8020 ENGLISH

IB Diploma Programme Physics

Giáo trình luyện thi chuyên sâu

Song ngữ Anh–Việt

8020 ENGLISH

5A/2 Trần Phú, P.4, Q.5, TP.HCM · luyenthi8020.com

Thầy Tường 0332 747 169 · Cô Nancy 0983 422 692

Lời mở đầu · Foreword

Cuốn sách này thuộc bộ giáo trình luyện thi chương trình quốc tế của **8020 English (8020 Education)** — trung tâm luyện thi trọng điểm **IELTS · SAT · AP · IB · A-level · IGCSE** và săn học bổng du học. Toàn bộ nội dung được biên soạn bởi đội ngũ **Giáo sư · Tiến sĩ · Thạc sĩ** tốt nghiệp các trường top đầu thế giới, bám sát khung chương trình chính thức, trình bày đầy đủ lý thuyết, ví dụ mẫu, hình minh hoạ và hệ thống bài tập có lời giải chi tiết.

This book is part of the 8020 English international exam-prep series: complete English theory with Vietnamese explanations, worked examples, illustrations and fully-solved practice, aligned to the official framework.

Thành tích 8020 English

9.0 IELTS cao nhất	**1570** SAT cao nhất
5/5 AP — điểm tuyệt đối	**90/100** IB Diploma
100% GV $\geq$ 8.0 IELTS / 1500 SAT	**CMU** Tiến sĩ ĐH #1 Hoa Kỳ về CS

> **Đội ngũ tiêu biểu:** Thầy Châu Cát Tường (Academic Head, ThS Western Sydney, top 1% IELTS 8.5) · TS Nguyễn Anh Huy (Carnegie Mellon, cựu kỹ sư Amazon) · TS Nguyễn Phước Trường (Toán, ĐH Klagenfurt) · cùng đội ngũ giảng viên SAT 1500–1600, IELTS 8.0–8.5.

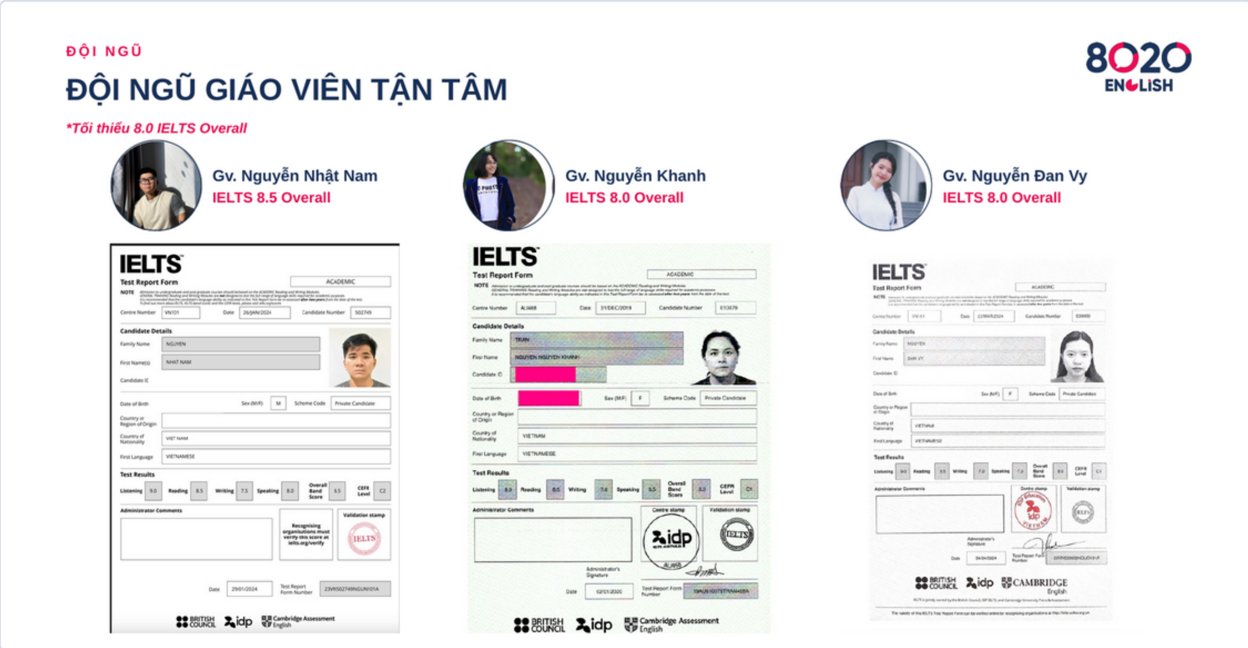
Kết quả học viên — điểm thi thật: IELTS 8.0–9.0 · SAT 1550 / 1570 / 1590 · AP 5/5 (Calculus BC, Microeconomics) · IB 90/100 (Economics) · Duolingo 110 · PTE 90 · TOEFL 120. **Bảng vàng SAT:** Khánh Đăng 1510 · Thảo Nguyên 1520 · Tâm Anh 1530.

"Cuối cùng đã đậu vào trường top như mong muốn." — Phan Thị Thanh Hằng, IELTS Overall 8.5.

