp.

Lời giải chi tiết

Take up the ramp as positive: $v_0 = +5.0$ m/s, $a = -1.0\,\text{m/s}^2$.
(a) At the highest point $v = 0$: $v^2 = v_0^2 + 2a\Delta x \Rightarrow 0 = 25 + 2(-1.0)\Delta x \Rightarrow \Delta x = \boxed{12.5\,\text{m}}$.

(b) $t = \dfrac{v - v_0}{a} = \dfrac{0 - 5.0}{-1.0} = \boxed{5.0\,\text{s}}$.
(c) The acceleration is constant and unchanged throughout, so $v = v_0 + at$ still applies at $t = 7.0\,\text{s}$: $v = 5.0 + (-1.0)(7.0) = \boxed{-2.0\,\text{m/s}}$ — the negative sign shows she is now moving back down the ramp at $2.0\,\text{m/s}$, $2.0\,\text{s}$ after momentarily stopping.

Bài tập luyện tập

A ball is thrown straight up at $14.7\,\text{m/s}$ from the edge of a $10\,\text{m}$ tall cliff. Find the total time until it strikes the ground at the base of the cliff.

Lời giải chi tiết

Take up as positive: $v_0 = +14.7\,\text{m/s}$. The ground is $10\,\text{m}$ below the launch point, so the net vertical displacement when it lands is $\Delta y = -10\,\text{m}$:

$$\Delta y = v_0 t - \tfrac{1}{2}gt^2 \;\Rightarrow\; -10 = 14.7t - 4.9t^2 \;\Rightarrow\; 4.9t^2 - 14.7t - 10 = 0.$$

Quadratic formula: $t = \dfrac{14.7 \pm \sqrt{14.7^2 + 4(4.9)(10)}}{2(4.9)} = \dfrac{14.7 \pm \sqrt{216.09 + 196}}{9.8} = \dfrac{14.7 \pm \sqrt{412.09}}{9.8} = \dfrac{14.7 \pm 20.3}{9.8}$.
Taking the positive root (the negative root is not physical): $t = \dfrac{35.0}{9.8} = \boxed{3.57\,\text{s}}$.

Bài tập luyện tập

A rock is thrown straight up at $12.0\,\text{m/s}$ from the edge of a $25\,\text{m}$ cliff. Find its speed the instant it strikes the ground at the base of the cliff.

(A) $20.0\,\text{m/s}$
(B) $22.1\,\text{m/s}$
(C) $25.2\,\text{m/s}$
(D) $29.4\,\text{m/s}$

Lời giải chi tiết

Take up as positive: $v_0 = +12.0\,\text{m/s}$, $\Delta y = -25\,\text{m}$ (net displacement down to the base of the cliff — this already accounts for both the rise above the edge and the full fall to the ground, so equation (4) applies directly without first finding the time of flight):

$$v^2 = v_0^2 + 2a\Delta y = (12.0)^2 + 2(-9.8)(-25) = 144 + 490 = 634 \;\Rightarrow\; v = \sqrt{634} = 25.2\,\text{m/s}.$$

(C).

Bài tập luyện tập

A ball is launched from ground level at $v_0 = 24.5\,\text{m/s}$, 53° above horizontal, on level ground. ($\sin 53^\circ = 0.80$, $\cos 53^\circ = 0.60$.) Find the two times at which the ball is at a height of $14.7\,\text{m}$ above the ground, and find the horizontal distance it covers between those two instants.

Lời giải chi tiết

$v_{y0} = 24.5(0.80) = 19.6\,\text{m/s}$, $v_{x0} = 24.5(0.60) = 14.7\,\text{m/s}$.
Set $y = v_{y0}t - \frac{1}{2}gt^2 = 14.7$:

$$14.7 = 19.6t - 4.9t^2 \;\Rightarrow\; 4.9t^2 - 19.6t + 14.7 = 0 \;\Rightarrow\; t^2 - 4.0t + 3.0 = 0 \;\Rightarrow\; (t-1.0)(t-3.0) = 0.$$

So $t = \boxed{1.0\,\text{s}}$ (on the way up) and $t = \boxed{3.0\,\text{s}}$ (on the way down) — symmetric about the peak at $t = 2.0\,\text{s}$.
Horizontal distance covered between these instants: $\Delta x = v_{x0}\,\Delta t = (14.7)(3.0 - 1.0) = (14.7)(2.0) = \boxed{29.4\,\text{m}}$.

Bài tập luyện tập

Car A moves due east at $30\,\text{m/s}$; car B moves due north at $40\,\text{m/s}$. Find the magnitude of car A's velocity relative to car B.

(A) $10\,\text{m/s}$
(B) $50\,\text{m/s}$
(C) $70\,\text{m/s}$
(D) $1200\,\text{m/s}$

Lời giải chi tiết

$\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B$. Taking east as $+x$ and north as $+y$: $\vec{v}_A = (30, 0)$, $\vec{v}_B = (0, 40)$, so $\vec{v}_{A/B} = (30, -40)$. Magnitude:

$$|\vec{v}_{A/B}| = \sqrt{30^2 + 40^2} = \sqrt{900 + 1600} = \sqrt{2500} = 50\,\text{m/s}.$$

(B).

Bài tập luyện tập

An object released from rest is timed at two points along its motion: at $t = 2.0\,\text{s}$, $x = 6.0\,\text{m}$; at $t = 5.0\,\text{s}$, $x = 37.5\,\text{m}$. Using the linearized x vs. t^2 method, find the object's acceleration.

(A) $1.5\,\text{m/s}^2$
(B) $2.0\,\text{m/s}^2$
(C) $3.0\,\text{m/s}^2$
(D) $6.0\,\text{m/s}^2$

Lời giải chi tiết

$t_1^2 = (2.0)^2 = 4.0\,\text{s}^2$, $t_2^2 = (5.0)^2 = 25.0\,\text{s}^2$. Slope of the x vs. t^2 line:

$$\text{slope} = \frac{37.5 - 6.0}{25.0 - 4.0} = \frac{31.5}{21.0} = 1.5\,\text{m/s}^2.$$

Since slope $= \frac{1}{2}a$: $a = 2(1.5) = 3.0\,\text{m/s}^2$. **(C)**.

Force and Translational Dynamics

Mục tiêu Unit: Ba định luật Newton; khối lượng vs trọng lượng; biểu đồ lực tự do (FBD); hệ vật nối dây qua ròng rọc; trọng lượng biểu kiến trong thang máy; ma sát nghỉ/trượt; mặt phẳng nghiêng; chuyển động tròn đều và lực hướng tâm.

2.1 Newton's First Law

Newton's first law (inertia): an object at rest stays at rest, and an object in motion continues at constant velocity in a straight line, unless acted on by a nonzero **net** external force. Equivalently: if $\sum \vec{F} = 0$, the object's velocity does not change (this includes the special case $v = 0$).

Inertia is an object's tendency to resist a change in its velocity; it is quantified by **mass** (a scalar, measured in kg). Mass is not the same as **weight** — weight is the force of gravity on an object, $W = mg$, and depends on location, while mass does not.

GIẢI THÍCH TIẾNG VIỆT

VN

Định luật 1 Newton (định luật quán tính): vật giữ nguyên trạng thái đứng yên hoặc chuyển động thẳng đều nếu **hợp lực** tác dụng lên nó bằng 0. **Khối lượng** m đặc trưng cho quán tính (không đổi theo vị trí), còn **trọng lượng** $W = mg$ là lực hấp dẫn tác dụng lên vật (phụ thuộc vào g tại nơi đó).

2.2 Newton's Second Law

Newton's second law: $\sum \vec{F} = m\vec{a}$, or $a = \dfrac{F_{net}}{m}$. The acceleration of an object is directly proportional to the net force acting on it, in the same direction as that net force, and inversely proportional to its mass.

Ví dụ 1 · Basic application of $F = ma$

A 5.0 kg block on a frictionless horizontal surface is pulled by a single horizontal force $F = 20$ N. Find its acceleration.

$$a = \frac{F_{net}}{m} = \frac{20}{5.0} = 4.0\,\text{m/s}^2.$$

2.3 Newton's Third Law

Newton's third law: if object A exerts a force on object B, then B exerts a force of equal magnitude and opposite direction on A. These two forces form an **action–reaction pair**: they act on *different* objects, are the *same type* of force, and occur *simultaneously*.

(!) Lỗi thường gặp: Action–reaction pairs act on **two different objects**, so they never cancel each other out when analyzing a single object's motion. A book's weight (Earth pulling the book) and the table's normal force (table pushing the book) both act on the *same* object (the book) and happen to be equal in a static case — but they are **not** a third-law pair. The true partner of the book's weight is the book's gravitational pull on the Earth.

Ví dụ 2 · Third law applied quantitatively

Skater A ($m_A = 60\,\text{kg}$) and skater B ($m_B = 40\,\text{kg}$) stand at rest on frictionless ice and push off from each other. If skater B moves away at $4.5\,\text{m/s}$, find skater A's speed.

By Newton's third law, the push force on A is equal and opposite to the push force on B at every instant, and both forces act for the *same* contact time Δt. Hence $m_A v_A = m_B v_B$ (equal magnitudes, opposite directions):

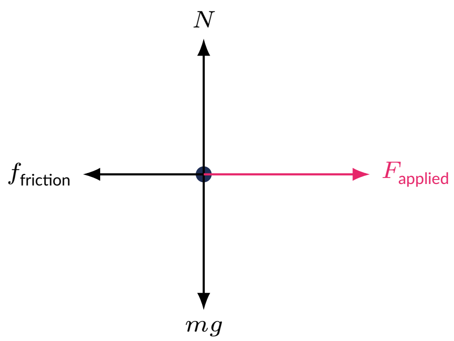
$$v_A = \frac{m_B v_B}{m_A} = \frac{(40)(4.5)}{60} = 3.0\,\text{m/s}.$$

GIẢI THÍCH TIẾNG VIỆT

VN Cặp lực định luật 3 Newton luôn tác dụng lên **hai vật khác nhau** nên **không bao giờ triệt tiêu nhau** khi phân tích chuyển động của một vật. Ví dụ 2 cho thấy hệ quả: vì lực đẩy giữa hai người trượt băng bằng nhau và tác dụng trong cùng thời gian, vật nhẹ hơn sẽ văng đi với tốc độ lớn hơn.

2.4 Free-Body Diagrams

A **free-body diagram** (FBD) isolates a single object and draws every external force acting on *it alone* as an arrow starting from its center. Common forces: weight mg (always straight down), normal force N (perpendicular to, and away from, the contact surface), tension T (along a string/rope, pulling), applied force, and friction f (along the surface, opposing relative sliding or impending sliding).



Free-body diagram of a block pulled across a rough horizontal surface.

(!) Lỗi thường gặp: The normal force is **not always equal to** mg. Whenever another force has a vertical component, or the surface is tilted, N must be solved for from $\sum F_\perp = 0$ (or $= ma_\perp$) in that problem, never assumed equal to weight by default.

2.5 Apparent Weight in an Elevator

A scale reads the **normal force** it exerts on you, not your true weight mg. In an accelerating elevator these differ.

Ví dụ 3 · Apparent weight

A $70\,\text{kg}$ person stands on a scale inside an elevator. Find the scale reading when the elevator (a) accelerates upward at $2.0\,\text{m/s}^2$, (b) accelerates downward at $2.0\,\text{m/s}^2$, (c) moves at constant velocity.

Taking up as positive, Newton's second law on the person: $N - mg = ma \Rightarrow N = m(g + a)$.
(a) $N = 70(9.8 + 2.0) = 70(11.8) = 826\,\text{N}$ (feels heavier).
(b) $N = 70(9.8 - 2.0) = 70(7.8) = 546\,\text{N}$ (feels lighter).
(c) $a = 0 \Rightarrow N = mg = 70(9.8) = 686\,\text{N}$ (equals true weight — constant velocity means zero net force, regardless of speed or direction of travel).

VN **GIẢI THÍCH TIẾNG VIỆT**

Cân trong thang máy đo **lực pháp tuyến** N, không phải trọng lượng thật mg. Khi thang máy tăng tốc **lên** ($a > 0$ theo chiều lên), $N > mg$ (cảm thấy nặng hơn); khi tăng tốc **xuống**, $N < mg$ (cảm thấy nhẹ hơn); khi chuyển động **đều** (kể cả đang đi lên hoặc xuống với vận tốc không đổi), $N = mg$ vì gia tốc bằng 0.

2.6 Systems of Connected Objects

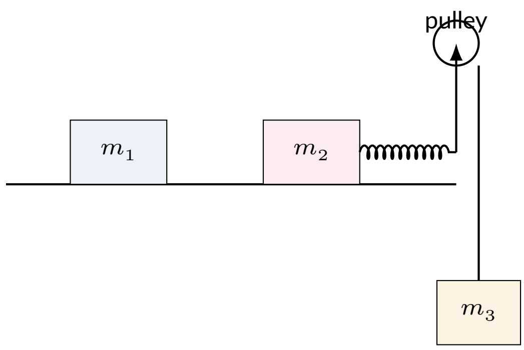
When two or more objects move together (connected by a string, or in contact), two approaches work: (1) treat the whole system as one object of total mass $\sum m$ to find the common acceleration, then (2) isolate a single object to find an internal force (tension or contact force).

Ví dụ 4 · Blocks in contact

Block A ($m_A = 3.0\,\text{kg}$) is in contact with block B ($m_B = 2.0\,\text{kg}$) on a frictionless floor. A horizontal force $F = 30\,\text{N}$ pushes on A, which in turn pushes B. Find the system's acceleration and the contact force between the blocks.

Whole system: $a = \dfrac{F}{m_A + m_B} = \dfrac{30}{5.0} = 6.0\,\text{m/s}^2$.

Isolate B (only the contact push from A acts on it horizontally): $F_{contact} = m_B a = (2.0)(6.0) = 12\,\text{N}$.



A pulley system: masses connected by a string change the direction of tension, coupling their motion.

Ví dụ 5 · Atwood machine

Two blocks hang from a light string over a frictionless, massless pulley: $m_1 = 4.0\,\text{kg}$ and $m_2 = 6.0\,\text{kg}$. Find the acceleration of the system and the string tension.

The heavier side determines the direction of motion. Net driving force $= (m_2 - m_1)g$; total mass being accelerated $= m_1 + m_2$:

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2} = \frac{(2.0)(9.8)}{10} = 1.96\,\text{m/s}^2.$$

Tension from m_1's equation ($T - m_1 g = m_1 a$, since m_1 accelerates upward):

$$T = m_1(g + a) = 4.0(9.8 + 1.96) = 4.0(11.76) = 47.0\,\text{N}.$$

Check with m_2 ($m_2 g - T = m_2 a$): $T = m_2(g - a) = 6.0(9.8 - 1.96) = 6.0(7.84) = 47.0\,\text{N}$ ✓.

2.7 Friction

Khái niệm cốt lõi

Static friction $f_s \leq \mu_s N$ prevents relative sliding from starting; it adjusts to match whatever is needed to keep the object stationary, up to a maximum value $\mu_s N$. **Kinetic friction** $f_k = \mu_k N$ acts once the object is sliding, always opposing the relative motion, and is essentially constant regardless of speed. Typically $\mu_s > \mu_k$ for a given pair of surfaces.

(!) Lỗi thường gặp: $f_s \leq \mu_s N$ is an **inequality**. Static friction only equals $\mu_s N$ right at the verge of slipping — for smaller applied forces, static friction is smaller too (just enough to keep the object at rest). Do not automatically set $f_s = \mu_s N$ unless the problem states the object is on the verge of moving.

VN **GIẢI THÍCH TIẾNG VIỆT**

Ma sát **nghỉ** f_s tự điều chỉnh để giữ vật đứng yên, chỉ đạt giá trị lớn nhất $\mu_s N$ khi vật sắp trượt. Ma sát **trượt** $f_k = \mu_k N$ có giá trị gần như không đổi một khi vật đã trượt, luôn ngược chiều chuyển động tương đối. Thông thường $\mu_s > \mu_k$.

Ví dụ 6 · Kinetic friction and acceleration

An 8.0 kg block on a horizontal floor ($\mu_k = 0.25$) is pushed with a horizontal force $F = 40$ N. Find its acceleration.

$N = mg = (8.0)(9.8) = 78.4$ N, so $f_k = \mu_k N = (0.25)(78.4) = 19.6$ N.

$F_{net} = F - f_k = 40 - 19.6 = 20.4\,\text{N} \Rightarrow a = \dfrac{20.4}{8.0} = 2.55\,\text{m/s}^2$.

Ví dụ 7 · Maximum static friction

A 12 kg crate rests on a floor with $\mu_s = 0.45$. Find the maximum horizontal force that can be applied without the crate starting to slide.

$$f_{s,\text{max}} = \mu_s mg = (0.45)(12)(9.8) = 52.9\,\text{N}.$$

Any applied force up to 52.9 N is exactly balanced by static friction; beyond it, the crate slides.

Ví dụ 8 · Friction with an angled applied force

A 5.0 kg block on a horizontal floor ($\mu_k = 0.30$) is pulled by a force $F = 40$ N directed $37°$ above horizontal. Find its acceleration.

Components: $F_x = F \cos 37° = 40(0.80) = 32$ N; $F_y = F \sin 37° = 40(0.60) = 24$ N (upward). The upward pull *reduces* the normal force: $N = mg - F_y = (5.0)(9.8) - 24 = 49 - 24 = 25$ N. $f_k = \mu_k N = (0.30)(25) = 7.5$ N.

$F_{net,x} = F_x - f_k = 32 - 7.5 = 24.5$ N $\Rightarrow a = \dfrac{24.5}{5.0} = 4.9$ m/s^2.

2.7.1 Applied Forces at an Angle

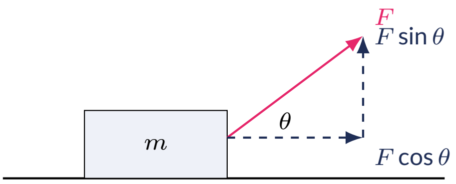
When a rope, handle, or push is applied at an angle to the horizontal rather than purely along it, the force's vertical component changes how hard the object presses on the floor — and therefore changes the normal force and the friction force as well.

Resolve an applied force F that makes angle θ with the horizontal into $F_x = F\cos\theta$ (horizontal, drives the motion) and $F_y = F\sin\theta$ (vertical). The vertical component shifts the normal force away from the simple case $N = mg$:

$$N = mg - F\sin\theta \qquad \text{(pulling up and forward, angle above horizontal)},$$

$$N = mg + F\sin\theta \qquad \text{(pushing down and forward, angle below horizontal)}.$$

Friction must always be computed from the *actual* normal force in that problem, $f_k = \mu_k N$ (or $f_{s,\max} = \mu_s N$) — never from μmg by default once an applied force has a vertical component. (If $F\sin\theta > mg$ while pulling upward, N cannot go negative — the object lifts off the floor, and $N = 0$ with no friction at all.)



An applied force F at angle θ above horizontal, resolved into $F\cos\theta$ (drives horizontal motion) and $F\sin\theta$ (reduces N if pulling upward, or increases N if pushing downward).

VN **GIẢI THÍCH TIẾNG VIỆT**

Khi lực tác dụng F hợp với phương ngang một góc θ, phân tích thành $F_x = F\cos\theta$ (nằm ngang, gây chuyển động) và $F_y = F\sin\theta$ (theo phương đứng). Nếu lực **kéo xiên lên** phía trước, thành phần đứng làm **giảm** lực pháp tuyến: $N = mg - F\sin\theta$, nên ma sát cũng giảm. Nếu lực **đẩy xiên xuống** phía trước, thành phần đứng làm **tăng** lực pháp tuyến: $N = mg + F\sin\theta$, nên ma sát cũng tăng. Luôn tính ma sát từ giá trị N thực tế của bài toán, tuyệt đối không mặc định dùng μmg.

Ví dụ 19 · Pulling up and forward at an angle

A 6.0 kg block on a horizontal floor ($\mu_k = 0.25$) is pulled by a rope exerting $F = 50$ N directed $37°$ above horizontal. Find its acceleration.

Components: $F_x = F\cos 37° = 50(0.80) = 40$ N; $F_y = F\sin 37° = 50(0.60) = 30$ N (upward). The upward pull *reduces* the normal force: $N = mg - F_y = (6.0)(9.8) - 30 = 58.8 - 30 = 28.8$ N.

$f_k = \mu_k N = (0.25)(28.8) = 7.2\,\text{N}$.
$F_{net,x} = F_x - f_k = 40 - 7.2 = 32.8\,\text{N} \Rightarrow a = \dfrac{32.8}{6.0} = 5.47\,\text{m/s}^2$.

Ví dụ 20 · Pushing down and forward at an angle

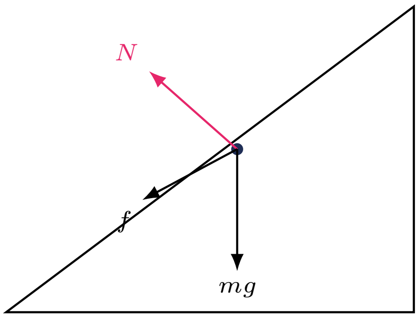
The same $6.0\,\text{kg}$ block ($\mu_k = 0.25$) is now pushed (not pulled) by the same-magnitude force $F = 50\,\text{N}$, directed 37° *below* horizontal — as when pushing a broom handle down and forward. Find its acceleration.

Components: $F_x = F\cos 37^\circ = 50(0.80) = 40\,\text{N}$; $F_y = F\sin 37^\circ = 50(0.60) = 30\,\text{N}$ (downward).
The downward push *increases* the normal force: $N = mg + F_y = 58.8 + 30 = 88.8\,\text{N}$.
$f_k = \mu_k N = (0.25)(88.8) = 22.2\,\text{N}$.
$F_{net,x} = F_x - f_k = 40 - 22.2 = 17.8\,\text{N} \Rightarrow a = \dfrac{17.8}{6.0} = 2.97\,\text{m/s}^2$.

Contrast: identical force magnitude ($50\,\text{N}$), angle (37°), mass, and μ_k give $a = 5.47\,\text{m/s}^2$ when pulling up but only $a = 2.97\,\text{m/s}^2$ when pushing down — nearly half. The horizontal driving component F_x is the same in both cases; the entire difference comes from how the vertical component F_y changes N, and hence the friction opposing the motion.

2.8 Inclined Planes

Resolve gravity into a component *along* the incline, $mg\sin\theta$, and a component *into* the incline, $mg\cos\theta$. The normal force balances the latter: $N = mg\cos\theta$ (on a simple incline with no other forces).



Block on a 37° incline: weight mg resolves into $mg\sin\theta$ along the slope and $mg\cos\theta$ into the slope; N balances the latter, friction f opposes sliding.

GIẢI THÍCH TIẾNG VIỆT
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Với mặt phẳng nghiêng, luôn phân tích trọng lực mg thành hai thành phần: **dọc theo mặt phẳng** ($mg\sin\theta$, gây trượt) và **vuông góc với mặt phẳng** ($mg\cos\theta$, bị triệt tiêu bởi N). Ma sát (nếu có) luôn nằm dọc theo mặt phẳng, ngược chiều chuyển động (hoặc chiều sắp trượt).

Ví dụ 9 · Frictionless incline

A block slides down a frictionless 37° incline. Find its acceleration along the slope.

$$a = g\sin\theta = (9.8)(0.60) = 5.88\,\mathrm{m/s^2}$$

— independent of the block's mass.

Ví dụ 10 · Incline with friction, sliding down

A $5.0\,\mathrm{kg}$ block slides down a 37° incline with $\mu_k = 0.20$. Find its acceleration.

$$a = g(\sin\theta - \mu_k\cos\theta) = 9.8(0.60 - 0.20(0.80)) = 9.8(0.60 - 0.16) = 9.8(0.44) = 4.31\,\mathrm{m/s^2}.$$

Ví dụ 11 · Incline with friction, sliding up and decelerating

A block is sent up a 37° incline ($\mu_k = 0.25$) with initial speed $8.4\,\mathrm{m/s}$. How far up the incline does it travel before momentarily stopping? (While moving up, both gravity's component and kinetic friction act down the slope.)

$$a = g(\sin\theta + \mu_k\cos\theta) = 9.8(0.60 + 0.25(0.80)) = 9.8(0.80) = 7.84\,\mathrm{m/s^2}\ \text{(deceleration)}.$$

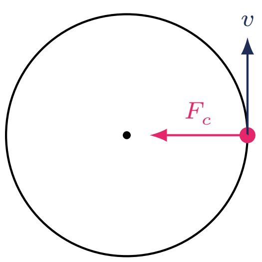
$$d = \frac{v_0^2}{2a} = \frac{(8.4)^2}{2(7.84)} = \frac{70.56}{15.68} = 4.5\,\mathrm{m}.$$

2.9 Uniform Circular Motion and Centripetal Force

An object moving at constant speed v around a circle of radius r continuously changes *direction*, so it accelerates even though its speed does not change. This **centripetal acceleration** always points toward the center:

$$a_c = \frac{v^2}{r}, \qquad F_c = ma_c = \frac{mv^2}{r}.$$

"Centripetal force" is **not** a new, separate type of force — it is the name given to whatever *net* force (tension, friction, gravity, normal force, or a combination) happens to point toward the center and cause circular motion.



Uniform circular motion: velocity is tangent to the circle, net (centripetal) force points toward the center, perpendicular to v.

(!) Lỗi thường gặp: Do not add a separate outward "centrifugal force" to a free-body diagram in an inertial (non-rotating) reference frame. The only forces present are the real, physical forces (gravity, tension, friction, normal, etc.); their vector sum equals mv^2/r, directed toward the center.

VN **GIẢI THÍCH TIẾNG VIỆT**

Lực hướng tâm **không phải** một loại lực mới — đó là **tên gọi** của hợp lực (lực căng dây, ma sát, trọng lực, lực pháp tuyến, hoặc tổng hợp của chúng) khi hợp lực đó hướng vào tâm quỹ đạo tròn. Không được tự thêm "lực li tâm" hướng ra ngoài vào biểu đồ lực tự do trong hệ quy chiếu đứng yên.

Ví dụ 12 · Car rounding a flat curve

A $1200\,\text{kg}$ car rounds a flat, unbanked curve of radius $50\,\text{m}$. If $\mu_s = 0.60$ between tires and road, find the maximum speed without skidding.

Static friction supplies the centripetal force: $\mu_s mg = \dfrac{mv^2}{r} \Rightarrow v = \sqrt{\mu_s gr}$.

$$v = \sqrt{(0.60)(9.8)(50)} = \sqrt{294} = 17.1\,\text{m/s}.$$

(Notice the mass cancels — the maximum safe speed does not depend on how heavy the car is.)

Ví dụ 13 · Vertical circle — minimum speed at the top

A ball on a string of length $0.80\,\text{m}$ is swung in a vertical circle. Find the minimum speed at the very top of the circle for the string to remain taut.

At the top, both gravity and tension point toward the center (downward): $T + mg = \dfrac{mv^2}{r}$. The string goes slack when $T \to 0$, so the minimum condition is $mg = \dfrac{mv_{\text{min}}^2}{r}$:

$$v_{\text{min}} = \sqrt{gr} = \sqrt{(9.8)(0.80)} = \sqrt{7.84} = 2.8\,\text{m/s}.$$

2.10 Newton's Law of Universal Gravitation

Every pair of objects in the universe attracts every other pair gravitationally. Newton showed that this attraction depends only on the two masses and the distance between their centers.

Newton's law of universal gravitation:

$$F_g = \frac{Gm_1m_2}{r^2}, \qquad G = 6.67 \times 10^{-11}\,\mathrm{N\cdot m^2/kg^2}.$$

F_g is always **attractive**, directed along the line joining the two centers; by Newton's third law, the force m_1 exerts on m_2 is equal in magnitude and opposite in direction to the force m_2 exerts on m_1.

Surface gravity of a planet:

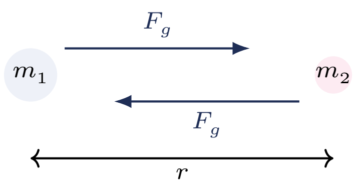
$$g = \frac{GM}{R^2} \qquad (\text{from setting } mg = F_g = \frac{GMm}{R^2}).$$

Weight vs. distance from center: weight falls off as $1/r^2$; at distance $r = nR$ from the center, weight is $\frac{1}{n^2}$ of the surface weight (e.g. at $r = 2R$, weight is $\frac{1}{4}$ of the surface value).

Circular orbit — gravity as the centripetal force:

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \implies v = \sqrt{\frac{GM}{r}}, \qquad T = 2\pi\sqrt{\frac{r^3}{GM}}.$$

Since $T^2 = \frac{4\pi^2}{GM}r^3$, every object orbiting the same central mass satisfies $T^2 \propto r^3$ — this is **Kepler's third law**.



Two masses pull on each other with equal and opposite forces of magnitude $F_g = Gm_1m_2/r^2$ (a Newton's-third-law pair), directed along the line joining their centers, separated by distance r.

Why $g \approx 9.8\,\mathrm{m/s^2}$ near Earth's surface. An object of mass m resting at Earth's surface has weight mg (Newton's second law) that *is* the gravitational force $\frac{GM_\oplus m}{R_\oplus^2}$ (universal gravitation) — the two expressions describe the same force, so $mg = \frac{GM_\oplus m}{R_\oplus^2}$, giving $g = \frac{GM_\oplus}{R_\oplus^2}$. Using $M_\oplus = 5.97 \times 10^{24}$ kg and $R_\oplus = 6.37 \times 10^6$ m:

$$g = \frac{(6.67 \times 10^{-11})(5.97 \times 10^{24})}{(6.37 \times 10^6)^2} = \frac{3.982 \times 10^{14}}{4.058 \times 10^{13}} = 9.81\,\mathrm{m/s^2} \approx 9.8\,\mathrm{m/s^2}.$$

So $g = 9.8\,\mathrm{m/s^2}$ is not a fundamental constant — it is a *consequence* of Earth's particular mass and radius, and a different planet or moon has a different surface gravity.

(!) Lỗi thường gặp: Do not confuse G and g. $G = 6.67 \times 10^{-11}\,\mathrm{N\cdot m^2/kg^2}$ is the **universal gravitational constant** — the same everywhere in the universe. $g = GM/R^2$ is the **local surface gravity** of one specific planet or moon; it depends on that body's mass and radius, and equals $\approx 9.8\,\mathrm{m/s^2}$ only for Earth.

VN **GIẢI THÍCH TIẾNG VIỆT**

Lực hấp dẫn giữa hai vật: $F_g = Gm_1m_2/r^2$, với $G = 6.67 \times 10^{-11}$ N · m^2/kg^2 là hằng số hấp dẫn (không đổi trong toàn vũ trụ). Gia tốc trọng trường tại bề mặt một hành tinh, $g = GM/R^2$, **phụ thuộc** vào khối lượng M và bán kính R của hành tinh đó — đây là lý do vì sao $g \approx 9.8$ m/s^2 chỉ đúng riêng cho Trái Đất. Trọng lượng giảm theo quy luật nghịch đảo bình phương khoảng cách: ở khoảng cách $r = nR$ tính từ tâm, trọng lượng chỉ còn $1/n^2$ so với ở bề mặt. Với vệ tinh bay theo quỹ đạo tròn, lực hấp dẫn đóng vai trò lực hướng tâm, cho tốc độ quỹ đạo $v = \sqrt{GM/r}$ và chu kỳ $T = 2\pi\sqrt{r^3/GM}$ (định luật Kepler thứ ba: $T^2 \propto r^3$).

Ví dụ 14 · Gravitational force between two objects

Two objects, $m_1 = 2000$ kg and $m_2 = 1500$ kg, have their centers separated by $r = 5.0$ m. Find the gravitational force between them.

$$F_g = \frac{Gm_1m_2}{r^2} = \frac{(6.67 \times 10^{-11})(2000)(1500)}{(5.0)^2} = \frac{(6.67 \times 10^{-11})(3.0 \times 10^6)}{25}.$$

$$F_g = \frac{2.00 \times 10^{-4}}{25} = 8.0e - 6\,\text{N}.$$

This is an extremely small force — gravity between everyday-sized objects is negligible; it becomes significant only when at least one mass is astronomically large.

Ví dụ 15 · g at an altitude above Earth's surface

Find g at an altitude of 400 km above Earth's surface, and express it as a fraction of surface gravity ($R_\oplus = 6.37 \times 10^6$ m).

Distance from Earth's center: $r = R_\oplus + h = 6.37 \times 10^6 + 0.40 \times 10^6 = 6.77e6$ m.

Since $g \propto 1/r^2$:

$$\frac{g_{alt}}{g_{surf}} = \left(\frac{R_\oplus}{r}\right)^2 = \left(\frac{6.37}{6.77}\right)^2 = (0.9409)^2 = 0.8853.$$

$$g_{alt} = (0.8853)(9.8) = 8.68\,\text{m/s}^2.$$

Even at this altitude (roughly the orbit of the International Space Station), gravity is still about 88.5% as strong as at the surface — nowhere near zero. ("Weightlessness" in orbit is not caused by an absence of gravity; see the discussion of vertical circles and orbital motion below.)

Ví dụ 16 · Orbital speed and period of a satellite

A satellite orbits Earth in a circle at the same altitude as Ví dụ 15, $r = 6.77e6$ m from Earth's center. Find its orbital speed and period.

Gravity supplies the centripetal force:

$$v = \sqrt{\frac{GM_\oplus}{r}} = \sqrt{\frac{(6.67 \times 10^{-11})(5.97 \times 10^{24})}{6.77 \times 10^6}} = \sqrt{\frac{3.982 \times 10^{14}}{6.77 \times 10^6}} = \sqrt{5.88 \times 10^7} = 7.67e3\,\text{m/s}.$$

Period, using $T = 2\pi r/v$:

$$T = \frac{2\pi(6.77 \times 10^6)}{7.67 \times 10^3} = 5.55e3\,\text{s} \approx 92.4\text{ minutes}.$$

This closely matches the real orbital period of the International Space Station (about 93 minutes) — a useful sanity check.

2.10.1 Circular Motion at an Angle: Banked Curves and Vertical Circles

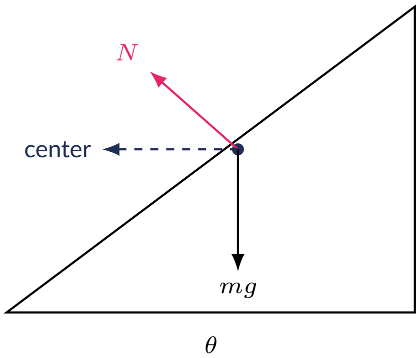
Not every centripetal-force problem involves a horizontal surface or a horizontal string. Two classic setups combine circular motion with forces at an angle: a car on a **banked curve**, and an object moving through a **vertical circle**.

Banked curve (frictionless case). On a curve banked at angle θ, the normal force N tilts inward and does double duty: its vertical component supports the object's weight, and its horizontal component supplies the centripetal force — no friction is needed at the "design speed."

$$N\cos\theta = mg \quad \text{(vertical equilibrium)}, \qquad N\sin\theta = \frac{mv^2}{r} \quad \text{(horizontal, toward center)}.$$

Dividing the second equation by the first eliminates both N and m:

$$\tan\theta = \frac{v^2}{rg} \quad \Longrightarrow \quad v = \sqrt{rg\tan\theta}.$$



Cross-section of a banked curve (viewed head-on, frictionless case): N is perpendicular to the road surface. Its horizontal component provides the centripetal force toward the center of the curve; its vertical component balances gravity.

Vertical circle. Consider a car on the inside of a vertical loop (or a ball on a string). At the **bottom**, the center of the circle is directly *above*, so the net inward (upward) force is $N - mg$:

$$N - mg = \frac{mv^2}{r} \quad \Longrightarrow \quad N = m\left(g + \frac{v^2}{r}\right) \qquad (\text{bottom},\ N > mg).$$

At the **top**, the center is directly *below*, so both N and mg point inward (downward):

$$N + mg = \frac{mv^2}{r} \quad \Longrightarrow \quad N = m\left(\frac{v^2}{r} - g\right) \qquad (\text{top}).$$

Since a track can only push (not pull), N cannot be negative — the object loses contact if v is too small. The **minimum speed at the top** occurs when $N = 0$, so gravity alone supplies the entire centripetal force:

$$mg = \frac{mv_{\text{min}}^2}{r} \quad \Longrightarrow \quad v_{\text{min}} = \sqrt{gr} \qquad (\text{minimum speed at the top of a vertical circle}).$$

GIẢI THÍCH TIẾNG VIỆT

VN

Trên khúc cua nghiêng **không ma sát**, thành phần ngang của lực pháp tuyến $N\sin\theta$ đóng vai trò lực hướng tâm, còn thành phần đứng $N\cos\theta$ cân bằng trọng lực, dẫn đến $\tan\theta = v^2/(rg)$. Trong chuyển động tròn theo phương đứng (vòng lặp), ở **đáy vòng**, N hướng vào tâm (lên trên) nên $N = m(g + v^2/r) > mg$; ở **đỉnh vòng**, cả N và mg đều hướng vào tâm (xuống dưới) nên

$N = m(v^2/r - g)$. Tốc độ tối thiểu ở đỉnh (khi $N = 0$, chỉ còn trọng lực làm lực hướng tâm) là $v_{\text{min}} = \sqrt{gr}$.

Ví dụ 17 · Banked curve, frictionless

A curve of radius $15\,\text{m}$ is banked at $\theta = 37^\circ$, engineered so that no friction is needed at the design speed. Find that design speed.

$$v = \sqrt{rg\tan\theta} = \sqrt{(15)(9.8)(0.75)} = \sqrt{110.25} = 10.5\,\text{m/s}.$$

At exactly this speed, the horizontal component of N alone provides the full centripetal force; a faster car would need friction to keep from sliding up and outward, and a slower car would need friction to keep from sliding down and inward.

Ví dụ 18 · Vertical loop — top and bottom

A roller-coaster car ($m = 500\,\text{kg}$) travels through a vertical circular loop of radius $10\,\text{m}$.

(a) Find the minimum speed at the top of the loop needed to maintain contact with the track.

$$v_{\text{min}} = \sqrt{gr} = \sqrt{(9.8)(10)} = \sqrt{98} = 9.90\,\text{m/s}.$$

(b) At the bottom of the loop, the car's speed is $20\,\text{m/s}$. Find the normal force from the track there.

$$N = m\left(g + \frac{v^2}{r}\right) = 500\left(9.8 + \frac{(20)^2}{10}\right) = 500(9.8 + 40) = 500(49.8) = 2.49e4\,\text{N}.$$

Notice N at the bottom ($2.49e4$ N) is far greater than the car's weight ($mg = 4900$ N) — this is the "pressed into your seat" sensation at the bottom of a loop.

2.11 Practice Problems

Bài tập luyện tập

An object moves at constant velocity in a straight line. What is the net force acting on it?

(A) Equal to its weight

(B) Zero

(C) Equal to ma, in the direction of motion

(D) Cannot be determined

Lời giải chi tiết

Constant velocity means $a = 0$, so by Newton's second law $F_{net} = ma = 0$. **(B).**

Bài tập luyện tập

A net force of 45 N acts on a 15 kg object initially at rest. Find its velocity after 4.0 s.

Lời giải chi tiết

$a = \dfrac{45}{15} = 3.0\,\text{m/s}^2$. $v = at = (3.0)(4.0) = \boxed{12\,\text{m/s}}$.

Bài tập luyện tập

Find the weight of an 8.0 kg object at Earth's surface.

(A) 8.0 N
(B) 19.6 N
(C) 78.4 N
(D) 9.8 N

Lời giải chi tiết

$W = mg = (8.0)(9.8) = 78.4$ N. **(C)**.

Bài tập luyện tập

A large truck collides head-on with a small car. Compare the magnitude of the force the truck exerts on the car to the force the car exerts on the truck.

(A) Truck exerts a larger force
(B) Car exerts a larger force
(C) The forces are equal in magnitude
(D) It depends on their relative speeds

Lời giải chi tiết

By Newton's third law, the two forces are always equal in magnitude and opposite in direction, regardless of the objects' relative size or speed (though their resulting accelerations differ, since $a = F/m$ and their masses differ). **(C)**.

Bài tập luyện tập

Skater A (60 kg) and skater B (40 kg) push off from rest on frictionless ice. Skater B moves away at 4.5 m/s. Find skater A's speed.

(A) 2.0 m/s
(B) 3.0 m/s
(C) 4.5 m/s
(D) 6.75 m/s

Lời giải chi tiết

$$m_A v_A = m_B v_B \Rightarrow v_A = \frac{(40)(4.5)}{60} = 3.0 \text{ m/s}.$$ **(B)**.

Bài tập luyện tập

A 60 kg person stands on a scale in an elevator accelerating downward at 3.0 m/s^2. Find the scale reading.

Lời giải chi tiết

$N = m(g - a) = 60(9.8 - 3.0) = 60(6.8) = \boxed{408 \text{ N}}$.

Bài tập luyện tập

An elevator moves upward at a constant 2.0 m/s. How does a passenger's apparent weight compare to their true weight?

(A) Greater
(B) Smaller
(C) Equal
(D) Zero

Lời giải chi tiết

Constant velocity means $a = 0$, so $N = mg$: apparent weight equals true weight. **(C)**.

Bài tập luyện tập

Blocks A ($2.0\,\text{kg}$) and B ($4.0\,\text{kg}$) are in contact on a frictionless floor. A force $F = 18\,\text{N}$ pushes on A, which pushes B. Find the system's acceleration and the contact force between the blocks.

Lời giải chi tiết

$a = \dfrac{18}{6.0} = \boxed{3.0\,\text{m/s}^2}$. Contact force on B: $F_{contact} = m_B a = (4.0)(3.0) = \boxed{12\,\text{N}}$.

Bài tập luyện tập

Blocks A ($3.0\,\text{kg}$, in front) and B ($2.0\,\text{kg}$, behind) are connected by a light string on a frictionless floor. A force $F = 20\,\text{N}$ pulls A forward. Find the system's acceleration and the string tension.

Lời giải chi tiết

$a = \dfrac{20}{5.0} = \boxed{4.0\,\text{m/s}^2}$. Isolating B: $T = m_B a = (2.0)(4.0) = \boxed{8.0\,\text{N}}$.

Bài tập luyện tập

An Atwood machine has $m_1 = 3.0\,\text{kg}$ and $m_2 = 7.0\,\text{kg}$. Find the acceleration of the system and the string tension.

Lời giải chi tiết

$a = \dfrac{(7.0-3.0)(9.8)}{10} = \dfrac{39.2}{10} = \boxed{3.92\,\text{m/s}^2}$.
$T = m_1(g+a) = 3.0(9.8+3.92) = 3.0(13.72) = \boxed{41.2\,\text{N}}$ (check: $m_2(g-a) = 7.0(5.88) = 41.2\,\text{N}$ ✓).

Bài tập luyện tập

A $10\,\text{kg}$ block ($\mu_k = 0.20$) is pushed horizontally with $F = 40\,\text{N}$. Find its acceleration.

(A) $1.96\,\text{m/s}^2$
(B) $2.04\,\text{m/s}^2$
(C) $2.94\,\text{m/s}^2$
(D) $4.00\,\text{m/s}^2$

Lời giải chi tiết

$N = mg = 98\,\text{N}$, $f_k = 0.20(98) = 19.6\,\text{N}$. $F_{net} = 40 - 19.6 = 20.4\,\text{N}$. $a = \dfrac{20.4}{10} = 2.04\,\text{m/s}^2$.

(B).

Bài tập luyện tập

A $15\,\text{kg}$ crate rests on a floor with $\mu_s = 0.50$. Find the maximum horizontal force applicable before the crate begins to slide.

Lời giải chi tiết

$f_{s,\text{max}} = \mu_s mg = (0.50)(15)(9.8) = \boxed{73.5\,\text{N}}$.

Bài tập luyện tập

A $5.0\,\text{kg}$ block on a horizontal floor ($\mu_k = 0.30$) is pulled by $F = 40\,\text{N}$ directed $37°$ *below* horizontal (pushing down and forward). Find the block's acceleration.

Lời giải chi tiết

$F_x = 40(0.80) = 32\,\text{N}$; $F_y = 40(0.60) = 24\,\text{N}$, now *downward*, so $N = mg + F_y = 49 + 24 = 73\,\text{N}$.
$f_k = 0.30(73) = 21.9\,\text{N}$. $F_{net,x} = 32 - 21.9 = 10.1\,\text{N}$. $a = \dfrac{10.1}{5.0} = \boxed{2.02\,\text{m/s}^2}$ — smaller than in the "pull up" case, because pushing down increases N and thus friction.

Bài tập luyện tập

A block slides down a frictionless $37°$ incline. Find its acceleration along the slope.

(A) $3.27\,\text{m/s}^2$
(B) $4.90\,\text{m/s}^2$
(C) $5.88\,\text{m/s}^2$
(D) $7.84\,\text{m/s}^2$

Lời giải chi tiết

$a = g\sin 37° = (9.8)(0.60) = 5.88\,\text{m/s}^2$. **(C)**.

Bài tập luyện tập

A block is released from rest at the top of a frictionless $37°$ incline of length $11.76\,\text{m}$ (measured along the slope). Find the time to reach the bottom and its speed there.

Lời giải chi tiết

$a = 5.88\,\text{m/s}^2$ (as above). $d = \frac{1}{2}at^2 \Rightarrow t = \sqrt{\dfrac{2d}{a}} = \sqrt{\dfrac{2(11.76)}{5.88}} = \sqrt{4.0} = \boxed{2.0\,\text{s}}$.
$v = at = (5.88)(2.0) = \boxed{11.76\,\text{m/s}}$.

Bài tập luyện tập

A $6.0\,\text{kg}$ block slides down a $37°$ incline with $\mu_k = 0.10$. Find its acceleration.

Lời giải chi tiết

$a = g(\sin\theta - \mu_k \cos\theta) = 9.8(0.60 - 0.10(0.80)) = 9.8(0.52) = \boxed{5.10\,\text{m/s}^2}$.

Bài tập luyện tập

A block is sent up a 37° incline ($\mu_k = 0.25$) with initial speed $9.8\,\text{m/s}$. Find how far up the incline it travels before momentarily stopping.

Lời giải chi tiết

$a = g(\sin\theta + \mu_k \cos\theta) = 9.8(0.60 + 0.20) = 9.8(0.80) = 7.84\,\text{m/s}^2$ (deceleration).

$d = \dfrac{v_0^2}{2a} = \dfrac{(9.8)^2}{2(7.84)} = \dfrac{96.04}{15.68} = \boxed{6.13\,\text{m}}$.

Bài tập luyện tập

Two forces act on a $4.0\,\text{kg}$ object: $F_1 = 12\,\text{N}$ east and $F_2 = 9.0\,\text{N}$ north. Find the object's acceleration (magnitude and direction).

Lời giải chi tiết

$F_{net} = \sqrt{12^2 + 9.0^2} = \sqrt{144 + 81} = \sqrt{225} = 15\,\text{N}$. $a = \dfrac{15}{4.0} = \boxed{3.75\,\text{m/s}^2}$.

Direction: $\theta = \arctan\left(\dfrac{9.0}{12}\right) = \boxed{37^\circ \text{ north of east}}$.

Bài tập luyện tập

A $1000\,\text{kg}$ car rounds a flat, unbanked curve of radius $40\,\text{m}$ with $\mu_s = 0.50$ between tires and road. Find its maximum safe speed.

(A) $7.0\,\text{m/s}$

(B) $14\,\text{m/s}$

(C) $19.6\,\text{m/s}$

(D) $28\,\text{m/s}$

Lời giải chi tiết

$v = \sqrt{\mu_s g r} = \sqrt{(0.50)(9.8)(40)} = \sqrt{196} = 14\,\text{m/s}$. **(B)**.

Bài tập luyện tập

A ball on a $0.50\,\text{m}$ string is swung in a vertical circle. Find the minimum speed at the top of the circle needed to keep the string taut.

Lời giải chi tiết

$v_{\text{min}} = \sqrt{gr} = \sqrt{(9.8)(0.50)} = \sqrt{4.9} = \boxed{2.21\,\text{m/s}}$.

Bài tập luyện tập

A 0.20 kg ball on a 0.50 m string moves at 4.0 m/s at the top of a vertical circle. Find the string tension at that point.

Lời giải chi tiết

At the top: $T+mg=\dfrac{mv^2}{r}\Rightarrow T=\dfrac{mv^2}{r}-mg=\dfrac{(0.20)(16)}{0.50}-(0.20)(9.8)=6.4-1.96=\boxed{4.44\text{ N}}$.

Bài tập luyện tập

A book rests on a table. Which pair of forces are true Newton's-third-law action–reaction partners?

(A) Weight of book, and normal force from table on book

(B) Weight of book, and the book's gravitational pull on the Earth

(C) Normal force from table on book, and weight of book

(D) None — the book is not accelerating, so there are no force pairs

Lời giải chi tiết

Third-law partners act on *different* objects and are the same type of force. The book's weight (Earth pulling book) pairs with the book pulling the Earth — both gravitational forces, on different objects. The normal force and weight both act on the *book itself*, so despite being equal in this static case, they are not a third-law pair. **(B)**.

Bài tập luyện tập

A 100 N traffic light is supported by two cables, each making $37°$ with the horizontal, in a symmetric "V" shape. Find the tension in each cable.

Lời giải chi tiết

By symmetry, both cables share the vertical support equally: $2T\sin 37°=W\Rightarrow T=\dfrac{100}{2(0.60)}=\dfrac{100}{1.2}=\boxed{83.3\text{ N}}$.

Bài tập luyện tập

A 20 kg crate sits on a truck bed ($\mu_s=0.40$). The truck accelerates forward at 3.0 m/s^2. Determine whether the crate slips, showing your reasoning.

Lời giải chi tiết

Force needed to accelerate the crate with the truck: $F_{needed}=ma=(20)(3.0)=60$ N.
Maximum static friction available: $f_{s,\text{max}}=\mu_s mg=(0.40)(20)(9.8)=78.4$ N.
Since 60 N $<$ 78.4 N, static friction can supply exactly the 60 N needed — $\boxed{\text{the crate does not slip}}$ and accelerates with the truck.

Bài tập luyện tập

A block $m_1 = 4.0\,\text{kg}$ on a frictionless 37° incline is connected by a string over a pulley at the top to a hanging mass $m_2 = 2.0\,\text{kg}$. Find the acceleration of the system and the string tension.

Lời giải chi tiết

Compare driving forces: $m_1 g \sin\theta = 4.0(9.8)(0.60) = 23.52\,\text{N}$ (pulls m_1 down the slope) vs. $m_2 g = 2.0(9.8) = 19.6\,\text{N}$ (pulls m_2 down). Since $23.52 > 19.6$, m_1 slides down and m_2 rises.
$a = \dfrac{m_1 g \sin\theta - m_2 g}{m_1 + m_2} = \dfrac{23.52 - 19.6}{6.0} = \dfrac{3.92}{6.0} = \boxed{0.653\,\text{m/s}^2}$.
$T = m_2(g + a) = 2.0(9.8 + 0.653) = 2.0(10.453) = \boxed{20.9\,\text{N}}$ (check via m_1: $23.52 - 4.0(0.653) = 23.52 - 2.61 = 20.9\,\text{N}$ ✓).

Bài tập luyện tập

In uniform circular motion, the net force (and acceleration) on the object points:

(A) In the direction of motion
(B) Away from the center
(C) Toward the center
(D) Nowhere, since it is zero

Lời giải chi tiết

Even though speed is constant, direction constantly changes; the net (centripetal) force points toward the center of the circle, perpendicular to the velocity. **(C)**.

Bài tập luyện tập

A $3.0\,\text{kg}$ block is placed on a 37° incline with $\mu_s = 0.70$, $\mu_k = 0.50$. Determine whether the block remains at rest when released; if not, find its acceleration.

Lời giải chi tiết

Gravity component along the incline: $mg\sin\theta = 3.0(9.8)(0.60) = 17.64\,\text{N}$.
Max static friction: $\mu_s mg\cos\theta = 0.70(3.0)(9.8)(0.80) = 16.46\,\text{N}$.
Since $17.64\,\text{N} > 16.46\,\text{N}$, the required holding force exceeds what static friction can provide — the block $\boxed{\text{slides}}$.
Kinetic friction: $\mu_k mg\cos\theta = 0.50(3.0)(9.8)(0.80) = 11.76\,\text{N}$. Net force $= 17.64 - 11.76 = 5.88\,\text{N}$.
$a = \dfrac{5.88}{3.0} = \boxed{1.96\,\text{m/s}^2}$.

Bài tập luyện tập

Which of the following does *not* belong on the free-body diagram of a block resting on an incline?

(A) Normal force perpendicular to the incline
(B) Weight, acting straight down
(C) Friction, acting along the incline surface
(D) The incline's own weight

Lời giải chi tiết

A free-body diagram shows only forces acting *on* the chosen object, not forces on other bodies such as the incline itself. **(D)**.

Bài tập luyện tập

A rocket in deep space (no air present) accelerates forward by expelling exhaust gases backward at high speed. Which explanation correctly applies Newton's third law to the forward thrust?

(A) The exhaust gases push against the surrounding air, and the air pushes the rocket forward

(B) The rocket exerts a backward force on the exhaust gases; by Newton's third law, the gases exert an equal and opposite forward force on the rocket

(C) The vacuum behind the rocket pulls it forward

(D) Thrust requires no reaction force, since the engine directly generates forward motion

Lời giải chi tiết

The rocket pushes the exhaust gases backward (action); by Newton's third law, the gases push the rocket forward with equal magnitude (reaction) — a pair of forces on two different objects (rocket and expelled gas), acting simultaneously. This is why rockets work even in a vacuum: thrust does not require anything external, like air, to push against. **(B)**.

Bài tập luyện tập

A 4.0 kg box on a frictionless floor is pulled by two horizontal ropes: one exerts 16 N due east, the other exerts 12 N due north. Find the box's acceleration (magnitude and direction).

Lời giải chi tiết

$F_{net} = \sqrt{16^2 + 12^2} = \sqrt{256 + 144} = \sqrt{400} = 20\,\text{N}$. $a = \dfrac{20}{4.0} = \boxed{5.0\,\text{m/s}^2}$.

Direction: $\theta = \arctan\left(\dfrac{12}{16}\right) = \boxed{37^\circ \text{ north of east}}$.

Bài tập luyện tập

A 50 kg person stands on a scale in an elevator. The elevator is moving upward but slowing down, with acceleration $2.5\,\text{m/s}^2$ directed downward (opposite the velocity). Find the scale reading.

Lời giải chi tiết

Taking up as positive, $a = -2.5\,\text{m/s}^2$. $N = m(g + a) = 50(9.8 - 2.5) = 50(7.3) = \boxed{365\,\text{N}}$.

Bài tập luyện tập

Three blocks are connected in a line by light strings on a frictionless floor: $m_1 = 2.0$ kg (front), $m_2 = 3.0$ kg (middle), $m_3 = 5.0$ kg (back). A horizontal force $F = 40$ N pulls m_1 forward. Find (a) the system's acceleration, (b) the tension between m_1 and m_2, (c) the tension between m_2

and m_3.

Lời giải chi tiết

(a) $a = \dfrac{F}{m_1 + m_2 + m_3} = \dfrac{40}{10} = \boxed{4.0\,\text{m/s}^2}$.
(b) The string between m_1 and m_2 must accelerate $m_2+m_3 = 8.0$ kg: $T_{12} = (8.0)(4.0) = \boxed{32\,\text{N}}$.
(c) The string between m_2 and m_3 must accelerate $m_3 = 5.0$ kg alone: $T_{23} = (5.0)(4.0) = \boxed{20\,\text{N}}$.

Bài tập luyện tập

A 20 kg crate rests on a floor with $\mu_s = 0.35$. A horizontal force of 60 N is applied. Determine whether the crate moves, and find the friction force acting on it.

Lời giải chi tiết

Maximum available static friction: $f_{s,\text{max}} = \mu_s mg = (0.35)(20)(9.8) = 68.6$ N.
Since the applied 60 N < 68.6 N, the crate $\boxed{\text{remains at rest}}$; static friction adjusts to exactly balance the applied force, so $f_s = \boxed{60\,\text{N}}$ (not the maximum value).

Bài tập luyện tập

A 4.0 kg block is placed on a frictionless 53° incline. Find (a) its acceleration along the incline, (b) the normal force on it.

Lời giải chi tiết

(a) $a = g\sin 53^\circ = (9.8)(0.80) = \boxed{7.84\,\text{m/s}^2}$.
(b) $N = mg\cos 53^\circ = (4.0)(9.8)(0.60) = \boxed{23.5\,\text{N}}$.

Bài tập luyện tập

A 1500 kg car rounds a flat, unbanked curve of radius 80 m. If $\mu_s = 0.40$ between tires and road, find the maximum speed without skidding.

Lời giải chi tiết

$v = \sqrt{\mu_s gr} = \sqrt{(0.40)(9.8)(80)} = \sqrt{313.6} = \boxed{17.7\,\text{m/s}}$.

Bài tập luyện tập

Two objects, $m_1 = 6.0 \times 10^2$ kg and $m_2 = 8.0 \times 10^2$ kg, have centers separated by 4.0 m. Find the gravitational force between them.

(A) $2.0e-6$ N
(B) $8.0e-6$ N
(C) $5.0e-5$ N
(D) $2.0e-8$ N

Lời giải chi tiết

$F_g = \frac{Gm_1m_2}{r^2} = \frac{(6.67\times10^{-11})(600)(800)}{(4.0)^2} = \frac{(6.67\times10^{-11})(4.8\times10^5)}{16} = 2.0e-6\,\text{N}$. **(A)**.

Bài tập luyện tập

At what distance from Earth's center is the gravitational acceleration equal to exactly $1/9$ of its surface value?

(A) R
(B) $2R$
(C) $3R$
(D) $9R$

Lời giải chi tiết

Since $g \propto 1/r^2$, we need $(R/r)^2 = 1/9 \Rightarrow R/r = 1/3 \Rightarrow r = 3R$. **(C)**.

Bài tập luyện tập

A satellite orbits Earth in a circle at $r = 8.0e6$ m from Earth's center ($M_\oplus = 5.97\times10^{24}$ kg). Find (a) its orbital speed, (b) its orbital period, in seconds and in hours.

Lời giải chi tiết

(a) $v = \sqrt{\frac{GM_\oplus}{r}} = \sqrt{\frac{(6.67\times10^{-11})(5.97\times10^{24})}{8.0\times10^6}} = \sqrt{4.98\times10^7} = \boxed{7.06e3\,\text{m/s}}$.

(b) $T = \frac{2\pi r}{v} = \frac{2\pi(8.0\times10^6)}{7.06\times10^3} = \boxed{7.12e3\,\text{s}} \approx \boxed{1.98\text{ hours}}$.

Bài tập luyện tập

A curve of radius 25 m is banked at 37°, engineered so that no friction is needed at the design speed. Find that speed.

Lời giải chi tiết

$v = \sqrt{rg\tan\theta} = \sqrt{(25)(9.8)(0.75)} = \sqrt{183.75} = \boxed{13.6\,\text{m/s}}$.

Bài tập luyện tập

A 1200 kg roller-coaster car travels through a vertical circular loop of radius 9.0 m. (a) Find the minimum speed at the top needed to maintain contact with the track. (b) If the actual speed at the top is 12 m/s, find the normal force there. (c) If the speed at the bottom is 22 m/s, find the normal force there.

Lời giải chi tiết

(a) $v_{\text{min}} = \sqrt{gr} = \sqrt{(9.8)(9.0)} = \sqrt{88.2} = \boxed{9.39\,\text{m/s}}$.

(b) $N = m\left(\frac{v^2}{r} - g\right) = 1200\left(\frac{144}{9.0} - 9.8\right) = 1200(16-9.8) = 1200(6.2) = \boxed{7440\,\text{N}}$ (positive, since 12 m/s > 9.39 m/s, so contact is maintained).

(c) $N = m\left(g + \frac{v^2}{r}\right) = 1200\left(9.8 + \frac{484}{9.0}\right) = 1200(9.8 + 53.8) = 1200(63.6) = \boxed{7.63e4\,\text{N}}$.

Bài tập luyện tập

Explain, using physics reasoning, why an astronaut in a circular low-Earth orbit (e.g., aboard the International Space Station at about 400 km altitude) feels weightless even though Earth's gravity is still acting on them with nearly its full surface strength.