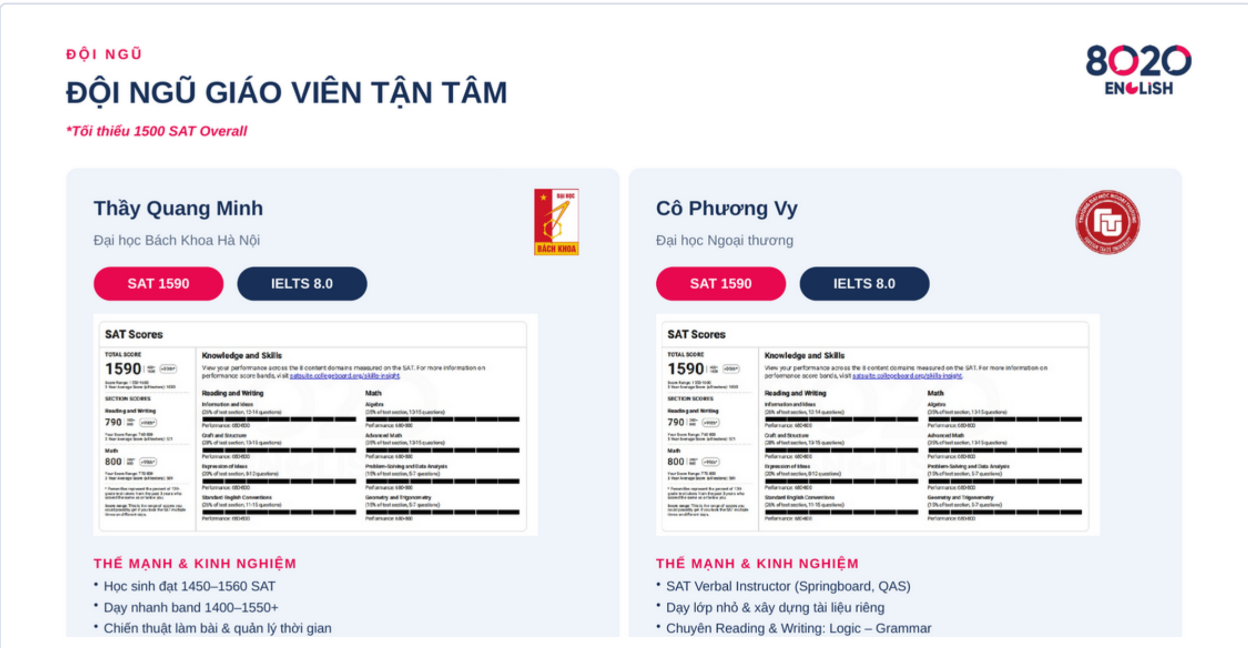
Cách dùng sách hiệu quả: mỗi Unit gồm (1) **Lý thuyết trọng tâm** tiếng Anh kèm **giải thích tiếng Việt**; (2) **Ví dụ mẫu** có lời giải; (3) **Hệ thống bài tập** kèm **lời giải chi tiết**. Hãy tự làm bài trước khi xem lời giải để đạt hiệu quả tối đa.

THÀNH TÍCH THỰC TẾ · 8020 ENGLISH

Điểm thi thật · Bằng & bảng điểm thật của giảng viên và học viên 8020 English — **Điểm thật · Thầy thật · Kết quả thật**



Đội ngũ giảng viên — IELTS 8.5 & 8.0 Overall (British Council · IDP · Cambridge)



Giảng viên SAT 1590 — ĐH Bách Khoa Hà Nội & ĐH Ngoại thương

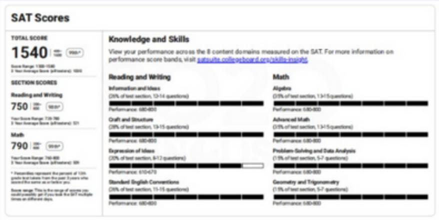
ĐỘI NGŨ

ĐỘI NGŨ GIÁO VIÊN TẬN TÂM

**Tối thiểu 1500 SAT Overall*

Cô Phương Thúy

SAT 1540


SAT Scores
TOTAL SCORE 1540
SECTION SCORES
Reading and Writing 750
Math 790
Knowledge and Skills

THẾ MẠNH & KINH NGHIỆM

- SAT 1540 (Top ~1%) – mạnh cả Verbal & Math
- Kinh nghiệm dạy SAT đa trình độ
- Dạy bản chất + chiến thuật + cá nhân hoá lộ trình

Cô Linh Đan

Đại học Ngoại thương

SAT 1500 IELTS 7.5


SAT Scores
TOTAL SCORE 1500
SECTION SCORES
Reading and Writing 710
Math 790
Knowledge and Skills

THẾ MẠNH & KINH NGHIỆM

- SAT 1500 & IELTS 7.5 (thi lần đầu)
- Chuyên Anh + FTU (CLC Kinh doanh quốc tế)
- Kinh nghiệm dạy thật (gia sư + trợ giảng IELTS)

Giảng viên SAT 1540 & 1500 Overall

KẾT QUẢ HỌC VIÊN

THÀNH TÍCH HỌC VIÊN

Bảng điểm thi thật – chất lượng được giám sát nghiêm ngặt. Một số học viên chỉ cần 1–2 khóa để đạt kết quả mong muốn.


Test Report Form
Candidate Details
Test Results
Overall Band Score 8.0
BRITISH COUNCIL idp CAMBRIDGE English


IELTS
Test Report Form
Candidate Details
Test Results
BRITISH COUNCIL idp Cambridge Assessment English


IELTS
Test Report Form
Candidate Details
Test Results
BRITISH COUNCIL idp Cambridge Assessment English

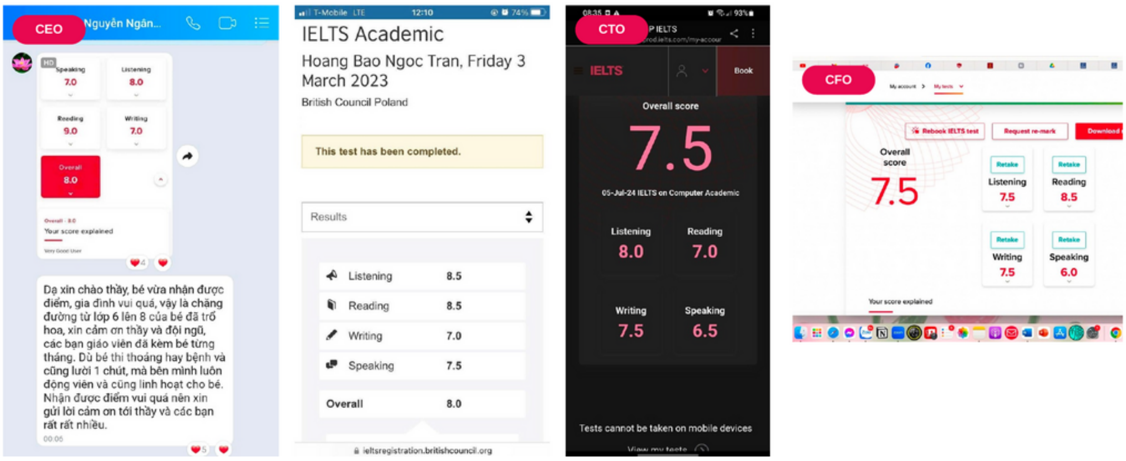
Học viên 8020 — IELTS 8.0 Overall (bảng điểm thật)

KẾT QUẢ HỌC VIÊN

THÀNH TÍCH HỌC VIÊN

8020 ENGLISH

Bảng điểm thi thật – chất lượng được giám sát nghiêm ngặt. Một số học viên chỉ cần 1–2 khóa để đạt kết quả mong muốn.



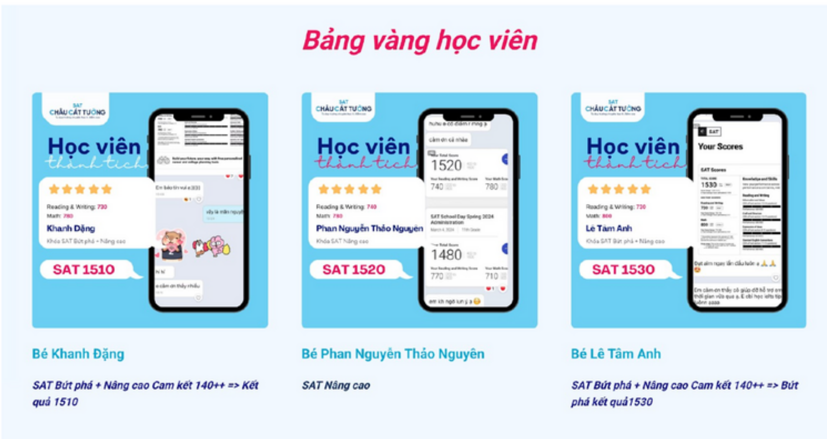
Học viên 8020 — IELTS 8.0 / 7.5 / 8.5 (báo điểm thật)

KẾT QUẢ HỌC VIÊN

BẢNG VÀNG HỌC VIÊN

8020 ENGLISH

SAT CHÂU CÁT TƯỜNG



KHÁNH ĐẶNG – SAT 1510
"SAT Bứt phá + Nâng cao cam kết 140++ → 1510"

THẢO NGUYÊN – SAT 1520
"SAT Nâng cao → 1520"

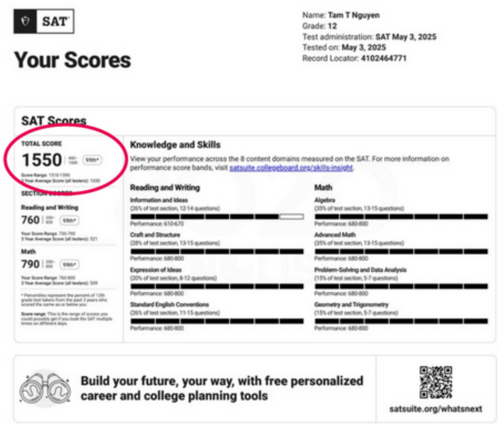
TÂM ANH – SAT 1530
"SAT Bứt phá + Nâng cao cam kết 140++ → 1530"