Lời giải chi tiết

Gravity has not disappeared: at 400 km altitude, g is about 88.5% of its surface value (see the g-at-altitude example earlier in this chapter) — only slightly weaker than at Earth's surface. In fact, gravity is essential here: it is precisely the force that supplies the centripetal acceleration keeping the astronaut and the spacecraft moving along a circular path instead of flying off in a straight line, as Newton's first law would otherwise require. "Weightlessness" refers to the absence of *apparent* weight — the normal force a scale or a floor would exert on the astronaut — not the absence of true gravitational force. Because gravity accelerates the astronaut and the surrounding spacecraft at exactly the same rate $g = GM/r^2$ (independent of mass), both fall toward Earth's center together, continuously "missing" the surface as they move forward: they are in continuous free fall. Since there is no relative acceleration between the astronaut and the cabin around them, there is no contact force pressing the astronaut against any surface, and hence no sensation of weight — the same reason a person feels momentarily weightless during any free fall, such as the instant an elevator cable is cut, or at the top of a sharp roller-coaster drop. The astronaut's true weight mg is still very much present; it is exactly what curves their path into a stable orbit rather than letting them travel in a straight line out into space.

Bài tập luyện tập

An object of mass m experiences a constant net force F, producing acceleration a. If the net force is tripled and the mass is doubled, what is the new acceleration, in terms of a?

(A) $0.67a$

(B) $1.5a$

(C) $3a$

(D) $6a$

Lời giải chi tiết

$a' = \frac{3F}{2m} = 1.5\left(\frac{F}{m}\right) = 1.5a$. **(B)**.

Bài tập luyện tập

An elevator cab, together with the passenger inside, has a total mass of 800 kg. The supporting cable accelerates the cab upward at $1.5\,\text{m/s}^2$. Find the tension in the cable.

Lời giải chi tiết

Taking up as positive and applying Newton's second law to the cab+passenger system: $T - mg = ma \Rightarrow T = m(g+a) = 800(9.8+1.5) = 800(11.3) = \boxed{9040\,\text{N}}$.

Bài tập luyện tập

A hockey puck slides across level ice at an initial speed of $10\,\text{m/s}$ and comes to rest after sliding $20\,\text{m}$, decelerated only by kinetic friction. Find μ_k between the puck and the ice.

(A) 0.128
(B) 0.255
(C) 0.510
(D) 2.50

Lời giải chi tiết

$v^2 = v_0^2 - 2ad \Rightarrow 0 = (10)^2 - 2a(20) \Rightarrow a = \dfrac{100}{40} = 2.5\,\text{m/s}^2$.

Since friction alone causes the deceleration, $a = \mu_k g \Rightarrow \mu_k = \dfrac{a}{g} = \dfrac{2.5}{9.8} = 0.255$. **(B).**

Bài tập luyện tập

A $10\,\text{kg}$ block is pushed up a 37° incline ($\mu_k = 0.20$) by a force F applied parallel to the incline surface, so that the block moves up at constant velocity. Find F.

Lời giải chi tiết

At constant velocity, F must balance both the gravity component along the incline and kinetic friction, both of which point down the slope while the block moves up:

$$F = mg\sin\theta + \mu_k mg\cos\theta = mg(\sin\theta + \mu_k\cos\theta) = (10)(9.8)(0.60 + 0.20(0.80)) = 98(0.76) = \boxed{74.5\,\text{N}}.$$

Bài tập luyện tập

Two point masses exert a gravitational force F on each other. If *both* masses are doubled and the distance between their centers is also doubled, the new gravitational force is:

(A) $\frac{1}{4}F$
(B) $\frac{1}{2}F$
(C) F (unchanged)
(D) $2F$

Lời giải chi tiết

$F' = \dfrac{G(2m_1)(2m_2)}{(2r)^2} = \dfrac{4Gm_1m_2}{4r^2} = \dfrac{Gm_1m_2}{r^2} = F$. The factor of 4 from doubling the two masses exactly cancels the factor of 4 from squaring the doubled distance. **(C).**

Bài tập luyện tập

Block $m_1 = 6.0\,\text{kg}$ rests on a 37° incline with $\mu_k = 0.15$, connected by a light string over a frictionless pulley at the top of the incline to a hanging block $m_2 = 5.0\,\text{kg}$. Released from rest, m_2 descends and m_1 slides up the incline. Find (a) the acceleration of the system, (b) the tension in the string.

Lời giải chi tiết

(a) Driving force: $m_2 g = 5.0(9.8) = 49.0\,\text{N}$. Opposing forces on m_1 (both point down the incline, since m_1 moves up): gravity component $m_1 g\sin\theta = 6.0(9.8)(0.60) = 35.28\,\text{N}$, and kinetic friction $\mu_k m_1 g\cos\theta = 0.15(6.0)(9.8)(0.80) = 7.056$ N.

$$a = \frac{m_2 g - m_1 g\sin\theta - \mu_k m_1 g\cos\theta}{m_1 + m_2} = \frac{49.0 - 35.28 - 7.056}{11.0} = \frac{6.664}{11.0} = \boxed{0.606\,\text{m/s}^2}.$$

(b) From m_2's equation ($m_2 g - T = m_2 a$): $T = m_2(g - a) = 5.0(9.8 - 0.606) = 5.0(9.194) = \boxed{46.0\,\text{N}}$ (check via m_1: $T = m_1 a + m_1 g\sin\theta + \mu_k m_1 g\cos\theta = 6.0(0.606) + 35.28 + 7.056 = 3.64 + 35.28 + 7.056 = 45.98$ N ✓).

Bài tập luyện tập

A 0.50 kg ball on a 0.60 m string moves in a vertical circle. At the bottom of the circle, its speed is 5.0 m/s. Find the tension in the string at that point.

Lời giải chi tiết

At the bottom, tension and the centripetal direction both point upward (toward the center), while gravity points down: $T - mg = \dfrac{mv^2}{r}$, so

$$T = m\left(g + \frac{v^2}{r}\right) = 0.50\left(9.8 + \frac{(5.0)^2}{0.60}\right) = 0.50(9.8 + 41.67) = 0.50(51.47) = \boxed{25.7\,\text{N}}.$$

Bài tập luyện tập

A curve of radius 20 m is banked (frictionless) so that a car traveling at 14 m/s needs no friction to stay on the road. Find the required banking angle θ.

Lời giải chi tiết

$$\tan\theta = \frac{v^2}{rg} = \frac{(14)^2}{(20)(9.8)} = \frac{196}{196} = 1.00 \Rightarrow \theta = \arctan(1.00) = \boxed{45^\circ}.$$

Bài tập luyện tập

A planet has mass $M = 3.0\times10^{23}$ kg and radius $R = 3.4\times10^{6}$ m. Find (a) the surface gravity of the planet, (b) the weight of an 80 kg astronaut standing on its surface.

Lời giải chi tiết

(a) $g = \dfrac{GM}{R^2} = \dfrac{(6.67\times10^{-11})(3.0\times10^{23})}{(3.4\times10^{6})^2} = \dfrac{2.00\times10^{13}}{1.156\times10^{13}} = \boxed{1.73\,\text{m/s}^2}$.

(b) $W = mg = (80)(1.73) = \boxed{138\,\text{N}}$.

Bài tập luyện tập

Block A (2.0 kg) sits on top of block B (3.0 kg), which is on a frictionless floor. A horizontal force $F = 15$ N is applied to B. There is enough friction between A and B that they move together with no slipping. Find the friction force that B exerts on A.

(A) $3.0\,\text{N}$

(B) $6.0\,\text{N}$

(C) $9.0\,\text{N}$

(D) $15\,\text{N}$

Lời giải chi tiết

Whole system: $a = \dfrac{F}{m_A + m_B} = \dfrac{15}{5.0} = 3.0\,\text{m/s}^2$. For block A alone, friction from B is the *only* horizontal force, and it must supply A's acceleration: $f = m_A a = (2.0)(3.0) = 6.0\,\text{N}$. **(B).**

Work, Energy, and Power

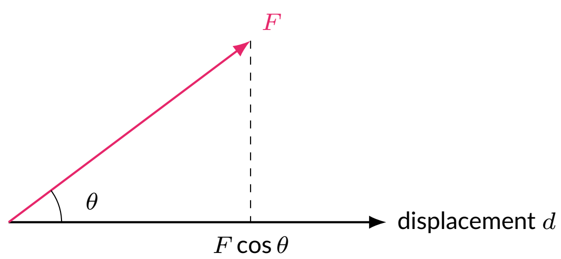
Mục tiêu Unit: Công của lực không đổi và lực biến thiên (đồ thị F–x); định lý công–động năng; thế năng trọng trường và thế năng đàn hồi; định luật bảo toàn cơ năng; công của lực không bảo toàn (ma sát); công suất trung bình và tức thời.

3.1 Work Done by a Constant Force

Work measures the transfer of energy to or from an object by a force acting through a displacement:

$$W = Fd\cos\theta,$$

where F is the force magnitude, d is the displacement magnitude, and θ is the angle *between* the force and the displacement vectors. Work is a **scalar**, measured in joules ($1\,\text{J} = 1\,\text{N}\cdot\text{m}$), and can be positive, negative, or zero.



Only the component of F *along* the displacement ($F\cos\theta$) does work; the perpendicular component does none.

Khái niệm cốt lõi

Only the force component **parallel** to the displacement does work. A force perpendicular to motion ($\theta = 90^\circ$) does **zero** work — this is why the normal force and gravity do no work on an object sliding along a level, horizontal surface. When $\theta < 90^\circ$, work is positive (energy added); when $\theta > 90^\circ$ (including 180°, directly opposing motion), work is negative (energy removed).

VN GIẢI THÍCH TIẾNG VIỆT

Công $W = Fd\cos\theta$ chỉ tính phần lực **cùng phương** với độ dịch chuyển. Lực vuông góc với chuyển động (như lực pháp tuyến N khi vật trượt ngang) **không sinh công**. Ma sát luôn sinh công **âm** khi cản trở chuyển động (ngược chiều với độ dịch chuyển, $\theta = 180^\circ$).

Ví dụ 1 · Work by a force at an angle

A rope pulls a crate with $F = 60\,\text{N}$ directed 53° above horizontal, dragging it $d = 8.0\,\text{m}$ across a horizontal floor. Find the work done by the rope. ($\sin 53^\circ = 0.80$, $\cos 53^\circ = 0.60$.)

$$W = Fd\cos 53° = (60)(8.0)(0.60) = 288\,\text{J}.$$

3.2 The Work–Energy Theorem

Kinetic energy is the energy of motion: $KE = \frac{1}{2}mv^2$.

Work–energy theorem: $W_{net} = \Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$. The *net* work done on an object (by *all* forces combined) equals its change in kinetic energy — regardless of how many individual forces contributed, or what path was taken.

Ví dụ 2 · Work–energy theorem, finding final speed

A 4.0 kg block starts at $v_0 = 3.0\,\text{m/s}$. A constant net force of 10 N acts on it, in the direction of motion, over a distance of 4.0 m. Find its final speed.

$W_{net} = Fd = (10)(4.0) = 40\,\text{J} = \Delta KE = \frac{1}{2}m(v_f^2 - v_0^2)$:

$40 = \frac{1}{2}(4.0)(v_f^2 - 9.0) \Rightarrow v_f^2 - 9.0 = 20 \Rightarrow v_f^2 = 29$ ...recompute: $v_f^2 = 9.0 + \frac{2(40)}{4.0} = 9.0 + 20 = 29.$

Careful bookkeeping: $\frac{1}{2}(4.0) = 2.0$, so $40 = 2.0(v_f^2 - 9.0) \Rightarrow v_f^2 - 9.0 = 20 \Rightarrow v_f^2 = 49 \Rightarrow v_f = 7.0\,\text{m/s}$.

Ví dụ 3 · Friction and the work–energy theorem

A block slides across a rough horizontal floor ($\mu_k = 0.20$) at initial speed $v_0 = 9.8\,\text{m/s}$ and comes to rest. Find the distance it slides. (Notice the mass will cancel.)

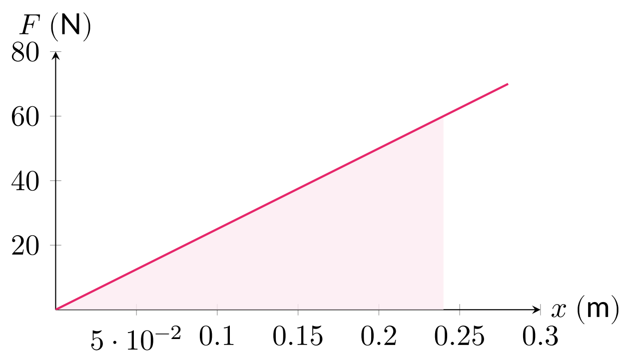
Only friction does (negative) work: $W_{fric} = -\mu_k mgd = \Delta KE = 0 - \frac{1}{2}mv_0^2$:

$$-\mu_k gd = -\frac{1}{2}v_0^2 \Rightarrow d = \frac{v_0^2}{2\mu_k g} = \frac{(9.8)^2}{2(0.20)(9.8)} = \frac{96.04}{3.92} = 24.5\,\text{m}.$$

3.3 Work Done by a Variable Force

When a force changes with position (like a spring), work equals the **area under the F-vs-x graph**. For an ideal spring obeying Hooke's law, $F = kx$, the graph is a straight line through the origin, and the area under it up to displacement x is a triangle:

$$W = \frac{1}{2}kx^2.$$



F–x graph for a spring with $k = 250\,\text{N/m}$. The shaded triangular area up to $x = 0.24\,\text{m}$ gives the work done: $W = \frac{1}{2}kx^2 = 7.2\,\text{J}$.

Ví dụ 4 · Work done stretching a spring

A spring with $k = 250\,\text{N/m}$ is stretched from its natural length by $x = 0.24\,\text{m}$. Find the work done on the spring.

$$W = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(250)(0.24)^2 = \tfrac{1}{2}(250)(0.0576) = 7.2\,\text{J}.$$

3.4 Potential Energy

Potential energy (PE) is energy stored due to an object's position or configuration, associated with a **conservative** force (one whose work is path-independent and fully recoverable).

Gravitational PE: $PE_g = mgh$, measured relative to a chosen reference height (only *changes* in PE_g matter physically).

Elastic (spring) PE: $PE_s = \frac{1}{2}kx^2$, where x is the displacement from the spring's natural (unstretched/uncompressed) length.

VN **GIẢI THÍCH TIẾNG VIỆT**

Thế năng chỉ phụ thuộc **vị trí/trạng thái** hiện tại, không phụ thuộc đường đi để đến đó — đây là đặc trưng của lực **bảo toàn** (trọng lực, lực đàn hồi). Mốc thế năng trọng trường có thể chọn tùy ý; chỉ **độ biến thiên** ΔPE_g mới có ý nghĩa vật lý.

3.5 Conservation of Mechanical Energy

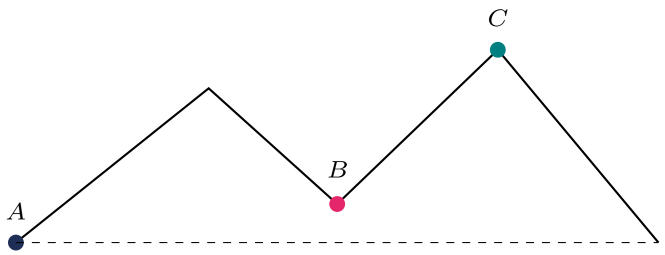
When only conservative forces (gravity, ideal springs) do work, total mechanical energy $E_{mech} = KE + PE$ is constant:

$$KE_i + PE_i = KE_f + PE_f.$$

If a nonconservative force (friction, air resistance, an applied push) also does work W_{nc}, the more general statement is

$$KE_i + PE_i + W_{nc} = KE_f + PE_f,$$

where $W_{nc} < 0$ for friction/drag (energy leaves the mechanical system, typically as heat).



Energy conservation along a frictionless track: $E_{mech} = KE + PE_g$ is the same at points A, B, and C, even though KE and PE_g individually trade off.

Ví dụ 5 · Elastic PE launching a block

A spring with $k = 400\,\text{N/m}$ is compressed $x = 0.15\,\text{m}$ and used to launch a $1.0\,\text{kg}$ block along a frictionless horizontal track. Find the block's launch speed.

$$PE_s = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(400)(0.15)^2 = \tfrac{1}{2}(400)(0.0225) = 4.5\,\text{J}.$$

All of it converts to kinetic energy: $4.5 = \frac{1}{2}(1.0)v^2 \Rightarrow v^2 = 9.0 \Rightarrow v = 3.0\,\text{m/s}$.

Ví dụ 6 · Frictionless hill

A cart of mass $500\,\text{kg}$ starts from rest at the top of a frictionless hill of height $10\,\text{m}$. Find its speed at the bottom.

$$mgh = \tfrac{1}{2}mv^2 \;\Rightarrow\; v = \sqrt{2gh} = \sqrt{2(9.8)(10)} = \sqrt{196} = 14\,\text{m/s}.$$

(Notice the mass cancels — the speed at the bottom of a frictionless drop never depends on mass.)

3.6 Energy with Nonconservative Forces

When friction acts, mechanical energy is not conserved — some is converted to thermal energy. The energy method (using W_{nc}) often replaces a multi-step kinematics calculation.

Ví dụ 7 · Incline with friction, solved by energy

A $1.0\,\text{kg}$ block starts from rest at the top of a 37° incline of length $L = 4.0\,\text{m}$ (measured along the slope), with $\mu_k = 0.25$. Find its speed at the bottom using energy methods.

Height dropped: $h = L\sin 37^\circ = 4.0(0.60) = 2.4\,\text{m}$. Gravity's work: $mgh = (1.0)(9.8)(2.4) = 23.52\,\text{J}$.

Friction force: $f_k = \mu_k mg\cos 37^\circ = (0.25)(1.0)(9.8)(0.80) = 1.96\,\text{N}$; work by friction: $W_{fric} = -f_k L = -(1.96)(4.0) = -7.84\,\text{J}$.

Energy balance: $KE_f = mgh + W_{fric} = 23.52 - 7.84 = 15.68\,\text{J} = \frac{1}{2}(1.0)v^2$:

$$v^2 = 31.36 \;\Rightarrow\; v = 5.6\,\text{m/s}.$$

Ví dụ 8 · Spring launch up a frictionless incline

A $0.50\,\text{kg}$ block, pressed against a spring ($k = 500\,\text{N/m}$) compressed $x = 0.20\,\text{m}$, is launched along a frictionless horizontal surface that leads onto a frictionless 37° incline. Find the maximum height (vertical) the block reaches on the incline.

$PE_s = \frac{1}{2}(500)(0.20)^2 = 10\,\text{J}$. All of it becomes gravitational PE at the highest point (no friction anywhere):

$$mgh = 10 \;\Rightarrow\; h = \frac{10}{(0.50)(9.8)} = \frac{10}{4.9} = 2.04\,\text{m}.$$

(!) Lỗi thường gặp: "Conservation of energy" does not mean KE and PE are each separately constant — it means their *sum* is constant (when no nonconservative force does work). When friction is present, do not forget the W_{nc} term; forgetting it is the most common energy-method error.

3.7 Power

Power is the rate at which work is done (or energy is transferred):

$$P = \frac{W}{t} \quad \text{(average power)}, \qquad P = Fv\cos\theta \quad \text{(instantaneous power, force at angle } \theta \text{ to velocity)}.$$

For a force acting along the direction of motion at constant velocity, $P = Fv$. SI unit: watt, $1\,\text{W} = 1\,\text{J/s}$.

VN **GIẢI THÍCH TIẾNG VIỆT**

Công suất đo **tốc độ sinh công** (hoặc truyền năng lượng). Công thức $P = W/t$ dùng khi biết tổng công và tổng thời gian (công suất trung bình); công thức $P = Fv\cos\theta$ dùng khi biết lực và vận tốc tức thời tại một thời điểm. Khi vật chuyển động thẳng đều dưới tác dụng của một lực song song với vận tốc, hai công thức cho cùng kết quả.

Ví dụ 9 · Power: two viewpoints

(a) A motor lifts an 80 kg load at a constant speed of 0.25 m/s. Find the motor's power output.
(b) A cyclist pedals with a constant forward force of 60 N while moving at 5.0 m/s. Find her power output, and the work she does in 10 s.

(a) At constant velocity, lifting force equals weight: $F = mg = (80)(9.8) = 784\,\text{N}$. $P = Fv = (784)(0.25) = 196\,\text{W}$.
(b) $P = Fv = (60)(5.0) = 300\,\text{W}$. $W = Pt = (300)(10) = 3000\,\text{J}$.

Ví dụ 10 · Average power over an acceleration

A 1000 kg car accelerates uniformly from rest to 20 m/s in 8.0 s. Find the average power delivered by the engine over this interval (ignore friction and air drag).

Average power equals total work (equal to ΔKE) divided by total time — this is valid even though the *instantaneous* power $P = Fv$ is changing throughout (constant F, but increasing v):

$$\Delta KE = \tfrac{1}{2}(1000)(20)^2 - 0 = 2.0 \times 10^5\,\text{J}.$$

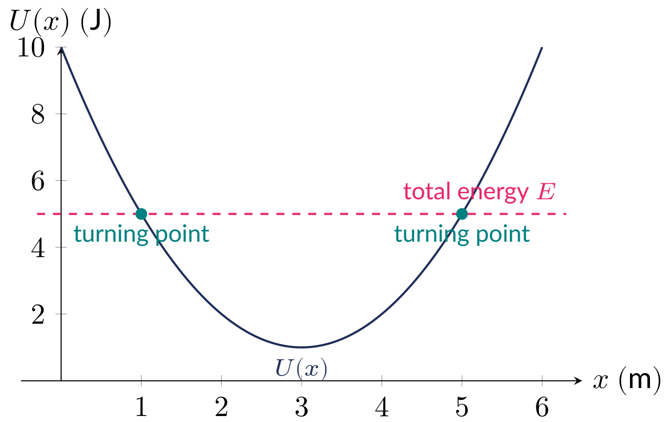
$$P_{avg} = \frac{\Delta KE}{t} = \frac{2.0 \times 10^5}{8.0} = 2.5 \times 10^4\,\text{W} = 25\,\text{kW}.$$

3.8 Energy Diagrams and Combined Circular-Motion Problems

A graph of potential energy U versus position x packs a surprising amount of information about a system's motion into a single curve — especially once the (constant) total mechanical energy E is drawn on the same axes as a horizontal line.

For a particle moving under a conservative force, with potential energy $U(x)$ and constant total mechanical energy E:

- At any position x, the kinetic energy equals the vertical gap between the two curves: $KE(x) = E - U(x)$.
- **Turning points** occur where $E = U(x)$, so $KE = 0$ and $v = 0$ — the object momentarily stops there and reverses direction.
- Motion is only possible where $E \geq U(x)$; any region where $U(x) > E$ is **classically forbidden** (it would require negative kinetic energy).
- The net force on the object equals the **negative slope** of $U(x)$: $F = -\dfrac{dU}{dx}$. Qualitatively, the object is always pushed *toward decreasing* U — a positive slope pushes it in the $-x$ direction, a negative slope pushes it in the $+x$ direction, exactly like a ball rolling downhill on the $U(x)$ curve itself.
- Where the slope of $U(x)$ is zero, the net force is zero: that position is an **equilibrium**. If $U(x)$ has a local minimum there, the equilibrium is **stable** (small displacements are pushed back toward it); if a local maximum, it is **unstable**.



A potential-energy "valley" $U(x) = (x-3)^2 + 1$ with total mechanical energy $E = 5.0$ J drawn as a horizontal line. The two marked points, where the line crosses the curve, are turning points; the object is confined between them, and the region outside is classically forbidden.

VN **GIẢI THÍCH TIẾNG VIỆT**

Đồ thị $U(x)$ cho biết gần như toàn bộ "bức tranh" định tính về chuyển động. Tại mỗi vị trí x, động năng bằng khoảng cách thẳng đứng giữa đường E (nằm ngang) và đường cong $U(x)$: $KE(x) = E - U(x)$. Vật chỉ có thể xuất hiện ở nơi $U(x) \leq E$; những điểm mà $U(x) = E$ là **điểm dừng** (turning point), nơi vận tốc bằng 0 và vật đổi chiều. Độ dốc của $U(x)$ cho biết chiều của lực tác dụng: vật luôn bị "đẩy" về phía U giảm, giống hệt một quả bóng lăn xuống dốc trên chính đồ thị đó. Đáy của "thung lũng" $U(x)$ (độ dốc bằng 0, U cực tiểu) là vị trí **cân bằng bền**.

Ví dụ 11 · Reading a potential-energy graph

A particle of mass 2.0 kg moves along the x-axis under a conservative force whose potential energy is $U(x) = (x-3)^2 + 1$ (in joules, with x in meters), shown in the figure above. The particle's total mechanical energy is $E = 5.0$ J. Find (a) the turning points of the motion, (b) the particle's speed at $x = 3.0$ m (the bottom of the well), and (c) the particle's speed at $x = 2.0$ m.

(a) Turning points satisfy $E = U(x)$:

$$5 = (x-3)^2 + 1 \;\Rightarrow\; (x-3)^2 = 4 \;\Rightarrow\; x - 3 = \pm 2 \;\Rightarrow\; x = 1.0\,\text{m or } x = 5.0\,\text{m}.$$

(b) At $x = 3.0$ m, $U = 1.0$ J, so $KE = E - U = 5.0 - 1.0 = 4.0$ J:

$$4.0 = \tfrac{1}{2}(2.0)v^2 \;\Rightarrow\; v^2 = 4.0 \;\Rightarrow\; v = 2.0\,\text{m/s}.$$

(c) At $x = 2.0$ m, $U = (2-3)^2 + 1 = 2.0$ J, so $KE = 5.0 - 2.0 = 3.0$ J:

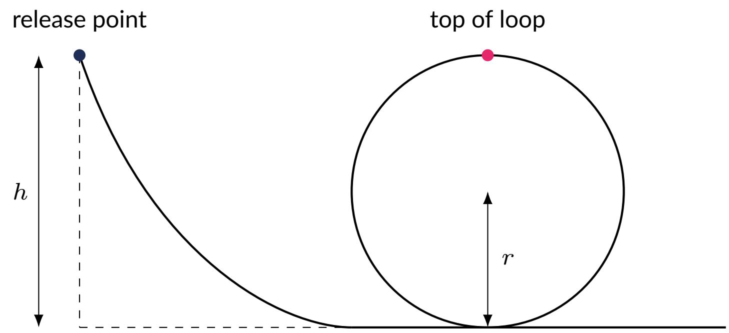
$$3.0 = \tfrac{1}{2}(2.0)v^2 \;\Rightarrow\; v^2 = 3.0 \;\Rightarrow\; v = 1.7\,\text{m/s}.$$

Circular-motion problems that begin with an energy-conservation phase (falling or sliding through some height) and end with a circular-motion phase (going around a loop, over a hill, through a valley) are among the most common combined AP Physics 1 problems. The extra ingredient needed is the centripetal force relation, $F_{net,c} = \dfrac{mv^2}{r}$, applied at the critical point of the circular path — almost always the **top** of a vertical loop.

At the **top of a vertical loop**, the track can only push inward (not pull), so contact is maintained only if the required centripetal force does not exceed what gravity (plus any normal force) can supply. The **minimum** speed to maintain contact occurs when the normal force drops to zero and gravity alone supplies the full centripetal force:

$$mg = \frac{mv_{top}^2}{r} \;\Rightarrow\; v_{top,min} = \sqrt{gr}.$$

Combined with energy conservation from a frictionless release height h (measured above the bottom of the loop) to the top of the loop (height $2r$ above the bottom), this leads to the classic minimum-height result $h_{min} = 2.5r$, derived below.



A block released from rest at height h above the bottom of a loop slides down a frictionless ramp and around a circular loop of radius r. At the top (height $2r$ above the bottom), gravity alone supplies the minimum centripetal force when $v_{top} = \sqrt{gr}$.

Ví dụ 12 · Loop-the-loop: minimum release height

A frictionless track leads into a vertical circular loop of radius $r = 2.0$ m. A small block is released from rest at height h above the bottom of the loop. Find the minimum value of h so the block completes the loop without losing contact with the track, and find its speed at the top of the loop under this condition.

Step 1 — minimum speed at the top. Contact is lost when the normal force reaches zero, so the minimum condition is gravity alone providing the centripetal force:

$$mg = \frac{mv_{top}^2}{r} \Rightarrow v_{top}^2 = gr = (9.8)(2.0) = 19.6\,\text{m}^2/\text{s}^2 \Rightarrow v_{top} = 4.4\,\text{m/s}.$$

Step 2 — energy conservation from release to the top. The top of the loop is a height $2r$ above the bottom, so measuring heights from the bottom of the loop:

$$mgh = mg(2r) + \tfrac{1}{2}mv_{top}^2.$$

The mass cancels. Substituting $v_{top}^2 = gr$:

$$gh = 2gr + \tfrac{1}{2}(gr) = 2.5gr \Rightarrow h = 2.5r = 2.5(2.0) = 5.0\,\text{m}.$$

Check: $gh = (9.8)(5.0) = 49.0$ m²/s²; and $2gr + \frac{1}{2}v_{top}^2 = (9.8)(4.0) + \frac{1}{2}(19.6) = 39.2 + 9.8 = 49.0$ m²/s². The two sides match, confirming $h_{min} = \boxed{5.0\,\text{m}}$ and $v_{top} = \boxed{4.4\,\text{m/s}}$.

(!) Lỗi thường gặp: The minimum-speed-at-the-top condition is $mg = mv_{top}^2/r$ — *not* $mg = mv_{top}^2/(2r)$ or some other variant. Also, the height used in the energy equation for the top of the loop is $2r$ above the bottom of the loop (the loop's *diameter*), not r — a common setup mistake.

Ví dụ 13 · Combining gravitational and elastic potential energy

A 1.0 kg block starts from rest at height $h = 2.0$ m on a frictionless ramp. The ramp leads onto a horizontal frictionless surface, where the block strikes and compresses a spring of force constant $k = 500$ N/m. Find the maximum compression of the spring.

All of the gravitational PE lost converts first to kinetic energy at the bottom, then entirely into elastic PE at maximum compression (the surface is frictionless throughout, so total mechanical energy is conserved from start to maximum compression):

$$mgh = \tfrac{1}{2}kx_{max}^2.$$

$$(1.0)(9.8)(2.0) = \tfrac{1}{2}(500)x_{max}^2 \Rightarrow 19.6 = 250\,x_{max}^2 \Rightarrow x_{max}^2 = 0.0784 \Rightarrow x_{max} = 0.28\,\text{m}.$$

3.8.1 Comparing the Energy Method and the Kinematics/Force Method

Many problems can be solved either by energy methods (this chapter) or by combining Newton's second law with the kinematics equations (Chapters 1–2). The next example solves the same problem both ways to confirm they agree.

Ví dụ 14 · Two routes to the same answer

A block starts from rest at the top of a frictionless incline of angle $30°$ and length $L = 10$ m (measured along the slope). Find its speed at the bottom (a) using energy conservation, and (b) using kinematics together with Newton's second law.

(a) Energy method. Vertical height dropped: $h = L\sin 30° = (10)(0.50) = 5.0$ m.

$$mgh = \tfrac{1}{2}mv^2 \Rightarrow v = \sqrt{2gh} = \sqrt{2(9.8)(5.0)} = \sqrt{98} = 9.9\,\text{m/s}.$$

(b) Kinematics/force method. Along a frictionless incline, the acceleration is $a = g\sin 30^\circ = (9.8)(0.50) = 4.9\,\mathrm{m/s^2}$. With $v_0 = 0$ and displacement $L = 10\,\mathrm{m}$ along the slope:

$$v^2 = v_0^2 + 2aL = 0 + 2(4.9)(10) = 98 \;\Rightarrow\; v = \sqrt{98} = 9.9\,\mathrm{m/s}.$$

Both methods give the identical result, $v = 9.9\,\mathrm{m/s}$, as they must.

The energy method is usually *faster* whenever a problem only asks for a speed (or height, or spring compression) and does not care about the path taken, the elapsed time, or intermediate positions along the way — it skips the need to find acceleration or resolve forces into components entirely. The kinematics/force method becomes *necessary* whenever the question asks for elapsed time, position as a function of time, or acceleration itself, since energy conservation alone contains no information about time.

VN **GIẢI THÍCH TIẾNG VIỆT**

Hai cách giải trên luôn cho cùng một đáp số vì cả hai đều xuất phát từ cùng định luật II Newton — phương pháp năng lượng chỉ là một "lối tắt" toán học. Nên dùng phương pháp năng lượng khi đề bài chỉ hỏi vận tốc, độ cao, hoặc độ biến dạng lò xo (không cần biết thời gian hay quỹ đạo chi tiết). Phải dùng động học (kèm định luật II Newton) khi đề bài hỏi về **thời gian**, gia tốc, hoặc vị trí tại một thời điểm cụ thể — những đại lượng này không xuất hiện trong phương trình bảo toàn năng lượng.

3.8.2 Power Requirements for Motion on an Incline

Many practical problems combine motion on an incline with the idea of power: an engine, motor, or muscle must supply enough power to move a vehicle or object up a slope while working against both gravity and a resisting force such as friction or air drag at the same time. The key difference from the flat-ground power problems earlier in this unit is that the driving force must now overcome an extra term — the weight component along the slope, $mg\sin\theta$.

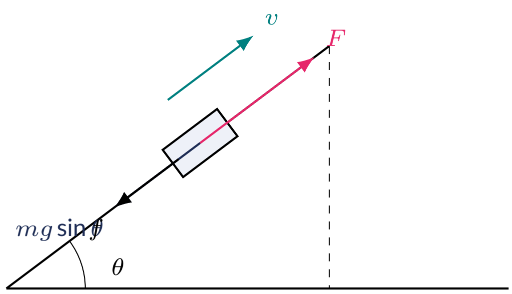
Constant velocity up an incline: the net force along the incline is zero, so the driving (engine/-motor) force exactly balances gravity's component along the slope plus any resisting force:

$$F = mg\sin\theta + f_{friction} \quad (\text{or } + F_{drag}), \qquad\qquad P = Fv.$$

Constant acceleration up an incline: apply Newton's second law along the incline first to find the required driving force,

$$F = ma + mg\sin\theta + f_{friction},$$

and only then compute the *instantaneous* power $P = Fv$ at whatever moment (or speed) the question asks about. Because v changes continuously while the object accelerates, this power is **not constant** — a single value of P describes only one instant, not the entire trip up the incline.



A vehicle climbs a rough incline of angle θ with velocity v directed up the slope. The engine supplies driving force F up the incline; gravity contributes a component $mg\sin\theta$ down the slope, and friction f also acts down the slope, opposing the motion.

VN **GIẢI THÍCH TIẾNG VIỆT**

Khi vật (xe, thùng hàng...) chuyển động **đều** (vận tốc không đổi) lên dốc, hợp lực dọc theo mặt dốc bằng 0, nên lực kéo của động cơ phải cân bằng đúng với thành phần trọng lực dọc dốc *và* lực ma sát/lực cản: $F = mg\sin\theta + f$. Công suất khi đó tính đơn giản bằng $P = Fv$. Khi vật chuyển động **nhanh dần đều** (gia tốc không đổi) lên dốc, phải dùng định luật II Newton trước để tìm lực kéo cần thiết ($F = ma + mg\sin\theta + f$), rồi mới nhân với vận tốc *tại đúng thời điểm* cần tính để ra công suất tức thời — công suất **không phải là hằng số** trong suốt quá trình tăng tốc, vì v thay đổi liên tục theo thời gian.

Ví dụ 15 · Constant-velocity power up an incline

A delivery truck of mass $m = 1200\,\text{kg}$ climbs a 37° incline at a constant speed $v = 20\,\text{m/s}$. A constant resistive force (rolling friction plus air drag) of $f = 600\,\text{N}$ opposes its motion. Find the power the engine must deliver.

At constant velocity, the net force along the incline is zero, so the driving force balances both gravity's component along the slope and the resistive force:

$$F = mg\sin 37^\circ + f = (1200)(9.8)(0.60) + 600 = 7056 + 600 = 7656\,\text{N}.$$

$$P = Fv = (7656)(20) = 153120\,\text{W} \approx 1.53\times 10^5\,\text{W} = 153\,\text{kW}.$$

Ví dụ 16 · Instantaneous power while accelerating up an incline

An $800\,\text{kg}$ car starts from rest at the bottom of a 37° incline and accelerates up the slope at a constant $a = 2.0\,\text{m/s}^2$. A constant resistive force $f = 400\,\text{N}$ (friction plus drag) opposes the motion throughout. Find the instantaneous power delivered by the engine at $t = 5.0\,\text{s}$.

Step 1 — velocity at $t = 5.0\,\text{s}$. Since the acceleration is constant,

$$v = v_0 + at = 0 + (2.0)(5.0) = 10\,\text{m/s}.$$

Step 2 — required driving force, from Newton's second law along the incline.

$$F - mg\sin 37^\circ - f = ma \;\Rightarrow\; F = ma + mg\sin 37^\circ + f.$$

$$F = (800)(2.0) + (800)(9.8)(0.60) + 400 = 1600 + 4704 + 400 = 6704\,\text{N}.$$

Step 3 — instantaneous power at $t = 5.0\,\text{s}$.

$$P = Fv = (6704)(10) = 67040\,\text{W} \approx 6.70\times 10^4\,\text{W} = 67.0\,\text{kW}.$$

Notice that although F itself happens to be constant here (since a and f are both constant), the power $P = Fv$ still *grows* as the car speeds up — the boxed value is only a snapshot valid at $t = 5.0\,\text{s}$, not the power delivered at every instant of the climb.

3.9 Practice Problems

Bài tập luyện tập

A force is applied exactly perpendicular to an object's displacement. How much work does it do?

(A) Fd
(B) $\frac{1}{2}Fd$
(C) Zero
(D) Cannot be determined

Lời giải chi tiết

$W = Fd\cos 90^\circ = 0$. **(C)**.

Bài tập luyện tập

A force $F = 60$ N, directed 53° above horizontal, moves an object 8.0 m horizontally. Find the work done by this force. ($\cos 53^\circ = 0.60$.)

Lời giải chi tiết

$W = Fd\cos 53^\circ = (60)(8.0)(0.60) = \boxed{288\,\text{J}}$.

Bài tập luyện tập

A 2.0 kg object moving at 3.0 m/s experiences a net force of 10 N along its direction of motion for a distance of 4.0 m. Find its final speed.

Lời giải chi tiết

$W = Fd = 40\,\text{J} = \Delta KE = \frac{1}{2}(2.0)(v_f^2 - 9.0) = 1.0(v_f^2 - 9.0)$.
$v_f^2 = 9.0 + 40 = 49 \Rightarrow v_f = \boxed{7.0\,\text{m/s}}$.

Bài tập luyện tập

A 5.0 kg block slides at 8.4 m/s across a rough floor ($\mu_k = 0.30$) and comes to rest. Find the distance it slides.

(A) 6.0 m
(B) 9.0 m
(C) 12.0 m
(D) 15.0 m

Lời giải chi tiết

$$d = \frac{v_0^2}{2\mu_k g} = \frac{(8.4)^2}{2(0.30)(9.8)} = \frac{70.56}{5.88} = 12.0\,\text{m. } \textbf{(C)}.$$

Bài tập luyện tập

A spring with $k = 250$ N/m is stretched $x = 0.24$ m from its natural length. Find the work done stretching it, using the area under the F–x graph.

Lời giải chi tiết

$W = \frac{1}{2}kx^2 = \frac{1}{2}(250)(0.24)^2 = \frac{1}{2}(250)(0.0576) = \boxed{7.2\,\text{J}}$.

Bài tập luyện tập

A spring with $k = 400$ N/m is compressed $x = 0.15$ m and used to launch a 1.0 kg block along a frictionless surface. Find the launch speed.

(A) 2.0 m/s

(B) 3.0 m/s

(C) 4.5 m/s

(D) 6.0 m/s

Lời giải chi tiết

$PE_s = \frac{1}{2}(400)(0.15)^2 = 4.5\,\text{J} = \frac{1}{2}(1.0)v^2 \Rightarrow v^2 = 9.0 \Rightarrow v = 3.0$ m/s. **(B).**

Bài tập luyện tập

A pendulum bob is released from rest at a height of 0.90 m above the lowest point of its swing (ignore air resistance). Find its speed at the lowest point.

Lời giải chi tiết

$v = \sqrt{2gh} = \sqrt{2(9.8)(0.90)} = \sqrt{17.64} = \boxed{4.2\,\text{m/s}}$.

Bài tập luyện tập

A 1.0 kg block starts from rest at the top of a $37°$ incline of length 4.0 m, with $\mu_k = 0.25$. Using energy methods, find its speed at the bottom.

Lời giải chi tiết

$h = 4.0(0.60) = 2.4$ m; $mgh = (1.0)(9.8)(2.4) = 23.52$ J.
$f_k = (0.25)(1.0)(9.8)(0.80) = 1.96$ N; $W_{fric} = -(1.96)(4.0) = -7.84$ J.
$KE_f = 23.52 - 7.84 = 15.68\,\text{J} = \frac{1}{2}(1.0)v^2 \Rightarrow v^2 = 31.36 \Rightarrow v = \boxed{5.6\,\text{m/s}}$.

Bài tập luyện tập

A motor lifts an 80 kg load at a constant 0.25 m/s. Find the motor's power output.

(A) 19.6 W

(B) 98 W

(C) 196 W

(D) 784 W

Lời giải chi tiết

$F = mg = 784$ N. $P = Fv = (784)(0.25) = 196$ W. **(C).**

Bài tập luyện tập

A crane does 1.2×10^5 J of work lifting a load in 30 s. Find its average power output.

Lời giải chi tiết

$P = \dfrac{1.2 \times 10^5}{30} = \boxed{4000\,\text{W}} = 4.0\,\text{kW}.$

Bài tập luyện tập

A person pulls a sled with a rope at 37° above horizontal, applying $80\,\text{N}$, while walking at a constant $1.5\,\text{m/s}$. Find the power she delivers.

Lời giải chi tiết

$P = Fv\cos 37^\circ = (80)(1.5)(0.80) = \boxed{96\,\text{W}}.$

Bài tập luyện tập

A ball thrown straight up is now on its way back down, still above the launch point. During this part of the motion, the work done by gravity is __ and the ball's kinetic energy is __.

(A) positive; increasing

(B) negative; decreasing

(C) positive; decreasing

(D) negative; increasing

Lời giải chi tiết

While falling, both gravity and displacement point downward, so $W_{gravity} > 0$. Positive net work means KE is **increasing**. **(A)**.

Bài tập luyện tập

A frictionless roller-coaster cart starts from rest at height $25\,\text{m}$. Find its speed when it passes a point at height $5.0\,\text{m}$ and a separate point at height $20\,\text{m}$.

Lời giải chi tiết

At $h = 5.0\,\text{m}$: $v = \sqrt{2g(25-5)} = \sqrt{2(9.8)(20)} = \sqrt{392} = \boxed{19.8\,\text{m/s}}$.

At $h = 20\,\text{m}$: $v = \sqrt{2g(25-20)} = \sqrt{2(9.8)(5.0)} = \sqrt{98} = \boxed{9.90\,\text{m/s}}$.

Bài tập luyện tập

A spring with $k = 600\,\text{N/m}$ is compressed $0.10\,\text{m}$ and launches a $0.30\,\text{kg}$ block along a frictionless horizontal surface onto a frictionless 37° incline. Find the maximum height and the distance traveled along the incline.

Lời giải chi tiết

$PE_s = \frac{1}{2}(600)(0.10)^2 = 3.0\,\text{J}$. $h = \dfrac{3.0}{(0.30)(9.8)} = \dfrac{3.0}{2.94} = \boxed{1.02\,\text{m}}.$

Distance along the slope: $L = \dfrac{h}{\sin 37^\circ} = \dfrac{1.02}{0.60} = \boxed{1.70\,\text{m}}.$

Bài tập luyện tập

If the compression of an ideal spring is doubled, how does its stored elastic potential energy change?

(A) Doubles
(B) Triples
(C) Quadruples
(D) Stays the same

Lời giải chi tiết

$PE_s = \frac{1}{2}kx^2 \propto x^2$. Doubling x multiplies PE_s by $2^2 = 4$. **(C).**

Bài tập luyện tập

A $70\,\text{kg}$ person climbs a flight of stairs of height $4.0\,\text{m}$ in $8.0\,\text{s}$. Find the work done against gravity and the average power output.

Lời giải chi tiết

$W = mgh = (70)(9.8)(4.0) = \boxed{2744\,\text{J}}$. $P = \dfrac{W}{t} = \dfrac{2744}{8.0} = \boxed{343\,\text{W}}$.

Bài tập luyện tập

Which force does negative work on a block that slides to a stop across a rough, level floor?

(A) Normal force
(B) Weight
(C) Kinetic friction
(D) None of these

Lời giải chi tiết

Kinetic friction opposes the sliding motion, so it acts opposite to displacement ($\theta = 180^\circ$), doing negative work. **(C).**

Bài tập luyện tập

A constant force F acts through two different displacements of equal magnitude d: one directed exactly parallel to F, and one directed at 60° to F. Find the ratio of the work done in the second case to the first.

Lời giải chi tiết

$W_\parallel = Fd\cos 0^\circ = Fd$. $W_{60^\circ} = Fd\cos 60^\circ = 0.50Fd$. Ratio $= \dfrac{W_{60^\circ}}{W_\parallel} = \boxed{0.50}$.

Bài tập luyện tập

A $2.0\,\text{kg}$ block slides down a curved frictionless ramp from a height of $3.0\,\text{m}$ onto a rough horizontal surface ($\mu_k = 0.40$) and slides until it stops. Find the distance traveled on the rough surface.

Lời giải chi tiết

Energy at the bottom of the ramp: $KE = mgh$. On the rough surface, $KE = \mu_k mgd$, so $mgh = \mu_k mgd \Rightarrow d = \dfrac{h}{\mu_k}$ — mass cancels entirely.

$$d = \frac{3.0}{0.40} = \boxed{7.5\,\text{m}}.$$

Bài tập luyện tập

Compared to a free-falling object with no air resistance dropped from the same height, an object experiencing air resistance (which does negative work) arrives at the ground with:

(A) A greater speed

(B) The same speed

(C) A smaller speed

(D) Cannot be determined

Lời giải chi tiết

Air resistance removes mechanical energy from the system, leaving less to convert to KE, so the final speed is smaller. **(C)**.

Bài tập luyện tập

A $1000\,\text{kg}$ car accelerates uniformly from rest to $20\,\text{m/s}$ in $8.0\,\text{s}$ (ignore friction and drag). Find the average power delivered by the engine.

Lời giải chi tiết

$\Delta KE = \frac{1}{2}(1000)(20)^2 = 2.0 \times 10^5\,\text{J}$. $P_{avg} = \dfrac{\Delta KE}{t} = \dfrac{2.0 \times 10^5}{8.0} = \boxed{2.5 \times 10^4\,\text{W}} = 25\,\text{kW}$.

Bài tập luyện tập

A spring with $k = 150\,\text{N/m}$ is stretched $0.40\,\text{m}$ from equilibrium. Find the elastic potential energy stored.

(A) $6.0\,\text{J}$

(B) $12\,\text{J}$

(C) $24\,\text{J}$

(D) $60\,\text{J}$

Lời giải chi tiết

$PE_s = \frac{1}{2}(150)(0.40)^2 = \frac{1}{2}(150)(0.16) = 12\,\text{J}$. **(B)**.

Bài tập luyện tập

For a conservative force such as gravity, how does the work done in moving an object between two fixed points depend on the path taken?

(A) It depends strongly on the path

(B) It is independent of the path — only start and end points matter

(C) It is always zero

(D) It always equals the total kinetic energy change alone

Lời giải chi tiết

This path-independence is the defining property of a conservative force, and is exactly why a potential energy function can be defined for it. **(B)**.

Bài tập luyện tập

A 3.0 kg block starts from rest at the top of a frictionless $37°$ incline of vertical height 4.9 m, slides down, then crosses a rough horizontal floor ($\mu_k = 0.50$) before stopping. Find the distance traveled on the horizontal floor.

Lời giải chi tiết

As in the earlier ramp problem, mass cancels: $mgh = \mu_k mgd \Rightarrow d = \dfrac{h}{\mu_k} = \dfrac{4.9}{0.50} = \boxed{9.8\,\text{m}}$.

Bài tập luyện tập

A box is pushed at constant velocity across a rough floor by a horizontal force equal in magnitude to the friction force. Find the net work done on the box over a distance d, and explain.

Lời giải chi tiết

Constant velocity means $\Delta KE = 0$, so by the work–energy theorem $W_{net} = \boxed{0}$. Individually, the applied force does $+Fd$ of work and friction does $-Fd$; these cancel exactly, leaving zero net work (though energy has still been converted to heat by friction).

Bài tập luyện tập

A motor rated at 750 W lifts a 60 kg load. Find the constant speed at which it can lift the load, assuming all power goes into lifting against gravity.

Lời giải chi tiết

$$P = mgv \Rightarrow v = \frac{P}{mg} = \frac{750}{(60)(9.8)} = \frac{750}{588} = \boxed{1.28\,\text{m/s}}.$$

Bài tập luyện tập

A 0.50 kg block is launched by a spring ($k = 500$ N/m, compressed 0.20 m) along a frictionless surface, then crosses a rough patch ($\mu_k = 0.20$) of length 2.0 m, then climbs a frictionless $37°$ ramp. Find (a) the block's kinetic energy right after leaving the spring, (b) its kinetic energy after crossing the rough patch, and (c) the maximum height it reaches on the ramp.

Lời giải chi tiết

(a) $KE_a = PE_s = \frac{1}{2}(500)(0.20)^2 = \boxed{10\,\text{J}}$.
(b) Friction work: $W_f = -\mu_k mgd = -(0.20)(0.50)(9.8)(2.0) = -1.96$ J. $KE_b = 10 - 1.96 = \boxed{8.04\,\text{J}}$.
(c) $h = \dfrac{KE_b}{mg} = \dfrac{8.04}{(0.50)(9.8)} = \dfrac{8.04}{4.9} = \boxed{1.64\,\text{m}}$.

Bài tập luyện tập

An object slides along a horizontal, frictionless floor. How much work does the normal force do on it?

(A) mgd
(B) Depends on speed
(C) Zero
(D) Cannot be determined

Lời giải chi tiết

The normal force is always perpendicular to horizontal displacement, so $W = Nd\cos 90^\circ = 0$. **(C)**.

Bài tập luyện tập

On a potential-energy graph $U(x)$ with the total mechanical energy E drawn as a horizontal line, what is the object's speed at a point where the E line crosses the $U(x)$ curve?

(A) It is maximum there
(B) It is zero there
(C) It equals $\sqrt{2E/m}$
(D) It cannot be determined from the graph

Lời giải chi tiết

Where $E = U(x)$, $KE = E - U = 0$, so the object is momentarily at rest — this is a turning point. **(B)**.

Bài tập luyện tập

A $0.50\,\text{kg}$ particle moves along the x-axis with potential energy $U(x) = (x-2)^2 + 3$ (in joules, x in meters) and total mechanical energy $E = 12\,\text{J}$. Find (a) the turning points of the motion, and (b) the particle's speed at $x = 2.0\,\text{m}$.

Lời giải chi tiết

(a) $12 = (x-2)^2 + 3 \Rightarrow (x-2)^2 = 9 \Rightarrow x - 2 = \pm 3 \Rightarrow x = \boxed{-1.0\,\text{m}}$ or $x = \boxed{5.0\,\text{m}}$.
(b) At $x = 2.0\,\text{m}$, $U = 3.0\,\text{J}$, so $KE = 12 - 3 = 9.0\,\text{J} = \frac{1}{2}(0.50)v^2 = 0.25v^2 \Rightarrow v^2 = 36 \Rightarrow v = \boxed{6.0\,\text{m/s}}$.

Bài tập luyện tập

A block is released from rest on a frictionless track and slides into a vertical circular loop of radius r. In terms of r, the minimum release height above the bottom of the loop needed for the block to maintain contact all the way around is:

(A) r
(B) $2r$
(C) $2.5r$
(D) $3r$

Lời giải chi tiết

Combining $mg = mv_{top}^2/r$ with energy conservation from height h to the top of the loop (height $2r$) gives $h_{min} = 2.5r$. **(C)**.

Bài tập luyện tập

A 0.20 kg toy car is released from rest on a frictionless track at height h above the bottom of a circular loop of radius $r = 0.50$ m. (a) Find the minimum height h for the car to complete the loop without leaving the track. (b) Find the car's speed at the top of the loop under this minimum condition. (c) Find the car's speed at the bottom of the loop, just before it enters the circular section.

Lời giải chi tiết

(a) $h_{min} = 2.5r = 2.5(0.50) = \boxed{1.25\,\text{m}}$ (the mass is not needed — it cancels).
(b) $v_{top} = \sqrt{gr} = \sqrt{(9.8)(0.50)} = \sqrt{4.9} = \boxed{2.2\,\text{m/s}}$.
(c) By energy conservation from the release point to the bottom of the loop: $v_{bottom} = \sqrt{2gh_{min}} = \sqrt{2(9.8)(1.25)} = \sqrt{24.5} = \boxed{4.9\,\text{m/s}}$.

Bài tập luyện tập

A 1.5 kg block slides from rest down a frictionless ramp from height $h = 2.0$ m onto a horizontal frictionless surface, where it compresses a spring of force constant $k = 600$ N/m. Find the maximum compression of the spring.

Lời giải chi tiết

$mgh = \frac{1}{2}kx_{max}^2$: $(1.5)(9.8)(2.0) = 29.4\,\text{J} = \frac{1}{2}(600)x_{max}^2 = 300x_{max}^2$.

$$x_{max}^2 = 0.098 \;\Rightarrow\; x_{max} = \boxed{0.31\,\text{m}}.$$

Bài tập luyện tập

A car's engine delivers constant power P while the car speeds up on level ground (ignore friction and air drag). As the car's speed increases, the forward force delivered by the engine:

(A) Increases
(B) Stays the same
(C) Decreases
(D) Is always zero

Lời giải chi tiết

Since $P = Fv$ is constant and v is increasing, $F = P/v$ must decrease. **(C).**

Bài tập luyện tập

A 3.0 kg object, initially at rest, is pushed by a force that is constant at 12 N from $x = 0$ to $x = 2.0$ m, then decreases linearly to zero from $x = 2.0$ m to $x = 5.0$ m. Using the area under the F–x graph, find the object's speed at $x = 5.0$ m.

Lời giải chi tiết

Rectangular area: $(12)(2.0) = 24$ J. Triangular area: $\frac{1}{2}(12)(3.0) = 18$ J. Total work: $W = 24 + 18 = 42\,\text{J} = \Delta KE$.

$$42 = \frac{1}{2}(3.0)v^2 = 1.5v^2 \;\Rightarrow\; v^2 = 28 \;\Rightarrow\; v = \boxed{5.3\,\text{m/s}}.$$

Bài tập luyện tập

A ball rolls up a frictionless hill with initial speed v_0 and rolls back down to its starting point at the bottom. A second, identical ball rolls up an identical hill with the same initial speed v_0, but this second hill has friction on the way up only (it is frictionless on the way back down, starting from wherever the ball turns around). Compare the two balls' speeds when they return to the bottom of their hills, and explain your reasoning using energy concepts.

Lời giải chi tiết

Ball 1 (frictionless throughout): mechanical energy is conserved for the whole round trip, so the ball returns to the bottom with exactly its initial speed, $v_{1,final} = v_0$.

Ball 2 (friction on the way up only): while climbing, friction converts some of the ball's kinetic energy into heat, so it reaches a maximum height h_2 that is *lower* than the height $h_1 = v_0^2/(2g)$ that ball 1 reaches. Rolling back down is frictionless, so for that portion of the trip mechanical energy is conserved: $\frac{1}{2}mv_{2,final}^2 = mgh_2$.

Since $h_2 < h_1$, ball 2 has less mechanical energy left to convert back into kinetic energy on the way down, so it returns to the bottom with a **smaller** speed than ball 1: $v_{2,final} < v_{1,final} = v_0$. The energy removed by friction during the ascent is permanently lost from the mechanical system (dissipated as heat) and can never be "given back" during the descent, even though the descent itself is frictionless.

Bài tập luyện tập

Two identical blocks are launched from rest by two different springs on a frictionless surface. Spring A has twice the force constant of spring B, but both springs are compressed by the same distance x. Compare the kinetic energies of the two blocks just after leaving their springs.

(A) They are equal

(B) Block A has twice the kinetic energy of block B

(C) Block A has four times the kinetic energy of block B

(D) Block B has twice the kinetic energy of block A

Lời giải chi tiết

$PE_s = \frac{1}{2}kx^2$; with x fixed, $PE_s \propto k$. Doubling k doubles the stored (and therefore released) energy. **(B)**.

Bài tập luyện tập

An elevator motor lifts a total load (car plus passengers) of mass 1000 kg at a constant speed of 2.0 m/s. Find the power required.

Lời giải chi tiết

At constant speed, $F = mg = (1000)(9.8) = 9800\,\text{N}$.

$$P = Fv = (9800)(2.0) = \boxed{1.96 \times 10^4\,\text{W}} = 19.6\,\text{kW}.$$

Bài tập luyện tập

A $4.0\,$kg block starts from rest at the top of a frictionless $30°$ incline and slides a distance of $6.0\,$m along the incline before reaching a horizontal surface with $\mu_k = 0.25$. Find (a) the block's speed at the bottom of the incline, and (b) how far it slides on the rough horizontal surface before stopping.

Lời giải chi tiết

$h = L \sin 30° = (6.0)(0.50) = 3.0\,\text{m}$.

(a) $v = \sqrt{2gh} = \sqrt{2(9.8)(3.0)} = \sqrt{58.8} = \boxed{7.7\,\text{m/s}}$.

(b) $mgh = \mu_k mgd \Rightarrow d = \dfrac{h}{\mu_k} = \dfrac{3.0}{0.25} = \boxed{12.0\,\text{m}}$ (mass cancels).

Bài tập luyện tập

At a point on a $U(x)$ graph where the curve has a local minimum, the net force on the object is __ and that position is a(n) __ equilibrium.

(A) zero; unstable

(B) zero; stable

(C) maximum; stable

(D) nonzero; unstable

Lời giải chi tiết

The slope of $U(x)$ is zero at a local minimum, so $F = -dU/dx = 0$. Small displacements away from a minimum increase U, meaning the force pushes the object back — a stable equilibrium. **(B)**.

Bài tập luyện tập

An elevator motor must lift a total load of $1200\,$kg through a height of $20\,$m in $15\,$s at constant speed. The motor is only 80% efficient (only 80% of its electrical input power becomes useful lifting power). Find (a) the useful power output required, and (b) the electrical power input required.

Lời giải chi tiết

(a) $P_{useful} = \dfrac{mgh}{t} = \dfrac{(1200)(9.8)(20)}{15} = \dfrac{235200}{15} = \boxed{1.57 \times 10^4\,\text{W}} = 15.7\,\text{kW}$.

(b) $P_{input} = \dfrac{P_{useful}}{0.80} = \dfrac{15680}{0.80} = \boxed{1.96 \times 10^4\,\text{W}} = 19.6\,\text{kW}$.

Bài tập luyện tập

A satellite moves in a perfectly circular orbit around Earth at constant speed. How much work does the gravitational force do on the satellite during one complete orbit?

(A) A large positive amount

(B) A large negative amount

(C) Zero

(D) It depends on the orbital radius

Lời giải chi tiết

In uniform circular motion, the (centripetal) gravitational force is always perpendicular to the velocity, so the instantaneous power $P = Fv\cos 90^\circ = 0$ at every point along the path; the total work over the full orbit is therefore **(C)** zero — consistent with the fact that the speed (and hence KE) never changes.

Bài tập luyện tập

A 2.0 kg object starts from rest at $x = 0$ on a frictionless horizontal surface. A force applied to it increases linearly from 0 to 40 N as x goes from 0 to 3.0 m, then remains constant at 40 N from $x = 3.0$ m to $x = 5.0$ m. Using the area under the F–x graph, find (a) the total work done on the object from $x = 0$ to $x = 5.0$ m, and (b) its speed at $x = 5.0$ m.

Lời giải chi tiết

(a) Triangular area (0 to 3.0 m): $\frac{1}{2}(3.0)(40) = 60$ J. Rectangular area (3.0 to 5.0 m): $(40)(2.0) = 80$ J. Total: $W = 60 + 80 = \boxed{140\,\text{J}}$.
(b) $W = \Delta KE = \frac{1}{2}(2.0)v^2 = 1.0\,v^2 \Rightarrow v^2 = 140 \Rightarrow v = \boxed{11.8\,\text{m/s}}$.

Bài tập luyện tập

An object's gravitational PE is first measured relative to the floor of a room, then re-measured relative to the ceiling, 3.0 m above the floor. Compare (i) the numerical value of PE at a single fixed position, and (ii) the physically meaningful change ΔPE between two fixed points, under the two choices of reference level.

(A) Both the PE value and ΔPE change

(B) The PE value changes, but ΔPE stays the same

(C) Neither the PE value nor ΔPE changes

(D) The PE value stays the same, but ΔPE changes

Lời giải chi tiết

$PE_g = mg(h - h_{ref})$ depends explicitly on the chosen reference height h_{ref}, so its value at a single position changes when the reference changes. But for any two fixed points, $\Delta PE = mg(h_A - h_B)$, and h_{ref} cancels out entirely — the change is reference-independent. **(B)**.