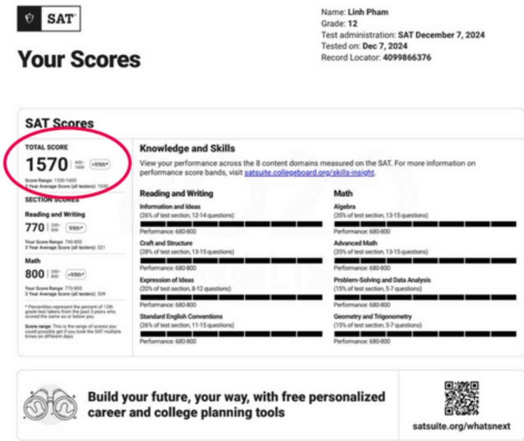
Bảng vàng SAT — Học viên đạt 1510 · 1520 · 1530

KẾT QUẢ HỌC VIÊN

TUTOR 1-1 – ĐIỂM SỐ ẤN TƯỢNG

8020 ENGLISH



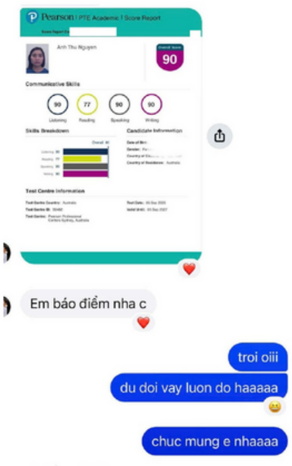


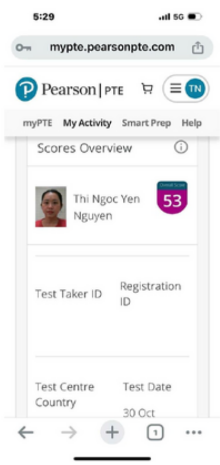
Học viên 8020 — SAT 1550 & 1570 (College Board)

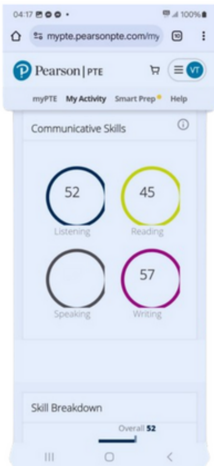
KẾT QUẢ HỌC VIÊN

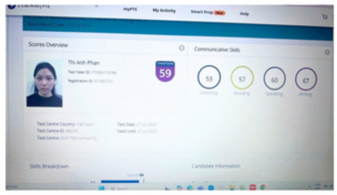
ĐIỂM SỐ PTE

8020 ENGLISH









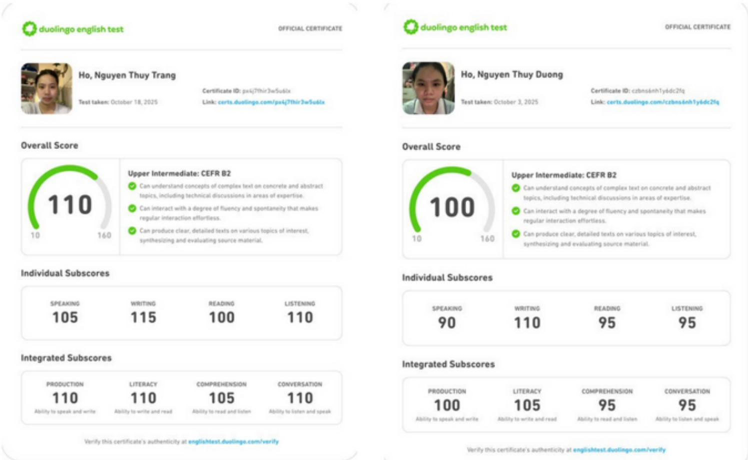
Bảng điểm PTE thật của học viên

Học viên 8020 — PTE Academic 90

KẾT QUẢ HỌC VIÊN

ĐIỂM SỐ DUOLINGO

8020 ENGLISH



Duolingo English Test – 110 & 100

Học viên 8020 — Duolingo English Test 110 & 100

Mục lục • Contents

Theme A: Space, Time and Motion

Mục tiêu Unit: Sau chương này, học sinh có thể: dựng và đọc hiểu đồ thị chuyển động, sử dụng thành thạo bốn phương trình chuyển động thẳng biến đổi đều (SUVAT) và áp dụng cho chuyển động rơi tự do cùng chuyển động ném ngang/ném xiên; phân tích chuyển động tròn đều qua chu kỳ, tần số, vận tốc góc và gia tốc hướng tâm; vận dụng ba định luật Newton, lực ma sát và định luật Hooke để giải các bài toán lực; sử dụng động lượng, xung lượng và định luật bảo toàn động lượng cho va chạm đàn hồi và va chạm mềm; áp dụng định luật vạn vật hấp dẫn của Newton cho chuyển động quỹ đạo tròn và định luật Kepler; tính công, động năng, thế năng, công suất và hiệu suất, đồng thời vận dụng định luật bảo toàn cơ năng; ở mức AHL: phân tích cân bằng của vật rắn qua mô-men lực, mô-men quán tính, động lượng góc và chuyển động lăn không trượt; và trình bày các khái niệm nền tảng của thuyết tương đối Galileo và thuyết tương đối hẹp Einstein, bao gồm giãn thời gian, co độ dài và sự tương đương khối lượng-năng lượng.

1.1 A.1 Kinematics

1.1.1 Position, Displacement, Velocity and Acceleration

Kinematics describes motion without reference to its cause. Every one-dimensional kinematic quantity needs a defined positive direction; once that sign convention is fixed, all quantities – positive or negative – are handled by ordinary algebra.

Position x is the location of an object relative to a chosen origin, measured along a chosen axis. **Distance** is the total length of the path travelled – a positive scalar that never decreases. **Displacement** $\Delta x = x_f - x_i$ is a vector: the straight-line change in position from initial to final point, which can be positive, negative, or (if the object returns to its starting point) zero, even though the distance travelled is not.

Quick check: a runner completes one full lap of a 400 m circular track, finishing exactly where she started. The distance she travelled is 400 m, but her displacement is 0 m – start and finish coincide. This simple case shows clearly that distance and displacement, though related, are genuinely different quantities.

Speed is the (scalar) rate at which distance is covered; **velocity** is the (vector) rate of change of displacement. Average velocity over a time interval is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i},$$

while instantaneous velocity is the average velocity taken over a vanishingly small time interval – graphically, the slope of the position-time (x–t) graph at that instant. Average speed is total distance divided by total time; in general average speed $\geq$ the magnitude of average velocity, with equality only when the motion never reverses direction.

Acceleration is the rate of change of velocity,

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i},$$

again a vector. An object speeds up when a and v point the same way, and slows down when they point in opposite directions – regardless of whether a itself is ”positive” or ”negative” in the chosen sign convention. A negative acceleration does not automatically mean "slowing down"; it means the acceleration vector points in the negative direction.

Choose one positive direction (e.g. right, or up) before writing any equation. Then:

- Displacement, velocity and acceleration in that direction are positive; opposite-direction values are negative.
- Speeding up: v and a have the *same* sign. Slowing down: v and a have *opposite* signs.
- average speed $= \dfrac{\text{total distance}}{\text{total time}}$; $\bar{v} = \dfrac{\text{displacement}}{\text{time}}$.

VN **GIẢI THÍCH TIẾNG VIỆT**

Quãng đường (distance) luôn là số dương và chỉ tăng, còn độ dịch chuyển (displacement) là một vector, có thể âm, dương, hoặc bằng 0 nếu vật quay lại đúng điểm xuất phát. Tốc độ trung bình = tổng quãng đường / tổng thời gian, còn vận tốc trung bình = độ dịch chuyển / thời gian – hai đại lượng này chỉ bằng nhau khi vật chuyển động theo một chiều duy nhất, không quay đầu. Mẹo nhớ dấu gia tốc: vật **tăng tốc** khi v và a **cùng dấu**; vật **giảm tốc** khi v và a **trái dấu** – không phải cứ $a < 0$ là vật đang chậm lại.

(!) Lỗi thường gặp: Nhiều học sinh mặc định "gia tốc âm nghĩa là vật chậm dần". Điều đó chỉ đúng khi vận tốc đang dương. Nếu một vật chuyển động theo chiều âm ($v < 0$) và có $a < 0$, vật đang **tăng tốc** theo chiều âm (đi nhanh dần về phía âm), không phải chậm lại. Luôn kiểm tra dấu của cả v và a cùng một lúc.

Ví dụ mẫu · Distance, displacement, speed and velocity

A hiker walks 300 m due east in 120 s, then turns around and walks 100 m due west in a further 80 s. Taking east as positive, find (a) the total distance travelled, (b) the displacement, (c) the average speed, and (d) the average velocity.

(a) Distance $= 300 + 100 = \boxed{400\,\text{m}}$.
(b) Displacement $= 300 - 100 = \boxed{200\,\text{m}}$ (east).
(c) Total time $= 120 + 80 = 200$ s. Average speed $= \dfrac{400}{200} = \boxed{2.0\,\text{m/s}}$.
(d) Average velocity $= \dfrac{200}{200} = \boxed{1.0\,\text{m/s}}$ east.
Note that (c) and (d) differ, precisely because the hiker reversed direction partway through the walk.