Bài tập luyện tập

A vertical spring with $k = 800$ N/m is compressed 0.20 m by a 0.40 kg ball resting on top of it, then released, launching the ball straight upward. Ignoring air resistance, find the maximum height the ball rises above its initial (compressed) position.

Lời giải chi tiết

All the spring's elastic PE converts to gravitational PE at the highest point:

$$PE_s = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(800)(0.20)^2 = \tfrac{1}{2}(800)(0.04) = 16\,\text{J}.$$

$$mgh = 16 \;\Rightarrow\; h = \frac{16}{(0.40)(9.8)} = \frac{16}{3.92} = \boxed{4.08\,\text{m}}.$$

Bài tập luyện tập

A $2.0\,\text{kg}$ block slides from rest down a frictionless incline from height $h_1 = 2.5\,\text{m}$, crosses a rough horizontal floor ($\mu_k = 0.30$) of length $4.0\,\text{m}$, and then climbs a second, frictionless incline. Find (a) the block's speed at the bottom of the first incline, (b) the energy dissipated by friction while crossing the floor, and (c) the maximum height it reaches on the second incline.

Lời giải chi tiết

(a) $v = \sqrt{2gh_1} = \sqrt{2(9.8)(2.5)} = \sqrt{49} = \boxed{7.0\,\text{m/s}}$.
(b) $f_k = \mu_k mg = (0.30)(2.0)(9.8) = 5.88\,\text{N}$. Energy dissipated: $W_{fric} = f_k d = (5.88)(4.0) = \boxed{23.52\,\text{J}}$.
(c) KE at the bottom of the first incline $= mgh_1 = (2.0)(9.8)(2.5) = 49\,\text{J}$. After crossing the floor: $KE = 49 - 23.52 = 25.48\,\text{J}$.

$$h_2 = \frac{25.48}{(2.0)(9.8)} = \frac{25.48}{19.6} = \boxed{1.30\,\text{m}}.$$

Bài tập luyện tập

An electric golf cart (combined mass $400\,\text{kg}$) travels up a 37° path at a constant speed of $2.0\,\text{m/s}$. A constant rolling-resistance force of $150\,\text{N}$ opposes its motion. Find the power the motor must deliver.

(A) $4.7\,\text{kW}$
(B) $5.0\,\text{kW}$
(C) $6.6\,\text{kW}$
(D) $10.0\,\text{kW}$

Lời giải chi tiết

At constant velocity, $F = mg\sin 37^\circ + f = (400)(9.8)(0.60) + 150 = 2352 + 150 = 2502\,\text{N}$.

$$P = Fv = (2502)(2.0) = 5004\,\text{W} \approx 5.0\,\text{kW}.$$

(B). (Forgetting the resistive force gives the too-small distractor $\approx 4.7\,\text{kW}$; using $\cos 37^\circ$ instead of $\sin 37^\circ$ gives the too-large distractor $\approx 6.6\,\text{kW}$.)

Bài tập luyện tập

A $500\,\text{kg}$ motorcycle starts from rest at the base of a 37° hill and accelerates up the slope at a constant $a = 1.5\,\text{m/s}^2$ (ignore friction and air resistance). Find the instantaneous power the engine delivers at the moment the motorcycle's speed reaches $8.0\,\text{m/s}$.

Lời giải chi tiết

Newton's second law along the incline: $F - mg\sin 37^\circ = ma \Rightarrow F = ma + mg\sin 37^\circ$.

$$F = (500)(1.5) + (500)(9.8)(0.60) = 750 + 2940 = 3690\,\text{N}.$$

$$P = Fv = (3690)(8.0) = 29520\,\text{W} \approx \boxed{2.95 \times 10^4\,\text{W}} = 29.5\,\text{kW}.$$

Bài tập luyện tập

A particle's potential energy $U(x)$ has a local **maximum** at $x = x_0$. If the particle is placed exactly at rest at x_0, what happens?

(A) It stays at rest indefinitely, in a stable equilibrium

(B) It stays at rest at that instant (the force is zero there), but the equilibrium is unstable — the slightest disturbance causes it to accelerate away and never return

(C) It oscillates back and forth through x_0 forever

(D) It accelerates away immediately, even with no disturbance at all

Lời giải chi tiết

At a local maximum, the slope of $U(x)$ is zero, so $F = -dU/dx = 0$ exactly at x_0 — the particle is momentarily in equilibrium. But any tiny displacement moves it into a lower-U region on one side, and the resulting force pushes it *further* away rather than back: this is an **unstable** equilibrium. **(B)**.

Bài tập luyện tập

A 0.30 kg ball is released from rest at height $h = 3.5$ m above the bottom of a frictionless vertical loop of radius $r = 1.0$ m. Find (a) the ball's speed at the top of the loop, and (b) the normal force exerted by the track on the ball at the top of the loop.

Lời giải chi tiết

(a) Energy conservation from the release point to the top of the loop (height $2r$ above the bottom):

$$mgh = mg(2r) + \tfrac{1}{2}mv_{top}^2 \;\Rightarrow\; v_{top}^2 = 2g(h - 2r).$$

$$v_{top}^2 = 2(9.8)(3.5 - 2.0) = 2(9.8)(1.5) = 29.4 \;\Rightarrow\; v_{top} = \boxed{5.42\,\text{m/s}}.$$

(b) At the top, both gravity and the normal force point downward, toward the center, together supplying the centripetal force:

$$N + mg = \frac{mv_{top}^2}{r} \;\Rightarrow\; N = \frac{(0.30)(29.4)}{1.0} - (0.30)(9.8) = 8.82 - 2.94 = \boxed{5.88\,\text{N}}.$$

Bài tập luyện tập

A 5.0 kg sled, initially at rest, is pulled across a horizontal snowy surface by a rope held at 53° above horizontal with tension 50 N, while a constant friction force of 15 N resists the motion. Find the sled's speed after it has moved 6.0 m. ($\cos 53^\circ = 0.60$.)

Lời giải chi tiết

$W_{tension} = Td\cos 53^\circ = (50)(6.0)(0.60) = 180$ J. $W_{fric} = -fd = -(15)(6.0) = -90$ J.

$$W_{net} = 180 - 90 = 90\,\text{J} = \Delta KE = \tfrac{1}{2}(5.0)v^2 = 2.5v^2 \;\Rightarrow\; v^2 = 36 \;\Rightarrow\; v = \boxed{6.0\,\text{m/s}}.$$

Linear Momentum

Mục tiêu Unit: Động lượng; xung lực và định lý xung lượng–động lượng (đồ thị F–t); định luật bảo toàn động lượng cho hệ kín; va chạm đàn hồi, va chạm mềm (không đàn hồi), và va chạm không đàn hồi tổng quát; va chạm 2 chiều; khối tâm và vận tốc khối tâm.

4.1 Momentum

Linear momentum: $\vec{p} = m\vec{v}$ — a vector, pointing in the same direction as velocity. SI unit: kg · m/s.

Ví dụ 1 · Momentum

Find the momentum of a 1500 kg car moving at 20 m/s.

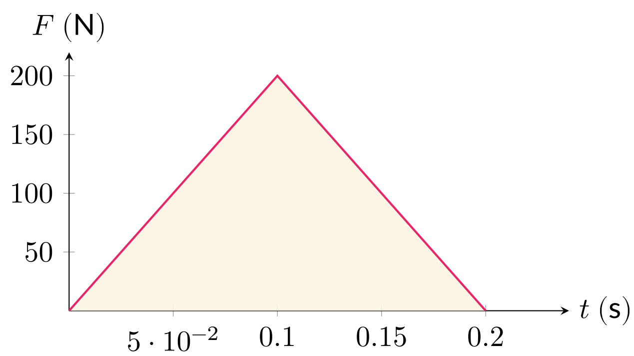
$$p = mv = (1500)(20) = 3.0 \times 10^4 \text{ kg} \cdot \text{m/s}.$$

4.2 Impulse and the Impulse–Momentum Theorem

Impulse: $\vec{J} = \vec{F}_{net}\,\Delta t$ (for constant force). **Impulse–momentum theorem:**

$$\vec{J} = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_i.$$

On a force-vs-time graph, impulse equals the *area* under the curve — this works even when the force is not constant.



Triangular F–t graph: impulse equals the shaded area, $J = \frac{1}{2}(\text{base})(\text{height})$.

Ví dụ 2 · Impulse from a constant force

A 300 N force acts on a 0.60 kg ball, initially at rest, for 0.040 s. Find the ball's final speed.

$$J = F\Delta t = (300)(0.040) = 12\,\mathrm{N\cdot s} = \Delta p = m\Delta v \;\Rightarrow\; \Delta v = \frac{12}{0.60} = 20\,\mathrm{m/s}.$$

Ví dụ 3 · Impulse from a triangular F–t graph

A force on a 2.0 kg object (initially at rest) rises linearly from 0 to 200 N over 0.10 s, then falls linearly back to 0 over the next 0.10 s. Find the object's velocity right after the force ends.

Impulse = area of the triangle = $\frac{1}{2}$(base)(height) = $\frac{1}{2}(0.20)(200) = 20\,\mathrm{N\cdot s}$.

$$\Delta v = \frac{J}{m} = \frac{20}{2.0} = 10\,\mathrm{m/s}.$$

GIẢI THÍCH TIẾNG VIỆT

VN

Xung lực $J = F\Delta t$ bằng **diện tích** dưới đồ thị F–t, kể cả khi lực thay đổi theo thời gian (đồ thị không phải hình chữ nhật). Định lý xung lượng–động lượng $J = \Delta p$ giải thích tại sao túi khí, đệm, hay thời gian va chạm kéo dài giúp **giảm lực trung bình**: với cùng Δp, Δt càng lớn thì $F_{avg} = \Delta p/\Delta t$ càng nhỏ.

4.3 Conservation of Momentum

Khái niệm cốt lõi

For an isolated system (zero net *external* force), total momentum is conserved:

$$\sum \vec{p}_{before} = \sum \vec{p}_{after}.$$

This holds during *any* interaction — collision, explosion, or recoil — elastic or not, as long as no outside force acts (or its effect is negligible over the interaction time). Internal forces (between the objects of the system) always cancel in pairs by Newton's third law and never change the system's total momentum.

GIẢI THÍCH TIẾNG VIỆT

VN

Động lượng của **hệ kín** (không chịu ngoại lực) luôn được bảo toàn, bất kể tương tác bên trong là va chạm đàn hồi, va chạm mềm, hay một vụ nổ. Đây là công cụ mạnh nhất để giải bài toán va chạm: viết $\sum p_{\text{trước}} = \sum p_{\text{sau}}$ theo từng phương, chú ý dấu (chiều) của vận tốc.

Ví dụ 4 · Recoil

An 800 kg cannon, initially at rest, fires a 4.0 kg shell at 300 m/s. Find the recoil speed of the cannon.

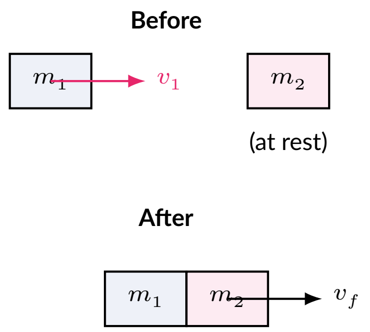
Total momentum starts and stays zero: $0 = MV + mv \Rightarrow V = -\dfrac{mv}{M} = -\dfrac{(4.0)(300)}{800} = -1.5\,\mathrm{m/s}$ (i.e. 1.5 m/s opposite to the shell).

4.4 Elastic and Inelastic Collisions

Perfectly inelastic: the objects stick together and share one common final velocity; momentum is conserved, but kinetic energy is *not* (some converts to heat, sound, or deformation).

Elastic: both momentum *and* total kinetic energy are conserved. A useful shortcut: in any 1D elastic collision, the **relative speed of separation equals the relative speed of approach**: $v_2' - v_1' = -(v_2 - v_1)$.

General inelastic (not "perfectly"): momentum is conserved but kinetic energy decreases, and the objects do *not* end up with a common velocity.



Perfectly inelastic collision: m_1 moving at v_1 strikes stationary m_2; afterward they move together at common velocity v_f.

Ví dụ 5 · Perfectly inelastic collision

A 3.0 kg cart moving at 5.0 m/s collides and sticks with a stationary 2.0 kg cart. Find the common final velocity and the kinetic energy lost.

Momentum: $(3.0)(5.0) + 0 = (5.0)v_f \Rightarrow v_f = 3.0$ m/s.
$KE_i = \frac{1}{2}(3.0)(5.0)^2 = 37.5$ J; $KE_f = \frac{1}{2}(5.0)(3.0)^2 = 22.5$ J. Energy lost $= 37.5 - 22.5 = 15$ J.

Ví dụ 6 · Elastic collision

$m_1 = 2.0$ kg moving at $v_1 = 6.0$ m/s strikes a stationary $m_2 = 4.0$ kg elastically. Find each object's velocity afterward.

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2}v_1 = \frac{2.0 - 4.0}{6.0}(6.0) = -2.0\text{ m/s}, \qquad v_2' = \frac{2m_1}{m_1 + m_2}v_1 = \frac{4.0}{6.0}(6.0) = 4.0\text{ m/s}.$$

Check momentum: before $= (2.0)(6.0) = 12$; after $= (2.0)(-2.0) + (4.0)(4.0) = -4.0 + 16 = 12$ ✓.
Check KE: before $= \frac{1}{2}(2.0)(36) = 36$ J; after $= \frac{1}{2}(2.0)(4.0) + \frac{1}{2}(4.0)(16) = 4.0 + 32 = 36$ J ✓.

(!) Lỗi thường gặp: Momentum is conserved in **every** collision inside an isolated system — elastic or not. Kinetic energy is conserved **only** in elastic collisions. Never assume KE is conserved unless the problem specifically states (or you have verified) that the collision is elastic.

4.5 Two-Dimensional Collisions

When motion occurs in a plane, apply conservation of momentum *separately* to the x- and y-components; the two components are independent, just as in projectile motion.

Ví dụ 7 · Perfectly inelastic collision in 2D

Car A (1500 kg) moving east at 12 m/s collides and sticks with car B (1000 kg) moving north at 9.0 m/s. Find the speed and direction of the wreckage immediately after the collision.

$p_x = (1500)(12) = 18000$ kg · m/s (east); $p_y = (1000)(9.0) = 9000$ kg · m/s (north). Total mass $M = 2500$ kg.

$$v_x = \frac{18000}{2500} = 7.2\,\text{m/s}, \qquad v_y = \frac{9000}{2500} = 3.6\,\text{m/s}.$$

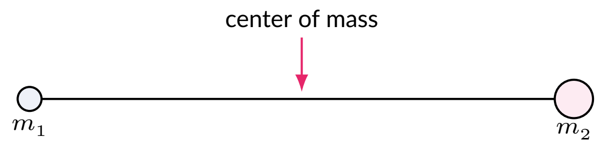
$$v = \sqrt{7.2^2 + 3.6^2} = \sqrt{51.84 + 12.96} = \sqrt{64.8} = 8.05\,\text{m/s}, \qquad \theta = \arctan\left(\frac{3.6}{7.2}\right) = 26.6^\circ \text{ north of east.}$$

4.6 Center of Mass

The **center of mass** of a system is the mass-weighted average position:

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}, \qquad v_{cm} = \frac{\sum m_i v_i}{\sum m_i}.$$

The total momentum of a system equals $p_{total} = Mv_{cm}$, where $M = \sum m_i$. For an isolated system, v_{cm} never changes — even during a collision — because internal forces cannot change total momentum.



The center of mass sits closer to the more massive object.

Ví dụ 8 · Center of mass position

Point mass $m_1 = 2.0$ kg sits at $x = 0$; $m_2 = 6.0$ kg sits at $x = 8.0$ m. Find the center of mass.

$$x_{cm} = \frac{(2.0)(0) + (6.0)(8.0)}{2.0 + 6.0} = \frac{48}{8.0} = 6.0\,\text{m}.$$

Ví dụ 9 · Center-of-mass velocity

$m_1 = 3.0$ kg moves at 4.0 m/s east; $m_2 = 2.0$ kg moves at 6.0 m/s west. Find the velocity of the center of mass.

Taking east as positive ($v_2 = -6.0$ m/s):

$$v_{cm} = \frac{(3.0)(4.0) + (2.0)(-6.0)}{5.0} = \frac{12 - 12}{5.0} = 0\,\text{m/s}.$$

The center of mass is momentarily at rest — total momentum of this system is exactly zero.

Ví dụ 10 · Impulse from a bounce

A 0.50 kg ball moving at 8.0 m/s toward a wall bounces straight back at 6.0 m/s; contact time is 0.020 s. Find the impulse delivered to the ball and the average force from the wall.

Taking the initial direction of motion as positive: $v_i = +8.0$ m/s, $v_f = -6.0$ m/s.

$$J = \Delta p = m(v_f - v_i) = (0.50)(-6.0 - 8.0) = (0.50)(-14) = -7.0\,\text{N}\cdot\text{s}$$

(magnitude $7.0\,\text{N}\cdot\text{s}$, directed away from the wall).

$$F_{avg} = \frac{|J|}{\Delta t} = \frac{7.0}{0.020} = 350\,\text{N}.$$

4.7 Explosions and the Ballistic Pendulum

An **explosion** is, in a real sense, a perfectly inelastic collision running backward in time: instead of several objects merging into one, a single object breaks apart into two or more pieces. The same law applies with no modification — total momentum before equals total momentum after — because the forces that drive the pieces apart (a spring, expanding gas, a chemical reaction) are entirely *internal* to the system and can never change its total momentum. If the object starts at rest, the total momentum stays at zero, so the momenta of the resulting fragments must add up (as vectors) to zero.

Explosions: momentum conservation applies exactly as in a collision. If the system starts at rest, $\sum \vec{p}_{after} = \vec{0}$, so the fragments' momenta must cancel as vectors. Unlike a collision, kinetic energy in an explosion is never conserved — it always *increases*, since stored energy (chemical, elastic, nuclear) converts into the kinetic energy of the flying fragments.

VN GIẢI THÍCH TIẾNG VIỆT

Một vụ nổ chính là va chạm mềm nhưng **chạy ngược thời gian**: một vật tách thành nhiều mảnh thay vì nhiều vật nhập thành một. Định luật bảo toàn động lượng vẫn áp dụng y hệt: nếu vật ban đầu đứng yên, tổng động lượng các mảnh sau vụ nổ phải bằng **không** — các vectơ động lượng của từng mảnh phải triệt tiêu lẫn nhau. Khác với va chạm, động năng trong vụ nổ luôn **tăng** (năng lượng hóa học hay năng lượng đàn hồi dự trữ chuyển thành động năng), không bao giờ giảm hay được bảo toàn nguyên vẹn.

Ví dụ 11 · Symmetric two-piece explosion

A 4.0 kg bomb at rest explodes into two equal fragments of 2.0 kg each, flying apart along the same line. The two fragments separate at a relative speed of 20 m/s. Find the speed of each fragment and the total kinetic energy released by the explosion.

Because the bomb starts at rest, total momentum stays zero: $mv_1 + mv_2 = 0$, and with equal masses this forces $v_1 = -v_2$ — the fragments move in *opposite directions at equal speed* v. Their relative speed of separation is then $v - (-v) = 2v$:

$$2v = 20\,\text{m/s} \;\Rightarrow\; v = 10\,\text{m/s (each fragment, opposite directions).}$$

Kinetic energy released (all created from stored energy, since $KE_i = 0$):

$$KE_f = \tfrac{1}{2}(2.0)(10)^2 + \tfrac{1}{2}(2.0)(10)^2 = 100 + 100 = 200\,\text{J}.$$

Ví dụ 12 · Three-piece explosion (vector components)

A 6.0 kg firework shell, initially at rest in the air, explodes into three fragments. Fragment A (1.0 kg) flies off at 9.0 m/s due east. Fragment B (2.0 kg) flies off at 6.0 m/s due north. Fragment

C has mass $3.0\,\text{kg}$ (the remainder). Find the velocity (magnitude and direction) of fragment C immediately after the explosion.

Total momentum before the explosion is zero, so the vector sum of all three final momenta must be zero: $\vec{p}_A + \vec{p}_B + \vec{p}_C = \vec{0}$. Take east as $+x$, north as $+y$.

x-direction: $p_{Ax} = (1.0)(9.0) = 9.0$, $p_{Bx} = 0$.

$$p_{Cx} = -(p_{Ax} + p_{Bx}) = -9.0\,\text{kg}\cdot\text{m/s} \;\Rightarrow\; v_{Cx} = \frac{-9.0}{3.0} = -3.0\,\text{m/s}.$$

y-direction: $p_{Ay} = 0$, $p_{By} = (2.0)(6.0) = 12.0$.

$$p_{Cy} = -(p_{Ay} + p_{By}) = -12.0\,\text{kg}\cdot\text{m/s} \;\Rightarrow\; v_{Cy} = \frac{-12.0}{3.0} = -4.0\,\text{m/s}.$$

Magnitude and direction:

$$v_C = \sqrt{(-3.0)^2 + (-4.0)^2} = \sqrt{9.0 + 16} = \sqrt{25} = 5.0\,\text{m/s}, \qquad \theta = \arctan\left(\frac{4.0}{3.0}\right) = 53^\circ \text{ south of west}.$$

Both velocity components are negative, so fragment C flies off into the quadrant opposite A and B — exactly what is needed for the three momenta to cancel as vectors.

The **ballistic pendulum** is a classic device for measuring a bullet's speed, which is otherwise very hard to time directly. A bullet of mass m and unknown speed v_0 is fired horizontally into a wood block of mass M hanging at rest from a string. The bullet embeds itself in the block (a perfectly inelastic collision), and the combined block+bullet swings up to some maximum height h, which is easy to measure. Analyzing the problem requires *splitting it into two separate stages*, each governed by a different conservation law.

Ballistic pendulum — the two-stage method.

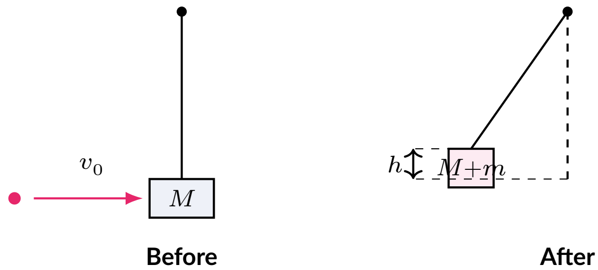
Stage 1 (the collision): the bullet embeds in the block in a fraction of a millisecond — far too fast for the string to swing or for gravity to do any measurable work. Treat this as an isolated, perfectly inelastic collision: momentum is conserved, kinetic energy is *not*.

$$mv_0 = (M + m)v.$$

Stage 2 (the swing): once the bullet is embedded, the combined block moves as a single rigid body on the string. Momentum is *not* conserved during the swing (the string tension is an external force on the block, with a net effect as it curves upward), but mechanical energy *is* conserved, since tension does no work (it is always perpendicular to the motion) and gravity is conservative.

$$\tfrac{1}{2}(M + m)v^2 = (M + m)gh \;\Rightarrow\; v = \sqrt{2gh}.$$

Combine the two stages — never conserve KE across Stage 1, and never conserve momentum across Stage 2 — to relate the bullet's initial speed v_0 to the observed swing height h.



Ballistic pendulum. **Before:** bullet (mass m, speed v_0) approaches a block (mass M) hanging at rest. **After:** the bullet has embedded in the block (momentum conserved, kinetic energy lost to heat/deformation), and the combined mass $M+m$ swings up to a maximum height h (mechanical energy conserved during the swing).

Ví dụ 13 · Full ballistic pendulum

A $10\,\text{g}$ bullet is fired into a $1.99\,\text{kg}$ wood block hanging at rest from a string (so the combined mass is $2.00\,\text{kg}$). The block+bullet swing up to a maximum height of $0.100\,\text{m}$. Find the bullet's initial speed.

Stage 2 first (energy conservation during the swing) — find the speed right after the collision:

$$v = \sqrt{2gh} = \sqrt{2(9.8)(0.100)} = \sqrt{1.96} = 1.4\,\text{m/s}.$$

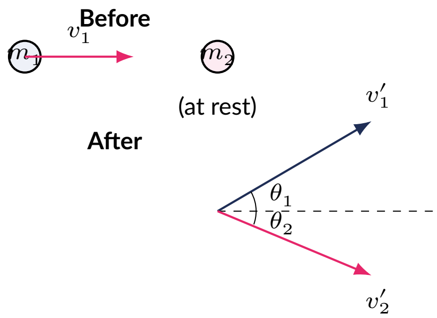
Stage 1 (momentum conservation during the collision) — use this v to find the bullet's initial speed:

$$mv_0 = (M+m)v \;\Rightarrow\; v_0 = \frac{(M+m)v}{m} = \frac{(2.00)(1.4)}{0.010} = \boxed{280\,\text{m/s}}.$$

Sanity check: $KE_{bullet,i} = \frac{1}{2}(0.010)(280)^2 = 392\,\text{J}$, but right after embedding, $KE = \frac{1}{2}(2.00)(1.4)^2 = 1.96\,\text{J}$ — over 99% of the kinetic energy was lost to heat and deformation *during the collision itself*. This is exactly what "perfectly inelastic" means, and it is why kinetic energy can never be assumed conserved across Stage 1.

4.7.1 Two-Dimensional Collisions in Detail

The 2D collision analyzed earlier (Ví dụ 7) was perfectly inelastic — the two objects stuck together, leaving only one unknown final velocity to find. In a general **glancing collision**, the objects do not stick: each goes its own way at some angle to the original line of motion, so there are two separate final velocities (four unknowns in total — two magnitudes and two directions). Momentum conservation in x and y supplies only two equations, so a glancing collision needs additional information — typically one final velocity given outright, as in the example below — to pin down a unique answer. The method is unchanged: resolve every velocity into components, conserve p_x and p_y separately, then recombine the unknown components into a magnitude and direction at the end.



A glancing 2D collision: m_1 approaches along the dashed line and strikes stationary m_2. Afterward, m_1 moves off at angle θ_1 above the original line of motion and m_2 moves off at angle θ_2 below it. Momentum is conserved separately along x and along y.

Ví dụ 14 · Glancing collision, one object initially at rest

$m_1 = 2.0$ kg moving at $v_1 = 9.0$ m/s east strikes a stationary $m_2 = 4.0$ kg. Afterward, m_1 moves off at $v_1' = 5.0$ m/s, at 37° above the original line of motion ($\sin 37^\circ = 0.60$, $\cos 37^\circ = 0.80$). Find m_2's velocity (magnitude and direction) after the collision.

Before: $p_x = m_1 v_1 = (2.0)(9.0) = 18$ kg · m/s, $p_y = 0$.

Components of m_1's final velocity:

$$v_{1x}' = v_1' \cos 37^\circ = (5.0)(0.80) = 4.0\,\text{m/s}, \qquad v_{1y}' = v_1' \sin 37^\circ = (5.0)(0.60) = 3.0\,\text{m/s}.$$

$$p_{1x}' = (2.0)(4.0) = 8.0, \qquad p_{1y}' = (2.0)(3.0) = 6.0 \ \ (\text{kg}\cdot\text{m/s}).$$

Conserve momentum in each direction to find m_2's final momentum:

$$p_{2x}' = p_x - p_{1x}' = 18 - 8.0 = 10.0, \qquad p_{2y}' = p_y - p_{1y}' = 0 - 6.0 = -6.0 \ \ (\text{kg}\cdot\text{m/s}).$$

$$v_{2x}' = \frac{10.0}{4.0} = 2.5\,\text{m/s}, \qquad v_{2y}' = \frac{-6.0}{4.0} = -1.5\,\text{m/s}.$$

$$v_2' = \sqrt{2.5^2 + 1.5^2} = \sqrt{8.5} = 2.92\,\text{m/s}, \qquad \theta_2 = \arctan\left(\frac{1.5}{2.5}\right) = 31^\circ \text{ below the original line of motion.}$$

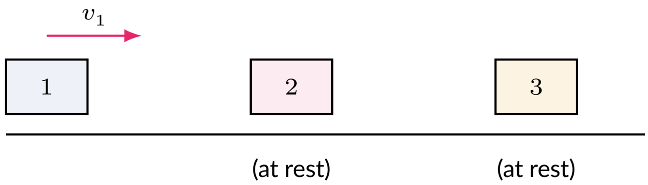
4.7.2 Sequential (Chain) Collisions

Not every collision problem involves just two objects. On a track (or any 1D system), three or more carts can interact one pair at a time: cart 1 strikes cart 2 first, and only afterward — now carrying whatever velocity that first collision gave it — does cart 2 strike cart 3. Problems like this are *not* solved by writing one giant conservation equation for all three carts at once; instead, each collision is its own separate, complete conservation-of-momentum problem, solved in the order the collisions actually happen. The final velocity produced by one stage becomes the *initial* velocity for the next stage.

Chain (multi-stage) collisions. Treat each collision as an independent momentum-conservation problem, solved in chronological order:

$$\text{Stage 1: } \sum p_{before,1} = \sum p_{after,1} \qquad \Longrightarrow \qquad \text{Stage 2: } \sum p_{before,2} = \sum p_{after,2}.$$

The object(s) entering Stage 2 start that stage with whatever velocity Stage 1 left them with — *never* their original velocity from before Stage 1. Each stage may be elastic, perfectly inelastic, or general inelastic independently of the others; classify and solve each stage on its own terms. As a check, the *total* momentum of the whole system, summed over every object, stays constant across all stages combined — equal to whatever it was at the very start — since only internal forces act throughout.



Three carts in a row on a frictionless track. Cart 1 approaches stationary cart 2 at v_1 (Stage 1); whatever velocity cart 2 picks up then carries it into a second collision with stationary cart 3 (Stage 2).

VN **GIẢI THÍCH TIẾNG VIỆT**

Va chạm dây chuyền (nhiều giai đoạn): khi ba vật va chạm lần lượt chứ không đồng thời, hãy tách bài toán thành **từng giai đoạn riêng biệt**, mỗi giai đoạn là một bài toán bảo toàn động lượng độc lập, giải theo đúng thứ tự thời gian xảy ra. Vận tốc **cuối** của giai đoạn trước chính là vận tốc **đầu** của giai đoạn sau — lỗi phổ biến nhất là dùng nhầm vận tốc ban đầu của vật thứ nhất (trước Giai đoạn 1) cho Giai đoạn 2, thay vì vận tốc thực tế vật đó có ngay trước khi bước vào giai đoạn đó.

Ví dụ 15 · Three-cart perfectly inelastic chain collision

On a frictionless track, cart 1 ($m_1 = 2.0$ kg, moving at $v_1 = 9.0$ m/s) collides with stationary cart 2 ($m_2 = 4.0$ kg) and sticks (Stage 1). The combined cart then collides with stationary cart 3 ($m_3 = 3.0$ kg) and sticks as well (Stage 2). Find the final common velocity of all three carts, and the total kinetic energy lost over both collisions combined.

Stage 1 (cart 1 + cart 2, momentum conservation):

$$m_1 v_1 = (m_1 + m_2)v_{12} \;\Rightarrow\; v_{12} = \frac{(2.0)(9.0)}{2.0 + 4.0} = \frac{18}{6.0} = 3.0\,\text{m/s}.$$

Stage 2 (combined cart + cart 3, momentum conservation) — using $v_{12} = 3.0$ m/s as the *initial* velocity of the 6.0-kg combined cart, *not* the original 9.0 m/s:

$$(m_1 + m_2)v_{12} = (m_1 + m_2 + m_3)v_f \;\Rightarrow\; v_f = \frac{(6.0)(3.0)}{6.0 + 3.0} = \frac{18}{9.0} = 2.0\,\text{m/s}.$$

Kinetic energy lost overall: only cart 1 has KE initially, and all three carts share v_f at the end:

$$KE_i = \tfrac{1}{2}(2.0)(9.0)^2 = 81\,\text{J}, \qquad KE_f = \tfrac{1}{2}(9.0)(2.0)^2 = 18\,\text{J}, \qquad \Delta KE = 81 - 18 = 63\,\text{J lost}.$$

Check, stage by stage: Stage 1 alone: $KE = \frac{1}{2}(6.0)(3.0)^2 = 27$ J, so it loses $81 - 27 = 54$ J. Stage 2 alone: $KE = 18$ J, so it loses $27 - 18 = 9$ J. Total lost: $54 + 9 = 63$ J ✓, matching the direct calculation.

Ví dụ 16 · Chain collision mixing two collision types

On the same track, cart 1 ($m_1 = 2.0$ kg, moving at $v_1 = 15$ m/s) collides with stationary cart 2 ($m_2 = 4.0$ kg) and sticks — Stage 1 is *perfectly inelastic*. The combined 1–2 cart (mass 6.0 kg) then collides *elastically* with stationary cart 3 ($m_3 = 2.0$ kg) — Stage 2. Find the velocity of the combined 1–2 cart and the velocity of cart 3 after Stage 2, and verify that the momentum of the whole three-cart system is the same at the end as it was at the very start.

Stage 1 (perfectly inelastic — momentum only):

$$m_1 v_1 = (m_1 + m_2)v_{12} \;\Rightarrow\; v_{12} = \frac{(2.0)(15)}{6.0} = \frac{30}{6.0} = 5.0\,\text{m/s}.$$

Stage 2 (elastic — the combined 6.0-kg cart, moving at $v_{12} = 5.0$ m/s, strikes stationary $m_3 = 2.0$ kg; apply the elastic-collision formulas with "m_1" → 6.0 kg and "v_1" → 5.0 m/s):

$$v'_{12} = \frac{6.0 - 2.0}{6.0 + 2.0}(5.0) = \frac{4.0}{8.0}(5.0) = 2.5\,\text{m/s}, \qquad v'_3 = \frac{2(6.0)}{8.0}(5.0) = \frac{12.0}{8.0}(5.0) = 7.5\,\text{m/s}.$$

Check — total momentum, start to finish: $p_i = (2.0)(15) = 30$ kg · m/s; $p_f = (6.0)(2.5) + (2.0)(7.5) = 15 + 15 = 30$ kg · m/s ✓, unchanged even though kinetic energy was lost in Stage 1 (inelastic) but conserved in Stage 2 (elastic). Each stage is classified and solved entirely on its own terms.

4.8 Practice Problems

Bài tập luyện tập

Find the momentum of a 1200 kg car moving at 25 m/s.

(A) 1.2×10^4 kg · m/s
(B) 2.4×10^4 kg · m/s
(C) 3.0×10^4 kg · m/s
(D) 4.8×10^4 kg · m/s

Lời giải chi tiết

$p = mv = (1200)(25) = 3.0 \times 10^4$ kg · m/s. **(C)**.

Bài tập luyện tập

A force of 150 N acts on a 0.50 kg object, initially at rest, for 0.080 s. Find its final velocity.

Lời giải chi tiết

$J = F\Delta t = (150)(0.080) = 12$ N·s. $v = \dfrac{J}{m} = \dfrac{12}{0.50} = \boxed{24\text{ m/s}}$.

Bài tập luyện tập

A constant force of 100 N acts on a 4.0 kg object, initially at rest, for 0.20 s. Find its final velocity.

(A) 2.5 m/s
(B) 5.0 m/s
(C) 8.0 m/s
(D) 10 m/s

Lời giải chi tiết

$J = (100)(0.20) = 20$ N·s. $v = \dfrac{20}{4.0} = 5.0$ m/s. **(B)**.

Bài tập luyện tập

A 4.0 kg rifle fires a 0.010 kg bullet at 400 m/s. Find the recoil speed of the rifle.

Lời giải chi tiết

$0 = MV + mv \Rightarrow V = -\dfrac{mv}{M} = -\dfrac{(0.010)(400)}{4.0} = \boxed{-1.0\text{ m/s}}$ (magnitude 1.0 m/s, opposite the bullet).

Bài tập luyện tập

A 1.5 kg ball moving at 8.0 m/s collides and sticks with a stationary 4.5 kg ball. Find the common velocity after the collision.

(A) 1.0 m/s
(B) 2.0 m/s
(C) 3.0 m/s
(D) 4.0 m/s

Lời giải chi tiết

$p_i = (1.5)(8.0) = 12\,\text{kg}\cdot\text{m/s}$. $v_f = \dfrac{12}{1.5+4.5} = \dfrac{12}{6.0} = 2.0\,\text{m/s}$. **(B)**.

Bài tập luyện tập

$m_1 = 1.0\,\text{kg}$ moving at $8.0\,\text{m/s}$ strikes a stationary $m_2 = 3.0\,\text{kg}$ elastically. Find each object's velocity afterward, and verify both momentum and kinetic energy are conserved.

Lời giải chi tiết

$v_1' = \dfrac{1.0-3.0}{4.0}(8.0) = \boxed{-4.0\,\text{m/s}}$; $\quad v_2' = \dfrac{2(1.0)}{4.0}(8.0) = \boxed{4.0\,\text{m/s}}$.
Momentum: before $= 8.0$; after $= 1.0(-4.0) + 3.0(4.0) = -4.0 + 12 = 8.0$ ✓.
KE: before $= \frac{1}{2}(1.0)(64) = 32\,\text{J}$; after $= \frac{1}{2}(1.0)(16) + \frac{1}{2}(3.0)(16) = 8.0 + 24 = 32\,\text{J}$ ✓.

Bài tập luyện tập

Before a collision: $m_1 = 3.0\,\text{kg}$ at $4.0\,\text{m/s}$, $m_2 = 2.0\,\text{kg}$ at rest. After: m_1 at $0.80\,\text{m/s}$, m_2 at $4.8\,\text{m/s}$ (same direction). Classify this collision, showing your work.

Lời giải chi tiết

Momentum: before $= 3.0(4.0) = 12$; after $= 3.0(0.80)+2.0(4.8) = 2.4+9.6 = 12$ ✓ conserved (as always).
KE: before $= \frac{1}{2}(3.0)(16) = 24\,\text{J}$; after $= \frac{1}{2}(3.0)(0.64) + \frac{1}{2}(2.0)(23.04) = 0.96 + 23.04 = 24.0\,\text{J}$ ✓ conserved.
Both are conserved $\Rightarrow$ $\boxed{\text{elastic}}$.

Bài tập luyện tập

Before a collision: $m_1 = 4.0\,\text{kg}$ at $6.0\,\text{m/s}$, $m_2 = 2.0\,\text{kg}$ at rest. After: m_1 at $3.0\,\text{m/s}$, m_2 at $6.0\,\text{m/s}$ (same direction). Classify the collision.

Lời giải chi tiết

Momentum: before $= 4.0(6.0) = 24$; after $= 4.0(3.0) + 2.0(6.0) = 12 + 12 = 24$ ✓ conserved.
KE: before $= \frac{1}{2}(4.0)(36) = 72\,\text{J}$; after $= \frac{1}{2}(4.0)(9.0) + \frac{1}{2}(2.0)(36) = 18 + 36 = 54\,\text{J}$ — decreased, so KE is *not* conserved. The two objects also end up with *different* final velocities (not moving together), so this is $\boxed{\text{inelastic, but not perfectly inelastic}}$.

Bài tập luyện tập

Car A ($1500\,\text{kg}$) moving east at $12\,\text{m/s}$ collides and sticks with car B ($1000\,\text{kg}$) moving north at $9.0\,\text{m/s}$. Find the speed and direction of the wreckage.

Lời giải chi tiết

$p_x = 1500(12) = 18000$; $p_y = 1000(9.0) = 9000$; $M = 2500\,\text{kg}$.
$v_x = \dfrac{18000}{2500} = 7.2\,\text{m/s}$; $v_y = \dfrac{9000}{2500} = 3.6\,\text{m/s}$.

$v = \sqrt{7.2^2 + 3.6^2} = \sqrt{64.8} = \boxed{8.05\,\text{m/s}}$, at $\theta = \arctan(3.6/7.2) = \boxed{26.6^\circ \text{ north of east}}$.

Bài tập luyện tập

Point masses $m_1 = 4.0\,\text{kg}$ at $x = 0$ and $m_2 = 6.0\,\text{kg}$ at $x = 10\,\text{m}$. Find the center of mass.

(A) $4.0\,\text{m}$

(B) $5.0\,\text{m}$

(C) $6.0\,\text{m}$

(D) $8.0\,\text{m}$

Lời giải chi tiết

$x_{cm} = \dfrac{4.0(0) + 6.0(10)}{10} = \dfrac{60}{10} = 6.0\,\text{m}$. **(C)**.

Bài tập luyện tập

$m_1 = 5.0\,\text{kg}$ moves at $3.0\,\text{m/s}$ east; $m_2 = 3.0\,\text{kg}$ moves at $4.0\,\text{m/s}$ west. Find the velocity of the center of mass.

Lời giải chi tiết

Taking east positive: $v_{cm} = \dfrac{5.0(3.0) + 3.0(-4.0)}{8.0} = \dfrac{15 - 12}{8.0} = \dfrac{3.0}{8.0} = \boxed{0.375\,\text{m/s east}}$.

Bài tập luyện tập

A force on a $3.0\,\text{kg}$ object (initially at rest) rises linearly from 0 to $150\,\text{N}$ over $0.10\,\text{s}$, then falls linearly to 0 over the next $0.10\,\text{s}$. Find the object's final velocity.

Lời giải chi tiết

$J = \frac{1}{2}(0.20)(150) = 15\,\text{N·s}$. $v = \dfrac{15}{3.0} = \boxed{5.0\,\text{m/s}}$.

Bài tập luyện tập

A $0.20\,\text{kg}$ ball hits a wall at $10\,\text{m/s}$ and bounces straight back at $8.0\,\text{m/s}$; contact time is $0.010\,\text{s}$. Find the average force exerted by the wall on the ball.

(A) $36\,\text{N}$

(B) $180\,\text{N}$

(C) $200\,\text{N}$

(D) $360\,\text{N}$

Lời giải chi tiết

$\Delta p = m(v_f - v_i) = (0.20)(-8.0 - 10) = (0.20)(-18) = -3.6\,\text{N·s}$ (magnitude $3.6\,\text{N·s}$). $F_{avg} = \dfrac{3.6}{0.010} = 360\,\text{N}$. **(D)**.

Bài tập luyện tập

For any collision between two objects in an isolated system (zero net external force), which quantity is *always* conserved?

(A) Kinetic energy only
(B) Momentum only
(C) Both momentum and kinetic energy
(D) Neither

Lời giải chi tiết

Momentum conservation follows directly from Newton's third law and requires no assumption about the collision type; kinetic energy is only conserved in elastic collisions. **(B)**.

Bài tập luyện tập

During a car crash, a 70 kg passenger's velocity changes from 20 m/s to 0 in 0.15 s (airbag + seatbelt). Find the average force on the passenger, and compare it to the force if the same stop happened in 0.015 s (no airbag).

Lời giải chi tiết

$J = \Delta p = (70)(0 - 20) = -1400$ N·s (magnitude 1400 N·s).
With airbag: $F_{avg} = \frac{1400}{0.15} = \boxed{9330\text{ N}}$.
Without airbag: $F_{avg} = \frac{1400}{0.015} = \boxed{93300\text{ N}}$ — ten times larger, since the same impulse is delivered over one-tenth the time.

Bài tập luyện tập

A 2.0 kg object at rest explodes into two fragments. One fragment, mass 0.50 kg, moves east at 12 m/s. Find the velocity of the other fragment (mass 1.5 kg).

Lời giải chi tiết

$0 = (0.50)(12) + (1.5)v_2 \Rightarrow v_2 = \frac{-6.0}{1.5} = \boxed{-4.0\text{ m/s}}$ (i.e. 4.0 m/s west).

Bài tập luyện tập

Cart A (2.0 kg) moves east at 6.0 m/s; cart B (3.0 kg) moves west at 4.0 m/s. They collide and stick. Find their common velocity after the collision.

Lời giải chi tiết

Taking east as positive ($v_B = -4.0$ m/s): $p = 2.0(6.0) + 3.0(-4.0) = 12 - 12 = 0$.
$v_f = \frac{0}{5.0} = \boxed{0}$ — the combined carts end up at rest.

Bài tập luyện tập

In a 1D elastic collision between two objects of *equal* mass, where the second is initially at rest, what happens after the collision?

(A) Both move off together at half the initial speed

(B) The moving object stops, and the initially-at-rest object moves off with the first object's original velocity

(C) Both objects reverse direction

(D) Nothing changes

Lời giải chi tiết

With $m_1 = m_2$: $v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 = 0$ and $v_2' = \frac{2m_1}{m_1 + m_2} v_1 = v_1$ — a complete exchange of velocities. **(B)**.

Bài tập luyện tập

A system of two objects interacts only through an internal spring (no external force), starting from rest. What happens to the velocity of the center of mass after the spring releases them?

(A) It moves in the direction of the more massive object

(B) It remains zero

(C) It equals the average of the two final speeds

(D) It becomes equal to the faster object's velocity

Lời giải chi tiết

With no external force, total momentum — and hence v_{cm} — cannot change. Since the system started at rest, v_{cm} stays zero even after the objects fly apart. **(B)**.

Bài tập luyện tập

A 3.0 kg object moving at 8.0 m/s collides perfectly inelastically with a stationary object of unknown mass m. The combined wreckage moves at 2.0 m/s. Find m.

Lời giải chi tiết

$p_i = (3.0)(8.0) = 24\,\text{kg}\cdot\text{m/s} = p_f = (3.0 + m)(2.0)$.
$24 = 6.0 + 2.0m \Rightarrow 2.0m = 18 \Rightarrow m = \boxed{9.0\,\text{kg}}$.

Bài tập luyện tập

A force-vs-time graph shows a constant 40 N force acting for 0.50 s. What physical quantity does the area under this graph represent?

(A) Work done

(B) Kinetic energy

(C) Impulse

(D) Power

Lời giải chi tiết

Area under an F–t graph is $F\Delta t = J$, the impulse. **(C)**.

Bài tập luyện tập

$m_1 = 3.0$ kg moving at 10 m/s collides elastically with a stationary $m_2 = 1.0$ kg. Using the fact that relative speed of separation equals relative speed of approach in an elastic collision, find v_1' and v_2'.

Lời giải chi tiết

Approach speed $= 10$ m/s, so separation requires $v_2' - v_1' = 10$.
Momentum: $3.0(10) = 3.0v_1' + 1.0v_2' \Rightarrow 30 = 3.0v_1' + v_2'$. Substituting $v_2' = v_1' + 10$:

$$30 = 3.0v_1' + v_1' + 10 = 4.0v_1' + 10 \Rightarrow v_1' = \boxed{5.0\,\text{m/s}}, \quad v_2' = \boxed{15\,\text{m/s}}.$$

Check (formula): $v_1' = \dfrac{3.0 - 1.0}{4.0}(10) = 5.0$ m/s ✓; $v_2' = \dfrac{2(3.0)}{4.0}(10) = 15$ m/s ✓.

Bài tập luyện tập

A 0.020 kg bullet moving at 300 m/s embeds in a 2.98 kg wood block hanging at rest from a string (ballistic pendulum), so the combined mass is 3.00 kg. Find (a) the velocity of the block+bullet right after impact, and (b) the maximum height the pendulum swings to.

Lời giải chi tiết

(a) Momentum: $(0.020)(300) = 6.0 = (3.00)v \Rightarrow v = \boxed{2.0\,\text{m/s}}$.

(b) Energy conservation *after* the (already-completed) collision: $\frac{1}{2}v^2 = gh \Rightarrow h = \dfrac{v^2}{2g} = \dfrac{4.0}{19.6} = \boxed{0.204\,\text{m}}$.

Bài tập luyện tập

In the ballistic-pendulum setup above, why is kinetic energy conserved during the pendulum's *swing* but not during the bullet's *embedding* into the block?

(A) The bullet-block collision is elastic

(B) The bullet-block collision is perfectly inelastic (energy converts to heat/deformation); the swing afterward involves only gravity acting on the now-rigid combined body, with no such dissipative process

(C) Energy is never conserved anywhere in this problem

(D) Momentum is not actually conserved during the collision

Lời giải chi tiết

(B). Perfectly inelastic collisions always dissipate kinetic energy, but once the objects move as one rigid unit, the subsequent swing is governed purely by conservative gravity, so mechanical energy is conserved from that point onward.

Bài tập luyện tập

A force on a 5.0 kg object (initially at rest) rises linearly from 0 to 60 N over 0.20 s, stays constant at 60 N for 0.30 s, then falls linearly to 0 over the next 0.10 s. Find the object's final velocity.

Lời giải chi tiết

Area $= \frac{1}{2}(0.20)(60) + (60)(0.30) + \frac{1}{2}(0.10)(60) = 6.0 + 18 + 3.0 = 27$ N·s.
$v = \dfrac{27}{5.0} = \boxed{5.4\,\text{m/s}}$.

Bài tập luyện tập

Two ice skaters, initially moving together at $2.0\,\text{m/s}$ ($m_1 = 50\,\text{kg}$, $m_2 = 70\,\text{kg}$), push apart such that the $50\,\text{kg}$ skater ends up moving at $5.0\,\text{m/s}$ in the same original direction. Find the $70\,\text{kg}$ skater's velocity afterward.

Lời giải chi tiết

Total initial momentum: $(50 + 70)(2.0) = 240\,\text{kg}\cdot\text{m/s}$.
$240 = 50(5.0) + 70v_2 = 250 + 70v_2 \Rightarrow 70v_2 = -10 \Rightarrow v_2 = \boxed{-0.143\,\text{m/s}}$ (moving slightly backward, opposite the original direction).

Bài tập luyện tập

A device initially at rest ejects a small mass to one side using a compressed spring, causing the larger remaining mass to recoil the other way. Which quantity is conserved during this event?

(A) Kinetic energy only
(B) Momentum only
(C) Both
(D) Neither

Lời giải chi tiết

No external horizontal force acts, so momentum (starting at zero) stays zero. Kinetic energy *increases*, since stored spring energy converts into motion — it is not conserved. **(B)**.

Bài tập luyện tập

A $0.50\,\text{kg}$ cart moving at $6.0\,\text{m/s}$ collides with a stationary $1.5\,\text{kg}$ cart; they do not stick. After the collision, the $0.50\,\text{kg}$ cart rebounds at $3.0\,\text{m/s}$. Find the $1.5\,\text{kg}$ cart's velocity, and determine whether the collision is elastic.

Lời giải chi tiết

Momentum: $(0.50)(6.0) = 3.0 = (0.50)(-3.0)+(1.5)v_2 \Rightarrow 3.0 = -1.5+1.5v_2 \Rightarrow v_2 = \boxed{3.0\,\text{m/s}}$.
KE before $= \frac{1}{2}(0.50)(36) = 9.0$ J. KE after $= \frac{1}{2}(0.50)(9.0) + \frac{1}{2}(1.5)(9.0) = 2.25 + 6.75 = 9.0$ J.
KE is conserved $\Rightarrow$ $\boxed{\text{the collision is elastic}}$.

Bài tập luyện tập

Point masses $m_1 = 2.0\,\text{kg}$ at $x = 0$, $m_2 = 3.0\,\text{kg}$ at $x = 4.0\,\text{m}$, and $m_3 = 5.0\,\text{kg}$ at $x = 8.0\,\text{m}$ lie along a line. Find the center of mass.

(A) $4.0\,\text{m}$
(B) $4.8\,\text{m}$
(C) $5.2\,\text{m}$
(D) $6.0\,\text{m}$

Lời giải chi tiết

$$x_{cm} = \frac{(2.0)(0) + (3.0)(4.0) + (5.0)(8.0)}{2.0 + 3.0 + 5.0} = \frac{0 + 12 + 40}{10} = \frac{52}{10} = 5.2\,\text{m}.$$ **(C)**.

Bài tập luyện tập

A 5.0 kg object moving at 4.0 m/s east is pushed apart internally (an internal spring releases) into a 2.0 kg piece and a 3.0 kg piece, with no external force acting. The 2.0 kg piece ends up moving at 7.0 m/s east. Find the velocity of the 3.0 kg piece.

Lời giải chi tiết

Momentum is conserved even though the object was not initially at rest: $p_i = (5.0)(4.0) = 20\,\text{kg}\cdot\text{m/s}$.

$p_i = (2.0)(7.0) + (3.0)v_2 \Rightarrow 20 = 14 + 3.0v_2 \Rightarrow v_2 = \dfrac{6.0}{3.0} = \boxed{2.0\,\text{m/s east}}$.

(Check: $2.0(7.0) + 3.0(2.0) = 14 + 6.0 = 20\ \checkmark$.)

Bài tập luyện tập

A firecracker at rest explodes into several fragments flying off in all directions. Which of the following *must* be true?

(A) All fragments have the same speed

(B) All fragments have the same kinetic energy

(C) The vector sum of all the fragments' momenta is zero

(D) The vector sum of all the fragments' velocities is zero

Lời giải chi tiết

Total momentum is conserved and was zero before the explosion (system at rest), so the fragment momenta — not their velocities or speeds individually — must sum to zero as vectors. **(C)**.

Bài tập luyện tập

A 10 kg device at rest explodes into three fragments. Fragment P (2.0 kg) moves at 12 m/s due east. Fragment Q (3.0 kg) moves at 8.0 m/s due south. Fragment R has mass 5.0 kg. Find the velocity (magnitude and direction) of fragment R.

Lời giải chi tiết

Take east $= +x$, north $= +y$. $p_{Px} = (2.0)(12) = 24.0$, $p_{Py} = 0$; $p_{Qx} = 0$, $p_{Qy} = (3.0)(-8.0) = -24.0$.

Total before $= 0 \Rightarrow p_{Rx} = -24.0$, $p_{Ry} = +24.0$.

$$v_{Rx} = \frac{-24.0}{5.0} = -4.8\,\text{m/s}, \qquad v_{Ry} = \frac{24.0}{5.0} = 4.8\,\text{m/s}.$$

$$v_R = \sqrt{4.8^2 + 4.8^2} = 4.8\sqrt{2} = \boxed{6.8\,\text{m/s}}, \qquad \theta = \arctan\left(\frac{4.8}{4.8}\right) = \boxed{45^\circ\ \text{north of west}}.$$

Bài tập luyện tập

A 25 g bullet is fired at 240 m/s into a 4.975 kg wood block hanging at rest from a string (combined mass 5.000 kg). Find the maximum height the block+bullet rise to.

Lời giải chi tiết

Stage 1 (momentum, collision): $v = \dfrac{mv_0}{M+m} = \dfrac{(0.025)(240)}{5.000} = \dfrac{6.0}{5.000} = 1.20$ m/s.

Stage 2 (energy, swing): $h = \dfrac{v^2}{2g} = \dfrac{(1.20)^2}{2(9.8)} = \dfrac{1.44}{19.6} = \boxed{0.0735\text{ m}}$ (about 7.35 cm).

Bài tập luyện tập

In a ballistic pendulum experiment, the same bullet (identical mass and initial speed) is fired into two different blocks hung from identical strings: block 1 has mass M_1, block 2 has a larger mass $M_2 > M_1$. Compared to block 1, block 2 will swing up to:

(A) A greater height

(B) The same height

(C) A smaller height

(D) It depends on the string length

Lời giải chi tiết

Stage 1 gives $v = \dfrac{mv_0}{M+m}$, which *decreases* as M increases (same m, v_0); Stage 2 gives $h = \dfrac{v^2}{2g}$, which decreases as v decreases. A heavier block therefore swings up to a smaller height for the same bullet. **(C)**.

Bài tập luyện tập

$m_1 = 3.0$ kg moving at $v_1 = 15$ m/s east strikes a stationary $m_2 = 6.0$ kg. Afterward, m_1 moves off at 10.0 m/s, at $37°$ above the original line of motion ($\sin 37° = 0.60$, $\cos 37° = 0.80$). Find m_2's velocity (magnitude and direction) after the collision.

Lời giải chi tiết

Before: $p_x = (3.0)(15) = 45.0$ kg · m/s, $p_y = 0$.
m_1 after: $v'_{1x} = 10.0(0.80) = 8.0$, $v'_{1y} = 10.0(0.60) = 6.0 \Rightarrow p'_{1x} = 3.0(8.0) = 24.0$, $p'_{1y} = 3.0(6.0) = 18.0$.
m_2 after: $p'_{2x} = 45.0 - 24.0 = 21.0$, $p'_{2y} = 0 - 18.0 = -18.0$.

$$v'_{2x} = \frac{21.0}{6.0} = 3.5\text{ m/s}, \qquad v'_{2y} = \frac{-18.0}{6.0} = -3.0\text{ m/s}.$$

$$v'_2 = \sqrt{3.5^2 + 3.0^2} = \sqrt{21.25} = \boxed{4.61\text{ m/s}}, \qquad \theta_2 = \arctan\left(\frac{3.0}{3.5}\right) = \boxed{40.6° \text{ below the original line of motion}}.$$

Bài tập luyện tập

An 80 kg astronaut is at rest (relative to their spacecraft) in deep space, holding a 2.0 kg wrench, also at rest. The astronaut throws the wrench directly away from themselves. Using conservation of momentum, explain what happens to the astronaut as a result, and compare the astronaut's final kinetic energy to the wrench's final kinetic energy. (You do not need exact numbers, but explain your reasoning fully.)

Lời giải chi tiết

The astronaut+wrench system starts at rest, so its total momentum is exactly zero. No external force acts on the system in deep space, so total momentum must stay zero after the throw. Since the wrench flies off with momentum $m_w v_w$ in one direction, the astronaut must recoil in the *exact opposite* direction with momentum of equal magnitude: $m_a v_a = m_w v_w$. Because the astronaut's mass is 40 times the wrench's ($80/2.0 = 40$), the astronaut's recoil speed is $1/40$ of the wrench's speed ($v_a = (m_w/m_a)v_w$) — small, but definitely nonzero: the astronaut *does* drift away, slowly, in the direction opposite the throw.

For kinetic energy, use $KE = p^2/(2m)$. Since the astronaut and wrench end up with equal-magnitude momenta but very different masses, their kinetic energies are *not* equal — KE is inversely proportional to mass for a fixed $|p|$:

$$\frac{KE_a}{KE_w} = \frac{p^2/(2m_a)}{p^2/(2m_w)} = \frac{m_w}{m_a} = \frac{2.0}{80} = \frac{1}{40}.$$

So the astronaut ends up with only $\frac{1}{40}$ of the wrench's kinetic energy. Almost all of the kinetic energy created by the throw (which came from the astronaut's own muscular effort) ends up in the light wrench, not in the heavy astronaut — even though the push the astronaut feels (impulse/momentum) is exactly as large as the push felt by the wrench.

Bài tập luyện tập

A 0.50 kg ball moving at 3.0 m/s receives an impulse of 4.0 N · s in the direction it is already moving. Find its final speed.

(A) 5.0 m/s
(B) 7.0 m/s
(C) 8.0 m/s
(D) 11 m/s

Lời giải chi tiết

$\Delta v = \dfrac{J}{m} = \dfrac{4.0}{0.50} = 8.0$ m/s. $v_f = v_i + \Delta v = 3.0 + 8.0 = 11$ m/s. **(D).**

Bài tập luyện tập

$m_1 = 5.0$ kg moving at 6.0 m/s collides *elastically* with a stationary $m_2 = 3.0$ kg. (a) Find each object's velocity after the collision. (b) Show that the velocity of the center of mass is the same before and after the collision.

Lời giải chi tiết

(a) $v_1' = \dfrac{5.0 - 3.0}{8.0}(6.0) = \boxed{1.5\text{ m/s}}$; $v_2' = \dfrac{2(5.0)}{8.0}(6.0) = \boxed{7.5\text{ m/s}}$.
Check momentum: before $= 5.0(6.0) = 30.0$; after $= 5.0(1.5) + 3.0(7.5) = 7.5 + 22.5 = 30.0$ ✓.
(b) $v_{cm,before} = \dfrac{30.0}{8.0} = 3.75$ m/s. $v_{cm,after} = \dfrac{5.0(1.5) + 3.0(7.5)}{8.0} = \dfrac{30.0}{8.0} = \boxed{3.75\text{ m/s}}$ — identical, confirming that internal collision forces (elastic or not) never change v_{cm}.

Bài tập luyện tập

A firework shell at rest explodes into several fragments. Compared to just before the explosion, the total kinetic energy of the system afterward:

(A) Decreases

(B) Stays exactly the same

(C) Increases

(D) Could increase, decrease, or stay the same, depending on the fragment masses

Lời giải chi tiết

$KE_i = 0$ (system at rest) and the fragments move afterward, so $KE_f > 0$ always. Stored energy (chemical/elastic) converts into kinetic energy; total KE always increases in an explosion. **(C)**.

Bài tập luyện tập

A $500\,\text{kg}$ cannon mounted on a frictionless cart moves at $2.0\,\text{m/s}$ east. It fires a $5.0\,\text{kg}$ shell, which leaves at $150\,\text{m/s}$ east (measured relative to the ground). Find the cannon's velocity immediately after firing.