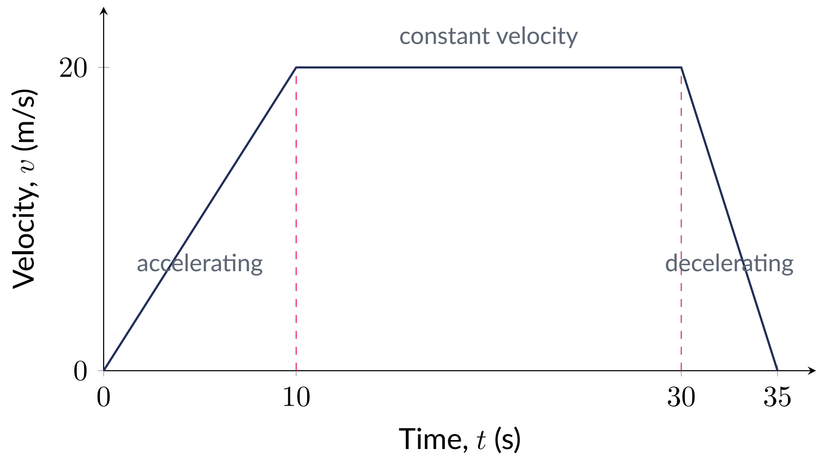
1.1.2 Motion Graphs

The three standard motion graphs – position-time (x–t), velocity-time (v–t), and acceleration-time (a–t) – encode the same motion in different but connected ways, related through the operations of slope (gradient) and area.

On an x–t graph, the slope equals the instantaneous velocity: a horizontal line means zero velocity (at rest), a constant nonzero slope means uniform velocity, and a changing slope means the object is accelerating. On a v–t graph, the slope equals the instantaneous acceleration, and the *area* between the curve and the time axis over an interval equals the *displacement* during that interval (area below the axis counts as negative displacement, to be subtracted). On an a–t graph, the area under the curve over an interval equals the *change in velocity*, Δv, during that interval.

Graph relationships

- Slope of x–t graph = velocity v.
- Slope of v–t graph = acceleration a.
- Area under v–t graph (between curve and time axis) = displacement Δx.
- Area under a–t graph = change in velocity Δv.



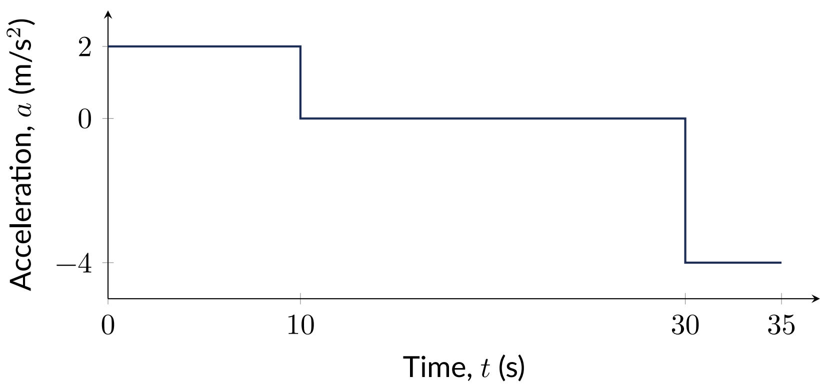
A velocity-time graph for a train: uniform acceleration (0–10 s), constant velocity (10–30 s), uniform deceleration (30–35 s). The area under the graph equals the total displacement.

Ví dụ mẫu · Displacement from a v-t graph

Using the train's motion graph above, find the total displacement over the 35 s journey.

Split the area under the graph into three simple shapes.
Phase 1 (0–10 s, triangle): $s_1 = \frac{1}{2}(10)(20) = 100\,\text{m}$.
Phase 2 (10–30 s, rectangle): $s_2 = (20)(20) = 400\,\text{m}$.
Phase 3 (30–35 s, triangle): $s_3 = \frac{1}{2}(5)(20) = 50\,\text{m}$.
Total displacement: $s = 100 + 400 + 50 = \boxed{550\,\text{m}}$.

The corresponding acceleration-time graph for the same journey is a simple step function, since the acceleration is constant (but different) within each of the three phases:



The acceleration-time graph for the same train journey: $a = +2.0\,\text{m/s}^2$ while accelerating (0–10 s), $a = 0$ while cruising (10–30 s), $a = -4.0\,\text{m/s}^2$ while decelerating (30–35 s).

The area under this a–t graph equals the change in velocity in each phase. Phase 1: area $= 2.0(10) = 20\,\text{m/s}$, matching the train's velocity change from 0 to $20\,\text{m/s}$. Phase 3: area $= -4.0(5) =$

-20 m/s, matching the change from 20 m/s back to 0 – confirming the area-under-a-t-graph rule stated above.

GIẢI THÍCH TIẾNG VIỆT

VN

Diện tích dưới đồ thị v–t chính là độ dịch chuyển – đây là kỹ năng đọc đồ thị quan trọng nhất của phần động học. Với đồ thị hình thang hoặc gồm nhiều đoạn thẳng, hãy chia thành các hình tam giác và hình chữ nhật đơn giản rồi cộng diện tích lại, như ví dụ tàu hoả ở trên (100 m + 400 m + 50 m = 550 m). Nếu đồ thị nằm dưới trục thời gian, diện tích đó ứng với độ dịch chuyển **âm**.

1.1.3 Equations of Motion for Uniform Acceleration (SUVAT)

For motion with constant acceleration a, five kinematic quantities are related: initial velocity u, final velocity v, acceleration a, displacement s, and time t – hence "SUVAT". Any three of the five determine the other two.

First equation. From the definition of (constant) acceleration, $a = \dfrac{v-u}{t}$, rearranging directly gives

$$v = u + at. \tag{1}$$

Second equation. Displacement equals average velocity multiplied by time; because acceleration is uniform, velocity increases linearly with time, so the average velocity over the interval is exactly the mean of the initial and final velocities:

$$\bar{v} = \frac{u+v}{2} \implies s = \bar{v}\,t = \frac{u+v}{2}t. \tag{4}$$

Substituting $v = u + at$ from (1) into (4):

$$s = \frac{u+(u+at)}{2}t = \frac{2u+at}{2}t = ut + \tfrac{1}{2}at^2. \tag{2}$$

Third equation. Eliminating t between (1) and (2) – solve (1) for $t = (v-u)/a$ and substitute into (2) – yields the time-independent form

$$v^2 = u^2 + 2as. \tag{3}$$

Equations of motion (constant acceleration a):

$$v = u + at \qquad s = ut + \tfrac{1}{2}at^2 \qquad v^2 = u^2 + 2as \qquad s = \tfrac{1}{2}(u+v)t$$

where u = initial velocity, v = final velocity, a = constant acceleration, s = displacement, t = time elapsed. Pick the equation that already contains the three known quantities and the one unknown you need – this avoids solving for an unwanted intermediate variable. These equations apply *only* while the acceleration is genuinely constant.

Equation (2), $s = ut + \frac{1}{2}at^2$, also explains the shape of the x–t graph under constant acceleration: starting from rest ($u = 0$), $s \propto t^2$, so the x–t graph is a parabola, not a straight line. Its slope – the velocity – grows linearly with time, consistent with $v = u + at$ and with the earlier statement that the slope of an x–t graph gives the velocity at every instant.

GIẢI THÍCH TIẾNG VIỆT

VN

Bốn công thức SUVAT chỉ áp dụng khi gia tốc **không đổi**. Mẹo chọn công thức nhanh: liệt kê 5 đại lượng u, v, a, s, t, gạch chân 3 đại lượng đã biết và 1 đại lượng cần tìm, rồi chọn công thức nào **không chứa** đại lượng còn lại (chưa biết và không cần tìm). Cách này giúp tránh giải phương trình bậc hai không cần thiết.

Ví dụ mẫu · Uniformly accelerated car

A car starts from rest and accelerates uniformly at $3.0\,\mathrm{m/s^2}$ for $6.0\,\mathrm{s}$. Find (a) its final velocity, and (b) the distance it travels.

(a) $v = u + at = 0 + 3.0(6.0) = \boxed{18\,\mathrm{m/s}}$.

(b) $s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}(3.0)(6.0)^2 = \boxed{54\,\mathrm{m}}$.

Check with $v^2 = u^2 + 2as$: $18^2 = 324$ and $2(3.0)(54) = 324$ – consistent.

Ví dụ mẫu · Stopping distance of a car

A car travels at a constant $25\,\mathrm{m/s}$. The driver's reaction time is $0.70\,\mathrm{s}$ (during which the car continues at constant velocity), after which the brakes produce a deceleration of $6.0\,\mathrm{m/s^2}$ until the car stops. Find the total stopping distance.

Thinking distance (constant velocity during reaction time): $s_1 = v\,t_{\text{reaction}} = 25(0.70) = 17.5\,\mathrm{m}$.

Braking distance: $v^2 = u^2 + 2as_2 \implies 0 = 25^2 + 2(-6.0)s_2 \implies s_2 = \dfrac{625}{12} = 52.1\,\mathrm{m}$.

Total stopping distance: $s = s_1 + s_2 = 17.5 + 52.1 = \boxed{69.6\,\mathrm{m}}$.

A typical alert driver's reaction time is around $0.2\,\mathrm{s}$ to $0.9\,\mathrm{s}$; fatigue, distraction (e.g. mobile phone use), or alcohol can more than double it, directly and proportionally increasing the thinking distance calculated above, even before accounting for any change in braking performance.

1.1.4 Free Fall and Vertical Motion

Near a planet's surface, in the absence of air resistance, every object – regardless of mass – falls with the same constant acceleration, the local gravitational field strength g. Near Earth's surface, $g = 9.81\,\mathrm{m/s^2}$, directed vertically downward. Free fall is simply SUVAT motion with $a = -g$ if "upward" is chosen positive (or $a = +g$ if "downward" is chosen positive) – the physics is identical; only the bookkeeping changes.

For an object thrown vertically upward with initial speed u, taking up as positive so $a = -g$:

- At the highest point, the instantaneous velocity is zero – but the acceleration is still $-g$ throughout, since gravity never switches off.
- Time to reach maximum height: from $v = u + at$ with $v = 0$: $0 = u - gt_{\text{up}} \implies t_{\text{up}} = \dfrac{u}{g}$.
- Maximum height above the launch point: from $v^2 = u^2 + 2as$ with $v = 0$: $0 = u^2 - 2gH \implies H = \dfrac{u^2}{2g}$.
- If the object lands back at the same height from which it was launched, by symmetry the total time of flight is $T = 2t_{\text{up}} = \dfrac{2u}{g}$, and it returns with speed u, directed downward.