Lời giải chi tiết

Before firing, cannon and shell move together: $p_i = (500 + 5.0)(2.0) = 1010\,\text{kg}\cdot\text{m/s}$.
After: $p_i = Mv + mv_{shell} \Rightarrow 1010 = 500V + (5.0)(150) = 500V + 750$.

$$500V = 260 \;\Rightarrow\; V = \boxed{0.52\,\text{m/s east}}.$$

The cannon does not reverse direction — it merely slows down from $2.0\,\text{m/s}$ to $0.52\,\text{m/s}$, since it was already moving the same way the shell was fired.

Bài tập luyện tập

A $0.40\,\text{kg}$ ball strikes the floor at $5.0\,\text{m/s}$, 37° below the horizontal ($v_x = 4.0\,\text{m/s}$, $v_y = -3.0\,\text{m/s}$), and bounces off at $5.0\,\text{m/s}$, 37° above the horizontal ($v_x = 4.0\,\text{m/s}$, $v_y = +3.0\,\text{m/s}$); the horizontal component of velocity is unchanged. Contact time with the floor is $0.020\,\text{s}$. Find the impulse delivered by the floor, and the average force (magnitude and direction).

Lời giải chi tiết

$\Delta p_x = m(v_{xf} - v_{xi}) = 0.40(4.0 - 4.0) = 0.$
$\Delta p_y = m(v_{yf} - v_{yi}) = 0.40(3.0 - (-3.0)) = 0.40(6.0) = 2.4\,\text{N}\cdot\text{s}.$
Since $\Delta p_x = 0$, the impulse is purely vertical: $J = \boxed{2.4\,\text{N}\cdot\text{s},\ \text{straight up}}$.
$F_{avg} = \dfrac{J}{\Delta t} = \dfrac{2.4}{0.020} = \boxed{120\,\text{N},\ \text{straight up (perpendicular to the floor)}}$.

Bài tập luyện tập

In a three-cart chain collision, cart 1 strikes cart 2 first; afterward, cart 2 (now moving) strikes cart 3. Which approach correctly finds the final velocities of all three carts?

(A) Apply conservation of momentum once to all three carts together, using only cart 1's original velocity as the sole initial velocity

(B) Apply conservation of momentum separately to each collision, in time order, using the velocity each cart actually has at the start of that particular collision

(C) Apply conservation of kinetic energy to the whole system at once, since momentum conservation only applies to two objects at a time

(D) Add the momenta of all three carts together at the end, then divide equally among the three masses

Lời giải chi tiết

Each collision must be treated as its own separate momentum-conservation problem, solved in the order the collisions occur; the velocity a cart carries into its next collision is whatever that cart's *previous* stage left it with, not its original velocity. **(B)**.

Bài tập luyện tập

Cart 1 ($m_1 = 3.0\,\text{kg}$, moving at $v_1 = 8.0\,\text{m/s}$) collides *elastically* with stationary cart 2 ($m_2 = 1.0\,\text{kg}$) — Stage 1. Cart 2 then collides with stationary cart 3 ($m_3 = 3.0\,\text{kg}$) and *sticks* — Stage 2. Find (a) the velocity of cart 1 after Stage 1, (b) cart 2's velocity right before it hits cart 3, and (c) the final combined velocity of cart 2+3 after Stage 2.

Lời giải chi tiết

(a)–(b) Stage 1, elastic collision formulas:

$$v_1' = \frac{3.0 - 1.0}{3.0 + 1.0}(8.0) = \frac{2.0}{4.0}(8.0) = \boxed{4.0\,\text{m/s}}, \qquad v_2' = \frac{2(3.0)}{4.0}(8.0) = \frac{6.0}{4.0}(8.0) = \boxed{12\,\text{m/s}}.$$

Check: momentum before $= 3.0(8.0) = 24$; after $= 3.0(4.0) + 1.0(12) = 12 + 12 = 24$ ✓. KE before $= \frac{1}{2}(3.0)(64) = 96\,\text{J}$; after $= \frac{1}{2}(3.0)(16) + \frac{1}{2}(1.0)(144) = 24 + 72 = 96\,\text{J}$ ✓.

(c) Stage 2 uses cart 2's velocity *from Stage 1* ($12\,\text{m/s}$), not its original 0:

$$m_2 v_2' = (m_2 + m_3)v_f \;\Rightarrow\; v_f = \frac{(1.0)(12)}{1.0 + 3.0} = \frac{12}{4.0} = \boxed{3.0\,\text{m/s}}.$$

Bài tập luyện tập

Cart 1 ($m_1 = 4.0\,\text{kg}$, moving at $v_1 = 6.0\,\text{m/s}$) collides with stationary cart 2 ($m_2 = 2.0\,\text{kg}$) and sticks (Stage 1). The combined cart then collides with a stationary cart 3 of unknown mass and sticks as well (Stage 2); the final velocity of all three carts together is $2.0\,\text{m/s}$. Find the mass of cart 3.

Lời giải chi tiết

Stage 1: $v_{12} = \dfrac{(4.0)(6.0)}{4.0 + 2.0} = \dfrac{24}{6.0} = 4.0\,\text{m/s}$.

Stage 2: total momentum entering Stage 2 is $(6.0)(4.0) = 24\,\text{kg}\cdot\text{m/s}$ (equal to the original $p_i = 24$, as expected, since no external force acts). With final velocity $2.0\,\text{m/s}$, the total final mass must be $\frac{24}{2.0} = 12\,\text{kg}$, so

$$m_3 = 12 - 6.0 = \boxed{6.0\,\text{kg}}.$$

Bài tập luyện tập

Two objects of equal mass m collide and stick together. Just before the collision, object A moves east at $6.0\,\text{m/s}$ and object B moves north at $6.0\,\text{m/s}$. What is the direction of the wreckage's velocity immediately after the collision?

(A) $30°$ north of east
(B) $45°$ north of east
(C) $60°$ north of east
(D) Due north

Lời giải chi tiết

With equal masses and equal speeds, $p_x = mv = p_y$, so $v_x = v_y$ for the wreckage regardless of the actual value of m or v — the direction is always arctan$(1) = 45°$ north of east. **(B).**

Bài tập luyện tập

Three point masses lie in the xy-plane: $m_1 = 2.0\,\text{kg}$ at $(0,0)$, $m_2 = 3.0\,\text{kg}$ at $(4.0,0)$ m, and $m_3 = 5.0\,\text{kg}$ at $(0,6.0)$ m. Find the coordinates of the center of mass.

Lời giải chi tiết

$$x_{cm} = \frac{(2.0)(0)+(3.0)(4.0)+(5.0)(0)}{2.0+3.0+5.0} = \frac{12}{10} = \boxed{1.2\,\text{m}}, \qquad y_{cm} = \frac{(2.0)(0)+(3.0)(0)+(5.0)(6.0)}{10} = \frac{30}{10} = \boxed{3.0$$

Center of mass: $(1.2,\ 3.0)$ m.

Bài tập luyện tập

A $10\,\text{kg}$ object moving east at $6.0\,\text{m/s}$ suddenly breaks into three fragments (an internal charge releases; no external force acts). Fragment A ($2.0\,\text{kg}$) flies off due north at $15\,\text{m/s}$. Fragment B ($3.0\,\text{kg}$) continues due east at $10\,\text{m/s}$. Fragment C has mass $5.0\,\text{kg}$. Find fragment C's velocity (magnitude and direction) immediately after the explosion.

Lời giải chi tiết

Take east $= +x$, north $= +y$. Before: $p_x = (10)(6.0) = 60\,\text{kg}\cdot\text{m/s}$, $p_y = 0$.
After: $p_{Ax} = 0$, $p_{Ay} = (2.0)(15) = 30$; $p_{Bx} = (3.0)(10) = 30$, $p_{By} = 0$.

$$p_{Cx} = 60-(0+30) = 30 \Rightarrow v_{Cx} = \frac{30}{5.0} = 6.0\,\text{m/s}, \qquad p_{Cy} = 0-(30+0) = -30 \Rightarrow v_{Cy} = \frac{-30}{5.0} = -6.0\,\text{m/s}.$$

$$v_C = \sqrt{6.0^2+6.0^2} = 6.0\sqrt{2} = \boxed{8.5\,\text{m/s}}, \qquad \theta = \arctan\left(\frac{6.0}{6.0}\right) = \boxed{45°\text{ south of east}}.$$

Bài tập luyện tập

A constant $20\,\text{N}$ force acts on a $2.0\,\text{kg}$ object, initially at rest, for $3.0\,\text{s}$; immediately afterward, a constant $10\,\text{N}$ force in the *opposite* direction acts for the next $4.0\,\text{s}$. Find the object's velocity at the end of the full $7.0\,\text{s}$ interval.

(A) $5.0\,\text{m/s}$
(B) $10\,\text{m/s}$
(C) $30\,\text{m/s}$
(D) $50\,\text{m/s}$

Lời giải chi tiết

$J_1 = (+20)(3.0) = +60\,\text{N·s}$; $J_2 = (-10)(4.0) = -40\,\text{N·s}$. $J_{total} = 60 - 40 = 20\,\text{N·s}$.
$v_f = \dfrac{J_{total}}{m} = \dfrac{20}{2.0} = 10\,\text{m/s}$ (in the direction of the first force). **(B)**.

Bài tập luyện tập

$m_1 = 3.0\,\text{kg}$ moves east at $v_1 = 4.0\,\text{m/s}$; $m_2 = 1.0\,\text{kg}$ moves west at $2.0\,\text{m/s}$ (so $v_2 = -2.0\,\text{m/s}$ taking east as positive). They collide elastically head-on. Using the general elastic-collision equations

$$v_1' = \frac{(m_1 - m_2)v_1 + 2m_2v_2}{m_1 + m_2}, \qquad v_2' = \frac{(m_2 - m_1)v_2 + 2m_1v_1}{m_1 + m_2},$$

find each object's velocity after the collision, and verify momentum and kinetic energy are conserved.

Lời giải chi tiết

$$v_1' = \frac{(3.0-1.0)(4.0)+2(1.0)(-2.0)}{4.0} = \frac{8.0-4.0}{4.0} = \boxed{1.0\,\text{m/s}}, \qquad v_2' = \frac{(1.0-3.0)(-2.0)+2(3.0)(4.0)}{4.0} = \frac{4.0}{4}$$

Momentum: before $= 3.0(4.0) + 1.0(-2.0) = 12 - 2.0 = 10.0$; after $= 3.0(1.0) + 1.0(7.0) = 3.0 + 7.0 = 10.0$ ✓.
KE: before $= \frac{1}{2}(3.0)(16) + \frac{1}{2}(1.0)(4.0) = 24 + 2.0 = 26.0\,\text{J}$; after $= \frac{1}{2}(3.0)(1.0) + \frac{1}{2}(1.0)(49) = 1.5 + 24.5 = 26.0\,\text{J}$ ✓.

Bài tập luyện tập

A ballistic pendulum consists of a $1.96\,\text{kg}$ wood block hanging at rest from a string of length $L = 0.80\,\text{m}$. A $40\,\text{g}$ bullet embeds in the block (combined mass $2.00\,\text{kg}$), and the block+bullet swing up until the string makes a maximum angle of $60°$ with the vertical ($\cos 60° = 0.50$). Find (a) the height h the block rises, (b) the speed of the block+bullet immediately after the bullet embeds, and (c) the bullet's initial speed v_0.

Lời giải chi tiết

(a) Geometry of the swing: $h = L - L\cos 60° = L(1 - \cos 60°) = (0.80)(1 - 0.50) = (0.80)(0.50) = \boxed{0.40\,\text{m}}$.
(b) Stage 2 (energy conservation during the swing): $v = \sqrt{2gh} = \sqrt{2(9.8)(0.40)} = \sqrt{7.84} = \boxed{2.8\,\text{m/s}}$.
(c) Stage 1 (momentum conservation during the collision): $v_0 = \dfrac{(M+m)v}{m} = \dfrac{(2.00)(2.8)}{0.040} =$

$\frac{5.6}{0.040} = \boxed{140\,\mathrm{m/s}}$.

Bài tập luyện tập

Two blocks connected by a compressed spring are held at rest on frictionless ice, then released. You know only the two masses m_1 and m_2 — not the spring constant or how far it was compressed. Which of the following can you determine from the masses alone?

(A) The actual speed of each block afterward

(B) The ratio of the two blocks' speeds afterward

(C) The total kinetic energy released by the spring

(D) The direction each block moves relative to the room's north wall

Lời giải chi tiết

The system starts at rest, so total momentum stays zero: $m_1 v_1 + m_2 v_2 = 0 \Rightarrow \frac{|v_1|}{|v_2|} = \frac{m_2}{m_1}$ — the speed *ratio* follows from the masses alone. The actual speeds and the total kinetic energy released both depend on the (unknown) spring energy, and the absolute directions depend on how the blocks happen to be oriented, not on the masses. **(B)**.

Torque and Rotational Dynamics

Mục tiêu Unit: Vị trí góc, vận tốc góc, gia tốc góc và bốn phương trình động học quay; mối liên hệ giữa đại lượng dài và đại lượng góc cho một điểm trên vật rắn quay; mô-men lực (torque) và cánh tay đòn; khối tâm; điều kiện cân bằng quay (tổng lực và tổng mô-men lực bằng 0 quanh mọi trục); mô-men quán tính của các vật rắn chuẩn và định lý trục song song; định luật II Newton cho chuyển động quay $\tau_{net} = I\alpha$, kể cả bài toán ròng rọc có khối lượng.

5.1 Rotational Kinematics

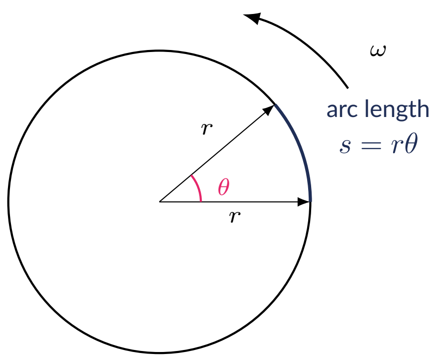
5.1.1 Angular Position, Velocity, and Acceleration

Angular position θ locates a rotating object relative to a chosen reference line, measured in **radians** (rad), where a full circle is 2π rad $= 360^\circ$. **Angular velocity** $\omega = \dfrac{\Delta\theta}{\Delta t}$ is the rate of change of angular position (rad/s); **angular acceleration** $\alpha = \dfrac{\Delta\omega}{\Delta t}$ is the rate of change of angular velocity (rad/s^2). By convention, counterclockwise rotation is taken as positive.

Every linear kinematics quantity has a rotational counterpart, and — because the defining relationships have exactly the same mathematical form — every linear kinematics equation carries over unchanged under the substitution $x \to \theta$, $v \to \omega$, $a \to \alpha$.

Linear quantity	Rotational quantity	Relationship
Position x	Angular position θ	rad
Velocity v	Angular velocity ω	rad/s
Acceleration a	Angular acceleration α	rad/s^2

Table 5.1: *
The linear–rotational analogy that underlies every equation in this chapter.



A point at radius r sweeps through angle θ, tracing arc length $s = r\theta$; angular velocity ω describes the rate and sense (here, counterclockwise) of rotation.

Khái niệm cốt lõi

Because θ, ω, α are defined by exactly the same ratios as x, v, a, *every* linear kinematics tool — the four constant-acceleration equations, graph-slope/area readings, sign conventions for speeding up/slowing down — applies to rotation with the direct substitution $x \to \theta,\ v \to \omega,\ a \to \alpha$. No new physics is required, only new symbols.

5.1.2 The Kinematics Equations for Constant Angular Acceleration

$$\omega = \omega_0 + \alpha t, \qquad \theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2, \qquad \theta = \theta_0 + \tfrac{1}{2}(\omega_0 + \omega)t, \qquad \omega^2 = \omega_0^2 + 2\alpha\,\Delta\theta.$$

Valid only while α is constant. As in linear kinematics, identify the three known quantities and the one you want among $(\Delta\theta, \omega_0, \omega, \alpha, t)$, then choose the equation that omits the unwanted fifth variable.

Ví dụ 1 · Spinning up a wheel

A wheel starts from rest and undergoes a constant angular acceleration of $\alpha = 3.0$ rad/s^2. Find (a) its angular velocity after 4.0 s, and (b) the angle it turns through in that time.

(a) $\omega = \omega_0 + \alpha t = 0 + (3.0)(4.0) = 12$ rad/s.
(b) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2}(3.0)(4.0)^2 = \frac{1}{2}(3.0)(16) = 24$ rad (about $24/(2\pi) = 3.8$ revolutions).

Ví dụ 2 · A fan blade slowing down

A fan blade rotating at $\omega_0 = 20$ rad/s is switched off and slows uniformly to rest after turning through 50 rad. Find its angular acceleration and the time it takes to stop.

$\omega^2 = \omega_0^2 + 2\alpha\,\Delta\theta \Rightarrow 0 = (20)^2 + 2\alpha(50) \Rightarrow \alpha = \dfrac{-400}{100} = -4.0$ rad/s^2.
$\omega = \omega_0 + \alpha t \Rightarrow 0 = 20 + (-4.0)t \Rightarrow t = 5.0$ s.

5.1.3 Period and Frequency

The **period** T is the time for one complete revolution; the **frequency** $f = 1/T$ is the number of revolutions per second (Hz). Since one revolution sweeps 2π rad,

$$\omega = 2\pi f = \frac{2\pi}{T}.$$

GIẢI THÍCH TIẾNG VIỆT

VN Vị trí góc θ, vận tốc góc ω, gia tốc góc α tương ứng **y hệt** với x, v, a trong chuyển động thẳng — chỉ đổi ký hiệu, công thức giữ nguyên dạng, kể cả bốn phương trình động học với gia tốc góc không đổi. Chu kỳ T là thời gian quay hết 1 vòng (2π rad); tần số $f = 1/T$ là số vòng/giây; liên hệ $\omega = 2\pi f = 2\pi/T$.

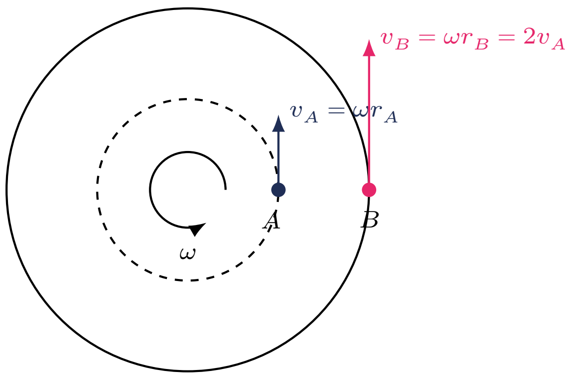
5.2 Relating Linear and Angular Quantities

Every point of a rigid rotating body shares the *same* ω and α at a given instant, but points at different distances r from the axis trace different arc lengths in the same time, so their *linear* quantities differ.

For a point at radius r on a rotating rigid body:

$$v = \omega r \text{ (tangential speed)}, \qquad a_t = \alpha r \text{ (tangential acceleration)}, \qquad a_c = \frac{v^2}{r} = \omega^2 r \text{ (centripetal/radial acceler}$$

a_t changes the *speed* of the point (exists only if $\alpha \neq 0$); a_c changes the *direction* of its velocity and exists whenever $\omega \neq 0$, even at constant angular speed.



Every point of a rigid rotating disk shares the same ω, but tangential speed $v = \omega r$ grows with distance from the axis: point B, twice as far from the center as A, moves twice as fast.

Ví dụ 3 · Tangential and centripetal acceleration on a rotating disk

At a certain instant, a disk rotates with $\omega = 4.0\,\text{rad/s}$ and $\alpha = 2.0\,\text{rad/s}^2$. Find the tangential speed, tangential acceleration, and centripetal acceleration of a point $r = 0.50\,\text{m}$ from the axis.

$v = \omega r = (4.0)(0.50) = 2.0\,\text{m/s}$.
$a_t = \alpha r = (2.0)(0.50) = 1.0\,\text{m/s}^2$.
$a_c = \omega^2 r = (4.0)^2(0.50) = (16)(0.50) = 8.0\,\text{m/s}^2$.
(The point's total acceleration magnitude is $\sqrt{a_t^2 + a_c^2} = \sqrt{1.0^2 + 8.0^2} = 8.06\,\text{m/s}^2$, dominated by the centripetal term.)

Ví dụ 4 · Comparing two points on a rotating rod

A rigid rod rotates about one end at $\omega = 6.0\,\text{rad/s}$. Compare the tangential speeds of points at $r_1 = 0.20\,\text{m}$ and $r_2 = 0.60\,\text{m}$ from the axis.

$v_1 = \omega r_1 = (6.0)(0.20) = 1.2\,\text{m/s}$.
$v_2 = \omega r_2 = (6.0)(0.60) = 3.6\,\text{m/s}$.
Both points share the same $\omega = 6.0\,\text{rad/s}$, but $r_2 = 3r_1$, so $v_2 = 3v_1$ — the outer point moves three times as fast even though it turns through the same angle in the same time.

(!) Lỗi thường gặp: A point moving in a circle at *constant* angular speed ($\alpha = 0$) still has acceleration: $a_c = \omega^2 r \neq 0$, because velocity is constantly changing *direction* even while its magnitude stays fixed. "Constant speed" in circular motion never means "zero acceleration" — it only means $a_t = 0$.

VN **GIẢI THÍCH TIẾNG VIỆT**

Mọi điểm trên một vật rắn quay có **cùng** ω và α tại một thời điểm, nhưng tốc độ dài $v = \omega r$, gia tốc tiếp tuyến $a_t = \alpha r$ và gia tốc hướng tâm $a_c = \omega^2 r$ của mỗi điểm phụ thuộc vào khoảng cách r đến trục quay — điểm càng xa trục thì v, a_t, a_c càng lớn. Lưu ý a_c tồn tại ngay cả khi vật quay đều ($\alpha = 0,\ \omega =$ hằng số), vì hướng vận tốc vẫn liên tục thay đổi.

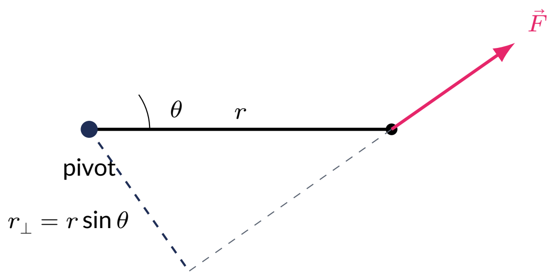
5.3 Torque

5.3.1 Defining Torque and the Lever Arm

Torque τ measures the effectiveness of a force at producing rotation about a chosen pivot. It depends not only on the size of the force but on *where* and at *what angle* it is applied.

$$\tau = rF\sin\theta = rF_\perp = r_\perp F,$$

where r is the distance from the pivot to the point of application of $\vec{F}$, θ is the angle between $\vec{r}$ and $\vec{F}$, $F_\perp = F\sin\theta$ is the component of $\vec{F}$ perpendicular to $\vec{r}$, and $r_\perp = r\sin\theta$ is the **lever arm** — the perpendicular distance from the pivot to the force's *line of action*. SI unit: N · m. By convention, torque that would rotate the object counterclockwise is positive, clockwise is negative.



Torque $\tau = rF\sin\theta = r_\perp F$: the lever arm $r_\perp$ is the perpendicular distance from the pivot to the line of action of $\vec{F}$ (dashed extension).

Ví dụ 5 · Torque from an angled force

A mechanic applies a force of 80 N to a wrench handle 0.25 m from the bolt, at an angle of 30° to the handle. Find the torque about the bolt. ($\sin 30^\circ = 0.50$.)

$$\tau = rF\sin\theta = (0.25)(80)(0.50) = 10\,\text{N}\cdot\text{m}.$$

5.3.2 Net Torque

When several forces act on a rigid body, each contributes its own torque about the chosen pivot; the **net torque** is their signed sum, $\sum\tau = \tau_1 + \tau_2 + \cdots$, respecting the counterclockwise-positive convention.

Ví dụ 6 · Net torque from two forces on a door

A door of width 0.90 m is pushed closed by a force $F_1 = 40$ N applied perpendicular to the door at its outer edge (0.90 m from the hinge), while a second force $F_2 = 25$ N pushes it open, applied perpendicular at 0.60 m from the hinge. Find the net torque about the hinge and state which way the door rotates.

Take opening (counterclockwise, as seen from above) as positive. Closing torque: $\tau_1 = -r_1F_1 = -(0.90)(40) = -36$ N · m. Opening torque: $\tau_2 = +r_2F_2 = +(0.60)(25) = +15$ N · m.

$$\sum\tau = -36 + 15 = -21\,\text{N}\cdot\text{m}.$$

The net torque is negative (clockwise): the door closes, with 21 N · m of net closing torque.

(!) Lỗi thường gặp: r in $\tau = rF\sin\theta$ is the distance from the pivot to the point where the force is *applied* — not the perpendicular lever arm itself. Always identify θ as the angle between $\vec{F}$ and the position vector $\vec{r}$ (not some other reference line), or equivalently locate $r_\perp$ directly by dropping a perpendicular from the pivot onto the force's line of action. Also, a force applied *at* the pivot, or one whose line of action passes *through* the pivot, produces **zero** torque about that pivot, no matter how large.

VN GIẢI THÍCH TIẾNG VIỆT

Mô-men lực $\tau = rF\sin\theta = r_\perp F$ đo mức độ hiệu quả của một lực trong việc làm vật quay quanh một trục (điểm tựa) đã chọn. $r_\perp$ (cánh tay đòn) là khoảng cách vuông góc từ trục quay đến **đường tác dụng** của lực — không phải khoảng cách đến điểm đặt lực. Quy ước: mô-men làm vật quay **ngược chiều kim đồng hồ** là dương, **cùng chiều kim đồng hồ** là âm. Khi có nhiều lực, mô-men lực tổng hợp $\sum\tau$ là tổng đại số (có dấu) của từng mô-men.

5.4 Center of Mass

The **center of mass** of a system is its mass-weighted average position.

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}, \qquad y_{cm} = \frac{\sum m_i y_i}{\sum m_i}.$$

For a uniform, symmetric object, the center of mass coincides with the geometric center. The center of mass of a system moves exactly as if the entire mass $M = \sum m_i$ were concentrated there and acted on by the net *external* force alone: $\sum \vec{F}_{ext} = M\vec{a}_{cm}$ — internal forces between parts of the system never affect this motion.

Ví dụ 7 · Center of mass of three point masses

Point masses $m_1 = 1.0$ kg at $x = 0$, $m_2 = 2.0$ kg at $x = 3.0$ m, and $m_3 = 3.0$ kg at $x = 6.0$ m lie along a line. Find the center of mass.

$$x_{cm} = \frac{(1.0)(0) + (2.0)(3.0) + (3.0)(6.0)}{1.0 + 2.0 + 3.0} = \frac{0 + 6.0 + 18}{6.0} = \frac{24}{6.0} = 4.0\,\text{m}.$$

VN GIẢI THÍCH TIẾNG VIỆT

Khối tâm là vị trí trung bình có trọng số khối lượng của một hệ. Với vật đồng chất, đối xứng, khối tâm trùng **tâm hình học**. Điều quan trọng: khối tâm của cả hệ chuyển động **y như một chất điểm** có khối lượng $M = \sum m_i$ chịu tác dụng của tổng ngoại lực ($\sum F_{ext} = Ma_{cm}$) — các nội lực (giữa các phần của hệ) hoàn toàn không ảnh hưởng đến chuyển động của khối tâm.

5.5 Rotational (Static) Equilibrium

5.5.1 Conditions for Equilibrium

A rigid object is in **static equilibrium** when both:

$$\sum \vec{F} = 0 \qquad \text{and} \qquad \sum \vec{\tau} = 0 \text{ (about } \textit{any} \text{ chosen pivot).}$$

If both conditions hold, the net torque is zero about *every* possible pivot, not just one — so the solver is free to choose whichever pivot makes the algebra simplest.

Chiến lược chọn trục quay

Choosing the pivot at the point of application of an *unknown* force makes that force's lever arm zero, so it vanishes entirely from the torque equation — even though it still appears in the force equations. This is the single most useful trick in equilibrium problems: pick the pivot to eliminate the unknown you care least about.

Ví dụ 8 · Seesaw balance

A massless seesaw is free to pivot at its center. A child weighing 300 N sits 1.5 m from the pivot. How far from the pivot must a 250 N child sit on the other side to balance the seesaw?

Balancing torques about the pivot: $(300)(1.5) = (250)d$.

$$d = \frac{450}{250} = 1.8\,\text{m}.$$

Ví dụ 9 · Hinged beam held by a cable

A uniform horizontal beam of length $L = 4.0$ m and weight $W = 200$ N is attached to a wall by a hinge at one end. A cable from the other end runs to the wall, making an angle of 37° with the beam and holding it horizontal. Find the cable tension and the force exerted by the hinge. ($\sin 37^\circ = 0.60$, $\cos 37^\circ = 0.80$.)

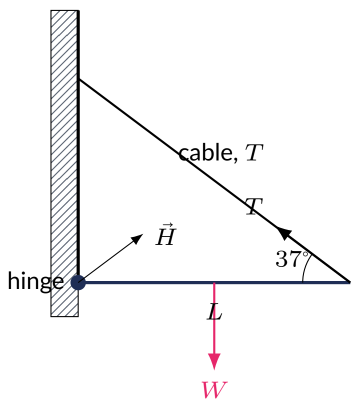
Torque about the hinge (eliminates the unknown hinge force entirely): the weight acts at the center, $L/2$ from the hinge; the cable's perpendicular component $T \sin 37^\circ$ acts at the far end, L from the hinge.

$$T \sin 37^\circ \cdot L = W \cdot \tfrac{L}{2} \;\Rightarrow\; T = \frac{W}{2 \sin 37^\circ} = \frac{200}{2(0.60)} = \frac{200}{1.2} = 166.7\,\text{N}.$$

Force balance on the beam: horizontally, the hinge must push outward to balance the cable's inward pull; vertically, it supports the remaining weight.

$$H_x = T \cos 37^\circ = (166.7)(0.80) = 133.3\,\text{N}, \qquad H_y = W - T \sin 37^\circ = 200 - (166.7)(0.60) = 200 - 100.0 = 100.0$$

$$H = \sqrt{H_x^2 + H_y^2} = \sqrt{133.3^2 + 100.0^2} = \sqrt{17769 + 10000} = \sqrt{27769} = 166.6\,\text{N}, \qquad \theta = \arctan\left(\frac{100.0}{133.3}\right) = 37^\circ$$



Uniform beam hinged to a wall, held horizontal by a cable at 37°; weight W acts at the beam's center, hinge force $\vec{H}$ acts at the pivot.

Ví dụ 10 · Horizontal beam on two supports with an off-center load

A uniform beam of length 6.0 m and weight 400 N rests horizontally on two supports, one at each end (A at $x = 0$, B at $x = 6.0$ m). A 600 N load sits 2.0 m from support A. Find the force exerted by each support.

Torque about A (eliminates the unknown F_A): the beam's weight acts at its center ($x = 3.0$ m), the load at $x = 2.0$ m, and F_B acts at $x = 6.0$ m.

$$F_B(6.0) = (400)(3.0) + (600)(2.0) = 1200 + 1200 = 2400 \;\Rightarrow\; F_B = \frac{2400}{6.0} = 400\,\text{N}.$$

Vertical force balance: $F_A + F_B = 400 + 600 = 1000$ N (total weight supported), so

$$F_A = 1000 - 400 = 600\,\text{N}.$$

Ví dụ 11 · Ladder against a frictionless wall

A uniform ladder of length $L = 5.0$ m and weight $W = 250$ N leans against a *frictionless* vertical wall, with its base on rough ground, making an angle $\theta = 53^\circ$ with the floor. ($\sin 53^\circ = 0.80$, $\cos 53^\circ = 0.60$.) Find the normal force from the wall, the normal force from the floor, the minimum friction force at the base, and the minimum coefficient of static friction needed to prevent slipping.

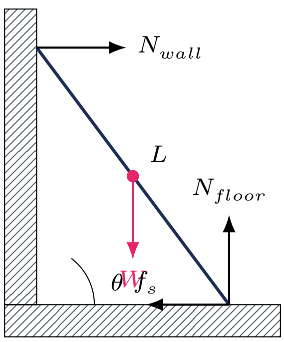
Since the wall is frictionless, it exerts only a horizontal normal force N_w at the top. The floor exerts a vertical normal force N_f and a horizontal friction force f_s at the base. The ladder's weight acts at its center.

Torque about the base (eliminates both N_f and f_s, since both act at the base): the weight's lever arm is the horizontal distance from the base to the center, $\frac{L}{2}\cos\theta$; the wall force's lever arm is the full height of the top, $L\sin\theta$.

$$N_w\,(L\sin\theta) = W\left(\tfrac{L}{2}\cos\theta\right) \;\Rightarrow\; N_w = \frac{W\cos\theta}{2\sin\theta} = \frac{(250)(0.60)}{2(0.80)} = \frac{150}{1.6} = 93.75\,\text{N}.$$

Force balance: vertically, $N_f = W = 250$ N (the wall contributes no vertical force). Horizontally, $f_s = N_w = 93.75$ N (friction balances the wall's outward push).

Minimum coefficient of static friction: $\mu_{min} = \dfrac{f_s}{N_f} = \dfrac{93.75}{250} = \boxed{0.375}$.



Ladder equilibrium: the frictionless wall supplies only a horizontal normal force N_{wall}; the rough floor supplies a vertical normal force N_{floor} and a horizontal friction force f_s that keeps the base from sliding outward.

(!) Lỗi thường gặp: Equilibrium requires $\sum \vec{\tau} = 0$ about *every* point, but you only ever need to *write* the torque equation about **one** strategically chosen pivot — provided $\sum \vec{F} = 0$ is also enforced separately. Placing the pivot at an unknown force's point of application removes that force from the torque equation (lever arm $= 0$); this is why the ladder problem is solved by taking torques about the *base*, and the hinged-beam problem about the *hinge*.

VN GIẢI THÍCH TIẾNG VIỆT

Vật ở trạng thái **cân bằng tĩnh** khi đồng thời $\sum F = 0$ **và** $\sum \tau = 0$ quanh **bất kỳ** trục nào ta chọn. Mẹo quan trọng: chọn trục quay đi qua điểm đặt của lực **chưa biết** mà ta không muốn tính — lực đó có cánh tay đòn bằng 0 nên biến mất khỏi phương trình mô-men (dù vẫn xuất hiện trong phương trình lực). Bài toán thang dựa tường (Ví dụ 11) là ví dụ kinh điển: tường trơn chỉ tạo phản lực ngang N_{wall}, còn sàn nhám tạo cả phản lực đứng N_{floor} lẫn lực ma sát f_s giữ chân thang không trượt.

5.6 Moment of Inertia

5.6.1 Point Masses and Extended Objects

Moment of inertia I measures a rigid body's resistance to *changes* in rotational motion about a specific axis — the rotational analog of mass. For a system of point masses,

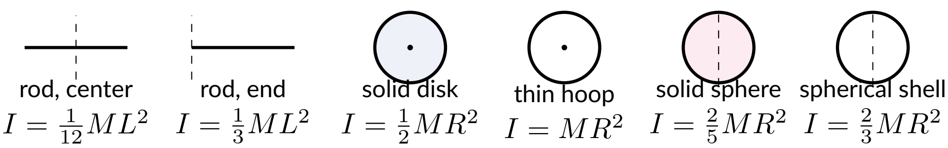
$$I = \sum m_i r_i^2,$$

where r_i is each mass's perpendicular distance from the axis. I depends on both *how much* mass there is and *how far* it sits from the axis — mass farther from the axis contributes far more to I (the distance is squared), even if the total mass is unchanged.

Object	Axis	Moment of inertia
Point mass m	distance r from axis	$I = mr^2$
Thin rod, length L	through center, $\perp$ to rod	$I = \frac{1}{12}ML^2$
Thin rod, length L	through one end, $\perp$ to rod	$I = \frac{1}{3}ML^2$
Solid disk/cylinder, radius R	through central axis	$I = \frac{1}{2}MR^2$
Thin hoop/ring, radius R	through central axis	$I = MR^2$
Solid sphere, radius R	through diameter	$I = \frac{2}{5}MR^2$
Thin spherical shell, radius R	through diameter	$I = \frac{2}{3}MR^2$

Table 5.2: *

Standard moments of inertia for common rigid bodies of uniform density and total mass M.



Moments of inertia for common shapes about the indicated axis (dashed line = rotation axis).

5.6.2 The Parallel-Axis Theorem

If I_{cm} is the moment of inertia about an axis through the center of mass, the moment of inertia about any *parallel* axis a distance d away is

$$I = I_{cm} + Md^2.$$

Ví dụ 12 · Verifying the parallel-axis theorem for a rod

A uniform rod has mass $M = 4.0\,\text{kg}$ and length $L = 2.0\,\text{m}$. Using the parallel-axis theorem and $I_{cm} = \frac{1}{12}ML^2$, find the moment of inertia about an axis through one end, and check it against $I_{end} = \frac{1}{3}ML^2$.

$$I_{cm} = \tfrac{1}{12}ML^2 = \tfrac{1}{12}(4.0)(2.0)^2 = \tfrac{1}{12}(4.0)(4.0) = 1.333\,\text{kg}\cdot\text{m}^2.$$

The end axis is $d = L/2 = 1.0\,\text{m}$ from the center of mass:

$$I_{end} = I_{cm} + Md^2 = 1.333 + (4.0)(1.0)^2 = 1.333 + 4.0 = 5.333\,\text{kg}\cdot\text{m}^2.$$

Direct formula check: $\frac{1}{3}ML^2 = \frac{1}{3}(4.0)(2.0)^2 = \frac{1}{3}(4.0)(4.0) = 5.333\,\text{kg}\cdot\text{m}^2$ ✓ — the two methods agree exactly.

(!) Lỗi thường gặp: Moment of inertia is *not* a fixed property of an object — it depends on the **axis of rotation**. The very same rod has $I = \frac{1}{12}ML^2$ about its center but $I = \frac{1}{3}ML^2$ about one end — four times larger — because, relative to the end axis, the mass is on average farther away. Always ask "about which axis?" before quoting a moment-of-inertia formula.

GIẢI THÍCH TIẾNG VIỆT

VN

Mô-men quán tính $I = \sum m_i r_i^2$ đo mức độ **"ì" trong chuyển động quay** — tương tự khối lượng trong chuyển động thẳng, nhưng phụ thuộc vào cách khối lượng phân bố quanh trục: khối lượng càng xa trục thì đóng góp vào I càng lớn (vì có r^2). I **không phải** đại lượng cố định của vật — nó thay đổi tuỳ trục quay, thể hiện qua bảng công thức chuẩn và **định lý trục song song** $I = I_{cm} + Md^2$: trục càng xa khối tâm, I càng lớn.

5.7 Newton's Second Law for Rotation

The rotational analog of $\sum F = ma$:

$$\sum \tau_{net} = I\alpha,$$

where I is the moment of inertia about the rotation axis. Just as net force divided by mass gives linear acceleration, net torque divided by moment of inertia gives angular acceleration.

Ví dụ 13 · Torque and angular acceleration of a flywheel

A solid disk-shaped flywheel ($I = \frac{1}{2}MR^2$) has mass $M = 8.0\,\text{kg}$ and radius $R = 0.25\,\text{m}$. Starting from rest, a constant net torque of $2.0\,\text{N}\cdot\text{m}$ is applied. Find the angular acceleration and the angular velocity after $4.0\,\text{s}$.

$$I = \tfrac{1}{2}MR^2 = \tfrac{1}{2}(8.0)(0.25)^2 = \tfrac{1}{2}(8.0)(0.0625) = 0.25\,\text{kg}\cdot\text{m}^2.$$

$$\alpha = \frac{\tau_{net}}{I} = \frac{2.0}{0.25} = 8.0\,\text{rad/s}^2.$$
$$\omega = \omega_0 + \alpha t = 0 + (8.0)(4.0) = 32\,\text{rad/s}.$$

5.7.1 Combining Linear and Rotational Dynamics: The Hanging-Mass Pulley

When a string wrapped around a pulley connects to a hanging mass, the string's tension exerts a torque on the pulley while the same tension appears in the hanging mass's linear $F = ma$ equation. The two motions are linked through $a = \alpha R$ (no slipping), which lets the tension be eliminated between the two equations.

Ví dụ 14 · Hanging mass on a string wrapped around a pulley

A solid disk-shaped pulley ($I = \frac{1}{2}M_pR^2$) has mass $M_p = 2.0$ kg and radius $R = 0.10$ m, mounted on a frictionless axle. A string wrapped around it holds a hanging mass $m = 1.0$ kg. Find the acceleration of the hanging mass and the tension in the string.

Hanging mass (Newton's second law, taking down as positive): $mg - T = ma$.
Pulley (Newton's second law for rotation, with $\alpha = a/R$):

$$TR = I\alpha = \left(\tfrac{1}{2}M_pR^2\right)\frac{a}{R} \;\Rightarrow\; T = \tfrac{1}{2}M_pa.$$

Substituting into the mass equation:

$$mg - \tfrac{1}{2}M_pa = ma \;\Rightarrow\; mg = a\big(m + \tfrac{1}{2}M_p\big) \;\Rightarrow\; a = \frac{mg}{m + \frac{1}{2}M_p} = \frac{(1.0)(9.8)}{1.0 + 1.0} = \frac{9.8}{2.0} = 4.9\,\text{m/s}^2.$$

$$T = \tfrac{1}{2}M_pa = \tfrac{1}{2}(2.0)(4.9) = 4.9\,\text{N}.$$

Check: $T = m(g - a) = (1.0)(9.8 - 4.9) = 4.9$ N ✓.

GIẢI THÍCH TIẾNG VIỆT

VN

Định luật II Newton cho chuyển động quay $\tau_{net} = I\alpha$ đóng vai trò y hệt $F = ma$ trong chuyển động thẳng. Với bài toán ròng rọc có khối lượng (dây vắt qua ròng rọc, nối vật treo), ta viết **hai** phương trình: $F = ma$ cho vật treo (chứa lực căng T) và $\tau = I\alpha$ cho ròng rọc (cũng chứa T qua $\tau = TR$), rồi nối hai phương trình bằng ràng buộc **không trượt** $a = \alpha R$ để khử T và giải ra gia tốc a.

5.7.2 Composite Objects and the Parallel-Axis Theorem in Depth

Many real rotating objects are not a single simple shape but an assembly of several simple shapes rigidly fastened together — a wrench (rod plus head), a physical pendulum (rod plus bob), a propeller (blades plus hub). Provided every piece is rigidly attached and the whole assembly rotates together about one common axis, the moment of inertia of the assembly is simply the **sum** of the moments of inertia of its pieces, each one computed about that *same* axis.

For a composite (compound) rigid object built from parts $1, 2, 3, \dots$ that all rotate rigidly about a shared axis,

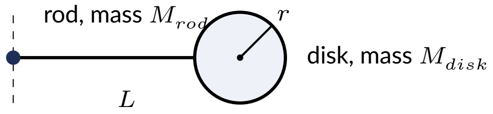
$$I_{total} = I_1 + I_2 + I_3 + \cdots,$$

where each I_i is the moment of inertia of piece i about the *common* rotation axis — not necessarily the piece's own natural axis. If a piece's natural axis (from the standard shape table) does not coincide with the shared rotation axis, first shift that piece's moment of inertia onto the shared axis using the parallel-axis theorem,

$$I_i = I_{i,cm} + m_i d_i^2,$$

where d_i is the distance from piece i's own center of mass to the shared axis. Each piece has its own d_i, but all pieces share the same final axis.

(!) Lỗi thường gặp: When summing moments of inertia for a composite object, every piece's contribution must be computed about the *same* axis. A common error is to look up each shape's moment of inertia about its own center from the standard table and add these "bare" values directly, forgetting to apply the parallel-axis theorem to any piece whose own center does not sit on the rotation axis.



A composite object: a uniform rod of length L and mass M_{rod} with a solid disk of radius r and mass M_{disk} rigidly attached at its far end. The assembly rotates as one piece about an axis through the rod's *near* end (dashed line), perpendicular to the rod — the disk's center therefore sits a distance L from that axis.

Ví dụ 15 · A dumbbell: moment of inertia about the center versus about one end

Two identical point masses, $m = 4.0$ kg each, sit at opposite ends of a very light (massless) rigid rod of length $L = 0.50$ m. Find the moment of inertia of this "dumbbell" about an axis through the midpoint of the rod (perpendicular to it), and about an axis through one end, and verify the second result using the parallel-axis theorem.

About the midpoint: by symmetry the midpoint is also the center of mass; each mass sits a distance $L/2 = 0.25$ m away.

$$I_{center} = m\left(\tfrac{L}{2}\right)^2 + m\left(\tfrac{L}{2}\right)^2 = 2m\left(\tfrac{L}{2}\right)^2 = 2(4.0)(0.25)^2 = 2(4.0)(0.0625) = 0.50\,\text{kg}\cdot\text{m}^2.$$

About one end: the mass at that end sits at $r = 0$ (contributing nothing) while the far mass sits at $r = L = 0.50$ m.

$$I_{end} = m(0)^2 + m(L)^2 = (4.0)(0.50)^2 = (4.0)(0.25) = 1.0\,\text{kg}\cdot\text{m}^2.$$

Parallel-axis check: the total mass is $M = 2m = 8.0$ kg, and the end axis is a distance $d = L/2 = 0.25$ m from the center of mass, so

$$I_{end} = I_{center} + Md^2 = 0.50 + (8.0)(0.25)^2 = 0.50 + (8.0)(0.0625) = 0.50 + 0.50 = 1.0\,\text{kg}\cdot\text{m}^2\ \checkmark.$$

The end-axis value is exactly double the center-axis value — moving the axis from the center of mass to either end always increases I, since $Md^2 \geq 0$.

Ví dụ 16 · Composite rod-and-disk: total moment of inertia about a specified axis

A uniform rod of mass $M_{rod} = 2.0$ kg and length $L = 0.80$ m has a solid disk of mass $M_{disk} = 1.5$ kg and radius $r = 0.20$ m rigidly attached at its far end, with the disk's center coinciding with the tip of the rod (as in the figure above). Find the total moment of inertia of the rod-plus-disk assembly about an axis through the rod's near end, perpendicular to the rod.

Rod, about its own end (its natural axis already coincides with the rotation axis, so no shift is needed):

$$I_{rod} = \tfrac{1}{3}M_{rod}L^2 = \tfrac{1}{3}(2.0)(0.80)^2 = \tfrac{1}{3}(2.0)(0.64) = 0.4267\,\text{kg}\cdot\text{m}^2.$$

Disk, about its own center (its natural axis, from the shape table):

$$I_{disk,cm} = \tfrac{1}{2}M_{disk}r^2 = \tfrac{1}{2}(1.5)(0.20)^2 = \tfrac{1}{2}(1.5)(0.04) = 0.030\,\text{kg}\cdot\text{m}^2.$$

The disk's center is *not* on the rotation axis — it sits a distance $d = L = 0.80$ m away (the full rod length), so the parallel-axis theorem shifts it onto the shared axis:

$$I_{disk} = I_{disk,cm} + M_{disk}d^2 = 0.030 + (1.5)(0.80)^2 = 0.030 + (1.5)(0.64) = 0.030 + 0.96 = 0.990\,\text{kg}\cdot\text{m}^2.$$

Total:

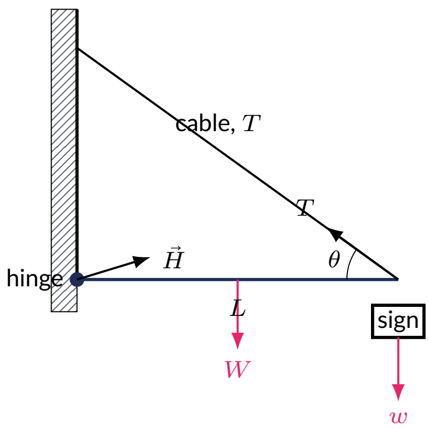
$$I_{total} = I_{rod} + I_{disk} = 0.4267 + 0.990 = 1.42\,\text{kg}\cdot\text{m}^2.$$

Note how the disk — despite being lighter than the rod — contributes more than twice the rod's share of I_{total}, simply because its mass is concentrated far from the axis.

5.7.3 Equilibrium with Multiple Cables at Angles

Real hinged structures — a shelf bracket, a traffic-light arm, a hanging sign — often carry more than one load: the beam's own weight *and* an additional object hanging from it, all supported by a single cable running to the wall at some angle θ. The hinge itself typically pushes or pulls in a direction that is not obvious in advance, so its horizontal and vertical components must be found separately from force equilibrium *after* the cable tension is known from torque equilibrium.

(!) Lỗi thường gặp: Never assume the hinge force points along the beam, along the cable, or in any other "obvious" direction. The hinge simply supplies whatever force is needed to satisfy $\sum F_x = 0$ and $\sum F_y = 0$ once every other force (weight, load, cable tension) is known — solve for H_x and H_y separately, then combine them with the Pythagorean theorem only at the very end if the magnitude and direction of $\vec{H}$ are requested.



A horizontal beam hinged to a wall and held up by a cable at angle θ; a sign of weight w hangs from the far end while the beam's own weight W acts at its center. The hinge force $\vec{H}$ has unknown direction — its components are found only *after* T is known.

Ví dụ 17 · Beam with a hanging sign, supported by an angled cable

A uniform horizontal beam of length $L = 4.0\,\text{m}$ and weight $W = 100\,\text{N}$ is attached to a wall by a hinge at one end. A cable runs from the far end of the beam to the wall, making an angle $\theta = 37^\circ$ with the beam ($\sin 37^\circ = 0.60$, $\cos 37^\circ = 0.80$). A sign of weight $w = 100\,\text{N}$ hangs from the far end of the beam, at the same point where the cable attaches. Find (a) the tension in the cable, and (b) the horizontal and vertical components of the force the hinge exerts on the beam.

(a) Torque about the hinge (this eliminates the unknown hinge force entirely, since its lever arm about the hinge is zero). The beam's weight acts at its center, a lever arm of $L/2$; the sign's weight and the cable's vertical component $T\sin\theta$ both act at the far end, a lever arm of L:

$$T\sin\theta \cdot L = W \cdot \tfrac{L}{2} + w \cdot L.$$

Dividing through by L:

$$T\sin\theta = \tfrac{W}{2} + w = \tfrac{100}{2} + 100 = 50 + 100 = 150 \;\Rightarrow\; T = \frac{150}{\sin 37^\circ} = \frac{150}{0.60} = 250\,\text{N}.$$

(b) Force balance on the beam. Horizontally, the cable's tension pulls the beam's end toward the wall, so the hinge must push outward (away from the wall) to balance it:

$$H_x = T\cos\theta = (250)(0.80) = 200\,\text{N}.$$

Vertically, the hinge and the cable's vertical component together support the beam's weight and the sign's weight:

$$H_y + T\sin\theta = W + w \;\Rightarrow\; H_y = W + w - T\sin\theta = 100 + 100 - (250)(0.60) = 200 - 150 = 50\,\text{N}.$$

The magnitude and direction of the hinge force follow if needed:

$$H = \sqrt{H_x^2 + H_y^2} = \sqrt{200^2 + 50^2} = \sqrt{40000 + 2500} = \sqrt{42500} = 206\,\text{N}, \qquad \arctan\left(\frac{50}{200}\right) = 14.0^\circ \text{ above horiz}$$

VN GIẢI THÍCH TIẾNG VIỆT

Với vật thể ghép từ nhiều hình khối đơn giản (thanh, đĩa, cầu, ...) cùng quay quanh **một trục chung**, mô-men quán tính tổng $I_{total} = I_1 + I_2 + \cdots$ là tổng các mô-men quán tính từng phần **tính quanh cùng trục đó** — nếu trục tự nhiên của một phần (tra theo bảng công thức chuẩn) không trùng trục quay chung, phải dùng định lý trục song song $I_i = I_{i,cm} + m_i d_i^2$ để dời trục trước khi cộng. Với bài toán cân bằng có dây cáp nghiêng góc θ và có thêm vật nặng treo ở đầu dầm: vẫn lấy mô-men quanh bản lề để tìm lực căng dây T trước (khử được lực bản lề chưa biết phương), sau đó mới dùng phương trình lực theo hai phương x, y để tìm từng thành phần của lực bản lề — **không được** giả định trước phương của lực bản lề.

5.8 Practice Problems

Bài tập luyện tập

A wheel accelerates from rest at $4.0\,\text{rad/s}^2$ for $3.0\,\text{s}$. Find its final angular velocity.

(A) $8.0\,\text{rad/s}$
(B) $12\,\text{rad/s}$
(C) $16\,\text{rad/s}$
(D) $24\,\text{rad/s}$

Lời giải chi tiết

$\omega = \omega_0 + \alpha t = 0 + (4.0)(3.0) = 12\,\text{rad/s}$. **(B).**

Bài tập luyện tập

For the wheel in the previous problem, find the angle it turns through during those $3.0\,\text{s}$, in radians and in revolutions.

Lời giải chi tiết

$\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}(4.0)(3.0)^2 = \frac{1}{2}(4.0)(9.0) = \boxed{18\,\text{rad}}$.

Revolutions $= \dfrac{18}{2\pi} = \boxed{2.9\,\text{rev}}$.

Bài tập luyện tập

A rotor slows from $30\,\text{rad/s}$ to $10\,\text{rad/s}$ over $4.0\,\text{s}$. Find its angular acceleration.

(A) $-2.5\,\text{rad/s}^2$
(B) $-5.0\,\text{rad/s}^2$
(C) $-7.5\,\text{rad/s}^2$
(D) $-10\,\text{rad/s}^2$

Lời giải chi tiết

$\alpha = \dfrac{10-30}{4.0} = \dfrac{-20}{4.0} = -5.0\,\text{rad/s}^2$. **(B).**

Bài tập luyện tập

A disk starts at $\omega_0 = 5.0\,\text{rad/s}$ with constant $\alpha = 1.5\,\text{rad/s}^2$. Find the angle it turns through after $6.0\,\text{s}$.

Lời giải chi tiết

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = (5.0)(6.0) + \frac{1}{2}(1.5)(6.0)^2 = 30 + \frac{1}{2}(1.5)(36) = 30 + 27 = \boxed{57\,\text{rad}}$.

Bài tập luyện tập

A turntable completes 45 revolutions per minute. Find its period T and its angular velocity ω in rad/s.

Lời giải chi tiết

$f = \frac{45}{60} = 0.75$ Hz, so $T = \frac{1}{f} = \boxed{1.33\,\text{s}}$.
$\omega = 2\pi f = 2\pi(0.75) = \boxed{4.71\,\text{rad/s}}$.

Bài tập luyện tập

A point at $r = 0.40$ m on a wheel spinning at $\omega = 10$ rad/s has what tangential speed?

(A) 2.5 m/s
(B) 4.0 m/s
(C) 10 m/s
(D) 25 m/s

Lời giải chi tiết

$v = \omega r = (10)(0.40) = 4.0$ m/s. **(B)**.

Bài tập luyện tập

For the point in the previous problem, if at that instant $\alpha = 2.0\,\text{rad/s}^2$, find its tangential acceleration and centripetal acceleration.

Lời giải chi tiết

$a_t = \alpha r = (2.0)(0.40) = \boxed{0.80\,\text{m/s}^2}$.
$a_c = \omega^2 r = (10)^2(0.40) = (100)(0.40) = \boxed{40\,\text{m/s}^2}$.

Bài tập luyện tập

Two points, A and B, lie on the same rotating rigid disk, with $r_B = 2r_A$. How do their angular velocities and tangential speeds compare?

(A) Both ω and v are equal
(B) ω equal; $v_B = 2v_A$
(C) $\omega_B = 2\omega_A$; v equal
(D) Both ω and v double

Lời giải chi tiết

A rigid body forces every point to share the same ω; tangential speed $v = \omega r$ scales with r, so $v_B = 2v_A$. **(B)**.

Bài tập luyện tập

Explain, in words, why every point of a spinning rigid body shares the same angular velocity but not necessarily the same linear (tangential) speed.

Lời giải chi tiết

Rigidity forces every point of the body to sweep through the *same angle* in the same time interval — that shared rate of angle-sweeping is precisely what $\omega = \Delta\theta/\Delta t$ measures, so ω is identical everywhere on the body. However, points at different radii trace circles of different circumference in that same time: a point at radius r covers arc length $s = r\theta$, so its *linear* speed $v = \omega r$ is proportional to r. Two points can therefore share ω while a point farther from the axis physically travels faster to keep pace with the same angular sweep.

Bài tập luyện tập

A force of 50 N is applied perpendicular to a wrench 0.20 m long. Find the torque produced about the bolt.

(A) 5 N · m
(B) 10 N · m
(C) 15 N · m
(D) 20 N · m

Lời giải chi tiết

$\tau = rF = (0.20)(50) = 10$ N · m (force already perpendicular, so $\sin\theta = 1$). **(B)**.

Bài tập luyện tập

A force of 60 N is applied at 30° to a rod, at a distance 0.30 m from the pivot. Find the torque about the pivot. ($\sin 30^\circ = 0.50$.)

Lời giải chi tiết

$\tau = rF\sin\theta = (0.30)(60)(0.50) = \boxed{9.0\text{ N}\cdot\text{m}}$.

Bài tập luyện tập

For a fixed force magnitude F and fixed distance r from the pivot, at what angle between $\vec{F}$ and $\vec{r}$ is the resulting torque maximized?

(A) 0°
(B) 45°
(C) 90°
(D) 180°

Lời giải chi tiết

$\tau = rF\sin\theta$ is maximized when $\sin\theta = 1$, i.e. $\theta = 90^\circ$ (force perpendicular to $\vec{r}$). **(C)**.

Bài tập luyện tập

Two forces act on a rod pivoted at one end: $F_1 = 20$ N perpendicular to the rod at $r_1 = 0.50$ m (tending to rotate it counterclockwise), and $F_2 = 15$ N perpendicular at $r_2 = 0.80$ m (tending to rotate it clockwise). Find the net torque and its direction.

Lời giải chi tiết

$\tau_1 = +r_1F_1 = +(0.50)(20) = +10$ N·m. $\tau_2 = -r_2F_2 = -(0.80)(15) = -12$ N·m.
$\sum\tau = 10 - 12 = \boxed{-2.0\text{ N}\cdot\text{m}}$ — net torque is clockwise.

Bài tập luyện tập

Explain why pushing a door very close to its hinges requires a much larger force to produce the same rotational effect as pushing near the doorknob.

Lời giải chi tiết

The torque produced by a push is $\tau = rF\sin\theta$, so for a fixed desired torque, $F = \tau/(r\sin\theta)$ — force needed is *inversely* proportional to the lever arm $r\sin\theta$. Near the hinges, r is small, so the same push produces very little torque; achieving the same torque as a push at the doorknob (where r is large) therefore requires a much larger force. This is exactly why door handles are placed as far from the hinge as possible.

Bài tập luyện tập

Masses 4.0 kg at $x = 0$ and 6.0 kg at $x = 5.0$ m. Find the center of mass.

Lời giải chi tiết

$$x_{cm} = \frac{(4.0)(0) + (6.0)(5.0)}{4.0 + 6.0} = \frac{30}{10} = \boxed{3.0\,\text{m}}.$$

Bài tập luyện tập

Two equal point masses m sit at $x = 0$ and $x = L$. Where is the center of mass?

(A) 0

(B) $L/4$

(C) $L/2$

(D) L

Lời giải chi tiết

$$x_{cm} = \frac{m(0) + m(L)}{2m} = \frac{L}{2}.$$ **(C).**

Bài tập luyện tập

Three masses lie in a plane: 1.0 kg at $(0, 0)$, 2.0 kg at $(4, 0)$, 3.0 kg at $(2, 3)$ (meters). Find x_{cm} and y_{cm}.

Lời giải chi tiết

$$x_{cm} = \frac{(1.0)(0) + (2.0)(4) + (3.0)(2)}{6.0} = \frac{0 + 8 + 6}{6.0} = \frac{14}{6.0} = \boxed{2.33\,\text{m}}.$$
$$y_{cm} = \frac{(1.0)(0) + (2.0)(0) + (3.0)(3)}{6.0} = \frac{9}{6.0} = \boxed{1.5\,\text{m}}.$$

Bài tập luyện tập

A 40 kg child sits 2.0 m from the pivot of a seesaw. How far from the pivot must a 50 kg child sit to balance it?

(A) 1.0 m

(B) 1.25 m

(C) 1.6 m

(D) 2.0 m

Lời giải chi tiết

Weight is proportional to mass and g cancels on both sides: $(40)(2.0) = (50)d \Rightarrow d = \frac{80}{50} = 1.6$ m. **(C)**.

Bài tập luyện tập

A uniform beam of length 3.0 m and weight 150 N is supported at both ends (A at $x = 0$, B at $x = 3.0$ m). A 100 N weight hangs 1.0 m from A. Find the force at each support.

Lời giải chi tiết

Torque about A: $F_B(3.0) = (150)(1.5) + (100)(1.0) = 225 + 100 = 325 \Rightarrow F_B = \frac{325}{3.0} = \boxed{108.3\,\text{N}}$.
$F_A = 150 + 100 - 108.3 = \boxed{141.7\,\text{N}}$.

Bài tập luyện tập

A uniform horizontal beam of length 3.0 m and weight 90 N is hinged to a wall; a cable at the far end makes 30° with the beam and holds it horizontal. Find the cable tension. ($\sin 30^\circ = 0.50$.)

Lời giải chi tiết

Torque about the hinge: $T \sin 30^\circ\,(3.0) = (90)(1.5) \Rightarrow T(0.50)(3.0) = 135 \Rightarrow 1.5T = 135 \Rightarrow T = \boxed{90\,\text{N}}$.

Bài tập luyện tập

For a rigid body in static equilibrium, if the net torque is zero about one particular pivot point, is it necessarily zero about every other point as well? Justify your answer.

(A) No — torque always depends on the pivot chosen

(B) Yes, always, provided the net force is also zero

(C) Yes, always, regardless of the net force

(D) Only if the body is perfectly symmetric

Lời giải chi tiết

(B). Shifting the pivot from point P to a new point P' changes each force's torque by an amount proportional to that force and the displacement $\vec{d}$ between P and P' (specifically, by $\vec{d} \times \vec{F}_i$ for each force). Summing over all forces, the total change in net torque is $\vec{d} \times \sum \vec{F}_i$. If $\sum \vec{F} = 0$, this extra term vanishes for *any* choice of $\vec{d}$, so the net torque is the same about every point — in particular, if it is zero about one point, it is zero about all of them. If $\sum F \neq 0$, this guarantee fails.

Bài tập luyện tập

A uniform 4.0 m ladder weighing 200 N leans against a frictionless wall at $\theta = 45^\circ$ above the floor ($\tan 45^\circ = 1$). Find the minimum coefficient of static friction at the base needed to prevent slipping.

(A) 0.25
(B) 0.375
(C) 0.50
(D) 0.75

Lời giải chi tiết

$N_w = \dfrac{W}{2\tan\theta} = \dfrac{200}{2(1)} = 100\,\text{N} = f_s$. $N_f = W = 200\,\text{N}$. $\mu_{min} = \dfrac{100}{200} = 0.50$. **(C).**

Bài tập luyện tập

If the ladder in the previous problem were instead set at a steeper angle (larger θ, closer to vertical), how would the minimum required coefficient of friction change?

(A) Increases
(B) Decreases
(C) Stays the same
(D) Depends on the ladder's weight

Lời giải chi tiết

Since $\mu_{min} = \dfrac{1}{2\tan\theta}$, increasing θ increases $\tan\theta$ and therefore *decreases* μ_{min} — a steeper ladder needs less friction to stay put, which matches everyday experience (a nearly vertical ladder is more stable against slipping than a shallow, nearly horizontal one). **(B).**

Bài tập luyện tập

Which has the greater moment of inertia about a central axis, for equal mass M and radius R: a solid disk or a thin hoop?

(A) The solid disk
(B) The thin hoop
(C) They are equal
(D) It depends on the rotation speed

Lời giải chi tiết

$I_{disk} = \frac{1}{2}MR^2$ versus $I_{hoop} = MR^2$. The hoop concentrates all its mass at radius R, while the disk spreads mass from 0 to R, giving it a smaller average r^2 and hence smaller I. **(B).**

Bài tập luyện tập

A uniform rod has mass 3.0 kg and length 1.5 m. Find its moment of inertia about its center, then use the parallel-axis theorem to find I about one end, and check the result against $I_{end} = \frac{1}{3}ML^2$.

Lời giải chi tiết

$I_{cm} = \frac{1}{12}ML^2 = \frac{1}{12}(3.0)(1.5)^2 = \frac{1}{12}(3.0)(2.25) = \boxed{0.5625\,\text{kg}\cdot\text{m}^2}$.

$d = L/2 = 0.75\,\text{m}$: $I_{end} = I_{cm} + Md^2 = 0.5625 + (3.0)(0.75)^2 = 0.5625 + 1.6875 = \boxed{2.25\,\text{kg}\cdot\text{m}^2}$.
Check: $\frac{1}{3}ML^2 = \frac{1}{3}(3.0)(2.25) = 2.25\,\text{kg}\cdot\text{m}^2$ ✓.

Bài tập luyện tập

Two $2.0\,\text{kg}$ point masses sit on opposite sides of an axis, each at $r = 0.30\,\text{m}$. Find the total moment of inertia.

(A) $0.09\,\text{kg}\cdot\text{m}^2$
(B) $0.18\,\text{kg}\cdot\text{m}^2$
(C) $0.36\,\text{kg}\cdot\text{m}^2$
(D) $0.72\,\text{kg}\cdot\text{m}^2$

Lời giải chi tiết

$I = \sum m_i r_i^2 = (2.0)(0.30)^2 + (2.0)(0.30)^2 = 2 \times (2.0)(0.09) = 2 \times 0.18 = 0.36\,\text{kg}\cdot\text{m}^2$. **(C)**.

Bài tập luyện tập

A torque of $12\,\text{N}\cdot\text{m}$ is applied to a wheel with $I = 3.0\,\text{kg}\cdot\text{m}^2$, initially at rest. Find its angular acceleration and its angular velocity after $5.0\,\text{s}$.

Lời giải chi tiết

$\alpha = \frac{\tau}{I} = \frac{12}{3.0} = \boxed{4.0\,\text{rad/s}^2}$.
$\omega = \alpha t = (4.0)(5.0) = \boxed{20\,\text{rad/s}}$.

Bài tập luyện tập

A net torque τ produces angular acceleration α in an object of moment of inertia I. If I is doubled while τ stays the same, what happens to α?

(A) Doubles
(B) Halves
(C) Quadruples
(D) Stays the same

Lời giải chi tiết

$\alpha = \tau/I$; doubling I with τ fixed halves α. **(B)**.

Bài tập luyện tập

A thin hoop-shaped pulley ($I = M_pR^2$) has mass $M_p = 1.0\,\text{kg}$ and radius $R = 0.15\,\text{m}$, mounted on a frictionless axle. A string wrapped around it holds a hanging mass $m = 2.0\,\text{kg}$. Find the acceleration of the hanging mass and the string tension.

Lời giải chi tiết

Hanging mass: $mg - T = ma$. Hoop: $TR = I\alpha = (M_pR^2)(a/R) \Rightarrow T = M_pa$.
Substituting: $mg - M_pa = ma \Rightarrow a = \frac{mg}{m + M_p} = \frac{(2.0)(9.8)}{2.0 + 1.0} = \frac{19.6}{3.0} = \boxed{6.53\,\text{m/s}^2}$.

$T = M_p a = (1.0)(6.53) = \boxed{6.53\,\text{N}}$. Check: $T = m(g-a) = (2.0)(9.8-6.53) = (2.0)(3.27) = 6.53\,\text{N}$ ✓.

Bài tập luyện tập

A solid disk and a thin hoop, both of the same mass M and radius R, are each mounted on a frictionless axle. The same constant torque τ is applied to each, starting from rest. Explain, using $\tau = I\alpha$, which one reaches a given angular speed first, and why.

Lời giải chi tiết

$I_{disk} = \frac{1}{2}MR^2$ is smaller than $I_{hoop} = MR^2$, because the hoop's mass sits entirely at radius R while the disk's mass is distributed closer to the axis on average. Since $\alpha = \tau/I$ for the same applied torque, the smaller moment of inertia gives the *larger* angular acceleration: the disk speeds up faster. Because $\omega = \alpha t$ starting from rest, the disk reaches any given angular speed in less time than the hoop.

Bài tập luyện tập

When solving a static equilibrium problem, why is it often strategic to choose the pivot point at the location where an unknown force is applied?

(A) It automatically makes the net force zero

(B) It eliminates that unknown force from the torque equation, since its lever arm about that point is zero

(C) It doubles the number of independent equations available

(D) It is required by Newton's third law

Lời giải chi tiết

A force applied exactly at the pivot has $r = 0$, so its torque $\tau = rF\sin\theta$ is automatically zero, regardless of the force's size or direction — it drops out of the torque equation entirely, leaving fewer unknowns to solve for. **(B)**.

Bài tập luyện tập

A uniform beam of length $5.0\,\text{m}$ and weight $300\,\text{N}$ rests on two supports at its ends. A $450\,\text{N}$ load is placed $1.0\,\text{m}$ from support A. Find the force at support B.

Lời giải chi tiết

Torque about A: $F_B(5.0) = (300)(2.5) + (450)(1.0) = 750 + 450 = 1200 \Rightarrow F_B = \dfrac{1200}{5.0} = \boxed{240\,\text{N}}$.

Bài tập luyện tập

A flywheel with $I = 0.40\,\text{kg}\cdot\text{m}^2$ is brought from rest to $20\,\text{rad/s}$ in $2.0\,\text{s}$ by a constant net torque. Find that torque.

(A) $2.0\,\text{N}\cdot\text{m}$

(B) $4.0\,\text{N}\cdot\text{m}$

(C) $8.0\,\text{N}\cdot\text{m}$

(D) $10\,\text{N}\cdot\text{m}$

Lời giải chi tiết

$\alpha = \dfrac{\Delta\omega}{\Delta t} = \dfrac{20}{2.0} = 10\,\text{rad/s}^2$. $\tau = I\alpha = (0.40)(10) = 4.0\,\text{N}\cdot\text{m}$. **(B)**.

Bài tập luyện tập

A grinding wheel accelerates uniformly from $\omega_0 = 5.0\,\text{rad/s}$ to $\omega = 25\,\text{rad/s}$ while turning through $60\,\text{rad}$. Find the time this takes.

(A) $2.0\,\text{s}$
(B) $3.0\,\text{s}$
(C) $4.0\,\text{s}$
(D) $5.0\,\text{s}$

Lời giải chi tiết

$\theta = \frac{1}{2}(\omega_0 + \omega)t \Rightarrow t = \dfrac{2\theta}{\omega_0 + \omega} = \dfrac{2(60)}{5.0 + 25} = \dfrac{120}{30} = 4.0\,\text{s}$. **(C)**.

Bài tập luyện tập

A CD spins at frequency $f = 5.0\,\text{Hz}$. Find its angular velocity ω, and the tangential speed of a point $r = 0.10\,\text{m}$ from the center.

Lời giải chi tiết

$\omega = 2\pi f = 2\pi(5.0) = \boxed{31.4\,\text{rad/s}}$.
$v = \omega r = (31.4)(0.10) = \boxed{3.14\,\text{m/s}}$.

Bài tập luyện tập

A rigid bar pivoted at one end has three forces applied simultaneously: $F_1 = 30\,\text{N}$ perpendicular to the bar at $r_1 = 0.40\,\text{m}$ (tending to rotate it counterclockwise), $F_2 = 50\,\text{N}$ applied directly at the pivot itself, and $F_3 = 20\,\text{N}$ perpendicular at $r_2 = 0.90\,\text{m}$ (tending to rotate it clockwise). Find the net torque about the pivot.

(A) $-6.0\,\text{N}\cdot\text{m}$
(B) $+6.0\,\text{N}\cdot\text{m}$
(C) $-18\,\text{N}\cdot\text{m}$
(D) $+30\,\text{N}\cdot\text{m}$

Lời giải chi tiết

F_2 is applied at the pivot itself, so $r = 0$ and it contributes *zero* torque no matter its magnitude or direction. $\tau_1 = +(0.40)(30) = +12\,\text{N·m}$. $\tau_3 = -(0.90)(20) = -18\,\text{N·m}$.

$$\sum \tau = 12 + 0 - 18 = -6.0\,\text{N}\cdot\text{m}.$$

(A).

Bài tập luyện tập

Four point masses sit at the corners of a rectangle in the xy-plane: $1.0\,\text{kg}$ at $(0,0)$, $2.0\,\text{kg}$ at $(4,0)$, $1.0\,\text{kg}$ at $(4,3)$, and $2.0\,\text{kg}$ at $(0,3)$, with all coordinates in meters. Find the center of mass of the system.

Lời giải chi tiết

Total mass $= 1.0 + 2.0 + 1.0 + 2.0 = 6.0$ kg.

$$x_{cm} = \frac{(1.0)(0) + (2.0)(4) + (1.0)(4) + (2.0)(0)}{6.0} = \frac{0+8+4+0}{6.0} = \frac{12}{6.0} = \boxed{2.0\,\text{m}}.$$

$$y_{cm} = \frac{(1.0)(0) + (2.0)(0) + (1.0)(3) + (2.0)(3)}{6.0} = \frac{0+0+3+6}{6.0} = \frac{9}{6.0} = \boxed{1.5\,\text{m}}.$$

Bài tập luyện tập

A uniform horizontal beam of length $L = 5.0$ m and weight $W = 80$ N is attached to a wall by a hinge at one end. A cable from the far end of the beam runs to the wall, making an angle $\theta = 53^\circ$ with the beam ($\sin 53^\circ = 0.80$, $\cos 53^\circ = 0.60$). A lamp of weight $w = 120$ N hangs from the far end of the beam, at the cable's attachment point. Find (a) the cable tension, and (b) the horizontal and vertical components of the hinge force.

Lời giải chi tiết

(a) Torque about the hinge: $T \sin\theta \cdot L = W \cdot \frac{L}{2} + w \cdot L$, so, dividing by L:

$$T \sin\theta = \frac{W}{2} + w = 40 + 120 = 160 \;\Rightarrow\; T = \frac{160}{0.80} = \boxed{200\,\text{N}}.$$

(b) Horizontally, the hinge pushes outward against the cable's inward pull:

$$H_x = T\cos\theta = (200)(0.60) = \boxed{120\,\text{N}}.$$

Vertically, the hinge plus the cable's vertical component support both weights:

$$H_y = W + w - T\sin\theta = 80 + 120 - (200)(0.80) = 200 - 160 = \boxed{40\,\text{N}}.$$

Bài tập luyện tập

Two point masses, 3.0 kg and 5.0 kg, are attached to opposite ends of a very light rigid rod of length 0.40 m. Find the moment of inertia of the system about an axis through the 3.0 kg mass, perpendicular to the rod.

(A) $0.48\,\text{kg}\cdot\text{m}^2$

(B) $0.64\,\text{kg}\cdot\text{m}^2$

(C) $0.80\,\text{kg}\cdot\text{m}^2$

(D) $1.28\,\text{kg}\cdot\text{m}^2$

Lời giải chi tiết

The 3.0 kg mass sits exactly at the axis ($r = 0$), contributing nothing. Only the 5.0 kg mass, at $r = 0.40$ m, contributes:

$$I = (3.0)(0)^2 + (5.0)(0.40)^2 = 0 + (5.0)(0.16) = 0.80\,\text{kg}\cdot\text{m}^2.$$

(C).

Bài tập luyện tập

A uniform rod of mass $M_{rod} = 3.0\,\text{kg}$ and length $L = 1.0\,\text{m}$ has a small solid sphere of mass $M_{sph} = 2.0\,\text{kg}$ and radius $R = 0.10\,\text{m}$ rigidly attached at its far end, with the sphere's center at the tip of the rod. Find the total moment of inertia of the assembly about an axis through the rod's near end, perpendicular to the rod. (Use $I_{sphere,cm} = \frac{2}{5}M_{sph}R^2$.)

Lời giải chi tiết

Rod about its own end: $I_{rod} = \frac{1}{3}M_{rod}L^2 = \frac{1}{3}(3.0)(1.0)^2 = 1.0\,\text{kg}\cdot\text{m}^2$.
Sphere about its own center: $I_{sph,cm} = \frac{2}{5}M_{sph}R^2 = \frac{2}{5}(2.0)(0.10)^2 = (0.4)(2.0)(0.01) = 0.008\,\text{kg}\cdot\text{m}^2$.
The sphere's center is a distance $d = L = 1.0\,\text{m}$ from the rotation axis, so by the parallel-axis theorem:

$$I_{sph} = I_{sph,cm} + M_{sph}d^2 = 0.008 + (2.0)(1.0)^2 = 0.008 + 2.0 = 2.008\,\text{kg}\cdot\text{m}^2.$$

$$I_{total} = I_{rod} + I_{sph} = 1.0 + 2.008 = \boxed{3.01\,\text{kg}\cdot\text{m}^2}.$$

Bài tập luyện tập

Two identical uniform rods, each of mass M and length L, are each subjected to the same net torque τ. Rod 1 rotates about an axis through its *center*; rod 2 rotates about an axis through one *end*. Without doing any further calculation beyond stating the ratio of their moments of inertia, explain which rod has the greater angular acceleration, and by what factor.

Lời giải chi tiết

From the standard shape table, $I_{center} = \frac{1}{12}ML^2$ for rotation about the center, while $I_{end} = \frac{1}{3}ML^2$ for rotation about one end. Since $\frac{1}{3}ML^2 = 4 \times \frac{1}{12}ML^2$, the end-pivoted rod has **four times** the moment of inertia of the center-pivoted rod: $I_{end} = 4I_{center}$. Newton's second law for rotation, $\tau = I\alpha$, means that for a *fixed* applied torque, angular acceleration is inversely proportional to moment of inertia: $\alpha = \tau/I$. Because rod 1 (pivoted at its center) has one-quarter the moment of inertia of rod 2 (pivoted at its end), rod 1 experiences **four times** the angular acceleration of rod 2: $\alpha_{center} = 4\,\alpha_{end}$. Physically, this is because pivoting at the end forces, on average, more of the rod's mass to sit farther from the axis than pivoting at the center does, and moment of inertia grows with the *square* of that distance — so even though both rods have identical mass and length and feel the identical torque, the end-pivoted rod is far more resistant to a change in its rotational motion.

Energy and Momentum of Rotating Systems

Mục tiêu Unit: Động năng quay $KE_{rot} = \frac{1}{2}I\omega^2$ và động năng tổng hợp của vật lăn không trượt; hệ thức ràng buộc $v_{cm} = \omega R$ và $a_{cm} = \alpha R$; bảo toàn cơ năng cho vật lăn xuống dốc, so sánh vận tốc của các hình dạng khác nhau và đối chiếu với vật trượt không ma sát; động lượng góc $L = I\omega$ (vật rắn) và $L = mvr\sin\theta$ (chất điểm); định luật bảo toàn động lượng góc; xung lượng góc $\tau\Delta t = \Delta L$; và bảng tương ứng đầy đủ giữa các đại lượng tịnh tiến và đại lượng quay.

6.1 Rotational Kinetic Energy

Any rotating rigid body has kinetic energy associated with its spin, even if its center of mass is not moving. Each small piece of mass m_i at distance r_i from the rotation axis moves in a circle at speed $v_i = \omega r_i$, so it carries kinetic energy $\frac{1}{2}m_i v_i^2 = \frac{1}{2}m_i\omega^2 r_i^2$. Summing over every piece of the body and factoring out the common ω^2 produces $\sum \frac{1}{2}m_i\omega^2 r_i^2 = \frac{1}{2}\omega^2 \sum m_i r_i^2 = \frac{1}{2}I\omega^2$, since $I \equiv \sum m_i r_i^2$ is exactly the definition of moment of inertia introduced in Unit 5.

$$KE_{rot} = \frac{1}{2}I\omega^2.$$

This is the exact rotational analog of translational kinetic energy $KE_{trans} = \frac{1}{2}mv^2$: mass m is replaced by moment of inertia I, and speed v is replaced by angular speed ω (in rad/s). If a rigid body simultaneously translates *and* rotates, its total kinetic energy is the sum of both contributions:

$$KE_{total} = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2,$$

where v_{cm} is the speed of the center of mass and I is the moment of inertia about the axis through the center of mass.

Object	Axis	Moment of inertia
Point mass	distance r from axis	$I = mr^2$
Thin rod	through center, $\perp$ to rod	$I = \frac{1}{12}ML^2$
Thin rod	through one end, $\perp$ to rod	$I = \frac{1}{3}ML^2$
Solid disk / cylinder	through center (symmetry axis)	$I = \frac{1}{2}MR^2$
Thin hoop / ring	through center (symmetry axis)	$I = MR^2$
Solid sphere	through center	$I = \frac{2}{5}MR^2$
Thin spherical shell	through center	$I = \frac{2}{3}MR^2$

Table 6.1: *

Moment of inertia for common shapes (recall from Unit 5). For an object of mass m and radius R rotating about its symmetry axis, it is convenient to define the dimensionless **shape factor** $c \equiv I/(mR^2)$ — used throughout this unit.

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Động năng quay $KE_{rot} = \frac{1}{2}I\omega^2$ là phiên bản "quay" của động năng tịnh tiến quen thuộc $\frac{1}{2}mv^2$: khối lượng m được thay bằng mô-men quán tính I, tốc độ v được thay bằng tốc độ góc ω. Nếu vật vừa tịnh tiến (khối tâm di chuyển) vừa quay quanh khối tâm, tổng động năng là **tổng của cả hai phần**: $KE_{total} = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2$ — đây chính là chìa khoá của toàn bộ phần "lăn không trượt" phía sau.

Ví dụ 1 · Rotational KE of a flywheel

A flywheel, modeled as a uniform solid disk of mass $M = 4.0$ kg and radius $R = 0.50$ m, spins at $\omega = 20$ rad/s about its central axis (it does not translate). Find its moment of inertia and its rotational kinetic energy.

$$I = \tfrac{1}{2}MR^2 = \tfrac{1}{2}(4.0)(0.50)^2 = \tfrac{1}{2}(4.0)(0.25) = 0.50\,\text{kg}\cdot\text{m}^2.$$
$$KE_{rot} = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}(0.50)(20)^2 = \tfrac{1}{2}(0.50)(400) = 100\,\text{J}.$$

6.2 Rolling Without Slipping

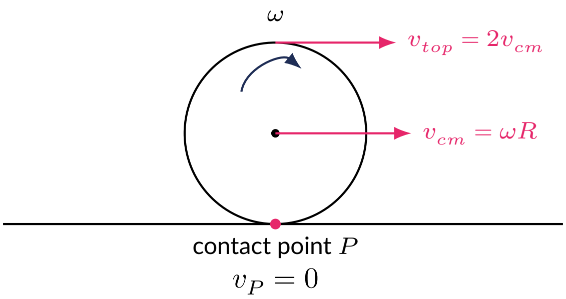
6.2.1 The Rolling Constraint

A wheel, ball, or cylinder **rolls without slipping** when the contact point between the object and the surface is instantaneously at rest — it does not skid or drag. This physical condition forces the object's translational speed and its rotational speed to be locked together.

For an object of radius R rolling without slipping:

$$v_{cm} = \omega R, \qquad a_{cm} = \alpha R.$$

The velocity of the contact point (bottom of the wheel) is exactly zero; the velocity of the topmost point is $2v_{cm}$; the center moves at v_{cm}. These follow because the contact point's velocity is the vector sum of the center's translational velocity v_{cm} (forward) and the point's rotational velocity ωR (backward, relative to the center) — for rolling without slipping these must exactly cancel.



A wheel rolling without slipping: the contact point P is instantaneously at rest, the center moves at $v_{cm} = \omega R$, and the top point moves at $2v_{cm}$.

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Khi vật **lăn không trượt**, điểm tiếp xúc với mặt đất tại mỗi thời điểm có vận tốc **bằng 0** — nó không trượt/không mài trên mặt đường. Điều này buộc vận tốc khối tâm và tốc độ góc phải liên hệ chặt chẽ: $v_{cm} = \omega R$ (và tương tự cho gia tốc, $a_{cm} = \alpha R$). Điểm trên cùng của bánh xe di chuyển nhanh gấp đôi khối tâm ($2v_{cm}$), vì nó vừa được "mang theo" bởi chuyển động tịnh tiến, vừa được "quay thêm" bởi chuyển động quay.

6.2.2 Total Kinetic Energy of a Rolling Object

Because rolling combines translation of the center of mass with rotation about the center of mass, the total kinetic energy is the sum described in Section 6.1, now with the rolling constraint substituted in.

Using $\omega = v_{cm}/R$ and $I = cMR^2$ (with shape factor c from the table above):

$$KE_{total} = \tfrac{1}{2}Mv_{cm}^2 + \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}Mv_{cm}^2 + \tfrac{1}{2}(cMR^2)\left(\frac{v_{cm}}{R}\right)^2 = \tfrac{1}{2}Mv_{cm}^2(1+c).$$

The factor $(1+c)$ shows exactly how much *extra* kinetic energy rolling carries compared to a non-rotating object with the same v_{cm}: a sliding block has $c = 0$ (no extra KE), while a hoop ($c = 1$) carries *double* the KE of a sliding block moving at the same speed.

GIẢI THÍCH TIẾNG VIỆT
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Tổng động năng của vật lăn không trượt gồm hai phần: động năng tịnh tiến của khối tâm và động năng quay quanh khối tâm. Thay $\omega = v_{cm}/R$ và $I = cMR^2$ vào, ta được công thức gọn: $KE_{total} = \frac{1}{2}Mv_{cm}^2(1+c)$. Hệ số $(1+c)$ cho biết vật lăn "tốn" bao nhiêu động năng vào việc quay so với một vật trượt (không quay, $c = 0$) có cùng tốc độ khối tâm.

Ví dụ 2 · Total KE of a rolling bowling ball

A bowling ball (uniform solid sphere), $M = 6.0$ kg, $R = 0.10$ m, rolls without slipping across the floor at $v_{cm} = 8.0$ m/s. Find its angular speed, its translational KE, its rotational KE, and its total KE.

Rolling constraint: $\omega = \dfrac{v_{cm}}{R} = \dfrac{8.0}{0.10} = 80$ rad/s.

Moment of inertia: $I = \frac{2}{5}MR^2 = \frac{2}{5}(6.0)(0.10)^2 = \frac{2}{5}(6.0)(0.010) = 0.024$ kg · m^2.

$$KE_{trans} = \tfrac{1}{2}Mv_{cm}^2 = \tfrac{1}{2}(6.0)(8.0)^2 = \tfrac{1}{2}(6.0)(64) = 192\text{ J}.$$
$$KE_{rot} = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}(0.024)(80)^2 = \tfrac{1}{2}(0.024)(6400) = 76.8\text{ J}.$$
$$KE_{total} = 192 + 76.8 = 268.8\text{ J}.$$

Check with the shortcut: $KE_{total} = \frac{1}{2}Mv_{cm}^2(1+c) = 192(1+0.4) = 192(1.4) = 268.8$ J — matches.

(!) Lỗi thường gặp: The single most common error in rolling problems is using only $\frac{1}{2}mv_{cm}^2$ for the kinetic energy of a rolling object and forgetting the rotational term $\frac{1}{2}I\omega^2$ entirely. A rolling object always has *more* total kinetic energy than a non-rotating object moving at the same v_{cm} — never forget the "$(1+c)$" factor.

6.3 Energy Conservation on an Incline: Which Shape Wins?

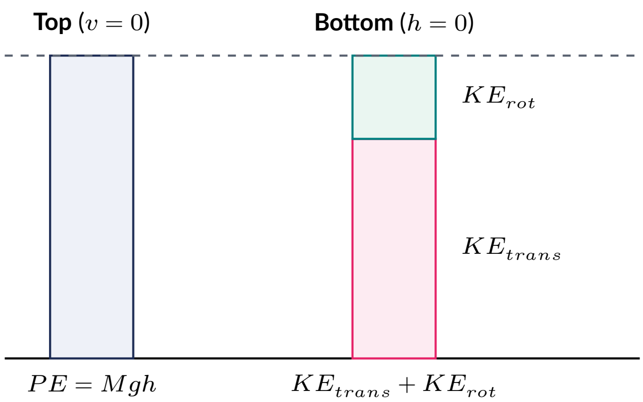
Consider an object of mass M and radius R released from rest at the top of an incline, a vertical height h above the bottom, and rolling without slipping the whole way down (static friction is present but, as shown below, it does no work). No energy is lost to sliding, so mechanical energy is conserved:

$$Mgh = KE_{total} = \tfrac{1}{2}Mv_{cm}^2(1+c).$$

Solving for the speed at the bottom:

$$v_{cm} = \sqrt{\frac{2gh}{1+c}}, \qquad c = \frac{I}{MR^2}.$$

Remarkably, M and R cancel completely — the final speed depends only on the shape factor c, i.e. only on *how the mass is distributed* relative to the radius, not on the actual mass or size of the object. **Smaller c means a larger final speed**, because less of the released potential energy is "diverted" into rotational KE, leaving more available for translational KE. For a frictionless block that *slides* (no rotation, $c = 0$), this reduces to the familiar $v = \sqrt{2gh}$ from Unit 3 — and that is always faster than any rolling object, since $1 + c > 1$ whenever $c > 0$.



Energy bar chart for a solid sphere rolling down: at the top, all energy is gravitational PE; at the bottom the *same total* has split into translational and rotational KE (about 71% / 29% for a solid sphere).

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Khi một vật lăn không trượt từ đỉnh dốc cao h, bảo toàn cơ năng cho $v_{cm} = \sqrt{2gh/(1+c)}$ — khối lượng M và bán kính R **triệt tiêu hoàn toàn**, chỉ còn phụ thuộc hệ số hình dạng $c = I/(MR^2)$. **c càng nhỏ, vật lăn xuống càng nhanh**, vì càng ít năng lượng "biến" thành động năng quay, để lại nhiều hơn cho động năng tịnh tiến. Vật trượt không ma sát (không quay, $c = 0$) luôn nhanh hơn mọi vật lăn, vì $1 + c > 1$.

Ví dụ 3 · Solid cylinder rolling down an incline

A solid cylinder is released from rest and rolls without slipping down an incline, descending a vertical height $h = 2.0$ m. Find its speed at the bottom.

For a solid cylinder, $c = \frac{1}{2}$:

$$v_{cm} = \sqrt{\frac{2gh}{1+c}} = \sqrt{\frac{2(9.8)(2.0)}{1.5}} = \sqrt{\frac{39.2}{1.5}} = \sqrt{26.13} = 5.11\,\text{m/s}.$$

Notice that neither the cylinder's mass nor its radius was needed.

Ví dụ 4 · Racing five objects down the same incline

A frictionless sliding block, a solid sphere, a solid cylinder (disk), a thin spherical shell, and a thin hoop are each released from rest at the top of the same incline, $h = 5.0$ m above the bottom (the four non-block objects roll without slipping). Rank their speeds at the bottom.

Using $v_{cm} = \sqrt{2gh/(1+c)}$ with $2gh = 2(9.8)(5.0) = 98\,\text{m}^2/\text{s}^2$:

Object	c	$1+c$	v_{cm}
Sliding block (no rotation)	0	1.00	$\sqrt{98} = 9.90\,\text{m/s}$
Solid sphere	$2/5$	1.40	$\sqrt{70} = 8.37\,\text{m/s}$
Solid cylinder / disk	$1/2$	1.50	$\sqrt{65.3} = 8.08\,\text{m/s}$
Thin spherical shell	$2/3$	1.67	$\sqrt{58.8} = 7.67\,\text{m/s}$
Thin hoop / ring	1	2.00	$\sqrt{49} = 7.00\,\text{m/s}$

The ranking from fastest to slowest is exactly the ranking of c from smallest to largest — mass distributed farther from the axis (larger c) means more of the released energy is locked into rotation, leaving less for translation.

6.3.1 Rolling Dynamics: The Role of Static Friction

Energy conservation tells us the speed at the bottom, but it is static friction, acting at the contact point, that supplies the torque needed to make the object spin up as it accelerates — without friction, the object would simply slide without rotating at all (like ice).

(!) Lỗi thường gặp: Static friction acting on an object rolling without slipping does **zero work**, because the contact point has zero velocity at every instant — a force does no work if its point of application does not move. Static friction changes *how* the energy is split between translation and rotation (via torque), but for ideal rolling it never removes mechanical energy from the system. This is different from *kinetic* friction during skidding, which does negative work and generates heat.

Ví dụ 5 · Minimum coefficient of friction for rolling without slipping

A solid sphere ($c = \frac{2}{5}$) rolls without slipping down an incline of angle $\theta = 30^\circ$. Find its linear acceleration and the minimum coefficient of static friction required to prevent slipping.

Applying Newton's second law along the incline ($Mg\sin\theta - f = Ma$) and the rotational form about the center ($fR = I\alpha = cMR^2 \cdot a/R \Rightarrow f = cMa$) simultaneously:

$$Mg\sin\theta = Ma + cMa = Ma(1+c) \;\Rightarrow\; a = \frac{g\sin\theta}{1+c} = \frac{(9.8)(0.50)}{1.4} = \frac{4.9}{1.4} = 3.5\,\text{m/s}^2.$$

Friction force: $f = cMa = (0.4)M(3.5) = 1.4M\,\text{N}$. Normal force: $N = Mg\cos\theta = (9.8)(0.866)M = 8.49M\,\text{N}$.

$$\mu_{\text{min}} = \frac{f}{N} = \frac{1.4M}{8.49M} = 0.165.$$

6.4 Angular Momentum

Angular momentum is the rotational analog of linear momentum. For a rigid body rotating about a fixed axis, it is defined analogously to $p = mv$:

$$L = I\omega \qquad \text{(rigid body about a fixed axis).}$$

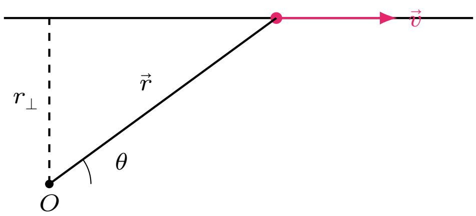
Units: kg · m^2/s (equivalently J · s). Like linear momentum, angular momentum is a vector; for rotation confined to a plane, its direction is given by the right-hand rule (curl the fingers in the direction of rotation — the thumb points along L), and it is positive/negative depending on whether the rotation is counterclockwise/clockwise, exactly like the sign convention used for ω and α in Unit 5.

A single particle need not move in a circle to have angular momentum about some chosen reference point — angular momentum is always defined *relative to a point*, just as torque is.

For a particle of mass m, velocity v, and position vector $\vec{r}$ (drawn from the reference point O to the particle), the angular momentum about O is

$$L = mvr\sin\theta = r_\perp mv,$$

where θ is the angle between $\vec{r}$ and $\vec{v}$, and $r_\perp = r\sin\theta$ is the **perpendicular distance** from O to the particle's line of motion (its extended velocity vector). This is the magnitude of the vector cross product $\vec{L} = \vec{r}\times\vec{p}$. A particle moving in a straight line at constant velocity — even one that never passes through O — still has a well-defined, and in fact *constant*, angular momentum about O, because $r_\perp$ never changes even as r and θ both change.



Angular momentum of a particle about point O: $L = mvr\sin\theta = mv\,r_\perp$, where $r_\perp$ is the perpendicular distance from O to the line of motion.

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Với vật rắn quay quanh trục cố định, động lượng góc là $L = I\omega$ — hoàn toàn tương tự $p = mv$, chỉ thay $m \to I$ và $v \to \omega$. Với **một chất điểm** (kể cả đang chuyển động thẳng đều, không quay quanh O nào cả), động lượng góc quanh một điểm O vẫn được định nghĩa: $L = mvr\sin\theta = mv\,r_\perp$, với $r_\perp$ là khoảng cách vuông góc từ O đến đường thẳng chứa quỹ đạo. Điều đặc biệt: nếu vật chuyển động thẳng đều (không có lực), $r_\perp$ không đổi nên L quanh O luôn **không đổi** — dù r và θ đều thay đổi theo thời gian.

Ví dụ 6 · Angular momentum of a particle moving in a straight line

A 2.0 kg ball moves at a constant $v = 6.0$ m/s along a straight path. The perpendicular distance from a fixed point O to this path is $r_\perp = 4.0$ m. Find the ball's angular momentum about O.

$$L = r_\perp mv = (4.0)(2.0)(6.0) = 48\ \text{kg}\cdot\text{m}^2/\text{s}.$$

This value stays the same at every instant of the ball's motion, since $r_\perp$ does not change even as the ball travels along its straight-line path.

Ví dụ 7 · Angular momentum of a spinning merry-go-round

A merry-go-round, modeled as a uniform solid disk of mass $M = 120$ kg and radius $R = 1.5$ m, spins at $\omega = 2.0$ rad/s. Find its moment of inertia and its angular momentum.

$$I = \tfrac{1}{2}MR^2 = \tfrac{1}{2}(120)(1.5)^2 = \tfrac{1}{2}(120)(2.25) = 135\,\text{kg}\cdot\text{m}^2.$$
$$L = I\omega = (135)(2.0) = 270\,\text{kg}\cdot\text{m}^2/\text{s}.$$

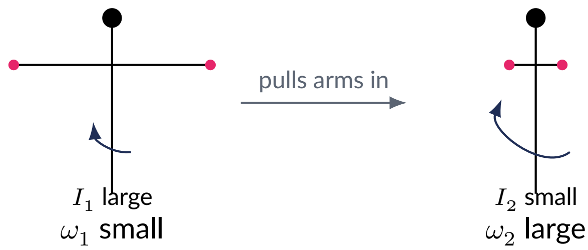
6.5 Conservation of Angular Momentum

Just as linear momentum is conserved when the net *external force* on a system is zero, angular momentum is conserved when the net *external torque* on a system is zero.

If $\tau_{net,ext} = 0$ on a system, its total angular momentum is constant:

$$L_i = L_f \qquad \Longleftrightarrow \qquad I_1\omega_1 = I_2\omega_2$$

for a system whose moment of inertia changes from I_1 to I_2 (e.g. because mass redistributes relative to the axis). Because $L = I\omega$ stays fixed, **a decrease in I must produce a proportional increase in** ω, and vice versa — this is why figure skaters, divers, and platform divers spin faster when they pull their limbs in.



Pulling the arms inward reduces moment of inertia; since $L = I\omega$ is conserved, angular velocity must increase.

GIẢI THÍCH TIẾNG VIỆT

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Khi mô-men xoắn ngoại lực tổng hợp bằng 0, động lượng góc L của hệ **được bảo toàn**: $I_1\omega_1 = I_2\omega_2$. Nếu I giảm (ví dụ vận động viên thu tay/chân lại gần trục quay), ω phải **tăng** tương ứng để giữ L không đổi — đây là lý do vận động viên trượt băng nghệ thuật quay nhanh hẳn lên khi họ thu tay vào người.

6.5.1 Classic Examples

Ví dụ 8 · Skater pulling her arms in

A figure skater spins at $\omega_1 = 2.0$ rad/s with her arms extended, giving her a moment of inertia $I_1 = 4.0$ kg · m². She pulls her arms in, reducing her moment of inertia to $I_2 = 1.0$ kg · m². Find her new angular velocity, and compare her rotational KE before and after.

Angular momentum is conserved (the ice exerts negligible torque about her spin axis):

$$I_1\omega_1 = I_2\omega_2 \;\Rightarrow\; \omega_2 = \frac{I_1\omega_1}{I_2} = \frac{(4.0)(2.0)}{1.0} = 8.0\,\text{rad/s}.$$

$$KE_1 = \tfrac{1}{2}I_1\omega_1^2 = \tfrac{1}{2}(4.0)(2.0)^2 = 8.0\,\text{J}, \qquad KE_2 = \tfrac{1}{2}I_2\omega_2^2 = \tfrac{1}{2}(1.0)(8.0)^2 = 32\,\text{J}.$$

Her rotational KE *quadruples* even though L stays fixed — the skater's own muscles do positive internal work pulling her arms inward against the effective outward pull of her spinning body, and that work becomes extra kinetic energy.

Ví dụ 9 · Reeling in a puck on a string

A 0.50 kg puck slides on frictionless ice in a circle of radius $r_1 = 0.80$ m at $v_1 = 4.0$ m/s, held by a string that passes through a hole in the table and is pulled from below. The string is reeled in until the radius is $r_2 = 0.20$ m. Find the puck's new speed.

The string tension always points toward the hole (radially), so it exerts **zero torque** about the hole — angular momentum about the hole is conserved. Treating the puck as a point mass, $L = mvr$, so:

$$mv_1r_1 = mv_2r_2 \;\Rightarrow\; v_2 = \frac{v_1r_1}{r_2} = \frac{(4.0)(0.80)}{0.20} = 16\,\text{m/s}.$$

Note $KE_1 = \frac{1}{2}(0.50)(4.0)^2 = 4.0$ J but $KE_2 = \frac{1}{2}(0.50)(16)^2 = 64$ J — sixteen times more. Unlike the skater's internal muscular work, here the *external* tension does positive work on the puck: as the puck spirals inward its displacement has a small component along the direction of tension, even though tension is always exactly perpendicular to the puck's instantaneous velocity (and hence exerts zero torque about the hole at every instant).

6.5.2 Rotational "Collisions": Two Bodies Coupling Together

When two rotating bodies on the same axis are brought together and lock into a common angular velocity (e.g. two disks pressed together by a clutch), the situation is the direct rotational analog of a **perfectly inelastic linear collision** from Unit 4: momentum (L, here) is conserved because there is no net external torque during the coupling, but kinetic energy is *not* conserved — energy is lost to friction/deformation as the bodies grip each other and match speeds.

Parallel to Unit 4: Linear Momentum

Compare directly to a block sliding into and sticking to a second block (perfectly inelastic collision): $m_1v_1 = (m_1 + m_2)v_f$, with kinetic energy lost. The rotational version simply replaces $m \to I$ and $v \to \omega$: $I_1\omega_1 = (I_1 + I_2)\omega_f$, again with kinetic energy lost. In both cases, *momentum* (linear or angular) survives the "collision" intact; *kinetic energy* does not, because internal forces (friction between the coupling surfaces) do negative work as the pieces merge.

Ví dụ 10 · Two disks coupling on a shared axle

Disk A has $I_A = 2.0\,\text{kg}\cdot\text{m}^2$ and spins at $\omega_A = 10$ rad/s. Disk B has $I_B = 6.0\,\text{kg}\cdot\text{m}^2$ and is initially at rest, on the same frictionless axle. The disks are pressed together and reach a common final angular velocity. Find ω_f and the fraction of kinetic energy lost.

Conservation of angular momentum (no external torque about the shared axle):

$$I_A\omega_A + I_B(0) = (I_A + I_B)\omega_f \;\Rightarrow\; \omega_f = \frac{(2.0)(10)}{2.0 + 6.0} = \frac{20}{8.0} = 2.5\,\text{rad/s}.$$

$$KE_i = \tfrac{1}{2}(2.0)(10)^2 = 100\,\text{J}, \qquad KE_f = \tfrac{1}{2}(8.0)(2.5)^2 = \tfrac{1}{2}(8.0)(6.25) = 25\,\text{J}.$$

Fraction lost: $\dfrac{100 - 25}{100} = 0.75$, i.e. **75%** of the initial kinetic energy is lost to the coupling — exactly analogous to how a perfectly inelastic linear collision loses kinetic energy while conserving momentum.

(!) Lỗi thường gặp: Do not confuse "angular momentum is conserved" with "kinetic energy is conserved." A zero net external torque guarantees L stays constant — it says *nothing* about kinetic energy. KE can increase (skater pulling arms in, string reeled in) or decrease (two disks coupling together) depending on whether internal forces do positive or negative work; only check KE conservation separately, the same way you already do for linear collisions.

6.6 Angular Impulse

Just as a net force applied over a time interval produces a linear impulse that changes linear momentum ($J = F\Delta t = \Delta p$, from Unit 4), a net torque applied over a time interval produces an **angular impulse** that changes angular momentum.

$$\tau\,\Delta t = \Delta L = I\omega_f - I\omega_i \qquad \text{(constant torque).}$$

For a torque that varies with time, the angular impulse is the *signed area* under a τ–t graph, exactly as the area under an F–t graph gives linear impulse:

$$\Delta L = \int \tau\,dt = \text{area under the } \tau\text{–}t \text{ graph.}$$

VN **GIẢI THÍCH TIẾNG VIỆT**

Xung lượng góc là phiên bản "quay" của xung lượng tuyến tính: $\tau\Delta t = \Delta L$ (so với $F\Delta t = \Delta p$). Nếu mô-men xoắn không đổi theo thời gian, xung lượng góc chính là $\tau\Delta t$; nếu τ thay đổi, xung lượng góc là **diện tích có dấu** dưới đồ thị τ–t — hoàn toàn giống cách đọc đồ thị F–t ở Unit 4.

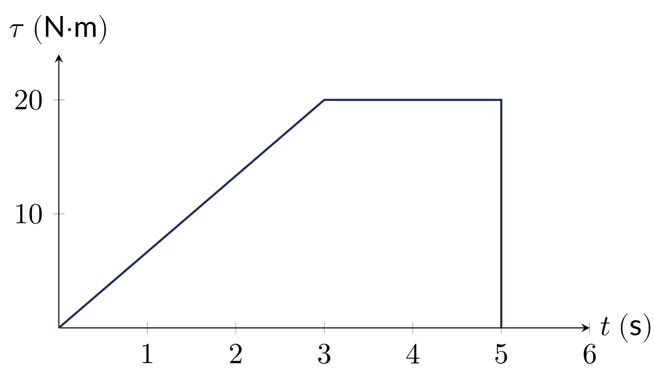
Ví dụ 11 · Angular impulse from a constant torque

A constant torque of $\tau = 15\,\text{N}\cdot\text{m}$ is applied to a wheel with $I = 3.0\,\text{kg}\cdot\text{m}^2$, initially at rest, for a duration $\Delta t = 4.0\,\text{s}$. Find the angular impulse delivered and the wheel's final angular velocity.

$$\text{Angular impulse} = \tau\Delta t = (15)(4.0) = 60\,\text{kg}\cdot\text{m}^2/\text{s} = \Delta L.$$

Since the wheel starts at rest, $L_i = 0$, so $L_f = 60\,\text{kg}\cdot\text{m}^2/\text{s}$, giving

$$\omega_f = \frac{L_f}{I} = \frac{60}{3.0} = 20\,\text{rad/s}.$$



τ–t graph: torque rises linearly from 0 to $20\,\text{N}\cdot\text{m}$ over 0–$3.0\,\text{s}$, then stays constant at $20\,\text{N}\cdot\text{m}$ from 3.0–$5.0\,\text{s}$.

Ví dụ 12 · Angular impulse from a τ–t graph

For the torque graph shown above, a rotor with $I = 5.0\,\text{kg}\cdot\text{m}^2$ starts at rest. Find its angular velocity at $t = 5.0\,\text{s}$.

Angular impulse = total signed area under the graph. Triangle (0–3.0 s):

$$\tfrac{1}{2}(3.0)(20) = 30\,\text{kg}\cdot\text{m}^2/\text{s}.$$

Rectangle (3.0–5.0 s):

$$(20)(5.0 - 3.0) = 40\,\text{kg}\cdot\text{m}^2/\text{s}.$$

Total angular impulse $= 30 + 40 = 70\,\text{kg}\cdot\text{m}^2/\text{s} = \Delta L$. Since $L_i = 0$:

$$\omega_f = \frac{L_f}{I} = \frac{70}{5.0} = 14\,\text{rad/s}.$$

6.7 Synthesis: The Translational–Rotational Analogy

Every rotational quantity introduced in Units 5 and 6 has a direct translational counterpart. Recognizing this parallel structure lets you transfer everything you already know about translational dynamics and energy directly into rotational problems.

Translational quantity	Rotational quantity
Mass m	Moment of inertia I
Position x	Angular position θ
Velocity v	Angular velocity ω
Acceleration a	Angular acceleration α
Force F	Torque τ
Newton's second law $F_{net} = ma$	$\tau_{net} = I\alpha$
Momentum $p = mv$	Angular momentum $L = I\omega$
Impulse $J = F\Delta t = \Delta p$	Angular impulse $\tau\Delta t = \Delta L$
Kinetic energy $KE = \frac{1}{2}mv^2$	Rotational KE $KE_{rot} = \frac{1}{2}I\omega^2$
Conservation of momentum ($F_{net,ext} = 0$)	Conservation of L ($\tau_{net,ext} = 0$)

Table 6.2: *

The full translational–rotational analogy. Every rotational law can be obtained from its translational counterpart by the substitutions $m \to I$, $x \to \theta$, $v \to \omega$, $a \to \alpha$, $F \to \tau$, $p \to L$.

Khái niệm cốt lõi

This analogy is not a coincidence — it follows directly from writing every translational quantity for each small piece of a rigid body and summing over the whole body using r as the "lever arm." It is also a powerful problem-solving check: if you are unsure which rotational formula to use, write down the translational formula for the same physical idea (momentum, energy, impulse, Newton's second law) and apply the substitutions above.

GIẢI THÍCH TIẾNG VIỆT

VN Mọi đại lượng "quay" đều có một đại lượng "tịnh tiến" tương ứng: khối lượng $m \leftrightarrow$ mô-men quán tính I; lực $F \leftrightarrow$ mô-men xoắn τ; động lượng $p = mv \leftrightarrow$ động lượng góc $L = I\omega$; động năng $\frac{1}{2}mv^2 \leftrightarrow \frac{1}{2}I\omega^2$; xung lượng $F\Delta t = \Delta p \leftrightarrow \tau\Delta t = \Delta L$. Khi không chắc công thức "quay" nào cần dùng, hãy viết công thức "tịnh tiến" tương ứng rồi thay các đại lượng theo bảng trên

— đây là mẹo kiểm tra rất hữu ích trong phòng thi.

6.8 Combined Angular-Momentum and Energy Problems

Some of the richest rotation problems combine conservation of angular momentum with a separate, independent check on kinetic energy — exactly the two-step approach already used for linear collisions in Unit 4. This section develops three closely related scenarios: an object *joining* a rotating system, an object *leaving* a rotating system, and a moving particle being *captured* by a rotating rigid body. In every case, the first step is to identify the axis about which angular momentum is conserved (justified by zero net external torque about that axis), and the second, separate step is to compute kinetic energy before and after to see whether — and how much — it changed.

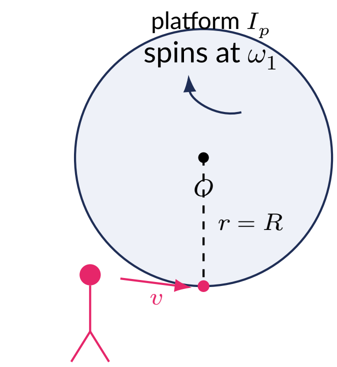
What is conserved, scenario by scenario:

- **An object joins a rotating system** (a person jumping onto a platform, a mud ball sticking to a rotating rod): angular momentum about the axis is conserved, but kinetic energy **decreases** — this is a *rotational perfectly inelastic collision*, the direct analog of two linear objects colliding and sticking together.
- **An object leaves a rotating system** (a person throwing a ball off a platform, an object flung outward from a rotating body): angular momentum of the whole system (thrower + thrown object together) is conserved, but kinetic energy **increases** — this is the rotational analog of an *explosion* (two objects, initially moving together, push apart), since some internal energy source (muscles, a spring) does positive work.
- **Mass redistributes within a single rotating body** (a skater pulling in her arms, a student walking along a rotating platform, Section 6.4): angular momentum is conserved, and kinetic energy changes according to the sign of the internal work done — this case was covered already in Section 6.4, and is included here only for completeness.

In every scenario, always check angular momentum conservation first (it requires only zero net external torque about the axis) and treat kinetic energy as a completely separate bookkeeping question.

VN GIẢI THÍCH TIẾNG VIỆT

Có ba dạng bài kết hợp bảo toàn động lượng góc với động năng: (1) **một vật ”nhập” vào hệ đang quay** (người nhảy lên bàn xoay, cục bùn dính vào đầu thanh đang quay) — động lượng góc bảo toàn nhưng động năng **giảm**, giống hệt va chạm mềm (hoàn toàn không đàn hồi) trong Unit 4; (2) **một vật ”tách” khỏi hệ đang quay** (người đứng trên bàn xoay ném một vật ra ngoài) — động lượng góc của toàn hệ vẫn bảo toàn nhưng động năng **tăng**, giống như một vụ ”nổ” (explosion) trong Unit 4, vì có công nội lực (cơ bắp) sinh thêm năng lượng; (3) **khối lượng tự phân bố lại bên trong một hệ** (vận động viên thu tay, người đi trên bàn xoay) — đã học ở mục 6.4. Trong mọi trường hợp, luôn kiểm tra bảo toàn L trước (chỉ cần mô-men xoắn ngoại lực bằng 0), rồi xét động năng như một bài toán hoàn toàn riêng biệt.



Top view: a person of mass m, running tangentially at speed v, jumps onto the rim (radius $r = R$) of a freely rotating platform (moment of inertia I_p, initial angular velocity ω_1). No external torque acts about the axis O during the jump, so angular momentum is conserved even though kinetic energy is not — a rotational perfectly inelastic collision.

6.8.1 An Object Joining a Rotating System: Rotational Perfectly Inelastic Collisions

Parallel to Unit 4: Elastic and Inelastic Collisions

Recall from Unit 4 that a *perfectly inelastic* linear collision — two objects colliding and sticking together — conserves momentum but loses kinetic energy: $m_1v_1 = (m_1+m_2)v_f$, with $KE_f < KE_i$. An object joining a rotating system behaves exactly the same way, with $m \to I$ and $v \to \omega$: angular momentum is conserved, $L_i = L_f$, but kinetic energy is not, because internal forces (the friction of the person's feet gripping the platform, or the mud deforming as it sticks) do negative work as the two pieces merge into one common angular velocity. Just as in Unit 4, an *elastic* rotational "collision" — one that conserves both L and KE — would require the two pieces to separate again afterward (bounce apart) rather than combine.

Ví dụ 13 · Student jumping onto a rotating platform

A circular platform (uniform disk), $M = 100$ kg, $R = 2.0$ m, is free to rotate on a frictionless bearing and is initially at rest. A $m = 40$ kg student runs tangent to the rim at $v = 3.0$ m/s and jumps on, landing right at the edge and sticking to the platform. Find the platform's angular velocity right after she lands, and compare the total kinetic energy just before and just after.

Moment of inertia of the platform: $I_p = \frac{1}{2}MR^2 = \frac{1}{2}(100)(2.0)^2 = 200\,\text{kg}\cdot\text{m}^2$.

Angular momentum about the central axis, just before landing (the student runs tangentially, so $\theta = 90^\circ$; the platform is at rest and contributes zero):

$$L_i = mvR\sin 90^\circ = (40)(3.0)(2.0) = 240\,\text{kg}\cdot\text{m}^2/\text{s}.$$

After landing, treat the student as a point mass at radius R, so the system's total moment of inertia is

$$I_f = I_p + mR^2 = 200 + (40)(2.0)^2 = 200 + 160 = 360\,\text{kg}\cdot\text{m}^2.$$

Angular momentum is conserved (no external torque about the axis):

$$\omega_f = \frac{L_i}{I_f} = \frac{240}{360} = 0.667\,\text{rad/s}.$$

Kinetic energy before (student only; the platform is at rest):

$$KE_i = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(40)(3.0)^2 = 180\,\text{J}.$$

Kinetic energy after:

$$KE_f = \tfrac{1}{2}I_f\omega_f^2 = \tfrac{1}{2}(360)(0.667)^2 = \tfrac{1}{2}(360)(0.445) = 80.1\,\text{J}.$$

Kinetic energy **decreases** from 180 J to about 80 J — a loss of roughly 100 J, or 55.6% — exactly as expected for a rotational perfectly inelastic collision: the student's feet and the platform surface do negative work on each other as she "sticks," converting some kinetic energy into heat and sound.

6.8.2 An Object Leaving a Rotating System: Rotational "Recoil"

Now reverse the process: a person standing on a rotating platform throws an object away from the axis. Just as an explosion splits one object at rest into two objects moving in opposite directions (each acquiring equal and opposite momentum, from Unit 4), throwing an object off a platform splits one system into two parts — the thrown object and the platform-plus-thrower — that must carry equal and opposite *changes* in angular momentum, so the total is exactly what it was before the throw.

Ví dụ 14 · Throwing a ball off-axis from a rotating platform

A student stands at rest on a frictionless rotating platform; together, student and platform have a combined moment of inertia $I_{sys} = 60\,\text{kg}\cdot\text{m}^2$ about the central axis O. She holds a ball of mass $m = 1.0\,\text{kg}$ at a distance $r = 1.0\,\text{m}$ from the axis and throws it horizontally; at the instant it leaves her hand, the ball moves at $v = 6.0\,\text{m/s}$, at an angle $\theta = 30^\circ$ to the line from O to the ball (i.e. not purely tangential). Find the angular velocity of the platform (with the student on it) immediately after the throw.

The ball's angular momentum about O, using $L = mvr\sin\theta$:

$$L_{ball} = mvr\sin\theta = (1.0)(6.0)(1.0)\sin 30^\circ = (1.0)(6.0)(1.0)(0.50) = 3.0\,\text{kg}\cdot\text{m}^2/\text{s}.$$

Before the throw, everything (student, platform, ball) is at rest, so the total angular momentum about O is zero. The platform's bearing is frictionless, so there is no net external torque about O during the throw — total angular momentum stays zero:

$$L_{ball} + I_{sys}\omega_f = 0 \;\Rightarrow\; \omega_f = -\frac{L_{ball}}{I_{sys}} = -\frac{3.0}{60} = -0.050\,\text{rad/s}.$$

The negative sign means the platform recoils in the direction *opposite* to the ball's angular momentum: the student and platform spin one way while the ball's angular momentum about O points the other way, so that the vector sum is still zero — the exact rotational analog of two objects recoiling apart in an explosion.

6.8.3 A Moving Particle Captured by a Rotating Body

When a small, fast-moving object strikes and sticks to the end of a larger rotating (or initially stationary) rigid body, its angular momentum just before impact is computed with the point-particle formula $L = mvr\sin\theta$, taken about the same axis as the rigid body's rotation; after impact, the combined object-plus-body system has a new, larger moment of inertia, $I_{total} = I_{body} + mr^2$ (treating the now-attached particle as a point mass at its sticking radius r).

Ví dụ 15 · Mud ball striking a rotating rod

A uniform rod of mass $M = 6.0\,\text{kg}$ and length $\ell = 1.0\,\text{m}$ is pivoted at one end on a frictionless horizontal axle and is initially at rest. A ball of mud, $m = 1.0\,\text{kg}$, moving at $v = 9.0\,\text{m/s}$ perpendicular to the rod, strikes and sticks to the rod's free end (a distance ℓ from the pivot). Find the angular velocity of the rod-plus-mud system immediately after impact, and compare

kinetic energy before and after.
Moment of inertia of the rod about the pivot: $I_{rod} = \frac{1}{3}M\ell^2 = \frac{1}{3}(6.0)(1.0)^2 = 2.0\,\text{kg}\cdot\text{m}^2$.
Angular momentum of the mud ball about the pivot, just before impact (it moves perpendicular to the rod, so $\theta = 90^\circ$, and $r = \ell$):

$$L_i = mv\ell \sin 90^\circ = (1.0)(9.0)(1.0) = 9.0\,\text{kg}\cdot\text{m}^2/\text{s}.$$

(The rod contributes zero angular momentum, since it starts at rest.) After the mud sticks, the system's total moment of inertia about the pivot is

$$I_f = I_{rod} + m\ell^2 = 2.0 + (1.0)(1.0)^2 = 3.0\,\text{kg}\cdot\text{m}^2.$$

Angular momentum is conserved (the only external forces — gravity and the pivot's force — act at or through the pivot, so they exert no torque about it):

$$\omega_f = \frac{L_i}{I_f} = \frac{9.0}{3.0} = 3.0\,\text{rad/s}.$$

Kinetic energy before (mud ball only):

$$KE_i = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(1.0)(9.0)^2 = 40.5\,\text{J}.$$

Kinetic energy after:

$$KE_f = \tfrac{1}{2}I_f\omega_f^2 = \tfrac{1}{2}(3.0)(3.0)^2 = 13.5\,\text{J}.$$

Kinetic energy drops from 40.5 J to 13.5 J — a loss of 66.7% — once again the signature of a perfectly inelastic "collision," this time between a linearly-moving particle and a rotating rigid body.

(!) Lỗi thường gặp: When an object joins a rotating body, its *own* contribution to the final moment of inertia is easy to forget. Always add mr^2 (treating the newly attached object as a point mass at its final distance r from the axis) to the rigid body's original moment of inertia to get I_f — using only the rigid body's original I will give a final angular velocity that is too large.

6.9 Practice Problems

Bài tập luyện tập

A solid disk and a thin hoop, with equal mass M and equal radius R, are released from rest at the top of the same incline and roll without slipping to the bottom. Which reaches the bottom first?

(A) The disk
(B) The hoop
(C) They arrive at the same time
(D) It depends on the incline angle

Lời giải chi tiết

The disk has $c = \frac{1}{2}$, smaller than the hoop's $c = 1$. A smaller c means both a larger final speed *and* a larger acceleration throughout the descent ($a = g\sin\theta/(1+c)$), so the disk arrives first regardless of the specific angle. **(A)**.

Bài tập luyện tập

A wheel with $I = 2.0\,\text{kg}\cdot\text{m}^2$, radius $R = 0.40\,\text{m}$, and mass $m = 5.0\,\text{kg}$ rolls without slipping at $v_{cm} = 6.0\,\text{m/s}$. Find its angular speed and its total kinetic energy.

Lời giải chi tiết

$\omega = v_{cm}/R = 6.0/0.40 = \boxed{15\,\text{rad/s}}$.
$KE_{rot} = \frac{1}{2}(2.0)(15)^2 = 225\,\text{J}$; $KE_{trans} = \frac{1}{2}(5.0)(6.0)^2 = 90\,\text{J}$.
$KE_{total} = 225 + 90 = \boxed{315\,\text{J}}$.

Bài tập luyện tập

A ball rolls without slipping across a floor. What is the instantaneous speed of the point on the ball currently touching the floor?

(A) v_{cm}
(B) Zero
(C) $2v_{cm}$
(D) $v_{cm}/2$

Lời giải chi tiết

Rolling without slipping means the contact point is instantaneously at rest. **(B)**.

Bài tập luyện tập

A solid sphere ($c = 2/5$) is released from rest and rolls without slipping down an incline, descending a vertical height $h = 3.5\,\text{m}$. Find its speed at the bottom.

(A) $5.94\,\text{m/s}$
(B) $6.50\,\text{m/s}$
(C) $7.00\,\text{m/s}$
(D) $8.29\,\text{m/s}$

Lời giải chi tiết

$v = \sqrt{2gh/(1+c)} = \sqrt{2(9.8)(3.5)/1.4} = \sqrt{68.6/1.4} = \sqrt{49} = 7.00\,\text{m/s}$. **(C)**.

Bài tập luyện tập

A solid sphere and a thin spherical shell, of equal mass and equal radius, are released together from rest at the top of an incline and both roll without slipping. Explain, in words, which one reaches the bottom first and why.

Lời giải chi tiết

The solid sphere reaches the bottom first. For any rolling object, $v_{cm} = \sqrt{2gh/(1+c)}$, where $c = I/(MR^2)$ is smaller when mass is concentrated closer to the axis. A solid sphere has $c = 2/5$, while a thin shell (mass concentrated at the outer radius) has the larger value $c = 2/3$. Because c is smaller for the solid sphere, a smaller fraction of the released gravitational potential energy is converted into rotational kinetic energy, leaving a larger fraction for translational kinetic energy — so the solid sphere has both a greater final speed and a greater acceleration throughout the trip. This conclusion does not depend on the common mass or radius, since both cancel out of the formula for v_{cm}.

Bài tập luyện tập

A frictionless block slides (without rotating) down a smooth incline of height h, while an identical-height incline is used for a hoop that rolls without slipping. Compare their final speeds at the bottom.

(A) The block is faster

(B) The hoop is faster

(C) They are equal

(D) Cannot be determined

Lời giải chi tiết

The block has $c = 0$, so $v_{block} = \sqrt{2gh}$; the hoop has $c = 1$, so $v_{hoop} = \sqrt{2gh/2} = \sqrt{gh}$. Since $\sqrt{2gh} > \sqrt{gh}$ for any $h > 0$, the sliding block is always faster than a rolling hoop released from the same height. **(A)**.

Bài tập luyện tập

A thin spherical shell, $M = 2.0\,\text{kg}$, $R = 0.15\,\text{m}$, rolls without slipping at $\omega = 12\,\text{rad/s}$. Find v_{cm}, KE_{trans}, KE_{rot}, and KE_{total}.

Lời giải chi tiết

$v_{cm} = \omega R = (12)(0.15) = \boxed{1.8\,\text{m/s}}$.
$I = \frac{2}{3}MR^2 = \frac{2}{3}(2.0)(0.15)^2 = 0.030\,\text{kg}\cdot\text{m}^2$.
$KE_{trans} = \frac{1}{2}(2.0)(1.8)^2 = \boxed{3.24\,\text{J}}$; $KE_{rot} = \frac{1}{2}(0.030)(12)^2 = \boxed{2.16\,\text{J}}$.
$KE_{total} = 3.24 + 2.16 = \boxed{5.40\,\text{J}}$.

Bài tập luyện tập

A $3.0\,\text{kg}$ particle moves at a constant $v = 5.0\,\text{m/s}$ along a straight line whose perpendicular distance from the origin O is $2.0\,\text{m}$. Find its angular momentum about O.

(A) $7.5\,\text{kg}\cdot\text{m}^2/\text{s}$

(B) $15\,\text{kg}\cdot\text{m}^2/\text{s}$

(C) $30\,\text{kg}\cdot\text{m}^2/\text{s}$

(D) $60\,\text{kg}\cdot\text{m}^2/\text{s}$

Lời giải chi tiết

$L = r_\perp mv = (2.0)(3.0)(5.0) = 30\,\text{kg}\cdot\text{m}^2/\text{s}$. **(C)**.

Bài tập luyện tập

A particle moves in a straight line at constant velocity, subject to zero net force. As it travels farther from a fixed reference point O (not on its path), its angular momentum about O:

(A) Increases

(B) Decreases

(C) Stays exactly the same

(D) Oscillates

Lời giải chi tiết

With zero net force, the particle experiences zero net torque about any point, so L about O is conserved. Equivalently, r and θ both change as the particle moves, but $r_\perp = r\sin\theta$ (the perpendicular distance to the line of motion) never changes, so $L = mvr_\perp$ stays fixed. **(C).**

Bài tập luyện tập

A merry-go-round, modeled as a solid disk, $M = 200\,\text{kg}$, $R = 2.0\,\text{m}$, spins at $\omega = 1.5\,\text{rad/s}$. Find its moment of inertia and angular momentum.

Lời giải chi tiết

$I = \frac{1}{2}MR^2 = \frac{1}{2}(200)(2.0)^2 = \boxed{400\,\text{kg}\cdot\text{m}^2}$.
$L = I\omega = (400)(1.5) = \boxed{600\,\text{kg}\cdot\text{m}^2/\text{s}}$.

Bài tập luyện tập

A $50\,\text{kg}$ student stands at the rim, $r_1 = 2.0\,\text{m}$, of a frictionless rotating platform with $I_{platform} = 300\,\text{kg}\cdot\text{m}^2$, spinning at $\omega_1 = 1.0\,\text{rad/s}$. She walks inward to $r_2 = 0.50\,\text{m}$ from the center. Treating her as a point mass, find the platform's new angular velocity.

Lời giải chi tiết

$I_1 = I_{platform} + mr_1^2 = 300 + (50)(2.0)^2 = 300 + 200 = 500\,\text{kg}\cdot\text{m}^2$.
$I_2 = I_{platform} + mr_2^2 = 300 + (50)(0.50)^2 = 300 + 12.5 = 312.5\,\text{kg}\cdot\text{m}^2$.
Angular momentum conserved: $I_1\omega_1 = I_2\omega_2 \Rightarrow \omega_2 = \dfrac{(500)(1.0)}{312.5} = \boxed{1.6\,\text{rad/s}}$.

Bài tập luyện tập

For the student and platform of the previous problem, as she walks toward the center, the system's total rotational kinetic energy:

(A) Increases
(B) Decreases
(C) Stays the same
(D) Cannot be determined

Lời giải chi tiết

$KE_1 = \frac{1}{2}(500)(1.0)^2 = 250\,\text{J}$; $KE_2 = \frac{1}{2}(312.5)(1.6)^2 = 400\,\text{J}$. Kinetic energy increases because the student does positive work on herself (and the platform) as her muscles pull her inward against the effective outward tendency of her rotating body — angular momentum is conserved, but kinetic energy is not. **(A).**

Bài tập luyện tập

Disk A, $I_A = 4.0\,\text{kg}\cdot\text{m}^2$, spins at $\omega_A = 8.0\,\text{rad/s}$. Disk B, $I_B = 12\,\text{kg}\cdot\text{m}^2$, is initially at rest on the same frictionless axle. They are pressed together and reach a common angular velocity. Find ω_f and the fraction of kinetic energy lost.

Lời giải chi tiết

$L_i = I_A\omega_A = (4.0)(8.0) = 32\,\text{kg}\cdot\text{m}^2/\text{s}$; $I_{tot} = 4.0 + 12 = 16\,\text{kg}\cdot\text{m}^2$.
$\omega_f = 32/16 = \boxed{2.0\,\text{rad/s}}$.
$KE_i = \frac{1}{2}(4.0)(8.0)^2 = 128\,\text{J}$; $KE_f = \frac{1}{2}(16)(2.0)^2 = 32\,\text{J}$.
Fraction lost $= \dfrac{128-32}{128} = \boxed{0.75\ (75\%)}$.

Bài tập luyện tập

Two disks are pressed together on a shared frictionless axle and lock into a common angular velocity, exactly as in the previous problem. This process is the rotational analog of which linear-momentum scenario?

(A) An elastic collision
(B) A perfectly inelastic collision
(C) An explosion (objects separating)
(D) Two isolated, non-interacting objects

Lời giải chi tiết

Momentum (L) is conserved but kinetic energy is lost as the two bodies merge into one common motion — this is the exact rotational parallel of a perfectly inelastic linear collision, where two objects collide and stick together. **(B)**.

Bài tập luyện tập

A constant torque $\tau = 8.0\,\text{N}\cdot\text{m}$ acts on a rotor with $I = 4.0\,\text{kg}\cdot\text{m}^2$, initially spinning at $\omega_i = 3.0\,\text{rad/s}$, for $\Delta t = 6.0\,\text{s}$, in the same direction as the initial spin. Find the final angular velocity.

Lời giải chi tiết

$L_i = I\omega_i = (4.0)(3.0) = 12\,\text{kg}\cdot\text{m}^2/\text{s}$.
Angular impulse $= \tau\Delta t = (8.0)(6.0) = 48\,\text{kg}\cdot\text{m}^2/\text{s} = \Delta L$.
$L_f = 12 + 48 = 60\,\text{kg}\cdot\text{m}^2/\text{s} \Rightarrow \omega_f = 60/4.0 = \boxed{15\,\text{rad/s}}$.

Bài tập luyện tập

A constant torque of $10\,\text{N}\cdot\text{m}$ acts on a rotor with $I = 5.0\,\text{kg}\cdot\text{m}^2$, starting from rest, for $4.0\,\text{s}$. Find its angular velocity at $t = 4.0\,\text{s}$.

(A) $2.0\,\text{rad/s}$
(B) $4.0\,\text{rad/s}$
(C) $8.0\,\text{rad/s}$
(D) $16\,\text{rad/s}$

Lời giải chi tiết

$\Delta L = \tau\Delta t = (10)(4.0) = 40\,\text{kg}\cdot\text{m}^2/\text{s}$. $\omega_f = \Delta L/I = 40/5.0 = 8.0\,\text{rad/s}$. **(C)**.

Bài tập luyện tập

A τ–t graph decreases linearly from $12\,\text{N}\cdot\text{m}$ at $t = 0$ to 0 at $t = 6.0\,\text{s}$. A rotor with $I = 3.0\,\text{kg}\cdot\text{m}^2$ starts at rest. Find its angular velocity at $t = 6.0\,\text{s}$.

Lời giải chi tiết

Angular impulse = area of the triangle $= \frac{1}{2}(6.0)(12) = 36\,\text{kg}\cdot\text{m}^2/\text{s} = \Delta L$.
$\omega_f = \Delta L/I = 36/3.0 = \boxed{12\,\text{rad/s}}$.

Bài tập luyện tập

Explain, in words, why angular momentum is conserved for a spinning figure skater who pulls her arms inward, but her rotational kinetic energy is not conserved. Identify the source of the extra energy.

Lời giải chi tiết

The only forces acting on the skater (gravity and the normal force from the ice) act essentially along her spin axis or through it, so they exert no net external torque about that axis. With zero net external torque, angular momentum $L = I\omega$ must remain exactly constant throughout the motion, which is why $I_1\omega_1 = I_2\omega_2$ applies. However, the conservation law for L places no restriction on kinetic energy. As the skater pulls her arms inward, her own muscles must do positive work against the outward tendency of her rotating limbs (in the rotating frame, this feels like pulling against a centrifugal effect); this internal muscular work adds mechanical energy to her body. Since I decreases while L stays fixed, $\omega = L/I$ increases, and because $KE_{rot} = \frac{1}{2}I\omega^2 = L^2/(2I)$, a smaller I with the same L directly produces a larger KE_{rot}. The extra kinetic energy comes from the chemical energy the skater's muscles convert into mechanical work, not from any external agent.

Bài tập luyện tập

Which pair correctly matches a translational quantity with its rotational analog?

(A) Momentum $p = mv \leftrightarrow$ angular momentum $L = I\omega$

(B) Force $F \leftrightarrow$ moment of inertia I

(C) Mass $m \leftrightarrow$ torque τ

(D) Impulse $J \leftrightarrow$ moment of inertia I

Lời giải chi tiết

Momentum's rotational analog replaces mass with moment of inertia and velocity with angular velocity: $L = I\omega$. **(A)**.

Bài tập luyện tập

A wheel experiences a torque of $20\,\text{N}\cdot\text{m}$, producing an angular acceleration of $4.0\,\text{rad/s}^2$. Starting from rest, the torque acts for $3.0\,\text{s}$. Find the wheel's moment of inertia, its final angular velocity, and its rotational kinetic energy gained.

Lời giải chi tiết

$I = \tau/\alpha = 20/4.0 = \boxed{5.0\,\text{kg}\cdot\text{m}^2}$.
$\omega_f = \alpha t = (4.0)(3.0) = \boxed{12\,\text{rad/s}}$.
$KE_{rot} = \frac{1}{2}(5.0)(12)^2 = \boxed{360\,\text{J}}$.
Check via angular impulse: $\Delta L = \tau t = (20)(3.0) = 60 = I\omega_f = (5.0)(12) = 60$ — matches.

Bài tập luyện tập

A figure skater spins with her arms extended, then pulls them in close to her body. Which of the following quantities **increases**?

(A) Her angular momentum

(B) Her moment of inertia

(C) Her angular velocity

(D) Her mass

Lời giải chi tiết

Angular momentum is conserved (unchanged) and moment of inertia and mass both decrease or stay fixed; only angular velocity increases, since $\omega = L/I$ with L fixed and I decreasing. **(C).**

Bài tập luyện tập

A 0.20 kg puck moves on frictionless ice in a circle of radius $r_1 = 1.2$ m at $v_1 = 3.0$ m/s, held by a string through a frictionless hole. The string is pulled in until the radius is $r_2 = 0.30$ m. Find the new speed, compare the initial and final kinetic energy, and explain (in words) how the kinetic energy can change even though the string tension is always perpendicular to the puck's velocity.

Lời giải chi tiết

Angular momentum about the hole is conserved because tension is always radial (zero torque about the hole): $v_2 = v_1 r_1/r_2 = (3.0)(1.2)/0.30 = \boxed{12\,\text{m/s}}$.
$KE_1 = \frac{1}{2}(0.20)(3.0)^2 = 0.90$ J; $KE_2 = \frac{1}{2}(0.20)(12)^2 = 14.4$ J — the kinetic energy increases sixteen-fold.
This is not a contradiction: tension is perpendicular to the velocity only at each *instant* of pure circular motion at fixed radius. While the radius is actively shrinking, the puck's path spirals slightly inward, so its actual displacement has a small component along the direction of the tension force; this component allows tension to do positive work on the puck over time even though, instant by instant, the torque about the hole (and hence the rate of change of L) stays zero. The net effect is that whoever pulls the string does positive mechanical work on the puck, and that work becomes the extra kinetic energy.

Bài tập luyện tập

For a particle in uniform circular motion (constant radius r, constant speed v) about a fixed center, which of the following is constant?

(A) Only its speed

(B) Only its angular momentum about the center

(C) Both its speed and its angular momentum about the center

(D) Neither

Lời giải chi tiết

Speed is constant by definition of uniform circular motion. The angle between $\vec{r}$ and $\vec{v}$ is always 90° for circular motion, so $L = mvr \sin 90^\circ = mvr$ is also constant, since m, v, and r are all fixed. **(C).**

Bài tập luyện tập

A solid cylinder ($c = 1/2$) rolls without slipping down an incline of angle $\theta = 20^\circ$ ($\sin 20^\circ = 0.342$, $\cos 20^\circ = 0.940$, $\tan 20^\circ = 0.364$). Find its linear acceleration and the minimum coefficient of static friction needed to prevent slipping.

Lời giải chi tiết

$$a = \frac{g\sin\theta}{1+c} = \frac{(9.8)(0.342)}{1.5} = \frac{3.352}{1.5} = \boxed{2.23\,\text{m/s}^2}.$$

$$\mu_{\text{min}} = \frac{c}{1+c}\tan\theta = \frac{0.5}{1.5}(0.364) = (0.333)(0.364) = \boxed{0.121}.$$

Bài tập luyện tập

For an object rolling without slipping down an incline (static friction present, no skidding), the work done by static friction on the object is:

(A) Positive

(B) Negative

(C) Zero

(D) Cannot be determined without μ

Lời giải chi tiết

The contact point has zero velocity at every instant during rolling without slipping, so the point of application of the friction force never moves — work $= F \cdot d$ requires displacement of the point of application, which is zero. **(C)**.

Bài tập luyện tập

A solid cylinder, $M = 4.0\,\text{kg}$, $R = 0.25\,\text{m}$, rolls without slipping across a floor at $v = 2.0\,\text{m/s}$, then rolls up a ramp without slipping and without friction losses. Find the maximum height it reaches before momentarily coming to rest.

Lời giải chi tiết

$c = \frac{1}{2}$ for a solid cylinder. Total kinetic energy converts entirely to gravitational PE:

$$KE_{total} = \tfrac{1}{2}Mv^2(1+c) = \tfrac{1}{2}(4.0)(2.0)^2(1.5) = 8.0(1.5) = 12\,\text{J}.$$

$$h = \frac{KE_{total}}{Mg} = \frac{12}{(4.0)(9.8)} = \frac{12}{39.2} = \boxed{0.306\,\text{m}}.$$

Bài tập luyện tập

A solid sphere and a thin hoop, of equal mass and equal radius, both roll without slipping and happen to have the *same* translational speed v_{cm} at some instant. Compare their total kinetic energies at that instant.

(A) Equal

(B) The sphere has more kinetic energy

(C) The hoop has more kinetic energy

(D) Cannot be determined without the radius

Lời giải chi tiết

$KE_{total} = \frac{1}{2}Mv_{cm}^2(1+c)$. At the same v_{cm}, the factor $(1+c)$ is larger for the hoop ($c=1$, so $1+c=2$) than for the sphere ($c=2/5$, so $1+c=1.4$), so the hoop has more total kinetic energy — it takes more energy for the hoop to reach the same translational speed, because more of its mass sits far from the axis. **(C)**.

Bài tập luyện tập

A 0.50 kg ball moves at a constant $v = 10$ m/s. At a certain instant, its position vector relative to origin O has magnitude $r = 6.0$ m, and the angle between the position vector and the velocity vector is $30°$. Find the ball's angular momentum about O at that instant.

Lời giải chi tiết

$L = mvr\sin\theta = (0.50)(10)(6.0)\sin 30° = (0.50)(10)(6.0)(0.50) = \boxed{15\,\text{kg}\cdot\text{m}^2/\text{s}}$.

Bài tập luyện tập

If a net external torque acts on a system for a very short time, delivering a large angular impulse, which quantity changes as a direct result?

(A) Its moment of inertia

(B) Its angular momentum

(C) Its mass

(D) Its angular position only

Lời giải chi tiết

Angular impulse is defined as $\tau\Delta t = \Delta L$, so a large angular impulse directly produces a large change in angular momentum. **(B)**.

Bài tập luyện tập

Explain, using the translational–rotational analogy, why $\tau_{net} = I\alpha$ is considered the rotational version of Newton's second law, and identify what plays the role of "inertia" (resistance to a change in motion) in the rotational case.

Lời giải chi tiết

Newton's second law for translation, $F_{net} = ma$, states that the net force needed to produce a given acceleration is proportional to the object's mass — mass measures an object's resistance to a change in *linear* velocity. The rotational law $\tau_{net} = I\alpha$ has exactly the same structure: the net torque needed to produce a given angular acceleration is proportional to the object's moment of inertia I. Just as F replaces with τ and a replaces with α, mass m is replaced by moment of inertia I, which therefore plays the role of "rotational inertia." Crucially, I depends not only on how much mass an object has but on *how that mass is distributed relative to the rotation axis* (as summarized by the shape factor $c = I/(MR^2)$) — two objects with identical mass can have very different resistance to angular acceleration if their mass is arranged differently, which has no analog in the purely translational case where only total mass matters.

Bài tập luyện tập

A hollow cylindrical shell (thin-walled pipe, $c = 1$) and a solid sphere ($c = 2/5$), released from rest at the same height on the same incline and both rolling without slipping, are timed over the same distance. The solid sphere takes 2.00 s to reach the bottom. Is the hollow shell's time greater than, less than, or equal to 2.00 s? Justify your answer without computing an exact value.

Lời giải chi tiết

The hollow shell's time is greater than 2.00 s. Both objects start from rest and accelerate down the same incline, with acceleration $a = g\sin\theta/(1+c)$. Since the shell's shape factor ($c = 1$) is larger than the sphere's ($c = 2/5$), the shell has a smaller acceleration at every instant of the descent. A smaller constant acceleration over the same distance, starting from the same rest condition, always requires more time (from $\Delta x = \frac{1}{2}at^2$, a smaller a requires a larger t^2 to reach the same Δx). Therefore the hollow shell arrives later than the solid sphere, so its time is greater than 2.00 s.

Bài tập luyện tập

A running person jumps onto the rim of a stationary, freely rotating platform (frictionless bearing) and sticks to it. Which of the following is conserved during the process?

(A) Both angular momentum and kinetic energy

(B) Angular momentum only

(C) Kinetic energy only

(D) Neither

Lời giải chi tiết

There is no external torque about the platform's axis during the jump (the person's feet exert only internal forces as they grip the rim), so angular momentum is conserved. However, this is a rotational perfectly inelastic collision: the person's feet and the platform surface do negative work on each other as they lock together, so kinetic energy is lost. **(B)**.

Bài tập luyện tập

A circular platform (uniform disk), $M = 120$ kg, $R = 2.0$ m, is free to rotate on a frictionless bearing and is initially at rest. A $m = 40$ kg person runs tangent to the rim at $v = 5.0$ m/s and jumps on, landing at the edge and sticking to the platform. Find (a) the platform's angular velocity right after landing and (b) the total kinetic energy just before and just after the jump.

Lời giải chi tiết

$I_p = \frac{1}{2}MR^2 = \frac{1}{2}(120)(2.0)^2 = 240\,\text{kg}\cdot\text{m}^2$.
$L_i = mvR = (40)(5.0)(2.0) = 400\,\text{kg}\cdot\text{m}^2/\text{s}$ (platform at rest contributes zero).
$I_f = I_p + mR^2 = 240 + (40)(2.0)^2 = 240 + 160 = 400\,\text{kg}\cdot\text{m}^2$.
(a) $\omega_f = L_i/I_f = 400/400 = \boxed{1.0\,\text{rad/s}}$.
(b) $KE_i = \frac{1}{2}mv^2 = \frac{1}{2}(40)(5.0)^2 = \boxed{500\,\text{J}}$; $KE_f = \frac{1}{2}I_f\omega_f^2 = \frac{1}{2}(400)(1.0)^2 = \boxed{200\,\text{J}}$.
Kinetic energy drops by 300 J, a loss of 60% — consistent with a rotational perfectly inelastic collision.

Bài tập luyện tập

A person stands on a frictionless rotating platform spinning at constant ω_0, holding a ball at distance r from the central axis O. She tosses the ball so that, at the instant it leaves her hand, its velocity is directed exactly along the line connecting O and the ball (purely radially, directly away from the axis). Compared to ω_0, the platform's angular velocity (with her still on it) immediately after the toss is:

(A) Larger

(B) Smaller

(C) Unchanged

(D) Reversed in direction

Lời giải chi tiết

Even though the ball's velocity is radial ($\theta = 0°$, so $L_{ball} = mvr \sin 0° = 0$ — it carries away *zero* angular momentum about O), the platform speeds up. Before the toss, the whole system (person + platform + ball, all rotating together) has total angular momentum $L_{total} = I_{total}\,\omega_0$, conserved because the frictionless bearing exerts no external torque about O. Since the ball leaves with $L_{ball} = 0$, the entire original L_{total} must now belong to the remaining person-plus-platform system alone: $L_{total} = I_{remain}\,\omega_{new}$. But $I_{remain} < I_{total}$, since the ball's own contribution mr^2 is no longer part of the rotating system, so $\omega_{new} = L_{total}/I_{remain} > \omega_0$ — the platform spins **faster**. **(A)**.

Bài tập luyện tập

A student together with a platform on which she stands (combined moment of inertia, including the ball she holds at $r = 1.0\,\text{m}$ from the axis, $I_{total} = 50\,\text{kg}\cdot\text{m}^2$) spins at $\omega_1 = 1.0\,\text{rad/s}$. She throws the ball, mass $m = 2.0\,\text{kg}$, so that it leaves her hand at $v = 4.0\,\text{m/s}$ tangentially ($\theta = 90°$), in the same rotational sense as the platform's spin. Find the platform's (with her still on it) angular velocity immediately after the throw.

Lời giải chi tiết

Total angular momentum before the throw: $L_{total} = I_{total}\omega_1 = (50)(1.0) = 50\,\text{kg}\cdot\text{m}^2/\text{s}$.
Ball's angular momentum after leaving her hand: $L_{ball} = mvr\sin\theta = (2.0)(4.0)(1.0)\sin 90° = 8.0\,\text{kg}\cdot\text{m}^2/\text{s}$.
Remaining moment of inertia of student + platform (ball's own contribution removed): $I_{remain} = I_{total} - mr^2 = 50 - (2.0)(1.0)^2 = 48\,\text{kg}\cdot\text{m}^2$.
Angular momentum conserved: $L_{total} = L_{ball} + I_{remain}\omega_2 \;\Rightarrow\; 50 = 8.0 + 48\,\omega_2 \;\Rightarrow\; \omega_2 = \dfrac{42}{48} = \boxed{0.875\,\text{rad/s}}$.
The platform slows down slightly, since the ball carries away angular momentum in the same rotational sense as the spin.

Bài tập luyện tập

A uniform rod, $M = 3.0\,\text{kg}$, $\ell = 0.60\,\text{m}$, is pivoted at one end on a frictionless axle and is initially at rest. A ball of mud, $m = 0.20\,\text{kg}$, moving at $v = 3.6\,\text{m/s}$ perpendicular to the rod, strikes and sticks to the rod's free end. Find the angular velocity of the rod-plus-mud system immediately after impact, and the percentage of kinetic energy lost.

Lời giải chi tiết

$I_{rod} = \frac{1}{3}M\ell^2 = \frac{1}{3}(3.0)(0.60)^2 = 0.36\,\text{kg}\cdot\text{m}^2$.
$L_i = mv\ell = (0.20)(3.6)(0.60) = 0.432\,\text{kg}\cdot\text{m}^2/\text{s}$ (rod contributes zero, since it starts at rest).
$I_f = I_{rod} + m\ell^2 = 0.36 + (0.20)(0.60)^2 = 0.36 + 0.072 = 0.432\,\text{kg}\cdot\text{m}^2$.
$\omega_f = L_i/I_f = 0.432/0.432 = \boxed{1.0\,\text{rad/s}}$.
$KE_i = \frac{1}{2}mv^2 = \frac{1}{2}(0.20)(3.6)^2 = 1.296\,\text{J}$; $KE_f = \frac{1}{2}I_f\omega_f^2 = \frac{1}{2}(0.432)(1.0)^2 = 0.216\,\text{J}$.
Fraction lost $= \dfrac{1.296 - 0.216}{1.296} = \boxed{0.833\ (83.3\%)}$.

Bài tập luyện tập

A hoop and a solid disk, of equal mass M and equal radius R, both roll without slipping. If the two objects happen to have equal rotational kinetic energy at some instant, which one has the greater translational speed v_{cm} at that instant?

(A) The hoop
(B) The disk
(C) They are equal
(D) Cannot be determined without R

Lời giải chi tiết

Using $\omega = v_{cm}/R$ and $I = cMR^2$: $KE_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}cMv_{cm}^2$. For equal KE_{rot} and equal M: $c_{disk}v_{disk}^2 = c_{hoop}v_{hoop}^2$. Since $c_{disk} = \frac{1}{2} < c_{hoop} = 1$, matching the same KE_{rot} requires $v_{disk}^2 = 2v_{hoop}^2$, i.e. $v_{disk} > v_{hoop}$ — the disk needs a larger translational speed to store the same amount of energy in rotation, because less of its mass sits far from the axis. **(B)**.

Bài tập luyện tập

An object of unspecified shape is released from rest at the top of an incline, a height $h_A = 4.9\,\text{m}$ above the bottom, and rolls without slipping down to the bottom. It then rolls without slipping up a second incline on the other side (also no energy lost to sliding) and momentarily comes to rest at a maximum height h_B. Find h_B, and explain why the object's shape does not need to be known.

Lời giải chi tiết

Since the object rolls without slipping throughout (static friction does zero work at every instant, per the pitfall in Section 6.3), no mechanical energy is lost anywhere in the trip. All of the energy released going down, Mgh_A, is fully available to be regained going back up: $Mgh_A = Mgh_B \Rightarrow h_B = h_A = \boxed{4.9\,\text{m}}$. The shape factor c affects *how* the energy is split between translational and rotational kinetic energy at any instant along the way, but by the time the object has come to rest again at the top of the second incline, all of that energy — translational and rotational alike — has converted back into gravitational PE, so c never enters the final height at all.

Bài tập luyện tập

A $2.0\,\text{kg}$ particle has a position vector of magnitude $r = 4.0\,\text{m}$ from point O, moving at $v = 5.0\,\text{m/s}$. At a certain instant, the angle between its position vector and its velocity vector is $37°$ ($\sin 37° = 0.60$). Find the particle's angular momentum about O at that instant.

(A) $12\,\text{kg}\cdot\text{m}^2/\text{s}$
(B) $24\,\text{kg}\cdot\text{m}^2/\text{s}$
(C) $40\,\text{kg}\cdot\text{m}^2/\text{s}$
(D) $60\,\text{kg}\cdot\text{m}^2/\text{s}$

Lời giải chi tiết

$L = mvr\sin\theta = (2.0)(5.0)(4.0)(0.60) = 24\,\text{kg}\cdot\text{m}^2/\text{s}$. **(B)**.

Bài tập luyện tập

A rotor with $I = 6.0\,\text{kg}\cdot\text{m}^2$ spins at $\omega_i = 15\,\text{rad/s}$. A constant braking torque of magnitude $9.0\,\text{N}\cdot\text{m}$, directed opposite to the spin, acts on it for $\Delta t = 4.0\,\text{s}$. Find the rotor's final angular velocity (magnitude and direction relative to the initial spin).

Lời giải chi tiết

$L_i = I\omega_i = (6.0)(15) = 90\,\text{kg}\cdot\text{m}^2/\text{s}$.
Angular impulse (opposing the spin, so negative) $= -\tau\Delta t = -(9.0)(4.0) = -36\,\text{kg}\cdot\text{m}^2/\text{s}$.
$L_f = 90 - 36 = 54\,\text{kg}\cdot\text{m}^2/\text{s} \Rightarrow \omega_f = 54/6.0 = \boxed{9.0\,\text{rad/s}}$, still in the **same rotational direction** as the initial spin (just slower), since the braking angular impulse ($36\,\text{kg}\cdot\text{m}^2/\text{s}$) is smaller in magnitude than L_i.

Bài tập luyện tập

In Scenario 1, a person standing still jumps tangentially onto the rim of a stationary, frictionless rotating platform and sticks to it. In Scenario 2, a person already spinning together with a platform throws a ball tangentially off the edge. In both scenarios, angular momentum about the central axis is conserved. Using **both** an angular-momentum argument and an energy/work argument, explain why the total kinetic energy of the system decreases in Scenario 1 but increases in Scenario 2.

Lời giải chi tiết

In both scenarios, the platform's bearing is frictionless, so the only torques involved (the push of the person's feet on the platform, the push of the person's hand on the ball) are *internal* to the system — there is no net *external* torque about the central axis in either case, which is why angular momentum about that axis is exactly conserved in both: $L_i = L_f$. Conservation of L, however, says nothing at all about kinetic energy; that must be tracked separately by asking what internal forces are doing.
In Scenario 1 (joining), the person's feet and the platform surface grip each other and are forced to match a single common angular velocity almost instantly. This is a rotational perfectly inelastic collision: exactly as when two linear objects collide and stick together, the internal contact forces between the person's feet and the platform do negative work on the system as they deform slightly and generate friction, heat, and sound while matching speeds. That negative internal work removes mechanical energy from the system, so the total kinetic energy after the jump is less than before — momentum-type conservation (L) survives, but energy does not.
In Scenario 2 (separating), the person's arm actively pushes the ball outward and forward, over a nonzero throwing distance, before the ball leaves her hand. This is the rotational analog of an explosion: the person's muscles supply chemical/internal energy and do *positive* work on the ball (and, by Newton's third law, an equal and opposite push back on themselves and the

platform) during the throw. That positive internal work adds mechanical energy to the system, so the combined kinetic energy of the ball and the remaining platform-plus-person, added together, is greater after the throw than before — even though their angular momenta, added together (with appropriate signs), still sum to the same constant total L.
The general lesson: conservation of angular momentum follows purely from zero net external torque and holds regardless of what the internal forces do; whether kinetic energy rises or falls depends entirely on whether those internal forces do positive work (an "explosion," Scenario 2) or negative work (a "sticking" collision, Scenario 1) — exactly the same logic used for linear collisions and explosions in Unit 4.

Oscillations

Mục tiêu Unit: Chuyển động tuần hoàn và dao động điều hòa (SHM); lực hồi phục $F = -kx$ và gia tốc $a = -\omega^2 x$; biên độ, chu kỳ, tần số, tần số góc; phương trình $x(t)$, $v(t)$, $a(t)$ và mối quan hệ pha; con lắc lò xo $T = 2\pi\sqrt{m/k}$; con lắc đơn $T = 2\pi\sqrt{L/g}$; năng lượng trong SHM; đọc và phân tích đồ thị dao động.

7.1 Periodic Motion and Simple Harmonic Motion

Periodic motion is any motion that repeats itself at regular time intervals: a swinging pendulum, a vibrating guitar string, a mass bobbing on a spring, even the orbit of a planet. The most important special case in this unit is **simple harmonic motion (SHM)**, the smooth back-and-forth motion produced whenever the restoring force pulling a system back toward equilibrium is directly proportional to displacement from that equilibrium.

7.1.1 Defining Simple Harmonic Motion

Khái niệm cốt lõi

A system undergoes **simple harmonic motion** when the net force on it (and therefore its acceleration) is proportional to its displacement x from equilibrium and always directed *opposite* to that displacement:

$$F = -kx \qquad \Longrightarrow \qquad a = \frac{F}{m} = -\frac{k}{m}x = -\omega^2 x,$$

where k is a positive constant (stiffness) and $\omega^2 \equiv k/m$. The minus sign is essential: it guarantees the force always points back toward $x = 0$, never away from it. Whenever a displaced object is pushed back, it builds up speed, overshoots equilibrium, decelerates on the far side, and reverses — producing smooth, repeating motion whose shape is exactly a cosine or sine curve. That is why SHM position vs. time is always **sinusoidal**.

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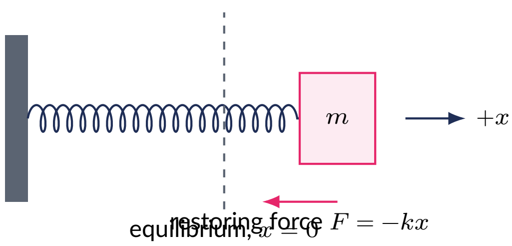
Dao động điều hòa (SHM) xảy ra khi lực hồi phục (và do đó gia tốc) tỉ lệ thuận với độ dịch chuyển x khỏi vị trí cân bằng và luôn hướng **ngược chiều** với x: $F = -kx$, suy ra $a = -\omega^2 x$ với $\omega^2 = k/m$. Dấu trừ đảm bảo lực luôn kéo vật về vị trí cân bằng. Chính vì lực luôn "kéo ngược" tỉ lệ với độ lệch mà đồ thị vị trí theo thời gian có dạng hình sin/cosin mượt và lặp lại.



A block of mass m on a spring of constant k, displaced a distance x to the right of equilibrium. The spring always pulls it back toward $x = 0$ with force $F = -kx$.

Ví dụ 1 · Restoring force, acceleration, and ω

A 0.50 kg block on a horizontal spring of constant $k = 200$ N/m is pulled 0.10 m from equilibrium and released. Find the restoring force and acceleration at the moment of release, and verify these are consistent with $a = -\omega^2 x$.

Force: $F = -kx = -(200)(0.10) = -20$ N (i.e., 20 N back toward equilibrium).

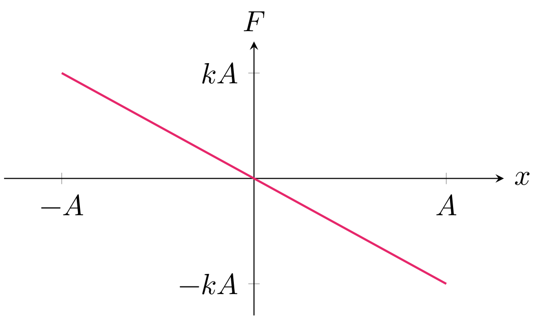
Acceleration: $a = \frac{F}{m} = \frac{-20}{0.50} = -40\,\text{m/s}^2$.

Angular frequency: $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{0.50}} = \sqrt{400} = 20$ rad/s.

Check: $-\omega^2 x = -(20)^2(0.10) = -(400)(0.10) = -40\,\text{m/s}^2$ — matches exactly.

7.1.2 Restoring Force and Hooke's Law Revisited

The straight-line relationship $F = -kx$ is **Hooke's law**, already familiar from spring forces. Graphing F against x for an ideal spring gives a straight line through the origin with slope $-k$: the further the spring is stretched or compressed, the harder it pulls or pushes back, always toward $x = 0$.



Restoring force vs. displacement for an ideal spring: $F = -kx$, a straight line through the origin with slope $-k$.

GIẢI THÍCH TIẾNG VIỆT

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Đồ thị F theo x của lò xo lý tưởng là một đường thẳng qua gốc tọa độ với độ dốc $-k$: độ lệch càng lớn thì lực hồi phục càng lớn, và luôn hướng về vị trí cân bằng ($x = 0$). Đây chính là điều kiện toán học định nghĩa dao động điều hòa — không phải mọi lực hồi phục đều tạo ra SHM, chỉ những lực tỉ lệ **tuyến tính** với x mới thỏa mãn.

7.1.3 Amplitude, Period, Frequency, and Angular Frequency

Amplitude A: the maximum displacement from equilibrium (m).
Period T: the time for one complete cycle (s).
Frequency $f = \frac{1}{T}$: the number of cycles per second (Hz).
Angular frequency $\omega = 2\pi f = \frac{2\pi}{T}$ (rad/s), and for SHM specifically $\omega = \sqrt{k/m}$ (spring) or $\omega = \sqrt{g/L}$ (pendulum, see below).

The symbol ω in SHM plays exactly the same mathematical role as the angular velocity of literal rotational motion (Unit 5) — both describe "radians of phase swept out per second" — but here ω describes a linear, back-and-forth oscillation, not an object physically going around a circle. Think of ω as the *rate at which the cosine/sine argument advances*, a bookkeeping tool borrowed from circular motion because sinusoidal functions and circular motion share the same underlying mathematics.

VN **GIẢI THÍCH TIẾNG VIỆT**

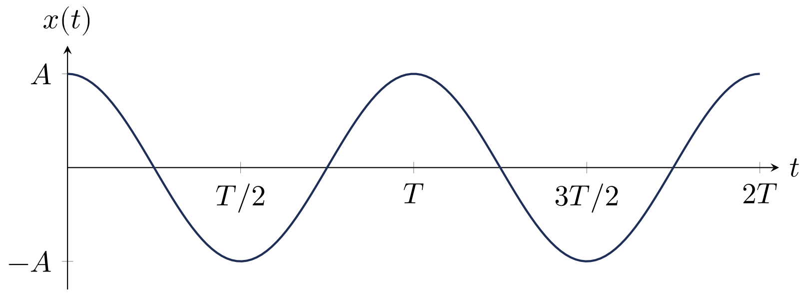
A là độ lệch xa nhất khỏi vị trí cân bằng; T là thời gian cho một chu kỳ đầy đủ; $f = 1/T$ là số chu kỳ mỗi giây; $\omega = 2\pi f = 2\pi/T$ là "tần số góc" — về mặt toán học giống hệt ω của chuyển động quay tròn (đơn vị rad/s, cùng vai trò trong công thức), nhưng ở đây vật **không thực sự quay** — nó chỉ dao động qua lại trên một đường thẳng (hoặc cung tròn nhỏ). ω chỉ đơn giản là tốc độ mà "pha" của hàm sin/cosin tiến triển theo thời gian.

7.2 Kinematics of Simple Harmonic Motion

If a block is released from rest at $x = +A$ at $t = 0$, its motion is fully described by:

$$x(t) = A\cos(\omega t), \qquad v(t) = -A\omega\sin(\omega t), \qquad a(t) = -A\omega^2\cos(\omega t) = -\omega^2 x(t).$$

(If the object instead starts somewhere other than $x = +A$, a phase constant ϕ is added: $x(t) = A\cos(\omega t + \phi)$.) Each equation is obtained from the one before it by the same slope relationship used throughout kinematics: v is the slope of the x–t graph, and a is the slope of the v–t graph.



Position vs. time for SHM released from rest at $x = +A$: $x(t) = A\cos(\omega t)$, shown over two full periods.

Ví dụ 2 · Position, velocity, and acceleration at a given time

A block executes SHM with $A = 0.50$ m and $T = 2.0$ s, released from rest at $x = +A$ at $t = 0$. Find x, v, and a at $t = 0.50$ s (one-quarter period).

$\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{2.0} = \pi = 3.14$ rad/s. At $t = 0.50$ s, $\omega t = \pi(0.50) = \pi/2 = 90^\circ$.

$x = A\cos(90^\circ) = 0.50(0) = 0$ m — the block is passing through equilibrium.

$v = -A\omega\sin(90^\circ) = -(0.50)(3.14)(1) = -1.57$ m/s — this is the block's maximum speed.

$a = -A\omega^2\cos(90^\circ) = 0$ — zero acceleration exactly where speed is maximum.

7.2.1 Maximum Speed and Maximum Acceleration

$$v_{\text{max}} = A\omega \quad \text{(occurs at } x = 0\text{)}, \qquad a_{\text{max}} = A\omega^2 \quad \text{(occurs at } x = \pm A\text{)}.$$

Speed is greatest where the restoring force (and acceleration) is zero — at equilibrium. Acceleration is greatest where speed is momentarily zero — at the extremes.

Ví dụ 3 · Maximum speed and maximum acceleration

An oscillator has amplitude $A = 0.40$ m and angular frequency $\omega = 5.0$ rad/s. Find v_{max} and a_{max}, and state where each occurs.

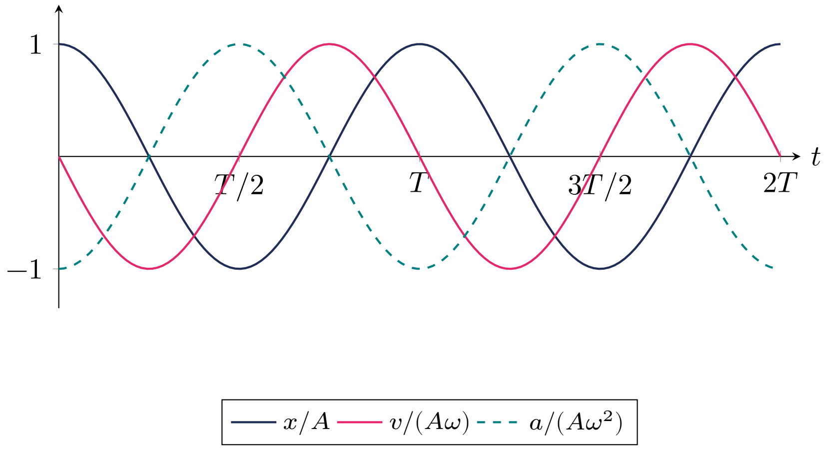
$v_{\text{max}} = A\omega = (0.40)(5.0) = 2.0$ m/s, occurring at $x = 0$.

$a_{max} = A\omega^2 = (0.40)(5.0)^2 = (0.40)(25) = 10\,\text{m/s}^2$, occurring at $x = \pm A = \pm 0.40\,\text{m}$.

VN **GIẢI THÍCH TIẾNG VIỆT**

Tốc độ đạt **cực đại** $v_{max} = A\omega$ khi vật đi qua vị trí cân bằng ($x = 0$), vì lúc đó lực hồi phục bằng 0 nhưng vật đã tích lũy đủ vận tốc suốt hành trình. Gia tốc đạt **cực đại** $a_{max} = A\omega^2$ tại hai biên $x = \pm A$, nơi độ lệch (và do đó lực hồi phục) lớn nhất nhưng vật tạm thời đứng yên ($v = 0$) trước khi đổi chiều.

7.2.2 Phase Relationships Among Position, Velocity, and Acceleration



Normalized position, velocity, and acceleration vs. time. Velocity leads position by 90°; acceleration is exactly out of phase (180°) with position.

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Ba đại lượng $x(t)$, $v(t)$, $a(t)$ lệch pha nhau: v **sớm pha** 90° so với x (khi x đạt cực trị thì $v = 0$; khi $x = 0$ thì $|v|$ cực đại), còn a **ngược pha hoàn toàn** (180°) với x vì $a = -\omega^2 x$ — a và x luôn trái dấu nhau tại mọi thời điểm.

(!) Lỗi thường gặp: Acceleration is **not** greatest where speed is greatest — it is the opposite. At equilibrium ($x = 0$) the restoring force is exactly zero, so $a = 0$ there even though speed is at its maximum. Acceleration is greatest in magnitude at the turning points $x = \pm A$, where the object is momentarily at rest.

7.3 Mass-Spring Systems

For an ideal spring of constant k holding a mass m (frictionless, horizontal or vertical):

$$\omega = \sqrt{\frac{k}{m}}, \qquad T = 2\pi\sqrt{\frac{m}{k}}.$$

The period depends **only** on m and k — it does **not** depend on amplitude A and does **not** depend on g.

A larger mass has more inertia, so it resists the spring's pull more and completes each cycle more slowly (T increases with m). A stiffer spring (larger k) restores the mass toward equilibrium more forcefully, so oscillations complete faster (T decreases with larger k). Because doubling the amplitude doubles *both* the distance to travel *and* the restoring force (and hence the speed built up), these two effects cancel exactly — a larger swing takes the same time as a smaller one.

VN **GIẢI THÍCH TIẾNG VIỆT**

Khối lượng m càng lớn thì quán tính càng lớn, lò xo "kéo" vật chậm hơn nên chu kỳ dài hơn. Độ cứng k càng lớn thì lực hồi phục càng mạnh, chu kỳ ngắn hơn. Biên độ A **không** xuất hiện trong công thức $T = 2\pi\sqrt{m/k}$ — dao động biên độ lớn đi quãng đường dài hơn nhưng cũng có lực (và tốc độ) lớn hơn tương ứng, hai hiệu ứng triệt tiêu nhau nên chu kỳ không đổi.

Ví dụ 4 · Period and frequency of a spring-block system

A 2.0 kg block is attached to a spring of constant $k = 800$ N/m. Find the period and frequency of oscillation.

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{2.0}{800}} = 2\pi\sqrt{0.0025} = 2\pi(0.050) = 0.314\,\text{s}.$$

$$f = \frac{1}{T} = \frac{1}{0.314} = 3.18\,\text{Hz}.$$

7.3.1 How Mass and Spring Constant Affect the Period

Ví dụ 5 · Effect of quadrupling the mass

A 0.25 kg block on a spring with $k = 100$ N/m has period T_1. If the mass is quadrupled to 1.0 kg (spring unchanged), find the new period T_2 and the ratio T_2/T_1.

$$T_1 = 2\pi\sqrt{\frac{0.25}{100}} = 2\pi\sqrt{0.0025} = 2\pi(0.050) = 0.314\,\text{s}.$$

$$T_2 = 2\pi\sqrt{\frac{1.0}{100}} = 2\pi\sqrt{0.010} = 2\pi(0.10) = 0.628\,\text{s}.$$

Ratio: $\dfrac{T_2}{T_1} = \dfrac{0.628}{0.314} = 2.0 = \sqrt{4}$ — quadrupling the mass doubles the period, exactly as predicted by $T \propto \sqrt{m}$.

(!) Lỗi thường gặp: Do not plug g into the mass-spring period formula, and do not expect a larger amplitude to change the period. $T = 2\pi\sqrt{m/k}$ contains only m and k — a common error is to treat a spring problem like a pendulum problem (adding g) or to assume "bigger swing = longer time," neither of which is true for an ideal spring.

7.3.2 Vertical Spring Systems

Hanging a mass vertically from a spring shifts the equilibrium position downward (the spring stretches until the spring force balances gravity, $kx_0 = mg$), but once the mass is set oscillating *about that new equilibrium*, the period formula $T = 2\pi\sqrt{m/k}$ is completely unchanged — gravity only relocates equilibrium, it does not alter how quickly the system oscillates around it.

Ví dụ 6 · Vertical spring: finding k, then the period

A 0.50 kg mass hung at rest from a vertical spring stretches it 0.049 m from its natural length. Find the spring constant k, and then the period of vertical oscillations about this equilibrium.

At equilibrium the spring force balances weight: $kx_0 = mg$.

$$k = \frac{mg}{x_0} = \frac{(0.50)(9.8)}{0.049} = \frac{4.9}{0.049} = 100\,\text{N/m}.$$

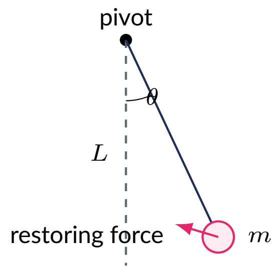
$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.50}{100}} = 2\pi\sqrt{0.0050} = 2\pi(0.0707) = 0.444\,\text{s}.$$

VN **GIẢI THÍCH TIẾNG VIỆT**

Với lò xo treo thẳng đứng, vị trí cân bằng **dịch xuống** một đoạn $x_0 = mg/k$ so với chiều dài tự nhiên do trọng lực, nhưng công thức chu kỳ $T = 2\pi\sqrt{m/k}$ **không đổi** — trọng lực chỉ dịch chuyển gốc tọa độ cân bằng, không ảnh hưởng đến tốc độ dao động quanh điểm cân bằng đó.

7.4 The Simple Pendulum

A **simple pendulum** is a point mass m on a light string of length L, swinging under gravity. When displaced by angle θ from vertical, the component of gravity along the arc (the restoring force) is $-mg\sin\theta$. For **small angles** (roughly $\theta < 15°$, measured in radians), $\sin\theta \approx \theta$, so the restoring force becomes approximately $-mg\theta$ — proportional to displacement along the arc, exactly the condition for SHM.



A simple pendulum displaced by angle θ from vertical equilibrium; the component of gravity along the arc, $-mg\sin\theta \approx -mg\theta$, restores the bob toward $\theta = 0$.

For small-angle oscillations, the acceleration cancels m from both the restoring force ($\propto m$) and the inertia ($\propto m$), leaving:

$$\omega = \sqrt{\frac{g}{L}}, \qquad T = 2\pi\sqrt{\frac{L}{g}}.$$

The period depends **only** on L and g — it does **not** depend on mass and does **not** depend on amplitude (for small angles).

VN **GIẢI THÍCH TIẾNG VIỆT**

Với góc lệch nhỏ, $\sin\theta \approx \theta$ (radian), lực hồi phục dọc theo cung là $-mg\theta$, tỉ lệ tuyến tính với độ lệch — thỏa điều kiện SHM. Áp dụng định luật II Newton: $a = F/m = -g\theta$ — khối lượng m **triệt tiêu** hoàn toàn vì nó xuất hiện như nhau ở cả lực hồi phục (tỉ lệ m) lẫn quán tính (tỉ lệ m). Do đó chu kỳ con lắc đơn $T = 2\pi\sqrt{L/g}$ chỉ phụ thuộc chiều dài L và gia tốc trọng trường g, hoàn toàn không phụ thuộc khối lượng hay biên độ (khi góc nhỏ).

Ví dụ 7 · Period of a simple pendulum

Find the period of a simple pendulum of length $L = 2.45$ m on Earth ($g = 9.8$ m/s^2).

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{2.45}{9.8}} = 2\pi\sqrt{0.25} = 2\pi(0.50) = \pi \approx 3.14\,\text{s}.$$

7.4.1 Contrasting the Pendulum and the Mass-Spring Oscillator

Property	Mass-spring system	Simple pendulum
Period formula	$T = 2\pi\sqrt{m/k}$	$T = 2\pi\sqrt{L/g}$
Depends on mass?	Yes ($T \propto \sqrt{m}$)	No
Depends on g?	No	Yes ($T \propto 1/\sqrt{g}$)
Depends on amplitude?	No	No (small angles only)

Table 7.1: *
Period dependence: mass-spring system vs. simple pendulum.

(!) Lỗi thường gặp: Students often assume mass affects *every* oscillator's period the same way. It does not: mass **does** change a spring's period ($T \propto \sqrt{m}$) but **cancels out completely** for a pendulum. Likewise, g never appears in the spring formula, but is essential to the pendulum formula. Always check which formula — and which variables — actually apply to the system in front of you.

Ví dụ 8 · Same pendulum, Earth vs. the Moon

The pendulum from Ví dụ 7 ($L = 2.45$ m) is taken to the Moon, where $g_{\text{Moon}} = g/6 = 1.63$ m/s^2. Find its period there and compare to Earth.

$$T_{\text{Moon}} = 2\pi\sqrt{\frac{2.45}{1.63}} = 2\pi\sqrt{1.50} = 2\pi(1.225) = 7.70\,\text{s}.$$

Compared to $T_{\text{Earth}} = 3.14$ s: ratio $= \frac{T_{\text{Moon}}}{T_{\text{Earth}}} = \sqrt{\frac{g_{\text{Earth}}}{g_{\text{Moon}}}} = \sqrt{6} = 2.449$, and indeed $3.14 \times 2.449 = 7.70$ s. Weaker gravity means a weaker restoring force for the same mass, so the pendulum swings more slowly — the period increases (mass never entered the calculation at all).

7.4.2 Designing a Pendulum for a Target Period

Ví dụ 9 · Length needed for a 1.0-second period

What length must a simple pendulum have so that $T = 1.0$ s on Earth?

Solve $T = 2\pi\sqrt{L/g}$ for L:

$$L = \frac{gT^2}{4\pi^2} = \frac{(9.8)(1.0)^2}{4\pi^2} = \frac{9.8}{39.48} = 0.248\,\text{m} = 24.8\,\text{cm}.$$

7.5 Energy in Simple Harmonic Motion

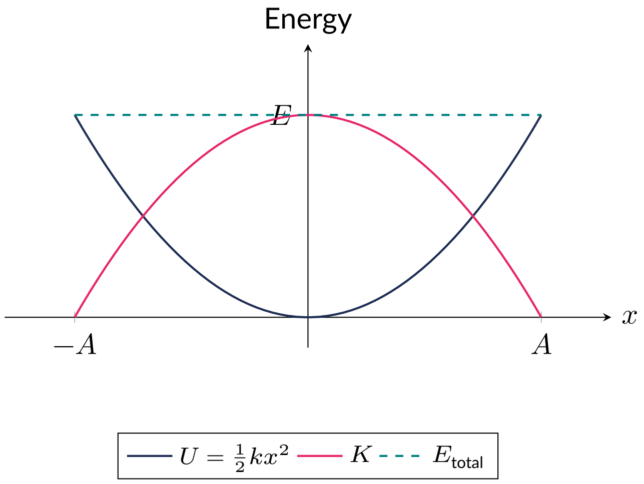
Because the restoring force is conservative, total mechanical energy in SHM is constant, continuously exchanging between potential energy (stored in the spring or in height, for a pendulum) and kinetic energy.

$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}mv_{\text{max}}^2 \quad \text{(constant, independent of position).}$$

At any position x: $\frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$, which solves to give the speed at any displacement:

$$v = \omega\sqrt{A^2 - x^2}.$$

At the extremes ($x = \pm A$): all potential energy, $v = 0$. At equilibrium ($x = 0$): all kinetic energy, $v = v_{\text{max}}$.



Potential energy U (rising parabola) and kinetic energy K (falling parabola) vs. position, plus the constant total mechanical energy E. At every x, $U + K = E$.

GIẢI THÍCH TIẾNG VIỆT VN

Cơ năng toàn phần $E = \frac{1}{2}kA^2$ là **hằng số**, không đổi trong suốt quá trình dao động (bỏ qua ma sát). Tại hai biên ($x = \pm A$), toàn bộ năng lượng là thế năng, $v = 0$. Tại vị trí cân bằng ($x = 0$), toàn bộ năng lượng là động năng, $v = v_{\text{max}}$. Ở vị trí trung gian, năng lượng được chia sẻ giữa thế năng $\frac{1}{2}kx^2$ và động năng $\frac{1}{2}mv^2$, luôn cộng lại đúng bằng E.

Ví dụ 10 · Total energy and maximum speed

A 0.50 kg block oscillates on a spring with $k = 200$ N/m and amplitude $A = 0.10$ m. Find the total mechanical energy and the maximum speed.

$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}(200)(0.10)^2 = \tfrac{1}{2}(200)(0.010) = 1.0\,\text{J}.$$

$$v_{\text{max}} = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(1.0)}{0.50}} = \sqrt{4.0} = 2.0\,\text{m/s}.$$

Check using ω: $\omega = \sqrt{k/m} = \sqrt{200/0.50} = 20$ rad/s, so $v_{\text{max}} = A\omega = (0.10)(20) = 2.0$ m/s — consistent.

Ví dụ 11 · Speed and energy split at a given position

For the system in Ví dụ 10 ($A = 0.10$ m, $\omega = 20$ rad/s, $k = 200$ N/m, $E = 1.0$ J), find (a) the speed at $x = 0.060$ m, and (b) the fraction of the total energy that is kinetic at that point.

(a) $v = \omega\sqrt{A^2 - x^2} = 20\sqrt{(0.10)^2 - (0.060)^2} = 20\sqrt{0.010 - 0.0036} = 20\sqrt{0.0064} = 20(0.080) = 1.6$ m/s.

(b) $U = \frac{1}{2}kx^2 = \frac{1}{2}(200)(0.060)^2 = \frac{1}{2}(200)(0.0036) = 0.36$ J. So $K = E - U = 1.0 - 0.36 = 0.64$ J, which is $64\,\%$ of the total energy (check: $\frac{1}{2}mv^2 = \frac{1}{2}(0.50)(1.6)^2 = \frac{1}{2}(0.50)(2.56) = 0.64$ J, matches).

(!) Lỗi thường gặp: The energy formula $E = \frac{1}{2}kA^2$ uses the **amplitude** A (the maximum displacement), not the current position x. A frequent error is to substitute whatever x is given in the problem into this formula — that instead gives only the potential energy at that point, $U = \frac{1}{2}kx^2$, which is less than the (constant) total energy unless $x = \pm A$.

7.6 Graphical Analysis of Oscillatory Motion

AP Physics 1 frequently gives a graph — x–t, v–t, or a–t — instead of a formula, and asks for A, T, f, ω, v_{max}, or a_{max}.

From an x–t graph: the peak height is A; the time for one full repeating cycle is T.
From a v–t graph: the peak height is $v_{max} = A\omega$; the time between successive zero-crossings (points where the object is momentarily at rest, at $x = \pm A$) is $T/2$.
From an a–t graph: the peak height is $a_{max} = A\omega^2$; a is exactly zero whenever $x = 0$.
Once T (hence $\omega = 2\pi/T$) and A are known, every other quantity follows from $v_{max} = A\omega$ and $a_{max} = A\omega^2$.

VN **GIẢI THÍCH TIẾNG VIỆT**

Đọc đồ thị SHM: trên đồ thị x–t, chiều cao đỉnh là A và thời gian lặp lại một chu trình đầy đủ là T. Trên đồ thị v–t, chiều cao đỉnh là v_{max}, còn khoảng thời gian giữa hai lần $v = 0$ liên tiếp là $T/2$ (nửa chu kỳ). Sau khi có T (suy ra ω) và A, mọi đại lượng còn lại tính được từ $v_{max} = A\omega$ và $a_{max} = A\omega^2$.

Ví dụ 12 · Reading an x–t graph

An x–t graph for an oscillator shows a peak displacement of 0.40 m and one full cycle every 8.0 s. Find ω, v_{max}, and a_{max}.

$A = 0.40$ m, $T = 8.0$ s, so $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{8.0} = 0.785$ rad/s.
$v_{max} = A\omega = (0.40)(0.785) = 0.314$ m/s.
$a_{max} = A\omega^2 = (0.40)(0.785)^2 = (0.40)(0.617) = 0.247$ m/s^2.

Ví dụ 13 · Reading a v–t graph

A v–t graph for an oscillator has a peak height of 0.60 m/s, and successive points where $v = 0$ are separated by 1.57 s. Find the period, angular frequency, and amplitude.

Time between successive zeros of v is half a period: $T/2 = 1.57$ s $\Rightarrow T = 3.14$ s $= \pi$ s.
$\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{\pi} = 2.0$ rad/s.

Since $v_{\text{max}} = A\omega$: $A = \frac{v_{\text{max}}}{\omega} = \frac{0.60}{2.0} = 0.30\,\text{m}$.

7.7 Vertical Springs, Initial Conditions, and Qualitative Damping

This section extends the mass-spring and SHM tools already developed to three practically important refinements: how gravity affects a vertical spring's equilibrium (but not its period), how to fully describe motion when an object starts away from $x = +A$ with some nonzero velocity, and what changes qualitatively once friction or drag is present.

7.7.1 Vertical Mass-Spring Systems: Deriving the Shifted Equilibrium

Let y denote the position of a hanging mass measured downward from the spring's *natural* (unstretched) length. Two forces act along this direction: gravity mg (downward) and the spring force $-ky$ (which opposes stretch and pulls upward). Newton's second law gives

$$m\ddot{y} = mg - ky.$$

At the new equilibrium the mass is momentarily unaccelerated ($\ddot{y} = 0$), so the spring force exactly balances gravity:

$$mg = kx_0 \qquad \Longrightarrow \qquad x_0 = \frac{mg}{k},$$

the distance the spring is *already* stretched once the mass hangs at rest — before any oscillation even begins. Now switch to a coordinate $x = y - x_0$ measured from this new equilibrium, so $y = x + x_0$:

$$m\ddot{x} = mg - k(x + x_0) = mg - kx - kx_0 = mg - kx - mg = -kx,$$

since $kx_0 = mg$ cancels the constant gravity term exactly. The result, $m\ddot{x} = -kx$, is *identical* to the horizontal spring equation — gravity has been completely "absorbed" into relocating the equilibrium point and no longer appears anywhere in the equation governing the oscillation.

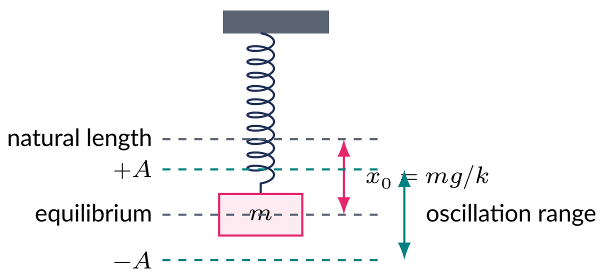
For a vertical mass-spring system, the new equilibrium is displaced a distance

$$x_0 = \frac{mg}{k}$$

below the spring's natural length. Motion **about this new equilibrium** is still simple harmonic, with the same

$$\omega = \sqrt{\frac{k}{m}}, \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

as a horizontal spring — completely unaffected by g. Gravity only relocates where "$x = 0$" is; it never appears in the period.



A mass m hanging in equilibrium from a vertical spring of constant k. Gravity stretches the spring by $x_0 = mg/k$ beyond its natural length, shifting the equilibrium position downward. The mass oscillates symmetrically about this *new* equilibrium, between $+A$ and $-A$, with the same period $T = 2\pi\sqrt{m/k}$ as a horizontal spring — gravity only relocates the center of oscillation.

VN **GIẢI THÍCH TIẾNG VIỆT**

Với lò xo treo thẳng đứng, đặt y đo từ đầu lò xo khi chưa giãn. Định luật II Newton: $m\ddot{y} = mg - ky$. Tại vị trí cân bằng mới, $\ddot{y} = 0$ nên $mg = kx_0$, suy ra $x_0 = mg/k$ — đây là độ giãn của lò xo ngay cả khi vật đứng yên, trước khi dao động. Đổi biến $x = y - x_0$ (đo từ vị trí cân bằng mới), phương trình trở thành $m\ddot{x} = -kx$ — giống hệt lò xo ngang. Trọng lực đã được "hấp thụ" hoàn toàn vào việc dịch chuyển gốc tọa độ cân bằng, không còn xuất hiện trong phương trình dao động, nên $T = 2\pi\sqrt{m/k}$ không đổi.

Ví dụ 14 · Vertical spring: released from the natural length

A 1.0 kg mass hangs from a vertical spring with $k = 49$ N/m. The mass is held so the spring is at its natural (unstretched) length and then released from rest. Find (a) the new equilibrium position x_0, (b) the amplitude of the resulting oscillation, and (c) the maximum speed reached.

(a) $x_0 = \dfrac{mg}{k} = \dfrac{(1.0)(9.8)}{49} = 0.20$ m.

(b) Since the mass is released *from rest* at the natural length, that starting point is itself a turning point of the motion. Measured from the new equilibrium, the natural length lies a distance x_0 away, so the amplitude is

$$A = x_0 = 0.20\,\text{m}.$$

(c) $\omega = \sqrt{k/m} = \sqrt{49/1.0} = 7.0$ rad/s, so

$$v_{\text{max}} = A\omega = (0.20)(7.0) = 1.4\,\text{m/s}.$$

Check by energy conservation: falling from the natural length to the new equilibrium (a drop of $x_0 = 0.20$ m), the mass loses gravitational PE $mgx_0 = (1.0)(9.8)(0.20) = 1.96$ J while the spring gains elastic PE $\frac{1}{2}kx_0^2 = \frac{1}{2}(49)(0.20)^2 = 0.98$ J. The difference becomes kinetic energy: $K = 1.96 - 0.98 = 0.98\,\text{J} = \frac{1}{2}mv_{\text{max}}^2 \Rightarrow v_{\text{max}} = \sqrt{2(0.98)/1.0} = 1.4$ m/s — matches exactly.

7.7.2 Determining the Phase Constant from Initial Conditions

So far every example has released the object from rest at $x = +A$ (giving $\phi = 0$) or from $x = 0$ moving at v_{max}. In general, an oscillator can start at *any* position $x_i = x(0)$ with *any* velocity $v_i = v(0)$, and the full solution $x(t) = A\cos(\omega t + \phi)$ must be matched to those two initial conditions to pin down both A and ϕ.

Evaluating the general solution and its derivative at $t = 0$:

$$x(0) = A\cos\phi = x_i, \qquad v(0) = -A\omega\sin\phi = v_i.$$

Solving the first for $\cos\phi = x_i/A$ and the second for $\sin\phi = -v_i/(A\omega)$, then using $\sin^2\phi + \cos^2\phi = 1$:

For $x(t) = A\cos(\omega t + \phi)$ with initial position $x_i = x(0)$ and initial velocity $v_i = v(0)$:

$$A = \sqrt{x_i^2 + \left(\frac{v_i}{\omega}\right)^2}, \qquad \tan\phi = -\frac{v_i}{\omega x_i}.$$

Because the tangent function repeats every $180°$, always check the individual signs of $\cos\phi = x_i/A$ and $\sin\phi = -v_i/(A\omega)$ separately to place ϕ in the correct quadrant.

VN **GIẢI THÍCH TIẾNG VIỆT**

Trường hợp tổng quát: vật xuất phát tại vị trí $x_i = x(0)$ bất kỳ với vận tốc $v_i = v(0)$ bất kỳ (không nhất thiết bằng 0). Thay $t = 0$ vào $x(t) = A\cos(\omega t + \phi)$ và $v(t) = -A\omega\sin(\omega t + \phi)$:

$x_i = A\cos\phi$ và $v_i = -A\omega\sin\phi$. Bình phương hai vế rồi cộng lại (dùng $\sin^2\phi + \cos^2\phi = 1$) cho $A = \sqrt{x_i^2 + (v_i/\omega)^2}$; chia hai phương trình cho nhau cho $\tan\phi = -v_i/(\omega x_i)$. Vì hàm tang lặp lại mỗi 180°, cần kiểm tra dấu của $\cos\phi$ và $\sin\phi$ riêng biệt để xác định đúng góc phần tư của ϕ.

Ví dụ 15 · Finding A, ϕ, and the equation of motion

An oscillator has $\omega = 4.0\,\text{rad/s}$. At $t = 0$ it is at $x_i = 0.30\,\text{m}$, moving with velocity $v_i = -1.6\,\text{m/s}$. Find the amplitude, the phase constant, and the complete equation of motion $x(t)$.

$$A = \sqrt{x_i^2 + \left(\frac{v_i}{\omega}\right)^2} = \sqrt{(0.30)^2 + \left(\frac{-1.6}{4.0}\right)^2} = \sqrt{0.090 + 0.160} = \sqrt{0.25} = 0.50\,\text{m}.$$

For the phase constant: $\cos\phi = x_i/A = 0.30/0.50 = 0.60$ and $\sin\phi = -v_i/(A\omega) = 1.6/[(0.50)(4.0)] = 0.80$. Both are positive, so ϕ lies in the first quadrant:

$$\phi = \arctan\left(\frac{0.80}{0.60}\right) = \arctan\left(\frac{4}{3}\right) = 53.1^\circ = 0.927\,\text{rad}.$$

Complete equation of motion:

$$x(t) = (0.50)\cos(4.0t + 0.927)\ \text{m} \qquad (t \text{ in s}).$$

Check at $t = 0$: $x(0) = 0.50\cos(0.927) = 0.50(0.60) = 0.30\,\text{m}$ — matches x_i. And $v(t) = -A\omega\sin(\omega t + \phi) = -2.0\sin(4.0t + 0.927)$, so $v(0) = -2.0\sin(0.927) = -2.0(0.80) = -1.6\,\text{m/s}$ — matches v_i. Both initial conditions are reproduced exactly.

Ví dụ 16 · Vertical spring with nonzero initial velocity

A $2.0\,\text{kg}$ mass hangs from a vertical spring with $k = 98\,\text{N/m}$. At equilibrium the spring is stretched $x_0 = mg/k = 0.20\,\text{m}$ beyond its natural length, and $\omega = \sqrt{k/m} = \sqrt{98/2.0} = 7.0\,\text{rad/s}$. At $t = 0$ the mass is pulled down an additional $0.050\,\text{m}$ below equilibrium and given an upward push so that, at that instant, it moves at $0.84\,\text{m/s}$ back toward equilibrium. Taking $x = 0$ at the new equilibrium with x positive *downward*, find the amplitude, the phase constant, and the equation of motion, and state the spring's minimum and maximum stretch during the motion.

Initial conditions relative to equilibrium: $x_i = 0.050\,\text{m}$ (below equilibrium) and $v_i = -0.84\,\text{m/s}$ (negative because the push is upward, opposite the positive-down direction).

$$A = \sqrt{x_i^2 + \left(\frac{v_i}{\omega}\right)^2} = \sqrt{(0.050)^2 + \left(\frac{-0.84}{7.0}\right)^2} = \sqrt{0.0025 + 0.0144} = \sqrt{0.0169} = 0.13\,\text{m}.$$

$\cos\phi = x_i/A = 0.050/0.13 = 0.385$ and $\sin\phi = -v_i/(A\omega) = 0.84/[(0.13)(7.0)] = 0.84/0.91 = 0.923$. Both positive, so ϕ is in the first quadrant:

$$\phi = \arctan\left(\frac{0.923}{0.385}\right) = \arctan(2.4) = 67.4^\circ = 1.176\,\text{rad}.$$

Equation of motion (relative to the new equilibrium, positive downward):

$$x(t) = (0.13)\cos(7.0t + 1.176)\ \text{m}.$$

Check: $x(0) = 0.13(0.385) = 0.050\,\text{m}$ — matches x_i. $v(t) = -A\omega\sin(\omega t+\phi) = -0.91\sin(7.0t + 1.176)$, so $v(0) = -0.91(0.923) = -0.84\,\text{m/s}$ — matches v_i.

Since $A = 0.13\,\text{m} < x_0 = 0.20\,\text{m}$, the spring never reaches its natural length: total stretch ranges from a minimum of $x_0 - A = 0.20 - 0.13 = 0.07\,\text{m}$ (top of the swing) to a maximum of $x_0 + A = 0.20 + 0.13 = 0.33\,\text{m}$ (bottom) — the spring stays stretched throughout the entire motion.

7.7.3 Damped Oscillations: A Qualitative Picture

Every real oscillator eventually loses energy to resistive forces — air resistance, internal friction in a spring, viscous drag in a fluid — collectively called **damping**. Unlike the ideal restoring force $-kx$, a damping force acts opposite to velocity and continuously removes mechanical energy from the system, cycle after cycle. Because $E = \frac{1}{2}kA^2$ ties the total energy directly to amplitude, a shrinking E means a shrinking A: the oscillator still swings back and forth at nearly the same rate, but each successive swing is a little smaller than the last, tracing out a curve that oscillates *within a decaying envelope* rather than between fixed limits $\pm A$.

Underdamped: light damping; the system still oscillates back and forth, but the amplitude shrinks cycle by cycle until the motion dies out. For *light* damping the period is only slightly longer than the undamped $T = 2\pi\sqrt{m/k}$ — timing is barely affected even though amplitude is clearly affected.
Critically damped: damping strong enough that the system returns to equilibrium *without oscillating at all*, and does so in the shortest possible time of any non-oscillating case.
Overdamped: damping stronger still; the system returns to equilibrium without oscillating, but more sluggishly than the critically damped case.

VN GIẢI THÍCH TIẾNG VIỆT

Trong thực tế, dao động luôn mất dần cơ năng do lực cản (ma sát, lực cản không khí, lực nhớt...). Vì $E = \frac{1}{2}kA^2$ nên năng lượng giảm đồng nghĩa biên độ A giảm dần theo thời gian — đồ thị x–t vẫn dao động qua lại nhưng "co lại" trong một đường bao giảm dần, không còn giữ biên độ không đổi. Có ba trường hợp: **tắt dần dưới mức tới hạn (underdamped)** — vẫn dao động qua lại nhưng biên độ giảm dần; **tắt dần tới hạn (critically damped)** — trở về vị trí cân bằng nhanh nhất mà không dao động; **tắt dần quá mức (overdamped)** — trở về cân bằng chậm hơn, cũng không dao động. Với lực cản nhẹ, chu kỳ gần như không đổi so với $T = 2\pi\sqrt{m/k}$ — chỉ biên độ bị ảnh hưởng rõ rệt, không phải chu kỳ.

(!) Lỗi thường gặp: Damping shrinks the **amplitude** over time — for light damping it does **not** meaningfully change the **period**. Do not assume a damped oscillator "speeds up" or "slows down" its swings just because the swings are getting smaller; under light (underdamped) damping, successive cycles still take almost exactly $T = 2\pi\sqrt{m/k}$ (or $T = 2\pi\sqrt{L/g}$), even as their size decays toward zero.

7.8 Practice Problems

Bài tập luyện tập

Which condition correctly defines simple harmonic motion?

(A) Net force is constant in magnitude and direction

(B) Net force is proportional to displacement and points in the *same* direction as displacement

(C) Net force is proportional to displacement and points *opposite* to displacement

(D) Net force is inversely proportional to displacement

Lời giải chi tiết

SHM requires a linear restoring force, $F = -kx$: proportional to x, opposite in direction. **(C).**

Bài tập luyện tập

A block-spring system has $k = 50$ N/m and $m = 2.0$ kg. Find its angular frequency.

(A) 2.5 rad/s

(B) 5.0 rad/s

(C) 10 rad/s

(D) 25 rad/s

Lời giải chi tiết

$\omega = \sqrt{k/m} = \sqrt{50/2.0} = \sqrt{25} = 5.0$ rad/s. **(B).**

Bài tập luyện tập

For the same system as above ($k = 50$ N/m, $m = 2.0$ kg), find the period and frequency.

Lời giải chi tiết

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{5.0} = \boxed{1.26\,\text{s}}. \quad f = \frac{1}{T} = \frac{1}{1.26} = \boxed{0.796\,\text{Hz}}.$$

Bài tập luyện tập

If the mass of a spring-block oscillator is doubled while k stays the same, the period:

(A) Doubles

(B) Halves

(C) Increases by a factor of $\sqrt{2}$

(D) Stays the same

Lời giải chi tiết

$T \propto \sqrt{m}$, so doubling m multiplies T by $\sqrt{2}$. **(C).**

Bài tập luyện tập

If the spring constant of a spring-block oscillator is quadrupled while m stays the same, by what factor does the period change?

Lời giải chi tiết

$T \propto 1/\sqrt{k}$. Quadrupling k gives $T_{\text{new}} = T_{\text{old}}/\sqrt{4} = T_{\text{old}}/2$ — the period is $\boxed{\text{halved}}$.

Bài tập luyện tập

Explain, using Newton's second law, why the period of a simple pendulum does not depend on the mass of the bob.

Lời giải chi tiết

For small angles, the restoring force along the arc is $F = -mg\sin\theta \approx -mg\theta$. Applying $a = F/m$ gives $a = -g\theta$: the mass m appears in both the restoring force (which is proportional to m, from gravity) and in the object's inertia (Newton's second law divides by m), so it cancels exactly. The resulting acceleration — and therefore the period $T = 2\pi\sqrt{L/g}$ — depends only on L and g, never on m. (This is the same reason all objects, heavy or light, free-fall with the same acceleration.)

Bài tập luyện tập

Find the length of a simple pendulum with period $T = 2.0$ s on Earth.

Lời giải chi tiết

$$L = \frac{gT^2}{4\pi^2} = \frac{(9.8)(2.0)^2}{4\pi^2} = \frac{39.2}{39.48} = \boxed{0.99\,\text{m}}.$$

Bài tập luyện tập

A pendulum clock keeps correct time on Earth. If moved to a planet where $g = 4.9\,\text{m/s}^2$ (half of Earth's) with its length unchanged, its period:

(A) Stays the same
(B) Increases by a factor of $\sqrt{2}$
(C) Decreases by a factor of $\sqrt{2}$
(D) Doubles

Lời giải chi tiết

$T \propto 1/\sqrt{g}$. Halving g multiplies T by $\sqrt{2}$. **(B).**

Bài tập luyện tập

Explain why both a spring-block system and a small-angle pendulum exhibit **isochronism** (period independent of amplitude), and explain why this breaks down for a pendulum swung through large angles.

Lời giải chi tiết

For small angles, $\sin\theta \approx \theta$, so the restoring force is exactly proportional to displacement no matter how large or small the (small-angle) amplitude is — doubling the amplitude doubles both the distance to travel and the restoring force/speed built up along the way, so these effects cancel and T stays fixed. The same cancellation happens for an ideal spring, since $F = -kx$ is exactly linear at any amplitude. For large pendulum angles, $\sin\theta$ noticeably differs from θ (the restoring force is smaller than the linear approximation predicts), so the motion is no longer true SHM, the force is no longer strictly proportional to displacement, and the period begins to depend on amplitude (larger swings take measurably longer than $2\pi\sqrt{L/g}$ predicts).

Bài tập luyện tập

A block-spring system has total energy $E = 3.0\,\text{J}$ and amplitude $A = 0.20\,\text{m}$. Find the spring constant k.

Lời giải chi tiết

$$E = \tfrac{1}{2}kA^2 \Rightarrow k = \frac{2E}{A^2} = \frac{2(3.0)}{(0.20)^2} = \frac{6.0}{0.040} = \boxed{150\,\text{N/m}}.$$

Bài tập luyện tập

For the system above ($E = 3.0\,\text{J}$), if the oscillating mass is $m = 1.5\,\text{kg}$, find the maximum speed.

Lời giải chi tiết

$$v_{\text{max}} = \sqrt{2E/m} = \sqrt{2(3.0)/1.5} = \sqrt{4.0} = \boxed{2.0\,\text{m/s}}.$$

Bài tập luyện tập

For any SHM oscillator, at $x = A/2$, what fraction of the total mechanical energy is kinetic?

(A) $25\,\%$
(B) $50\,\%$
(C) $75\,\%$
(D) $87.5\,\%$

Lời giải chi tiết

$U = \frac{1}{2}kx^2 = \frac{1}{2}k(A/2)^2 = \frac{1}{4}\left(\frac{1}{2}kA^2\right) = E/4$. So $K = E - E/4 = 3E/4 = 75\,\%$ of E. **(C)**.

Bài tập luyện tập

In SHM, the speed of the oscillating object is maximum at:

(A) $x = +A$
(B) $x = -A$
(C) $x = 0$
(D) $x = A/2$

Lời giải chi tiết

Speed is maximum at equilibrium, where all energy is kinetic. **(C)**.

Bài tập luyện tập

In SHM, the magnitude of the acceleration is maximum at:

(A) $x = 0$
(B) $x = \pm A$
(C) $x = \pm A/2$
(D) Acceleration is constant everywhere

Lời giải chi tiết

Acceleration is proportional to displacement, so it is largest at the extremes. **(B)**.

Bài tập luyện tập

A student claims: "Acceleration must be greatest at the equilibrium position, because that's where the object moves fastest." Explain what is wrong with this reasoning, and state where the acceleration is actually greatest.

Lời giải chi tiết

This confuses velocity with acceleration — they are governed by different relationships and are 90° out of phase in SHM. Acceleration depends on *displacement*, not speed: $a = -\omega^2 x$. At equilibrium, $x = 0$, so the net restoring force — and hence the acceleration — is exactly zero, even though speed is at its maximum there. Acceleration is greatest in magnitude at the turning points $x = \pm A$, where displacement (and thus the restoring force) is largest, even though the object is momentarily at rest there. Speed and acceleration never peak at the same location in SHM.

Bài tập luyện tập

An x–t graph shows amplitude $0.50\,\text{m}$ and period $6.0\,\text{s}$. Find ω, v_{max}, and a_{max}.

Lời giải chi tiết

$\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{6.0} = \boxed{1.05\,\text{rad/s}}$. $v_{\text{max}} = A\omega = (0.50)(1.05) = \boxed{0.524\,\text{m/s}}$. $a_{\text{max}} = A\omega^2 = (0.50)(1.10) = \boxed{0.548\,\text{m/s}^2}$.

Bài tập luyện tập

On a v–t graph for an SHM oscillator, the acceleration at a given instant equals:

(A) The value of v at that instant

(B) The area under the curve up to that instant

(C) The slope of the curve at that instant

(D) The amplitude of the curve

Lời giải chi tiết

Acceleration is the rate of change of velocity — the slope of the v–t graph, exactly as in linear kinematics. **(C)**.

Bài tập luyện tập

In SHM, the phase difference between the velocity curve and the position curve is:

(A) 0°

(B) 90°

(C) 180°

(D) 270°

Lời giải chi tiết

$v(t) = -A\omega\sin(\omega t)$ leads $x(t) = A\cos(\omega t)$ by a quarter cycle. **(B)**.

Bài tập luyện tập

In SHM, the phase relationship between acceleration and position is:

(A) In phase (0°)

(B) 90° out of phase

(C) Exactly opposite (180°)

(D) No fixed relationship

Lời giải chi tiết

Since $a = -\omega^2 x$, a and x always have opposite signs and equal-shaped curves — exactly 180° out of phase. **(C)**.

Bài tập luyện tập

A 0.40 kg mass hung from a vertical spring stretches it 0.098 m at rest. Find the spring constant and the period of vertical oscillations.

Lời giải chi tiết

$$k = \frac{mg}{x_0} = \frac{(0.40)(9.8)}{0.098} = \frac{3.92}{0.098} = \boxed{40\,\text{N/m}}.$$
$$T = 2\pi\sqrt{m/k} = 2\pi\sqrt{0.40/40} = 2\pi\sqrt{0.010} = 2\pi(0.10) = \boxed{0.628\,\text{s}}.$$

Bài tập luyện tập

Compared to an identical spring-block system oscillating horizontally, the same system hung vertically (same m, same k) has a period that is:

(A) Longer, because gravity adds extra restoring force

(B) Shorter, because gravity speeds up the motion

(C) Exactly the same, because gravity only shifts the equilibrium position

(D) Undefined — gravity prevents periodic motion when a spring is vertical

Lời giải chi tiết

Gravity shifts where equilibrium is but does not change $T = 2\pi\sqrt{m/k}$. **(C)**.

Bài tập luyện tập

A spring-block system with $m = 0.20$ kg oscillates with period $T = 0.2\pi$ s ≈ 0.628 s. Find the spring constant k.

Lời giải chi tiết

$$k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(0.20)}{(0.2\pi)^2} = \frac{4\pi^2(0.20)}{0.04\pi^2} = \frac{0.80}{0.040} = \boxed{20\,\text{N/m}}.$$

Bài tập luyện tập

Two pendulums swing at the same location: pendulum A has length 1.0 m, pendulum B has length 4.0 m. Find the ratio $T_A : T_B$.

(A) $1 : 1$
(B) $1 : 2$
(C) $1 : 4$
(D) $4 : 1$

Lời giải chi tiết

$T \propto \sqrt{L}$, so $T_A : T_B = \sqrt{1.0} : \sqrt{4.0} = 1 : 2$. **(B)**.

Bài tập luyện tập

Two pendulums at the same location have lengths $L_1 = 0.90$ m and $L_2 = 3.6$ m. Find T_2/T_1.

Lời giải chi tiết

$\dfrac{T_2}{T_1} = \sqrt{\dfrac{L_2}{L_1}} = \sqrt{\dfrac{3.6}{0.90}} = \sqrt{4.0} = \boxed{2.0}$ — pendulum B's period is twice pendulum A's.

Bài tập luyện tập

A mass-spring system and a simple pendulum both have period exactly 2.0 s on Earth's surface. If the entire apparatus is moved to the Moon ($g_{\text{Moon}} \approx g/6$), explain what happens to each system's period, and why they behave differently.

Lời giải chi tiết

Spring: $T = 2\pi\sqrt{m/k}$ contains no g at all, so the spring-block period is completely unaffected by the change in gravity — it remains 2.0 s on the Moon, because the restoring force comes entirely from the spring itself, not from gravity.
Pendulum: $T = 2\pi\sqrt{L/g}$; with g reduced to $g/6$, $T_{\text{Moon}} = 2\pi\sqrt{6L/g} = \sqrt{6} \cdot T_{\text{Earth}} \approx 2.449(2.0) = \boxed{4.9\,\text{s}}$. The pendulum's restoring force comes from gravity, so weaker gravity means a weaker restoring force acting on the same inertia (mass), and the bob accelerates back toward equilibrium more slowly — the period increases by a factor of $\sqrt{6}$.

Bài tập luyện tập

An oscillator's position is given by $x(t) = 0.25\cos(3.0t)$ (SI units, angle in radians). Find A, ω, T, f, v_{max}, and a_{max}.

Lời giải chi tiết

$A = \boxed{0.25\,\text{m}}$, $\omega = \boxed{3.0\,\text{rad/s}}$. $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{3.0} = \boxed{2.09\,\text{s}}$. $f = \dfrac{\omega}{2\pi} = \dfrac{3.0}{6.28} = \boxed{0.477\,\text{Hz}}$.
$v_{\text{max}} = A\omega = (0.25)(3.0) = \boxed{0.75\,\text{m/s}}$. $a_{\text{max}} = A\omega^2 = (0.25)(9.0) = \boxed{2.25\,\text{m/s}^2}$.

Bài tập luyện tập

For the oscillator $x(t) = 0.25\cos(3.0t)$ of the previous problem, which expression correctly gives $v(t)$?

(A) $v(t) = 0.75\cos(3.0t)$
(B) $v(t) = -0.75\sin(3.0t)$
(C) $v(t) = -0.75\cos(3.0t)$
(D) $v(t) = 0.25\sin(3.0t)$

Lời giải chi tiết

$v(t) = -A\omega\sin(\omega t) = -(0.25)(3.0)\sin(3.0t) = -0.75\sin(3.0t)$. **(B).**

Bài tập luyện tập

At $t = 0$, a block undergoing SHM with amplitude $A = 0.40$ m is passing through $x = 0$ moving at 2.0 m/s. Find the angular frequency and period.

Lời giải chi tiết

At $x = 0$ the speed equals v_{max}, so $v_{max} = 2.0$ m/s. Since $v_{max} = A\omega$: $\omega = \frac{v_{max}}{A} = \frac{2.0}{0.40} = \boxed{5.0\,\text{rad/s}}$.

$T = \frac{2\pi}{\omega} = \frac{2\pi}{5.0} = \boxed{1.26\,\text{s}}$.

Bài tập luyện tập

An oscillator has $a_{max} = 8.0$ m/s^2 and $v_{max} = 2.0$ m/s. Find its amplitude.

(A) 0.25 m
(B) 0.50 m
(C) 1.0 m
(D) 2.0 m

Lời giải chi tiết

Since $a_{max} = A\omega^2$ and $v_{max} = A\omega$: $\frac{a_{max}}{v_{max}} = \omega = \frac{8.0}{2.0} = 4.0$ rad/s. Then $A = \frac{v_{max}}{\omega} = \frac{2.0}{4.0} = 0.50$ m. **(B).**

Bài tập luyện tập

For the oscillator in the previous problem ($A = 0.50$ m, $\omega = 4.0$ rad/s), find its speed at $x = 0.30$ m.

Lời giải chi tiết

$v = \omega\sqrt{A^2 - x^2} = 4.0\sqrt{(0.50)^2 - (0.30)^2} = 4.0\sqrt{0.25 - 0.090} = 4.0\sqrt{0.16} = 4.0(0.40) = \boxed{1.6\,\text{m/s}}$.

Bài tập luyện tập

A spring-block oscillator's amplitude is doubled (same m, same k). Which quantity below remains unchanged?

(A) Maximum speed
(B) Maximum acceleration
(C) Total mechanical energy
(D) Period

Lời giải chi tiết

$\omega = \sqrt{k/m}$ depends only on m and k, not A, so $T = 2\pi/\omega$ is unaffected by amplitude. **(D).**

Bài tập luyện tập

Using $v_{\text{max}} = A\omega$, $a_{\text{max}} = A\omega^2$, and $E = \frac{1}{2}kA^2$, explain quantitatively what happens to the maximum speed, maximum acceleration, total energy, and period of a spring-block oscillator if the amplitude is doubled (mass and spring constant unchanged).

Lời giải chi tiết

Since $\omega = \sqrt{k/m}$ depends only on m and k, doubling A does not change ω or T — the **period stays the same.** Maximum speed $v_{\text{max}} = A\omega$ is directly proportional to A, so it **doubles.** Maximum acceleration $a_{\text{max}} = A\omega^2$ is also directly proportional to A (with ω fixed), so it **doubles** as well. Total energy $E = \frac{1}{2}kA^2$ depends on A^2, so doubling A **quadruples** the total mechanical energy.

Bài tập luyện tập

For a vertical mass-spring system oscillating about its (gravity-shifted) equilibrium, the period depends on:

(A) m, k, and g
(B) m and g only
(C) m and k only
(D) k and g only

Lời giải chi tiết

Gravity only relocates the equilibrium position ($x_0 = mg/k$); the period itself is still $T = 2\pi\sqrt{m/k}$, which contains only m and k. **(C).**

Bài tập luyện tập

A 0.80 kg mass hangs from a vertical spring with $k = 80$ N/m. (a) Find how far the spring stretches at equilibrium. (b) Find the period of oscillation. (c) If the mass is instead held at the spring's natural (unstretched) length and released from rest, find the amplitude and the maximum speed of the resulting motion.

Lời giải chi tiết

(a) $x_0 = \dfrac{mg}{k} = \dfrac{(0.80)(9.8)}{80} = \dfrac{7.84}{80} = \boxed{0.098\,\text{m}}$.

(b) $\omega = \sqrt{k/m} = \sqrt{80/0.80} = \sqrt{100} = 10\,\text{rad/s}$, so $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{10} = \boxed{0.628\,\text{s}}$.

(c) Released from rest at the natural length, that point is a turning point, so $A = x_0 = \boxed{0.098\,\text{m}}$. Then $v_{\text{max}} = A\omega = (0.098)(10) = \boxed{0.98\,\text{m/s}}$.

Bài tập luyện tập

An oscillator's position and velocity are both nonzero at $t = 0$. Compared to an identical oscillator released from rest at $x = +A$, this oscillator's subsequent motion:

(A) Follows a completely different amplitude formula and is no longer periodic

(B) Is still SHM with the same ω, but described by a nonzero phase constant ϕ

(C) Is no longer sinusoidal, since it did not start at a turning point

(D) Has a period that depends on the specific values of x_i and v_i

Lời giải chi tiết

Any set of initial conditions in a linear restoring-force system produces SHM with the same $\omega = \sqrt{k/m}$; only A and ϕ adjust to match x_i and v_i, not the period. **(B)**.

Bài tập luyện tập

An oscillator has angular frequency $\omega = 5.0\,\text{rad/s}$. At $t = 0$ it is at $x_i = 0.16\,\text{m}$, moving with velocity $v_i = -1.5\,\text{m/s}$. Find the amplitude, the phase constant, and the complete equation of motion $x(t)$.

Lời giải chi tiết

$$A = \sqrt{x_i^2 + \left(\frac{v_i}{\omega}\right)^2} = \sqrt{(0.16)^2 + \left(\frac{-1.5}{5.0}\right)^2} = \sqrt{0.0256 + 0.090} = \sqrt{0.1156} = \boxed{0.34\,\text{m}}.$$

$\cos\phi = x_i/A = 0.16/0.34 = 0.471$ and $\sin\phi = -v_i/(A\omega) = 1.5/[(0.34)(5.0)] = 1.5/1.7 = 0.882$. Both positive, so ϕ is in the first quadrant:

$$\phi = \arctan\left(\frac{0.882}{0.471}\right) = \arctan(1.875) = \boxed{61.9^\circ = 1.081\,\text{rad}}.$$

Equation of motion: $x(t) = (0.34)\cos(5.0t + 1.081)$ m. *Check:* $x(0) = 0.34(0.471) = 0.16\,\text{m}$, matching x_i; $v(t) = -(0.34)(5.0)\sin(5.0t + 1.081) = -1.7\sin(5.0t + 1.081)$, so $v(0) = -1.7(0.882) = -1.5\,\text{m/s}$, matching v_i — both initial conditions reproduced.

Bài tập luyện tập

Find the period of a simple pendulum of length $L = 0.90\,\text{m}$ on Earth.

(A) 0.95 s

(B) 1.5 s

(C) 1.9 s

(D) 3.8 s

Lời giải chi tiết

$T = 2\pi\sqrt{L/g} = 2\pi\sqrt{0.90/9.8} = 2\pi\sqrt{0.0918} = 2\pi(0.303) = 1.9$ s. **(C)**.

Bài tập luyện tập

A pendulum clock keeps correct time on Earth. If the clock is brought to the Moon, where g is about $\frac{1}{6}$ of its value on Earth, explain — using the pendulum period formula, and without plugging in the specific factor of $\frac{1}{6}$ — whether the clock will now run fast, run slow, or keep correct time. Justify your reasoning.

Lời giải chi tiết

The period of a pendulum is $T = 2\pi\sqrt{L/g}$. The clock's gears are built and calibrated so that each swing (each period T) is counted as marking off a fixed, correct amount of real time, based on the pendulum's length and Earth's gravity. On the Moon, the pendulum's length L is unchanged, but g is smaller. Since $T \propto 1/\sqrt{g}$, a smaller g produces a **larger** T — each individual swing physically takes **longer** to complete than it did on Earth, because the restoring force (which comes from gravity) is weaker while the inertia of the bob is unchanged. However, the clock mechanism still advances by a fixed amount for every swing, as if each swing still took the Earth value of T. Since every real swing now takes longer than the mechanism assumes, fewer swings occur per actual hour, so the clock's hands advance more slowly than real time elapses. The clock will run **slow** — it will fall behind correct time.

Bài tập luyện tập

For any SHM oscillator, at $x = A/2$, what is the ratio of kinetic energy to potential energy, $K : U$?

(A) $1 : 4$

(B) $1 : 3$

(C) $3 : 1$

(D) $4 : 1$

Lời giải chi tiết

$U = \frac{1}{2}kx^2 = \frac{1}{2}k(A/2)^2 = \frac{1}{4}\left(\frac{1}{2}kA^2\right) = E/4$, so $K = E - U = 3E/4$. Thus $K : U = (3E/4) : (E/4) = 3 : 1$. **(C)**.

Bài tập luyện tập

A v–t graph for an SHM oscillator shows a peak speed of 3.2 m/s, with successive points where $v = 0$ separated by $\pi/4$ s (≈ 0.785 s). Find the period, the angular frequency, and the amplitude.

Lời giải chi tiết

Time between successive zeros of v is half a period: $T/2 = \pi/4\,\text{s} \Rightarrow T = \boxed{\pi/2\,\text{s} \approx 1.57\,\text{s}}$.

$\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{\pi/2} = \boxed{4.0\,\text{rad/s}}$.

Since $v_{\text{max}} = A\omega$: $A = \dfrac{v_{\text{max}}}{\omega} = \dfrac{3.2}{4.0} = \boxed{0.80\,\text{m}}$.

Bài tập luyện tập

An oscillator is displaced from equilibrium and released from rest. If the system is **critically damped**, its subsequent motion is best described as:

(A) Oscillating indefinitely with constant amplitude

(B) Oscillating back and forth with slowly shrinking amplitude

(C) Returning to equilibrium without oscillating, in the minimum possible time of any non-oscillating case

(D) Never returning to equilibrium

Lời giải chi tiết

Critical damping is the boundary case: just enough resistance to prevent any overshoot/oscillation, while returning to equilibrium as quickly as possible among all non-oscillating (overdamped or critical) cases. **(C)**.

Bài tập luyện tập

Two identical spring-block systems are pulled to the same displacement and released simultaneously: System 1 oscillates in open air (very light damping); System 2 oscillates fully submerged in a thick oil bath (heavy, overdamping). Describe qualitatively how the x–t graphs of the two systems differ.

Lời giải chi tiết

System 1 (light damping, underdamped) still oscillates back and forth through equilibrium many times, but each successive peak is slightly smaller than the last — its x–t graph looks like the usual cosine curve trapped inside a slowly shrinking envelope, and its period stays very close to the undamped value $T = 2\pi\sqrt{m/k}$. System 2 (heavy, overdamping) never crosses equilibrium more than once — the resistive force from the oil removes energy so quickly that the block simply glides back toward $x = 0$ without overshooting or oscillating at all; its x–t graph is a smooth, non-oscillating curve that approaches zero more slowly than a critically damped system would. In short, System 1's graph keeps its wave-like shape while shrinking in height; System 2's graph never waves at all.

Bài tập luyện tập

A mass hangs in equilibrium from a vertical spring, already stretched a distance $x_0 = mg/k$ beyond the spring's natural length. It is then pulled down an *additional* distance d (beyond that equilibrium) and released from rest. What is the amplitude of the resulting oscillation?

(A) x_0

(B) d

(C) $x_0 + d$

(D) $x_0 - d$

Lời giải chi tiết

Amplitude is measured from the equilibrium position, and since the mass is released from rest, its starting point *is* the turning point. That starting point is a distance d from equilibrium, so $A = d$ — the equilibrium stretch x_0 does not enter the amplitude at all (it only shifts where "equilibrium" is located). **(B)**.

Bài tập luyện tập

A simple pendulum of length $L = 1.2$ m is released from a small angle. Find its period. Then state what happens to the period if the release angle is doubled (while remaining a small angle).

Lời giải chi tiết

$T = 2\pi\sqrt{L/g} = 2\pi\sqrt{1.2/9.8} = 2\pi\sqrt{0.1224} = 2\pi(0.350) = \boxed{2.2\,\text{s}}$.
Doubling the release angle (while it remains small) does **not** change the period: for small angles the restoring force is proportional to displacement, so the pendulum is isochronous and T stays at $\approx 2.2\,\text{s}$ regardless of amplitude.

Bài tập luyện tập

In simple harmonic motion, which pair of quantities is always exactly 180° out of phase with each other?

(A) x and v

(B) v and a

(C) x and a

(D) x and t

Lời giải chi tiết

Since $a = -\omega^2 x$, acceleration and position always have opposite sign at every instant — exactly 180° out of phase. **(C)**.

Bài tập luyện tập

A $0.50\,\text{kg}$ mass hangs from a vertical spring with $k = 50\,\text{N/m}$. (a) Find the equilibrium stretch x_0. (b) Find the period of oscillation. (c) The mass is pulled down $0.10\,\text{m}$ below equilibrium and released from rest. Find the total mechanical energy of the oscillation (measured relative to the equilibrium position) and the maximum speed.

Lời giải chi tiết

(a) $x_0 = \dfrac{mg}{k} = \dfrac{(0.50)(9.8)}{50} = \dfrac{4.9}{50} = \boxed{0.098\,\text{m}}$.

(b) $\omega = \sqrt{k/m} = \sqrt{50/0.50} = \sqrt{100} = 10\,\text{rad/s}$, so $T = \dfrac{2\pi}{10} = \boxed{0.628\,\text{s}}$.

(c) Released from rest $0.10\,\text{m}$ below equilibrium, that point is a turning point, so $A = \boxed{0.10\,\text{m}}$. Then $E = \frac{1}{2}kA^2 = \frac{1}{2}(50)(0.10)^2 = \boxed{0.25\,\text{J}}$, and $v_{\text{max}} = A\omega = (0.10)(10) = 1.0\,\text{m/s}$; check: $v_{\text{max}} = \sqrt{2E/m} = \sqrt{2(0.25)/0.50} = \sqrt{1.0} = \boxed{1.0\,\text{m/s}}$ — matches.

Fluids

Mục tiêu Unit: Khối lượng riêng và áp suất; áp suất tuyệt đối theo độ sâu $P = P_0 + \rho g h$ và áp suất kế (gauge); nguyên lý Pascal và hệ thống thủy lực; lực đẩy Archimedes, điều kiện vật chìm/nổi và trọng lượng biểu kiến; phương trình liên tục $A_1 v_1 = A_2 v_2$; phương trình Bernoulli, hiệu ứng Venturi và định luật Torricelli.

8.1 Density and Pressure

8.1.1 Density

The **density** of a substance is its mass per unit volume,

$$\rho = \frac{m}{V},$$

with SI unit kg/m^3. Density is an intrinsic property of a material — it does not depend on how much of the substance is present, only on what the substance *is*. Two blocks of the same metal, one twice the size of the other, have the same density even though their masses and volumes both differ. Fresh water has density $\rho_w = 1000$ kg/m^3; this is the default value used throughout this chapter unless a problem states otherwise.

Material	Density (kg/m^3)
Air (sea level)	≈ 1.2
Ice	920
Fresh water	1000
Seawater	1025
Aluminum	2700
Iron / steel	≈ 7800
Lead	11000

Table 8.1: *
Typical densities used throughout this chapter.

8.1.2 Pressure and Pascal's Insight

Pressure at a point in a fluid is the perpendicular (normal) force per unit area that the fluid exerts on any surface it touches:

$$P = \frac{F}{A},$$

with SI unit the pascal, Pa $=$ N/m^2. Because a fluid at rest cannot sustain a sideways (shear) force, the force it exerts on any surface is always directed **perpendicular** to that surface, no matter how the surface is tilted. Moreover, at any single point inside a static fluid the pressure is the same in **every direction** — it pushes equally hard up, down, and sideways on an infinitesimal test surface placed at that point. This is Pascal's insight, and it underlies every other result in this chapter.

Khái niệm cốt lõi

Pressure is a **scalar**, not a vector — it has no single direction of its own. What *is* directional is the force pressure produces on a particular surface: that force always points into the surface, perpendicular to it. A submerged object therefore feels pressure pushing inward on every face, regardless of how the faces are oriented.

$$\rho = \frac{m}{V}, \qquad P = \frac{F}{A}\ [\mathrm{Pa} = \mathrm{N/m^2}].$$

Pressure at a point in a static fluid is the same in every direction; the force a fluid exerts on a surface always acts perpendicular to that surface.

VN **GIẢI THÍCH TIẾNG VIỆT**

Khối lượng riêng $\rho = m/V$ là đặc trưng của chất liệu, không phụ thuộc lượng chất đang xét. **Áp suất** $P = F/A$ là lực vuông góc trên một đơn vị diện tích, đơn vị pascal (Pa $=$ N/m^2). Điểm mấu chốt của Pascal: tại một điểm trong chất lưu tĩnh, áp suất **như nhau theo mọi hướng**, và lực do áp suất gây ra trên một bề mặt luôn **vuông góc** với bề mặt đó, bất kể bề mặt nghiêng thế nào.

Ví dụ 1 · Computing density

A metal block measures 3.0 cm $\times$ 4.0 cm $\times$ 5.0 cm and has mass 0.540 kg. Find its density.
Volume: $V = (3.0)(4.0)(5.0) = 60\,\mathrm{cm^3} = 6.0 \times 10^{-5}\,\mathrm{m^3}$.

$$\rho = \frac{m}{V} = \frac{0.540}{6.0 \times 10^{-5}} = 9000\,\mathrm{kg/m^3}.$$

This is close to the density of copper ($8960\,\mathrm{kg/m^3}$), consistent with a dense metal block.

Ví dụ 2 · Pressure depends on area, not just force

A 600 N dancer balances her full weight on one stiletto heel of area $1.0\,\mathrm{cm^2}$. An elephant of weight 4.0×10^4 N stands on one foot of area $0.10\,\mathrm{m^2}$ ($1000\,\mathrm{cm^2}$). Compare the pressure each exerts on the ground.
Heel: $A = 1.0\,\mathrm{cm^2} = 1.0 \times 10^{-4}\,\mathrm{m^2}$,

$$P_{\text{heel}} = \frac{600}{1.0 \times 10^{-4}} = 6.0 \times 10^6\ \mathrm{Pa}.$$

Elephant's foot:

$$P_{\text{foot}} = \frac{4.0 \times 10^4}{0.10} = 4.0 \times 10^5\ \mathrm{Pa}.$$

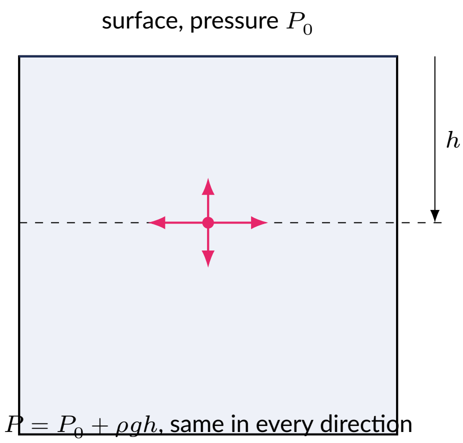
The heel's pressure is about 15 times larger than the elephant's, even though the elephant's *force* is nearly 70 times bigger — because pressure depends on force *per unit area*, and the heel's area is enormously smaller.

8.2 Pressure Variation with Depth

Consider a static, incompressible fluid open to the atmosphere at its top surface, where the pressure is P_0. To find the pressure at a depth h below that surface, isolate an imaginary vertical column of fluid of cross-sectional area A extending from the surface down to depth h. The column's weight,

ρAhg, must be supported by the difference between the (larger) pressure pushing up on its bottom face and the (smaller) pressure pushing down on its top face:

$$(P - P_0)A = \rho Ahg \quad \Rightarrow \quad P = P_0 + \rho gh.$$



Pressure at depth h pushes equally in every direction and grows linearly with depth.

8.2.1 Absolute Pressure vs. Gauge Pressure

$P = P_0 + \rho gh$ gives the **absolute pressure** — the true total pressure at that point, including the atmosphere pushing down from above. Most pressure gauges (tire gauges, blood-pressure cuffs, dive computers) are built to read zero when exposed to the open atmosphere; they report the **gauge pressure**, the amount *above* atmospheric:

$$P_{\text{gauge}} = P - P_0 = \rho gh.$$

$$P_{\text{abs}} = P_0 + \rho gh, \qquad P_{\text{gauge}} = P_{\text{abs}} - P_0 = \rho gh.$$

Absolute pressure is what a submerged object actually experiences; gauge pressure is what most instruments display. A gauge pressure of zero corresponds to an absolute pressure of P_0 (atmospheric), not to a vacuum.

(!) Lỗi thường gặp: Watch for the word "absolute" vs. "gauge" in a problem. If a question gives a gauge-pressure reading and asks for absolute pressure (or vice-versa), the atmospheric term P_0 must be added or subtracted — forgetting it is one of the most common scoring errors on fluid-pressure problems. When a problem simply says "pressure at depth h" without specifying, assume it wants absolute pressure using $P = P_0 + \rho gh$.

VN GIẢI THÍCH TIẾNG VIỆT

Áp suất tuyệt đối $P = P_0 + \rho gh$ là áp suất thật tại một điểm, tính luôn cả áp suất khí quyển P_0 đè lên từ trên. **Áp suất kế (gauge)** là phần **vượt trên** áp suất khí quyển: $P_{\text{gauge}} = \rho gh$. Đồng hồ đo áp suất (bánh xe, bình lặn...) thường hiển thị áp suất gauge (bằng 0 khi ở ngoài không khí), **không phải** áp suất tuyệt đối — cần cộng/trừ P_0 khi đề bài chuyển đổi giữa hai loại.

8.2.2 The Hydrostatic Paradox

Notice that $P = P_0 + \rho gh$ depends only on the depth h, the fluid's density ρ, and g — it does **not** depend on the shape of the container, its cross-sectional area, or the total volume (and hence total weight) of fluid present. A narrow test tube and a wide swimming pool, both filled with water to the same depth, have exactly the same pressure at the bottom. This counter-intuitive result is sometimes called the **hydrostatic paradox**.

(!) Lỗi thường gặp: Do not assume a wider or heavier column of fluid produces more pressure at a given depth. Pressure at depth h is set entirely by h, ρ, and g; a skinny tube and a massive tank filled to the same depth with the same fluid press equally hard on their bottoms.

Ví dụ 3 · Absolute pressure at depth

Find the absolute pressure at a depth of 3.0 m in a swimming pool. Use $P_0 = 1.0 \times 10^5$ Pa.

$P = P_0 + \rho g h = 1.0 \times 10^5 + (1000)(9.8)(3.0) = 1.0 \times 10^5 + 29400 = 1.294e5$ Pa $\approx 1.29 \times 10^5$ Pa.

Ví dụ 4 · Gauge vs. absolute pressure

A scuba diver's gauge reads a gauge pressure of 1.96×10^5 Pa. Find her depth below the surface and the absolute pressure she experiences.

$P_{\text{gauge}} = \rho g h \Rightarrow h = \dfrac{P_{\text{gauge}}}{\rho g} = \dfrac{1.96 \times 10^5}{(1000)(9.8)} = \dfrac{1.96 \times 10^5}{9800} = 20.0$ m.

Absolute pressure: $P_{\text{abs}} = P_0 + P_{\text{gauge}} = 1.0 \times 10^5 + 1.96 \times 10^5 = 2.96e5$ Pa.

8.3 Pascal's Principle and Hydraulic Systems

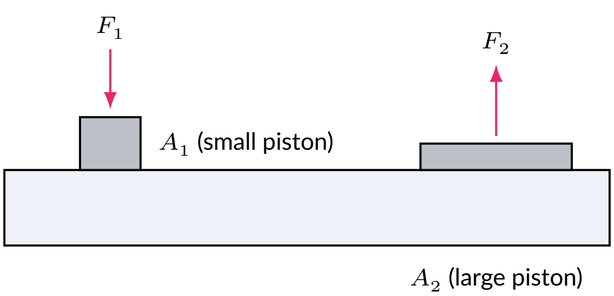
Pascal's principle: a pressure change applied anywhere to an enclosed, incompressible fluid is transmitted **undiminished** to every point of the fluid and to the walls of its container. This is the operating principle of a hydraulic lift.

8.3.1 Force Amplification

Consider a sealed hydraulic system with two pistons of different areas, A_1 (small) and A_2 (large), in contact with the same enclosed fluid. A force F_1 applied to the small piston creates a pressure increase $\Delta P = F_1/A_1$. By Pascal's principle, this same pressure increase is transmitted to the large piston, producing an upward force $F_2 = \Delta P \cdot A_2$. Combining (and neglecting any height difference between the two pistons):

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}.$$

Because $A_2 > A_1$, the force on the large piston is **amplified**: $F_2 = F_1\left(\dfrac{A_2}{A_1}\right)$. This is exactly how a small hand pump can lift a car on an automotive hydraulic lift.



A hydraulic lift: enclosed, incompressible fluid transmits pressure undiminished, so $F_1/A_1 = F_2/A_2$.

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}, \qquad F_2 = F_1\left(\frac{A_2}{A_1}\right).$$

A hydraulic lift multiplies force by the ratio of piston areas; the larger the area ratio, the larger the mechanical advantage $F_2/F_1 = A_2/A_1$.

8.3.2 Volume Conservation and Energy

A hydraulic lift does **not** create energy. Because the fluid is incompressible, the volume pushed out from under the small piston as it moves down a distance d_1 must exactly equal the volume that flows in under the large piston as it rises a distance d_2:

$$A_1 d_1 = A_2 d_2 \quad\Rightarrow\quad d_2 = d_1\left(\frac{A_1}{A_2}\right).$$

Since $A_2 > A_1$, the large piston moves a much **smaller** distance than the small piston. Multiplying the force relation by the distance relation shows work is conserved:

$$F_1 d_1 = \left(F_1\frac{A_2}{A_1}\right)\left(d_1\frac{A_1}{A_2}\right) = F_1 d_1 = F_2 d_2.$$

The work you put in by pushing the small piston down equals the work delivered by the large piston lifting its load — a hydraulic lift trades force for distance, exactly like a lever or a ramp.

$$A_1 d_1 = A_2 d_2 \quad \text{(volume conservation)}, \qquad F_1 d_1 = F_2 d_2 \quad \text{(work in = work out)}.$$

VN **GIẢI THÍCH TIẾNG VIỆT**

Nguyên lý Pascal: áp suất tác dụng lên chất lưu kín, không nén được sẽ truyền **nguyên vẹn** đến mọi điểm. Trong hệ thủy lực, $F_1/A_1 = F_2/A_2$ nên pít-tông lớn nhận lực lớn hơn — đây là cách một lực nhỏ có thể nâng cả một chiếc xe. Tuy nhiên hệ thủy lực **không tạo ra năng lượng**: do thể tích chất lưu bảo toàn ($A_1 d_1 = A_2 d_2$), pít-tông nhận lực lớn hơn lại di chuyển quãng đường **ngắn hơn** tương ứng, sao cho công vào bằng công ra ($F_1 d_1 = F_2 d_2$).

Ví dụ 5 · Hydraulic lift: force ratio

A hydraulic lift has a small piston of area $A_1 = 5.0\,\text{cm}^2$ and a large piston of area $A_2 = 250\,\text{cm}^2$. A force of 200 N is applied to the small piston. Find the force delivered by the large piston.

$$F_2 = F_1\left(\frac{A_2}{A_1}\right) = 200\left(\frac{250}{5.0}\right) = 200(50) = 1.0e4\,\text{N}.$$

This $1.0e4$ N (about 1020 kg-weight) is enough to lift a small car.

Ví dụ 6 · Hydraulic lift: piston displacement and energy check

For the same system as Ví dụ 5, the small piston is pushed down $d_1 = 0.60$ m. Find how far the large piston rises, and verify that work in equals work out.

Volume conservation: $A_1 d_1 = A_2 d_2 \Rightarrow d_2 = d_1\left(\frac{A_1}{A_2}\right) = 0.60\left(\frac{5.0}{250}\right) = 0.60(0.02) = 0.012\,\text{m} = 1.2\,\text{cm}$.
Work in: $W_1 = F_1 d_1 = (200)(0.60) = 120\,\text{J}$. Work out: $W_2 = F_2 d_2 = (1.0 \times 10^4)(0.012) = 120\,\text{J}$.

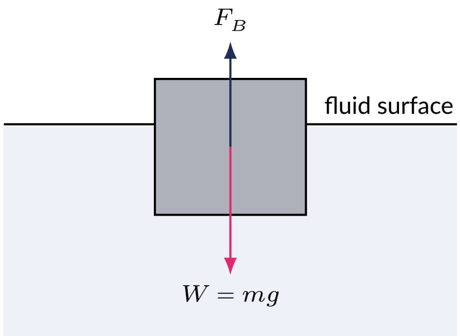
The two works match exactly — the $50\times$ force amplification is paid for by a $50\times$ reduction in distance moved.

8.4 Buoyancy and Archimedes' Principle

Archimedes' principle: any object partially or fully submerged in a fluid experiences an upward **buoyant force** equal in magnitude to the weight of the fluid the object displaces:

$$F_B = \rho_{\text{fluid}}\, g\, V_{\text{displaced}}.$$

Crucially, F_B depends only on the density of the **fluid** and the **volume displaced** — never on the object's own density, mass, or shape. A block of lead and a block of wood of the same volume, fully submerged, feel exactly the same buoyant force; what differs is their own weight, which determines whether that buoyant force is enough to hold them up.



Buoyant force $F_B = \rho_{\text{fluid}}\, g\, V_{\text{displaced}}$ acts upward; weight $W = mg$ acts downward.

$$F_B = \rho_{\text{fluid}}\, g\, V_{\text{displaced}}.$$

F_B never depends on the submerged object's own density or shape — only on the fluid's density and how much volume is pushed aside.

(!) Lỗi thường gặp: The single most common error in buoyancy problems is plugging the **object's** density into $F_B = \rho g V$ instead of the **fluid's** density. The buoyant force is the weight of fluid displaced — it is computed with ρ_{fluid}, always. The object's own density is needed only to find the object's actual weight, for comparison with F_B.

VN GIẢI THÍCH TIẾNG VIỆT

Nguyên lý Archimedes: lực đẩy nổi F_B bằng **trọng lượng của phần chất lưu bị vật chiếm chỗ**, $F_B = \rho_{\text{fluid}} g V_{\text{displaced}}$ — luôn dùng khối lượng riêng của **chất lưu**, tuyệt đối không dùng khối lượng riêng của vật. Đây là lỗi sai phổ biến nhất trong các bài toán lực đẩy Archimedes.

8.4.1 Fully Submerged Objects: Sink, Float, or Neutral

For an object that is completely underwater, $V_{\text{displaced}} = V_{\text{object}}$, so the buoyant force is $F_B = \rho_{\text{fluid}} g V_{\text{object}}$, while the object's actual weight is $W = \rho_{\text{object}} g V_{\text{object}}$. Comparing the two:

- If $\rho_{\text{object}} > \rho_{\text{fluid}}$: $W > F_B$, net force downward — the object **sinks**.
- If $\rho_{\text{object}} < \rho_{\text{fluid}}$: $W < F_B$, net force upward — the object **rises** toward the surface, where it will float partially submerged.
- If $\rho_{\text{object}} = \rho_{\text{fluid}}$: $W = F_B$ exactly — the object is **neutrally buoyant** and stays suspended at whatever depth it is placed.

GIẢI THÍCH TIẾNG VIỆT

VN

Với vật **chìm hoàn toàn**, so sánh khối lượng riêng vật với khối lượng riêng chất lưu quyết định vật chìm, nổi hay lơ lửng: vật đặc hơn chất lưu ($\rho_{\text{vật}} > \rho_{fluid}$) thì chìm; vật loãng hơn ($\rho_{\text{vật}} < \rho_{fluid}$) thì nổi lên; bằng nhau thì lơ lửng cân bằng ở mọi độ sâu.

Ví dụ 7 · Sink or float?

A solid aluminum cube ($\rho = 2700\,\text{kg/m}^3$) has volume $2.0 \times 10^{-4}\,\text{m}^3$ and is fully submerged in water. Find its weight, the buoyant force on it, and determine whether it sinks or rises.

Weight: $W = \rho_{\text{obj}} V g = (2700)(2.0 \times 10^{-4})(9.8) = 5.292\,\text{N}$.
Buoyant force: $F_B = \rho_{\text{water}} V g = (1000)(2.0 \times 10^{-4})(9.8) = 1.96\,\text{N}$.
Since $W > F_B$ (aluminum is denser than water), the net force is $5.292 - 1.96 = 3.33\,\text{N}$ downward — the cube **sinks**.

8.4.2 Floating Objects

An object floats when it is only **partially** submerged and in equilibrium, so the buoyant force exactly balances its weight: $F_B = W$. Writing this out with V_{sub} as the submerged volume and V_{total} as the object's total volume,

$$\rho_{\text{fluid}}\, g\, V_{\text{sub}} = \rho_{\text{object}}\, g\, V_{\text{total}} \quad \Rightarrow \quad \frac{V_{\text{sub}}}{V_{\text{total}}} = \frac{\rho_{\text{object}}}{\rho_{\text{fluid}}}.$$

The fraction of an object's volume that sits below the surface equals the ratio of its own density to the fluid's density.

$$\frac{V_{\text{sub}}}{V_{\text{total}}} = \frac{\rho_{\text{object}}}{\rho_{\text{fluid}}} \qquad \text{(floating equilibrium)}.$$

GIẢI THÍCH TIẾNG VIỆT

VN

Với vật **nổi**, điều kiện cân bằng $F_B = W$ cho tỉ lệ thể tích chìm: $V_{\text{sub}}/V_{\text{total}} = \rho_{\text{vật}}/\rho_{\text{fluid}}$. Vật càng đặc (gần bằng khối lượng riêng chất lưu) thì càng chìm sâu; vật càng nhẹ thì phần nổi trên mặt càng lớn.

Ví dụ 8 · Floating iceberg

An iceberg ($\rho_{\text{ice}} = 920\,\text{kg/m}^3$) floats in seawater ($\rho_{\text{sea}} = 1025\,\text{kg/m}^3$). Find the fraction of its volume that is submerged.

$$\frac{V_{\text{sub}}}{V_{\text{total}}} = \frac{\rho_{\text{ice}}}{\rho_{\text{sea}}} = \frac{920}{1025} = 0.898.$$

About 89.8% of the iceberg's volume lies below the waterline, with only 10.2% visible above — the familiar "tip of the iceberg."

8.4.3 Apparent Weight

When a submerged (or partially submerged) object hangs from a scale or a string, the scale does not read the object's true weight — it reads the **apparent weight**, reduced by the buoyant force:

$$W_{\text{apparent}} = W_{\text{actual}} - F_B.$$

This is how the classic "crown problem" works: an object's weight can be measured in air, then measured again while submerged; the difference between the two readings is exactly the buoyant force, which can be used to find the object's volume and hence its density.

$$W_{\text{apparent}} = W_{\text{actual}} - F_B = W_{\text{actual}} - \rho_{\text{fluid}} g V_{\text{object}}.$$

GIẢI THÍCH TIẾNG VIỆT

VN

Trọng lượng biểu kiến là số chỉ trên cân/lực kế khi vật bị nhúng trong chất lưu, luôn **nhỏ hơn** trọng lượng thật vì bị lực đẩy Archimedes "gánh bớt": $W_{\text{apparent}} = W_{\text{actual}} - F_B$. Đo hai lần (ngoài không khí và khi nhúng chìm) rồi lấy hiệu số chính là cách xác định F_B và từ đó suy ra thể tích/khối lượng riêng của vật.

Ví dụ 9 · Apparent weight of a submerged stone

A stone of mass $2.0\,\text{kg}$ and volume $8.0 \times 10^{-4}\,\text{m}^3$ hangs from a spring scale, fully submerged in water. Find the scale reading.

Actual weight: $W = mg = (2.0)(9.8) = 19.6\,\text{N}$.
Buoyant force: $F_B = \rho_{\text{water}} V g = (1000)(8.0 \times 10^{-4})(9.8) = 7.84\,\text{N}$.

$$W_{\text{apparent}} = 19.6 - 7.84 = 11.8\,\text{N}.$$

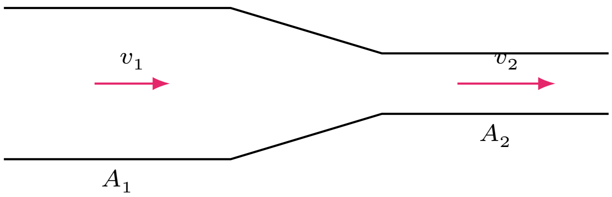
8.5 Fluid Flow and the Continuity Equation

For the rest of this chapter, fluid is treated as **incompressible** (constant density) and flow as **steady** — the velocity at each fixed point in space does not change with time, so the fluid follows smooth paths called **streamlines**.

The **volume flow rate** Q through a cross-section of a pipe is the volume of fluid passing that cross-section per unit time, $Q = Av$, with SI unit m^3/s. Because the fluid is incompressible and cannot be created or destroyed inside the pipe, the volume flow rate must be the **same** at every cross-section along a single pipe with no branches — this is the **continuity equation**:

$$A_1 v_1 = A_2 v_2.$$

Wherever the pipe's cross-sectional area shrinks, the fluid speed must increase to keep the same volume moving past every point each second — and vice-versa. The **mass flow rate**, $\rho A v$ (SI unit kg/s), is likewise constant along the pipe for a single incompressible fluid.



Continuity: $A_1 v_1 = A_2 v_2$ — the fluid speeds up where the pipe narrows.

$$Q = Av \ \ [\text{m}^3/\text{s}], \qquad A_1 v_1 = A_2 v_2 \quad (\text{continuity}), \qquad \dot{m} = \rho A v \ \ [\text{kg/s}].$$

GIẢI THÍCH TIẾNG VIỆT

VN

Với dòng chảy ổn định, không nén được, **lưu lượng thể tích** $Q = Av$ không đổi dọc theo ống (không tạo thêm hay mất bớt chất lưu): $A_1v_1 = A_2v_2$ — chỗ ống **hẹp** thì tốc độ chất lưu **tăng**, chỗ ống **rộng** thì tốc độ **giảm**. Lưu lượng khối lượng $\dot{m} = \rho Av$ cũng không đổi với một loại chất lưu duy nhất.

Ví dụ 10 · Continuity in a narrowing pipe

Water flows through a pipe with cross-sectional area $A_1 = 6.0\,\text{cm}^2$ at speed $v_1 = 2.0\,\text{m/s}$. The pipe narrows to $A_2 = 2.0\,\text{cm}^2$. Find the speed in the narrow section.

$$v_2 = \frac{A_1v_1}{A_2} = \frac{(6.0)(2.0)}{2.0} = 6.0\,\text{m/s}.$$

Ví dụ 11 · Mass flow rate

Water flows through a pipe of cross-sectional area $3.0 \times 10^{-3}\,\text{m}^2$ at $2.0\,\text{m/s}$. Find the volume flow rate and the mass flow rate.

$$Q = Av = (3.0 \times 10^{-3})(2.0) = 6.0e - 3\,\text{m}^3/\text{s}.$$

$$\dot{m} = \rho Q = (1000)(6.0 \times 10^{-3}) = 6.0\,\text{kg/s}.$$

8.6 Bernoulli's Equation

Bernoulli's equation is a statement of energy conservation for a fluid, valid along a single streamline for flow that is steady, non-viscous (no internal friction losses), and incompressible:

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2.$$

Each term has units of energy per unit volume (equivalently, pressure): P plays the role of a "pressure energy," $\frac{1}{2}\rho v^2$ is a kinetic-energy density, and $\rho g y$ is a gravitational potential-energy density. Their sum stays constant as the fluid moves along a streamline — exactly parallel to KE + PE = const for a single particle, but written per unit volume for a continuous fluid.

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2.$$

If two of the three terms are equal at both points (e.g. equal heights), Bernoulli's equation reduces to a simple relationship between the remaining two.

(!) Lỗi thường gặp: Bernoulli's equation only applies along a streamline for flow that is steady, incompressible, and effectively free of viscous (friction) losses. It should not be applied blindly to turbulent flow, to flow with significant pumps/fans adding or removing energy between the two points, or to compressible gases moving at high speed. On the AP exam, apply it only where the problem clearly describes smooth, steady flow of an (effectively) incompressible fluid.

GIẢI THÍCH TIẾNG VIỆT

VN

Phương trình Bernoulli là phát biểu **bảo toàn năng lượng** cho chất lưu chảy ổn định, không nhớt, không nén được, dọc theo một đường dòng: $P + \frac{1}{2}\rho v^2 + \rho g y =$ hằng số. Ba số hạng tương ứng "năng lượng áp suất," động năng và thế năng trọng trường tính trên một đơn vị thể tích. Chỉ áp dụng khi dòng chảy **êm, ổn định**, không có bơm/quạt bơm thêm năng lượng giữa hai điểm đang xét.

8.6.1 Same-Height Flow: The Venturi Effect

When both points lie at the same height ($y_1 = y_2$), the height terms cancel and Bernoulli's equation simplifies to

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2.$$

Combined with continuity ($A_1 v_1 = A_2 v_2$), this shows that wherever a horizontal pipe narrows, the fluid speeds up *and* its pressure drops. This inverse relationship between speed and pressure at constant height is the **Venturi effect**.

Ví dụ 12 · The Venturi effect

A horizontal pipe carries water at pressure $P_1 = 1.5 \times 10^5$ Pa and speed $v_1 = 1.0\,\text{m/s}$ through a wide section of area $A_1 = 8.0\,\text{cm}^2$. It narrows to $A_2 = 2.0\,\text{cm}^2$. Find the speed and pressure in the narrow section.

Continuity: $v_2 = \dfrac{A_1 v_1}{A_2} = \dfrac{(8.0)(1.0)}{2.0} = 4.0\,\text{m/s}$.

Bernoulli (same height, $y_1 = y_2$):

$$P_2 = P_1 + \tfrac{1}{2}\rho\left(v_1^2 - v_2^2\right) = 1.5\times10^5 + \tfrac{1}{2}(1000)\left(1.0^2 - 4.0^2\right) = 1.5\times10^5 + 500(-15) = 1.425e5\,\text{Pa}.$$

Pressure **drops** in the narrow, faster-moving section, exactly as the Venturi effect predicts.

8.6.2 Height Differences: Torricelli's Law

Now consider a large tank, open to the atmosphere, with a small hole in its side at depth h below the top surface. Apply Bernoulli's equation between the top surface (point 1) and the hole (point 2). Because the tank's cross-sectional area is much larger than the hole's, the top surface drops so slowly that $v_1 \approx 0$. Both the surface and the hole are open to the atmosphere, so $P_1 = P_2 = P_0$. Taking the hole as $y = 0$ and the surface as $y = h$:

$$P_0 + 0 + \rho g h = P_0 + \tfrac{1}{2}\rho v_2^2 + 0 \quad\Rightarrow\quad v_2 = \sqrt{2gh}.$$

This is **Torricelli's law**: the efflux speed from a hole at depth h below an open surface is exactly the speed an object would reach falling freely through the same height h — a striking energy-conservation connection between a fluid squirting out a hole and a simple falling object.

$$v = \sqrt{2gh} \qquad \text{(Torricelli's law — efflux speed from a hole at depth } h\text{)}.$$

GIẢI THÍCH TIẾNG VIỆT

VN

Định luật Torricelli: tốc độ chất lưu phun ra từ một lỗ nhỏ ở độ sâu h dưới mặt thoáng (hở với khí quyển, bể đủ lớn để mặt thoáng gần như không hạ) bằng đúng tốc độ một vật rơi tự do đạt được sau khi rơi cùng độ cao h: $v = \sqrt{2gh}$. Đây là hệ quả trực tiếp của bảo toàn năng lượng khi áp dụng Bernoulli với $P_1 = P_2 = P_0$ và $v_1 \approx 0$.

Ví dụ 13 · Torricelli's law

A large open water tank has a small hole 1.25 m below the water surface. Find the speed of the water as it exits the hole.

$$v = \sqrt{2gh} = \sqrt{2(9.8)(1.25)} = \sqrt{24.5} = 4.95\,\text{m/s}.$$

8.6.3 Combining Continuity and Bernoulli

Many problems involve both a height change *and* an area change between the two points of interest. The standard approach: use **continuity** first to find the unknown speed from the known areas, then substitute into **Bernoulli's equation** to solve for the remaining unknown (usually a pressure).

Ví dụ 14 · Bernoulli with both height and area change

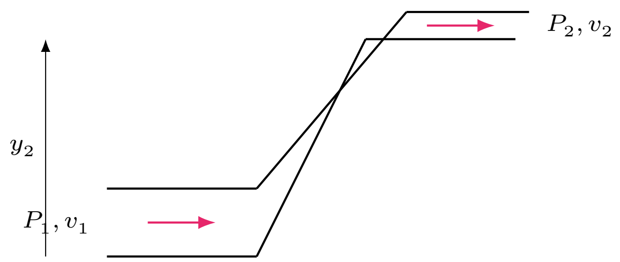
Water enters a vertical pipe at the ground floor of a building with speed $v_1 = 4.0$ m/s and absolute pressure $P_1 = 4.0 \times 10^5$ Pa. The pipe rises $y_2 = 5.0$ m to the second floor, where its cross-sectional area is half of what it was at the ground floor ($A_2 = \frac{1}{2}A_1$). Find the water's speed and pressure at the second floor.

Continuity: $A_1v_1 = A_2v_2 = \left(\frac{1}{2}A_1\right)v_2 \Rightarrow v_2 = 2v_1 = 8.0$ m/s.
Bernoulli (taking $y_1 = 0$):

$$P_2 = P_1 + \tfrac{1}{2}\rho\left(v_1^2 - v_2^2\right) - \rho g y_2 = 4.0 \times 10^5 + 500(16 - 64) - (1000)(9.8)(5.0)$$

$$= 4.0 \times 10^5 - 24000 - 49000 = 3.27e5\,\text{Pa}.$$

Both the rise in height and the increase in speed act to lower the pressure at the second floor, from 4.00×10^5 Pa down to 3.27×10^5 Pa.



A pipe that both rises through height y_2 and narrows: Bernoulli's equation links pressure, speed, and height between the two labeled points.

8.6.4 Qualitative Application: Lift on an Airfoil

An airplane wing (airfoil) is shaped so that air flowing over its curved top surface moves faster, on average, than air flowing along its flatter bottom surface. Treating the two flows as approximately steady streamlines at roughly the same height, Bernoulli's equation implies that the faster-moving air above the wing is at **lower** pressure than the slower air below it. This pressure difference, acting over the wing's surface area, produces a net upward force — **lift**. (The full aerodynamic picture also involves the wing's angle of attack and the downward deflection of air beneath it, which go beyond a strict Bernoulli treatment, but the speed–pressure relationship above captures the essential mechanism at the level of this course.)

GIẢI THÍCH TIẾNG VIỆT

VN

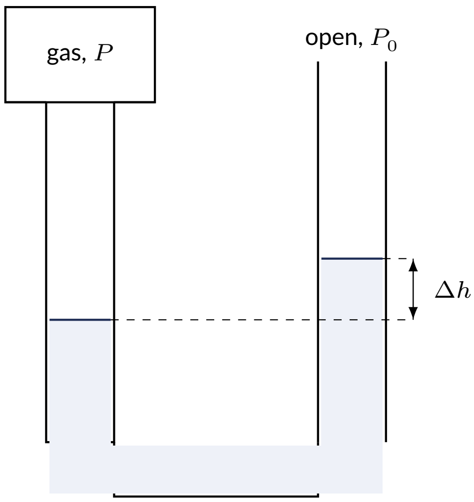
Ứng dụng định tính: trên cánh máy bay, dòng khí phía trên (mặt cong) chảy **nhanh hơn** dòng khí phía dưới (mặt phẳng hơn). Theo Bernoulli, nơi tốc độ lớn hơn thì áp suất **nhỏ hơn**, nên áp suất phía dưới cánh lớn hơn phía trên, tạo ra lực nâng hướng lên. Đây là cách giải thích định tính, đơn giản hoá — lực nâng thực tế còn phụ thuộc góc tấn và các hiệu ứng khác.

8.7 Manometers and Multi-Layer Fluids

8.7.1 U-Tube Manometers

A **manometer** is a device that measures pressure by balancing an unknown pressure against a column of liquid whose height can be read directly. The simplest version is the **open U-tube manometer**: a U-shaped tube partially filled with a liquid (commonly mercury or water), with one arm connected to the gas or system whose pressure is being measured and the other arm left open to the atmosphere.

Because the liquid inside the tube is a single connected body of static fluid, any two points at the **same height** within it must have the same pressure — otherwise the fluid at that height would still be flowing to equalize the difference. This is the balancing principle behind every manometer, and it is really just $P_1 + \rho g h_1 = P_2 + \rho g h_2$ applied to two points at a common reference height inside the connected liquid.



A U-tube manometer: the left arm connects to the gas, the right arm is open to the atmosphere. Since $P > P_0$ here, the liquid is pushed down on the gas side and up on the open side, with $P = P_0 + \rho g \Delta h$.

To use this principle, pick the reference height at the **lower** of the two liquid surfaces. Suppose the gas pressure P exceeds atmospheric pressure P_0: the gas pushes the liquid down in the connected arm and up in the open arm, until the two surfaces settle with a height difference Δh, the open arm's surface being higher. At the height of the lower (gas-side) surface, the pressure on the gas side is simply P itself; reaching that same height on the open side means descending an extra depth Δh through liquid open to the atmosphere, so the pressure there is $P_0 + \rho g \Delta h$. Setting the two equal,

$$P = P_0 + \rho g \Delta h.$$

If instead $P < P_0$, the same reasoning with the roles reversed gives $P = P_0 - \rho g \Delta h$, and now it is the **gas-side** surface that sits higher. Either way, the rule is the same: **the liquid stands higher on the lower-pressure side**, and the pressure difference between the two sides is $\rho g \Delta h$.

(!) Lỗi thường gặp: The most common manometer error is losing track of *which* side is higher. Before writing an equation, decide physically which surface must be pushed down: the side with the larger pressure pushes its own liquid down and forces the liquid up on the other side. The side with the **lower** liquid level is always the side with the **higher** pressure.

Ví dụ 15 · Reading a mercury manometer

A mercury U-tube manometer ($\rho_{\text{Hg}} = 13600\,\text{kg/m}^3$) has one arm connected to a sealed gas tank and the other arm open to the atmosphere ($P_0 = 1.0 \times 10^5$ Pa). The mercury level in the open arm sits 0.150 m higher than the level in the arm connected to the tank. Find the absolute pressure of the gas.

The open arm's mercury is higher, so the tank-side (gas) pressure exceeds atmospheric: $P = P_0 + \rho g \Delta h$.

$$P = 1.0 \times 10^5 + (13600)(9.8)(0.150) = 1.0 \times 10^5 + 19992 = 1.19992e5\ \text{Pa} \approx 1.20 \times 10^5\ \text{Pa}.$$

The gauge pressure of the gas is $\rho g \Delta h \approx 2.00e4$ Pa, about 0.20 atmosphere above atmospheric.

8.7.2 Pressure in Layered (Immiscible) Fluids

Many real containers hold two or more **immiscible** fluids — fluids that do not mix — stacked in layers by density, with the least dense fluid floating on top. A familiar example is a layer of oil floating on water. To find the pressure at the bottom of such a container, apply $P = P_0 + \rho g h$ one layer at a time, using each layer's *own* thickness and *own* density, and add the contributions in sequence from the open surface downward.

Consider a top layer of fluid 1 (density ρ_1, depth h_1) resting on fluid 2 (density ρ_2, depth h_2), open to the atmosphere. At the interface between the two fluids — a depth h_1 below the surface — the pressure is found exactly as for a single fluid: $P_{\text{interface}} = P_0 + \rho_1 g h_1$. Continuing downward through the second fluid to the bottom of the container adds one more term, built from fluid 2's own density and thickness:

$$P_{\text{bottom}} = P_{\text{interface}} + \rho_2 g h_2 = P_0 + \rho_1 g h_1 + \rho_2 g h_2.$$

Each layer contributes its own $\rho g h$ term to the running total; the layers simply stack.

(!) Lỗi thường gặp: Do not multiply the *combined* depth $h_1 + h_2$ by a single density. Each fluid layer contributes $\rho g h$ using **its own** thickness and **its own** density; compute the layers separately, from the top surface down, and add the results.

Ví dụ 16 · Pressure beneath two fluid layers

An open container holds a layer of oil ($\rho_{\text{oil}} = 800\,\text{kg/m}^3$) of depth $h_1 = 0.30$ m floating on top of a layer of water ($\rho_{\text{water}} = 1000\,\text{kg/m}^3$) of depth $h_2 = 0.50$ m. Take $P_0 = 1.0 \times 10^5$ Pa. Find (a) the pressure at the oil–water interface, and (b) the pressure at the bottom of the container.

(a) $P_{\text{interface}} = P_0 + \rho_{\text{oil}} g h_1 = 1.0 \times 10^5 + (800)(9.8)(0.30) = 1.0 \times 10^5 + 2352 = 1.02352e5$ Pa $\approx 1.02 \times 10^5$ Pa.

(b) $P_{\text{bottom}} = P_{\text{interface}} + \rho_{\text{water}} g h_2 = 1.02352 \times 10^5 + (1000)(9.8)(0.50) = 1.02352 \times 10^5 + 4900 = 1.07252e5$ Pa $\approx 1.07 \times 10^5$ Pa.

$$P = P_0 \pm \rho g \Delta h \quad \text{(manometer; liquid stands higher on the lower-pressure side)},$$

$$P_{\text{bottom}} = P_0 + \rho_1 g h_1 + \rho_2 g h_2 + \cdots \quad \text{(stacked immiscible fluid layers)}.$$

Both results follow from the same idea: at a fixed height inside one connected, static fluid, pressure is the same everywhere; crossing into a different fluid adds that fluid's own $\rho g h$ contribution.

VN **GIẢI THÍCH TIẾNG VIỆT**

Áp kế ống chữ U (manometer) dựa trên nguyên lý: hai điểm cùng độ cao trong một khối chất lưu tĩnh, liên thông với nhau thì có **cùng áp suất**. Nếu áp suất khí P lớn hơn áp suất khí quyển P_0, cột chất lưu bên hở sẽ **cao hơn** bên nối với khí một đoạn Δh, và $P = P_0 + \rho g \Delta h$; ngược lại nếu $P < P_0$ thì cột bên nối với khí cao hơn, $P = P_0 - \rho g \Delta h$ — bên nào có mực chất lưu **thấp hơn** thì bên đó có áp suất **cao hơn**. Với **chất lưu nhiều lớp** không hòa tan (như dầu nổi trên nước), áp suất tại đáy cộng dồn từng lớp theo khối lượng riêng và độ dày riêng của lớp đó: $P_{\text{đáy}} = P_0 + \rho_1 g h_1 + \rho_2 g h_2 + \cdots$.

8.7.3 Objects Floating at a Fluid–Fluid Interface

When an object floats at the boundary between two immiscible fluid layers of different density, part of its volume displaces the upper fluid and part displaces the lower fluid. The total buoyant force is the **sum of both contributions**:

$$F_B = \rho_1 g V_1 + \rho_2 g V_2, \qquad V_1 + V_2 = V_{\text{total}},$$

where V_1 is the volume of the object sitting in the upper fluid and V_2 is the volume sitting in the lower fluid. Floating equilibrium still requires $F_B = W$, exactly as with a single fluid — only now the buoyant force is built from a **volume-weighted average** of the two fluids' densities. Equivalently, floating equilibrium is reached when this weighted-average density of displaced fluid equals the object's own density.

Ví dụ 17 · Floating between oil and water

A block of density $\rho_{\text{block}} = 850\,\text{kg/m}^3$ and volume $V = 2.0 \times 10^{-4}\,\text{m}^3$ is placed in a container holding oil ($\rho_{\text{oil}} = 800\,\text{kg/m}^3$) floating on top of water ($\rho_{\text{water}} = 1000\,\text{kg/m}^3$). Because the block is denser than the oil but less dense than the water, it sinks through the oil layer and comes to rest straddling the interface, with its upper portion in oil and its lower portion in water. Find the fraction of the block's volume in each fluid.

Let f be the fraction of the block's volume in the oil (above the interface); then $(1-f)$ is the fraction in the water. Floating equilibrium requires the combined buoyant force from both fluids to equal the block's weight:

$$\rho_{\text{oil}} g (fV) + \rho_{\text{water}} g[(1-f)V] = \rho_{\text{block}} g V.$$

Dividing by gV and solving for f:

$$\rho_{\text{oil}} f + \rho_{\text{water}}(1-f) = \rho_{\text{block}} \quad \Rightarrow \quad f = \frac{\rho_{\text{water}} - \rho_{\text{block}}}{\rho_{\text{water}} - \rho_{\text{oil}}} = \frac{1000-850}{1000-800} = \frac{150}{200} = 0.75.$$

So 75% of the block's volume sits in the oil layer, and the remaining 25% sits in the water layer. Check: with $V_{\text{oil}} = 0.75V = 1.5 \times 10^{-4}\,\text{m}^3$ and $V_{\text{water}} = 0.25V = 5.0 \times 10^{-5}\,\text{m}^3$,

$$F_B = (800)(9.8)(1.5 \times 10^{-4}) + (1000)(9.8)(5.0 \times 10^{-5}) = 1.176 + 0.490 = 1.666\,\text{N},$$

which matches the block's actual weight, $W = \rho_{\text{block}} V g = (850)(2.0 \times 10^{-4})(9.8) = 1.666\,\text{N}$, exactly.

8.8 Practice Problems

Bài tập luyện tập

A block has mass 4.5 kg and volume 5.0×10^{-3} m^3. Find its density.

(A) 225 kg/m^3
(B) 450 kg/m^3
(C) 900 kg/m^3
(D) 1800 kg/m^3

Lời giải chi tiết

$\rho = m/V = 4.5/5.0 \times 10^{-3} = 900$ kg/m^3. **(C)**.

Bài tập luyện tập

A box exerts a force of 300 N on the floor through a rectangular base measuring 0.50 m $\times$ 0.40 m. Find the pressure the box exerts on the floor.

Lời giải chi tiết

$A = (0.50)(0.40) = 0.20$ m^2. $P = F/A = 300/0.20 = \boxed{1500\text{ Pa}}$.

Bài tập luyện tập

Without changing its weight, which action would **increase** the pressure a block exerts on a table?

(A) Resting it on a larger face
(B) Resting it on a smaller face
(C) Painting it a different color
(D) Raising the table's height

Lời giải chi tiết

Pressure $= F/A$ with F (weight) fixed; a smaller contact area gives a larger pressure. **(B)**.

Bài tập luyện tập

Explain, in terms of pressure, why a sharp knife cuts through material more easily than a dull knife pressed with the same force.

Lời giải chi tiết

A sharp knife's edge has a much smaller contact area than a dull knife's. Since $P = F/A$, applying the same force F over the sharp edge's tiny area produces a far larger pressure than the same force spread over the dull edge's wider, rounded area. It is this larger pressure — not a larger force — that allows the sharp knife to exceed the material's threshold for cutting.

Bài tập luyện tập

Find the absolute pressure at a depth of 10 m in fresh water ($P_0 = 1.0 \times 10^5$ Pa).

(A) 1.0×10^5 Pa
(B) 1.49×10^5 Pa
(C) 1.98×10^5 Pa
(D) 2.98×10^5 Pa

Lời giải chi tiết

$P = P_0 + \rho g h = 1.0 \times 10^5 + (1000)(9.8)(10) = 1.0 \times 10^5 + 98000 = 1.98 \times 10^5$ Pa. **(C).**

Bài tập luyện tập

Find the gauge pressure at the bottom of a water tank 12 m deep.

Lời giải chi tiết

$P_{\text{gauge}} = \rho g h = (1000)(9.8)(12) = \boxed{1.18e5\,\text{Pa}}$ (to 3 sig. figs.).

Bài tập luyện tập

Two open containers — one narrow and tall, one wide and short — are both filled with water to the same depth $h = 0.80$ m. Compare the pressure at the bottom of each container, and explain your reasoning.

Lời giải chi tiết

Absolute pressure at depth h in a static fluid is $P = P_0 + \rho g h$, which depends only on the depth h, the fluid's density ρ, and g — not on the container's shape, cross-sectional area, or the total volume/weight of fluid it holds (the hydrostatic paradox). Since both containers hold the same fluid (water) to the same depth, the pressure at the bottom is identical for both: $P = 1.0 \times 10^5 + (1000)(9.8)(0.80) = 1.0 \times 10^5 + 7840 = \boxed{1.08e5\,\text{Pa}}$ for each container, regardless of width.

Bài tập luyện tập

A hydraulic gauge reads a gauge pressure of 3.0×10^5 Pa. What is the absolute pressure ($P_0 = 1.0 \times 10^5$ Pa)?

(A) 2.0×10^5 Pa

(B) 3.0×10^5 Pa

(C) 4.0×10^5 Pa

(D) 3.0×10^{10} Pa

Lời giải chi tiết

$P_{\text{abs}} = P_0 + P_{\text{gauge}} = 1.0 \times 10^5 + 3.0 \times 10^5 = 4.0 \times 10^5$ Pa. **(C).**

Bài tập luyện tập

Which expression correctly gives the **absolute** pressure at depth h below an open water surface?

(A) $\rho g h$

(B) $P_0 + \rho g h$

(C) $P_0 - \rho g h$

(D) $P_0 \rho g h$

Lời giải chi tiết

Absolute pressure includes the atmospheric term added to the depth term: $P = P_0 + \rho g h$. **(B).**

Bài tập luyện tập

A hydraulic press has a small piston of area $8.0\,\text{cm}^2$ and a large piston of area $400\,\text{cm}^2$. A force of 150 N is applied to the small piston. Find the force delivered by the large piston.

Lời giải chi tiết

$F_2 = F_1(A_2/A_1) = 150(400/8.0) = 150(50) = \boxed{7500\,\text{N}}$.

Bài tập luyện tập

In the hydraulic press of the previous problem, the small piston moves down 20 cm. How far does the large piston move up?

(A) 0.40 cm
(B) 4.0 cm
(C) 20 cm
(D) 1000 cm

Lời giải chi tiết

$A_1d_1 = A_2d_2 \Rightarrow d_2 = d_1(A_1/A_2) = 20(8.0/400) = 20(0.02) = 0.40$ cm. **(A)**.

Bài tập luyện tập

Using conservation of energy, explain why the large piston of a hydraulic lift cannot move up by the same distance the small piston moves down, even though it feels a much larger force.

Lời giải chi tiết

No hydraulic system can create energy, so the work done pushing the small piston must equal the work delivered by the large piston: $F_1d_1 = F_2d_2$. Because the large piston's area is bigger, Pascal's principle makes F_2 much larger than F_1 ($F_2 = F_1A_2/A_1$). For the work products F_1d_1 and F_2d_2 to stay equal while $F_2 \gg F_1$, the large piston's displacement d_2 must be proportionally smaller than d_1 ($d_2 = d_1A_1/A_2$). This is also required by volume conservation of the incompressible fluid, $A_1d_1 = A_2d_2$: the volume of fluid vacated by the small piston must exactly fill the space made available by the large piston, and since $A_2 > A_1$, that same volume corresponds to a smaller rise d_2.

Bài tập luyện tập

A hydraulic car lift has a small piston of radius 1.0 cm and a large piston of radius 15 cm. Find the mechanical advantage F_2/F_1 and the force needed on the small piston to lift a 1200 kg car.

Lời giải chi tiết

Since area scales as r^2: $\dfrac{F_2}{F_1} = \dfrac{A_2}{A_1} = \left(\dfrac{15}{1.0}\right)^2 = \boxed{225}$.
Weight of car: $F_2 = (1200)(9.8) = 11760$ N. Required force: $F_1 = F_2/225 = 11760/225 = \boxed{52.3\,\text{N}}$.

Bài tập luyện tập

An object floats with 60% of its volume submerged in water. Find the object's density.

(A) $400\,\text{kg/m}^3$
(B) $600\,\text{kg/m}^3$
(C) $1000\,\text{kg/m}^3$
(D) $1667\,\text{kg/m}^3$

Lời giải chi tiết

$\rho_{\text{obj}} = (V_{\text{sub}}/V_{\text{total}})\rho_{\text{fluid}} = (0.60)(1000) = 600\,\text{kg/m}^3$. **(B)**.

Bài tập luyện tập

A solid cube of side 0.10 m and density $2500\,\text{kg/m}^3$ is fully submerged in water. Find the buoyant force, the net force on the cube, and state whether it sinks or floats.

Lời giải chi tiết

$V = (0.10)^3 = 1.0 \times 10^{-3}\,\text{m}^3$. Weight: $W = (2500)(1.0 \times 10^{-3})(9.8) = 24.5\,\text{N}$. Buoyant force: $F_B = (1000)(1.0 \times 10^{-3})(9.8) = \boxed{9.8\,\text{N}}$.
Net force $= 24.5 - 9.8 = \boxed{14.7\,\text{N}}$ downward; since $\rho_{\text{obj}} > \rho_{\text{water}}$, the cube **sinks**.

Bài tập luyện tập

An object with density less than water is held fully submerged and then released. What happens?

(A) It sinks to the bottom
(B) It rises until it floats, partly out of the water
(C) It remains neutrally buoyant at that depth
(D) It oscillates up and down forever

Lời giải chi tiết

Since $\rho_{\text{obj}} < \rho_{\text{water}}$, the buoyant force exceeds the object's weight while fully submerged, so it accelerates upward until it breaks the surface and settles into floating equilibrium with only part of its volume submerged. **(B)**.

Bài tập luyện tập

A stone of mass 3.0 kg and volume $1.2 \times 10^{-3}\,\text{m}^3$ hangs from a spring scale, fully submerged in water. Find the scale reading (apparent weight).

Lời giải chi tiết

$W = mg = (3.0)(9.8) = 29.4\,\text{N}$. $F_B = \rho_{\text{water}} V g = (1000)(1.2 \times 10^{-3})(9.8) = 11.76\,\text{N}$.
$W_{\text{apparent}} = 29.4 - 11.76 = \boxed{17.6\,\text{N}}$.

Bài tập luyện tập

A crown weighs 20 N in air and 17.6 N when fully submerged in water. Find the buoyant force and the crown's volume.

(A) 2.4×10^{-4} m^3

(B) 2.0×10^{-3} m^3

(C) 1.76×10^{-3} m^3

(D) 4.9×10^{-4} m^3

Lời giải chi tiết

$F_B = 20 - 17.6 = 2.4$ N. $V = F_B/(\rho_{\text{water}} g) = 2.4/[(1000)(9.8)] = 2.4/9800 = 2.4 \times 10^{-4}$ m^3. **(A)**.

Bài tập luyện tập

A student claims that a heavy steel ship floats because "steel is somehow lighter than water" when shaped into a ship. Explain what is wrong with this reasoning and give the correct explanation for why a steel ship floats.

Lời giải chi tiết

The claim is incorrect: the density of steel itself (≈ 7800 kg/m^3) is unchanged by shaping it into a hull — it is still about 7.8 times denser than water, and a solid block of steel would sink. What actually keeps a ship afloat is its **shape**: a hull encloses a large hollow volume filled mostly with air, so the ship's **overall average density** (total mass of steel plus air, divided by the total volume the hull encloses, including the empty interior) is less than the density of water. As the ship settles into the water, it displaces a large volume of water; floating equilibrium is reached once the buoyant force — the weight of that displaced water, $F_B = \rho_{\text{water}} g V_{\text{displaced}}$ — equals the ship's total weight. This has nothing to do with the density of the steel changing; it is entirely a consequence of the hull's shape enclosing enough empty volume.

Bài tập luyện tập

Two solid spheres of equal volume — one wood ($\rho = 600$ kg/m^3), one lead ($\rho = 11000$ kg/m^3) — are fully submerged in water and released. Compare the buoyant force on each while both are still submerged.

(A) Equal, since F_B depends only on displaced volume

(B) Lead has the larger F_B

(C) Wood has the larger F_B

(D) Cannot be determined without more information

Lời giải chi tiết

$F_B = \rho_{\text{water}} g V_{\text{displaced}}$ depends only on the fluid's density and the volume displaced — since both spheres have equal volume and are both fully submerged in the same water, they experience exactly the same buoyant force. (Their *net* forces still differ enormously, because their weights differ.) **(A)**.

Bài tập luyện tập

Water flows through a horizontal pipe of area 4.0×10^{-3} m^2 at 3.0 m/s. The pipe narrows to an area of 1.0×10^{-3} m^2. Find the speed in the narrow section and the volume flow rate.

Lời giải chi tiết

$Q = A_1 v_1 = (4.0 \times 10^{-3})(3.0) = \boxed{1.2 \times 10^{-2}\,\text{m}^3/\text{s}}$.
$v_2 = Q/A_2 = (1.2 \times 10^{-2})/(1.0 \times 10^{-3}) = \boxed{12\,\text{m/s}}$.

Bài tập luyện tập

For the pipe in the previous problem, find the mass flow rate.

(A) $1.2\,\text{kg/s}$
(B) $12\,\text{kg/s}$
(C) $120\,\text{kg/s}$
(D) $1200\,\text{kg/s}$

Lời giải chi tiết

$\dot{m} = \rho Q = (1000)(1.2 \times 10^{-2}) = 12\,\text{kg/s}$. **(B)**.

Bài tập luyện tập

Explain, using the continuity equation, why a river's current speeds up where the riverbed narrows. Then explain what would happen to the speed if the river also became shallower at that same narrow point.

Lời giải chi tiết

For steady, incompressible flow, the volume flow rate $Q = Av$ must be the same at every cross-section along the river — no water is created or destroyed along the way. If the river narrows, its cross-sectional area A decreases; to keep $Q = Av$ constant, the speed $v = Q/A$ must increase. If the river also becomes shallower at that same location, the cross-sectional area shrinks by even more (both width and depth are reduced), so by the same reasoning the speed must increase by an even larger factor than from narrowing alone.

Bài tập luyện tập

In a horizontal pipe with a Venturi-style constriction, compared to the wide section, in the narrow section the fluid speed is ___ and the pressure is ___.

(A) higher; higher
(B) higher; lower
(C) lower; higher
(D) lower; lower

Lời giải chi tiết

By continuity, a smaller area means higher speed. By Bernoulli's equation at constant height, higher speed corresponds to lower pressure. **(B)**.

Bài tập luyện tập

A horizontal pipe carries water at pressure 2.0×10^5 Pa and speed $2.0\,\text{m/s}$ through a section of area $6.0 \times 10^{-3}\,\text{m}^2$. It narrows to $2.0 \times 10^{-3}\,\text{m}^2$. Find the speed and pressure in the narrow section.

Lời giải chi tiết

$v_2 = A_1 v_1 / A_2 = (6.0 \times 10^{-3})(2.0)/(2.0 \times 10^{-3}) = \boxed{6.0\,\text{m/s}}$.
$P_2 = P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) = 2.0 \times 10^5 + 500(4.0 - 36) = 2.0 \times 10^5 - 16000 = \boxed{1.84e5\,\text{Pa}}$.

Bài tập luyện tập

A large open tank has a small hole 0.45 m below the water surface. Find the efflux speed.

(A) 1.5 m/s
(B) 3.0 m/s
(C) 4.4 m/s
(D) 8.8 m/s

Lời giải chi tiết

$v = \sqrt{2gh} = \sqrt{2(9.8)(0.45)} = \sqrt{8.82} = 2.97$ m/s ≈ 3.0 m/s. **(B)**.

Bài tập luyện tập

A large open water tank has a small hole 1.8 m below the surface. (a) Find the water's exit speed. (b) If the hole's area is 5.0×10^{-5} m^2, find the volume flow rate through the hole. Assume the tank's surface area is much larger than the hole's, so the surface descends negligibly slowly.

Lời giải chi tiết

(a) $v = \sqrt{2gh} = \sqrt{2(9.8)(1.8)} = \sqrt{35.28} = \boxed{5.94\,\text{m/s}}$.
(b) $Q = Av = (5.0 \times 10^{-5})(5.94) = \boxed{2.97 \times 10^{-4}\,\text{m}^3/\text{s}}$.

Bài tập luyện tập

Air flows faster over the curved top of an airfoil than under its flatter bottom. According to Bernoulli's principle, this results in:

(A) Higher pressure above, net downward force
(B) Lower pressure above, net upward force (lift)
(C) Equal pressure on both sides, no net force
(D) A force that depends only on the airfoil's material

Lời giải chi tiết

Faster flow above the wing corresponds to lower pressure there (Bernoulli, roughly constant height); the higher pressure underneath then produces a net upward force. **(B)**.

Bài tập luyện tập

Water flows steadily through a pipe that rises 4.0 m from the ground floor to the second floor of a building, while the pipe's cross-sectional area shrinks to one-third of its original value. At the ground floor, the water has speed 1.5 m/s and gauge pressure 3.0×10^5 Pa. Find the speed and gauge pressure at the second floor.

Lời giải chi tiết

Continuity: $A_2 = A_1/3 \Rightarrow v_2 = 3v_1 = 3(1.5) = \boxed{4.5\,\text{m/s}}$.
Bernoulli (gauge pressure works the same way here, since P_0 appears identically on both sides and cancels):

$$P_2 = P_1 + \tfrac{1}{2}\rho(v_1^2 - v_2^2) - \rho g\,\Delta y = 3.0\times10^5 + 500(2.25 - 20.25) - (1000)(9.8)(4.0)$$

$$= 3.0\times10^5 - 9000 - 39200 = \boxed{2.52e5\,\text{Pa}}.$$

Bài tập luyện tập

Which pair of relationships would you use to relate pressure, speed, and cross-sectional area at two points along a horizontal streamline of an ideal fluid?

(A) $F = ma$ and $v = v_0 + at$

(B) The continuity equation and Bernoulli's equation

(C) Archimedes' principle and Pascal's principle

(D) Hooke's law and Newton's third law

Lời giải chi tiết

Continuity ($A_1v_1 = A_2v_2$) connects area and speed; Bernoulli's equation then connects speed to pressure (and height, if it varies). **(B)**.

Bài tập luyện tập

An open-tube mercury manometer ($\rho_{\text{Hg}} = 13600\,\text{kg/m}^3$) is connected to a gas source. The mercury column on the side connected to the gas is $0.080\,\text{m}$ higher than the column on the open side. If atmospheric pressure is 1.0×10^5 Pa, what is the absolute pressure of the gas?

(A) 8.9×10^4 Pa

(B) 1.0×10^5 Pa

(C) 1.1×10^5 Pa

(D) 2.1×10^5 Pa

Lời giải chi tiết

The gas-side column is *higher*, so the gas-side surface is on the lower-pressure side: $P = P_0 - \rho g\Delta h = 1.0\times10^5 - (13600)(9.8)(0.080) = 1.0\times10^5 - 10662.4 = 89337.6$ Pa $\approx 8.9\times10^4$ Pa. **(A)**.

Bài tập luyện tập

An open tank contains a $0.40\,\text{m}$ layer of oil ($\rho = 800\,\text{kg/m}^3$) on top of a $0.60\,\text{m}$ layer of water ($\rho = 1000\,\text{kg/m}^3$). Find (a) the gauge pressure at the oil–water interface, and (b) the gauge pressure at the bottom of the tank.

Lời giải chi tiết

(a) $P_{\text{interface,gauge}} = \rho_{\text{oil}}gh_1 = (800)(9.8)(0.40) = \boxed{3136\,\text{Pa}} \approx 3.14\times10^3$ Pa.
(b) $P_{\text{bottom,gauge}} = P_{\text{interface,gauge}} + \rho_{\text{water}}gh_2 = 3136 + (1000)(9.8)(0.60) = 3136 + 5880 = \boxed{9016\,\text{Pa}} \approx 9.02\times10^3$ Pa.

Bài tập luyện tập

A horizontal pipe carries water at pressure 2.5×10^5 Pa and speed 1.5 m/s through a section of area 5.0×10^{-3} m^2. It narrows to 1.0×10^{-3} m^2. Find (a) the volume flow rate, (b) the speed in the narrow section, and (c) the pressure in the narrow section.

Lời giải chi tiết

(a) $Q = A_1 v_1 = (5.0 \times 10^{-3})(1.5) = \boxed{7.5 \times 10^{-3}\,\text{m}^3/\text{s}}$.
(b) $v_2 = Q/A_2 = (7.5 \times 10^{-3})/(1.0 \times 10^{-3}) = \boxed{7.5\,\text{m/s}}$.
(c) $P_2 = P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) = 2.5 \times 10^5 + 500(1.5^2 - 7.5^2) = 2.5 \times 10^5 + 500(2.25 - 56.25) = 2.5 \times 10^5 - 27000 = \boxed{2.23e5\,\text{Pa}}$.

Bài tập luyện tập

A weather balloon filled with helium has volume 0.50 m^3 and a total mass (fabric plus gas) of 0.30 kg. The surrounding air has density 1.2 kg/m^3. Find the net force on the balloon, and state whether it rises, sinks, or hovers.

Lời giải chi tiết

Buoyant force from the surrounding air: $F_B = \rho_{\text{air}} g V = (1.2)(9.8)(0.50) = 5.88$ N. Weight: $W = mg = (0.30)(9.8) = 2.94$ N.
Net force $= F_B - W = 5.88 - 2.94 = \boxed{2.94\,\text{N}}$ upward; since $F_B > W$, the balloon **rises**.

Bài tập luyện tập

A solid block floats in water with 70% of its volume submerged. Explain, using Archimedes' principle, what would happen to the submerged fraction if the same block were placed in a denser liquid such as glycerin instead — and why — without computing the exact new fraction.

Lời giải chi tiết

In floating equilibrium the buoyant force must equal the block's fixed weight, and $F_B = \rho_{\text{fluid}} g V_{\text{sub}}$. Because glycerin is denser than water, each cubic meter of glycerin displaced supplies *more* buoyant force than the same volume of water would. Since the block's weight does not change, a **smaller** submerged volume of the denser glycerin is now enough to generate that same buoyant force. Equivalently, from $V_{\text{sub}}/V_{\text{total}} = \rho_{\text{block}}/\rho_{\text{fluid}}$, increasing ρ_{fluid} while ρ_{block} stays fixed makes the ratio **smaller**. So the submerged fraction **decreases** — the block floats higher, with more of it above the surface, in glycerin than it did in water.

Bài tập luyện tập

An open water-filled U-tube manometer ($\rho = 1000$ kg/m^3) is connected to a container of compressed air. The water level in the arm connected to the air is 0.25 m lower than the level in the open arm. Find the gauge pressure of the air.

(A) 2.45×10^2 Pa
(B) 2.45×10^3 Pa
(C) 2.45×10^4 Pa
(D) 2.45×10^5 Pa

Lời giải chi tiết

The air-side level is lower, so the air pressure exceeds atmospheric: $P_{\text{gauge}} = \rho g \Delta h = (1000)(9.8)(0.25) = 2450$ Pa $= 2.45 \times 10^3$ Pa. **(B)**.

Bài tập luyện tập

A block of plastic with density $960\,\text{kg/m}^3$ and volume $1.0 \times 10^{-3}\,\text{m}^3$ floats at the interface between a layer of oil ($\rho = 800\,\text{kg/m}^3$) on top and a denser liquid ($\rho = 1200\,\text{kg/m}^3$) below. Find the fraction of the block's volume in each fluid.

Lời giải chi tiết

Let f be the fraction of the block's volume in the oil (upper fluid). Floating equilibrium gives

$$f = \frac{\rho_{\text{lower}} - \rho_{\text{block}}}{\rho_{\text{lower}} - \rho_{\text{oil}}} = \frac{1200 - 960}{1200 - 800} = \frac{240}{400} = \boxed{0.60}.$$

So 60% of the block's volume ($6.0 \times 10^{-4}\,\text{m}^3$) sits in the oil, and the remaining 40% ($4.0 \times 10^{-4}\,\text{m}^3$) sits in the denser lower liquid.

Bài tập luyện tập

Two identical U-tube manometers — one filled with water, one filled with mercury — are each connected to identical gas tanks at the same unknown gauge pressure. Which manometer shows the larger height difference Δh between its two arms?

(A) The water manometer, since $\Delta h = P_{\text{gauge}}/(\rho g)$ and water has the smaller ρ

(B) The mercury manometer, since it is denser

(C) Both show the same Δh regardless of the fluid

(D) This cannot be determined without knowing the actual pressure

Lời giải chi tiết

Solving $P_{\text{gauge}} = \rho g \Delta h$ for the height difference gives $\Delta h = P_{\text{gauge}}/(\rho g)$: for a fixed gauge pressure, Δh is **inversely proportional** to the fluid's density. Water is far less dense than mercury, so the same gauge pressure produces a much larger Δh in the water manometer. **(A)**.

Bài tập luyện tập

A container holds a layer of oil floating on top of a layer of water. Compared to a second container filled with water alone to the same total depth, the pressure at the bottom of the oil-and-water container is:

(A) Higher

(B) Lower

(C) The same

(D) Impossible to say without the exact densities

Lời giải chi tiết

Replacing the top portion of an all-water column with the same thickness of oil (which is *less* dense than water) reduces the $\rho g h$ contribution from that top layer, since $\rho_{\text{oil}} < \rho_{\text{water}}$; every other term is unchanged. So the bottom pressure in the layered container is **lower** than in the all-water container of the same total depth. **(B)**.

Ôn tập nhanh:

- Luôn định nghĩa chiều dương trước khi gán dấu cho x, v, a trong động học; v và a cùng dấu $\Rightarrow$ tăng tốc, trái dấu $\Rightarrow$ giảm tốc.
- Vẽ đầy đủ giản đồ lực (FBD) trước khi áp dụng $\sum F = ma$; phân tích lực theo hai trục vuông góc khi vật chuyển động trên mặt nghiêng hoặc chịu nhiều lực.
- Định lý công – năng $W_{\text{net}} = \Delta KE$; cơ năng $E = KE + PE$ bảo toàn khi không có lực ma sát/lực cản sinh công.
- Động lượng của một hệ kín (tổng ngoại lực bằng 0) luôn bảo toàn, $\sum \vec{p}_i = \sum \vec{p}_f$, dù va chạm đàn hồi hay không đàn hồi.
- Mô-men lực dùng cánh tay đòn **vuông góc** với lực, $\tau = rF\sin\theta$; vật cân bằng quay khi $\sum \tau = 0$.
- Vật lăn không trượt có $v = \omega r$; một phần cơ năng "chuyển" thành động năng quay $\frac{1}{2}I\omega^2$, nên vật lăn xuống dốc chậm hơn vật trượt không ma sát cùng độ cao.
- Chu kỳ dao động điều hòa (con lắc lò xo, con lắc đơn góc nhỏ) **không phụ thuộc biên độ**, chỉ phụ thuộc m, k hoặc L, g.
- Hai công thức chất lưu dùng nhiều nhất: $P = P_0 + \rho g h$ (áp suất theo độ sâu, tuyệt đối so với gauge) và $A_1 v_1 = A_2 v_2$ (phương trình liên tục); lực đẩy Archimedes $F_B = \rho_{\text{fluid}} g V_{\text{displaced}}$ luôn dùng khối lượng riêng của **chất lưu**, không phải của vật.
- Luôn kiểm tra lại đơn vị và số chữ số có nghĩa của đáp số trước khi kết luận.

Đồng hành cùng 8020 English

Cảm ơn bạn đã học cùng bộ giáo trình quốc tế 8020. **Điểm thật · Thầy thật · Kết quả thật** — nhiều học viên chỉ cần 1–2 khoá để đạt kết quả mong muốn.

> Chương trình đào tạo tại 8020 English
> **IELTS** Academic/General · **SAT** · PTE · Duolingo · TOEFL | **AP** (College Board) · **IB** Diploma · **Cambridge** IGCSE / A-Level | săn học bổng du học.

Vì sao chọn 8020 English?

- **100% giáo viên** đạt tối thiểu 8.0 IELTS hoặc 1500 SAT — đội ngũ Giáo sư · Tiến sĩ · Thạc sĩ tốt nghiệp trường top đầu thế giới (Carnegie Mellon — #1 Hoa Kỳ về Khoa học Máy tính).
- **Kèm 1–1** mọi độ tuổi; lớp sĩ số nhỏ 2–5 bạn (đa số dưới 10 bạn/lớp).
- Lộ trình cá nhân hoá, theo sát tiến độ từng học viên; lớp OFFLINE tại TP.HCM & ONLINE toàn quốc.

Bảng vàng học viên

AP 5/5 — Calculus AB/BC	**AP** 4–5 — Biology, Chemistry	**IB** 90/100 — Economics
SAT 1590 / 1570 / 1550	**IELTS** 8.0–9.0	**Duolingo** 110 · **PTE** 90

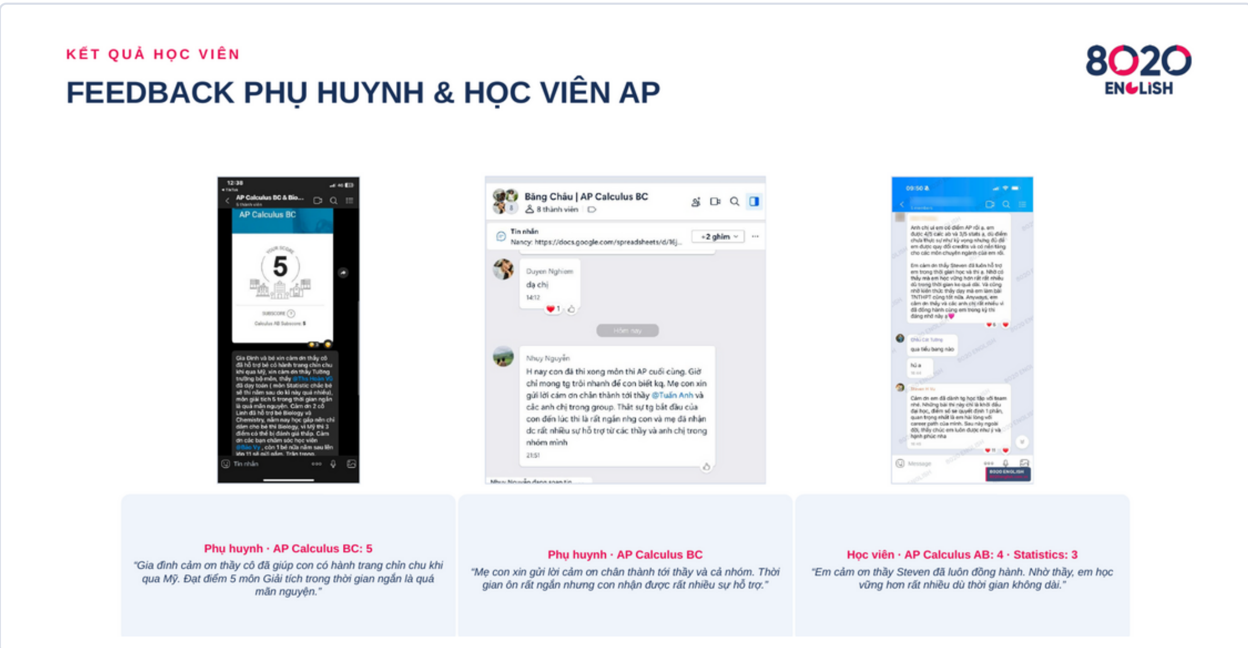
Cảm nhận học viên & phụ huynh

> *"Bạn đã cải thiện được tư duy Writing — kỹ năng từng là nỗi sợ lớn nhất — và cuối cùng đã đậu vào trường top như mong muốn. Thái độ học tập cực kỳ nghiêm túc; ngay cả khi nằm viện vẫn cố gắng học đều đặn."*
>
> — Phan Thị Thanh Hằng · IELTS Overall 8.5

> *"Cần tất cả kỹ năng trên 7.5 để xét vào trường top 1 Anh Quốc về Y học (chương trình UKFPO của NHS) — và đã đạt mục tiêu sau khi học Writing 1–1."*
>
> — Trần Hoàng Bảo Ngọc · IELTS Overall 8.0 (Writing 6.0 → 7.5)

> *"Gia đình và bé xin cảm ơn thầy cô đã giúp con có hành trang chỉn chu khi qua Mỹ. Đạt điểm 5 môn Giải tích trong thời gian ngắn là quá mãn nguyện."*
>
> — Phụ huynh · AP Calculus BC 5/5

> *"Em cảm ơn thầy đã luôn đồng hành. Nhờ thầy, em học vững hơn rất nhiều dù thời gian ôn không dài."*
>
> — Học viên · AP Calculus AB 4



Feedback phụ huynh & học viên AP — nhiều môn đạt điểm 4 & 5

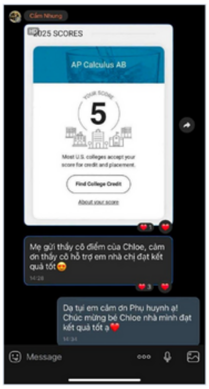
Điểm thi thật của học viên

KẾT QUẢ HỌC VIÊN

ĐIỂM SỐ AP

8020 ENGLISH

AP Biology: 4 · AP Chemistry: 4



AP Calculus AB: 5

AP Calculus BC: 5

AP Chemistry: 5

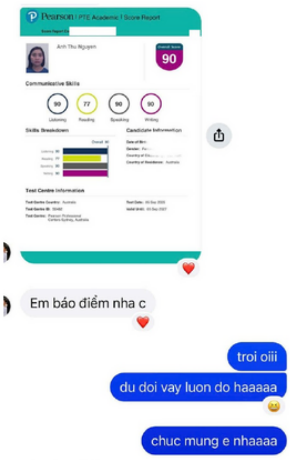
Bảng điểm AP thật của học viên – nhiều môn đạt điểm 4 & 5

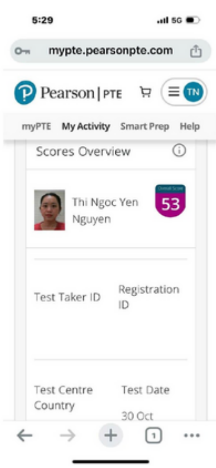
Bảng điểm AP thật — Calculus AB/BC 5/5 · Biology & Chemistry 4–5 (College Board)

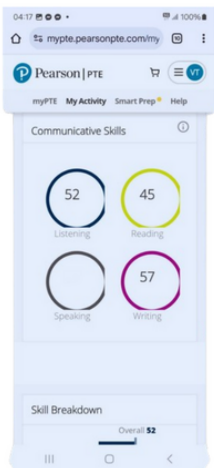
KẾT QUẢ HỌC VIÊN

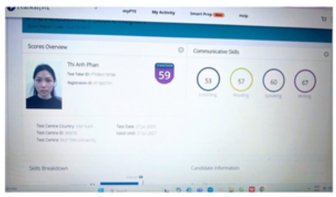
ĐIỂM SỐ PTE

8020 ENGLISH







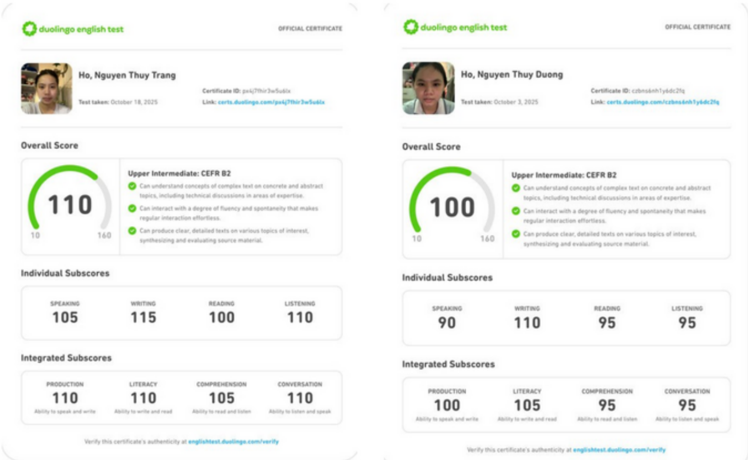


Bảng điểm PTE thật của học viên

Học viên 8020 — PTE Academic 90

KẾT QUẢ HỌC VIÊN

ĐIỂM SỐ DUOLINGO



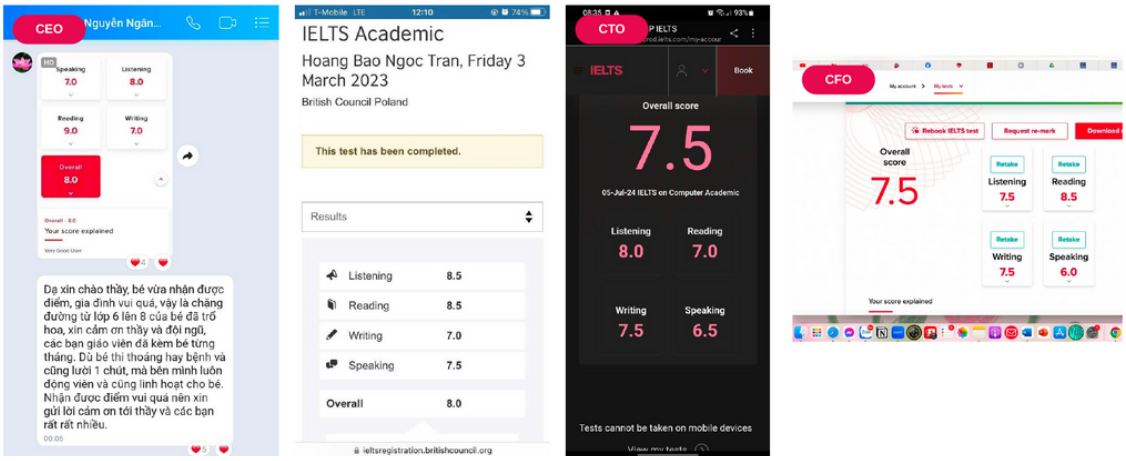
Duolingo English Test – 110 & 100

Học viên 8020 — Duolingo English Test 110 & 100

KẾT QUẢ HỌC VIÊN

THÀNH TÍCH HỌC VIÊN

Bảng điểm thi thật – chất lượng được giám sát nghiêm ngặt. Một số học viên chỉ cần 1–2 khóa để đạt kết quả mong muốn.



Học viên 8020 — IELTS 8.0 / 7.5 / 8.5 (báo điểm thật)