Newton's second law explains directly why, in the absence of air resistance, all objects fall with the same acceleration regardless of their mass: applying $F_{\text{net}} = ma$ to a falling object gives $mg = ma$, and the mass cancels exactly, leaving $a = g$ independent of m. A heavier object experiences a proportionally larger gravitational force, but also has a proportionally larger inertia resisting acceleration – the two effects cancel perfectly.

In this course, free-fall problems are idealised by neglecting air resistance, so $a = g$ exactly, for the entire fall. In reality, air resistance grows with speed and eventually balances gravity, so a real falling object (a skydiver, a raindrop) stops accelerating and reaches a constant **terminal velocity**; the idealised constant-g treatment developed here is an excellent approximation only for reasonably dense objects falling over modest distances.

Vertical motion under gravity (up positive, $a = -g$, $g = 9.81\,\text{m/s}^2$):

$$t_{\text{up}} = \frac{u}{g} \qquad H = \frac{u^2}{2g} \qquad T_{\text{flight (same level)}} = \frac{2u}{g}$$

The acceleration is $-g$ at every instant of the flight, including at the highest point, where $v = 0$.

VN GIẢI THÍCH TIẾNG VIỆT

Tại điểm cao nhất của quỹ đạo ném thẳng đứng, vận tốc $v = 0$ nhưng gia tốc **vẫn là** $g \approx 9.81\,\text{m/s}^2$ hướng xuống – trọng lực không hề "biến mất" chỉ vì vật đang đứng yên tức thời. Đây là lý do vật rơi tự do luôn có đồ thị v–t là một đường thẳng có độ dốc không đổi, kể cả khi đi qua điểm $v = 0$.

(!) Lỗi thường gặp: Lỗi rất phổ biến: cho rằng $a = 0$ tại điểm cao nhất vì $v = 0$ ở đó. Vận tốc bằng 0 chỉ là một **thời điểm tức thời** vật đổi chiều chuyển động; gia tốc trọng trường vẫn tác dụng liên tục và không đổi trong suốt chuyến bay.

Ví dụ mẫu · Ball thrown vertically upward

A ball is thrown vertically upward from ground level at $15\,\text{m/s}$. Find (a) the time to reach maximum height, (b) the maximum height, and (c) the total time of flight before it returns to the ground.

(a) $t_{\text{up}} = \dfrac{u}{g} = \dfrac{15}{9.81} = \boxed{1.53\,\text{s}}$.

(b) $H = \dfrac{u^2}{2g} = \dfrac{15^2}{2(9.81)} = \dfrac{225}{19.62} = \boxed{11.5\,\text{m}}$.

(c) $T = 2t_{\text{up}} = \boxed{3.06\,\text{s}}$.

1.1.5 Projectile Motion

Independence of horizontal and vertical motion

In projectile motion (no air resistance), the horizontal and vertical components of the motion are completely independent of one another, except that they share the same elapsed time t. Horizontally there is no horizontal force, so the horizontal velocity component is constant throughout the flight ($a_x = 0$). Vertically, the only force is gravity, so the vertical component undergoes constant acceleration $a_y = -g$ (up positive), exactly as in one-dimensional free fall. Each component can therefore be analysed with ordinary SUVAT equations applied separately, linked only through t.

Horizontal launch. An object launched horizontally with speed u from height h above level ground has purely vertical initial motion in the falling sense: it falls under gravity exactly as if dropped from rest, so $h = \frac{1}{2}gt^2 \implies t = \sqrt{2h/g}$ – the time of flight is set entirely by the vertical drop, independent of u. Horizontally, the speed stays at u throughout, so the range is $x = ut = u\sqrt{2h/g}$.

Quick check: a ball rolls off a table $h = 20\,\text{m}$ high with horizontal speed $u = 15\,\text{m/s}$. Time to land: $t = \sqrt{2(20)/9.81} = 2.02\,\text{s}$; horizontal range: $x = 15(2.02) = 30.3\,\text{m}$.

Angled launch. Now launch at speed u, angle θ above the horizontal, from level ground (landing at the same height). Components: $u_x = u\cos\theta$ (constant throughout), $u_y = u\sin\theta$ (initial vertical component). The vertical position at time t is

$$y(t) = u\sin\theta\, t - \tfrac{1}{2}gt^2.$$

Time of flight: setting $y=0$ (besides the trivial $t=0$): $t\left(u\sin\theta-\frac{1}{2}gt\right)=0 \implies T=\dfrac{2u\sin\theta}{g}$.

Range: $R=u_xT=u\cos\theta\cdot\dfrac{2u\sin\theta}{g}=\dfrac{2u^2\sin\theta\cos\theta}{g}=\dfrac{u^2\sin 2\theta}{g}$, using $2\sin\theta\cos\theta=\sin 2\theta$.

Maximum height: occurs at $t=T/2=\dfrac{u\sin\theta}{g}$ (the midpoint of the symmetric flight). Substituting into $y(t)$:

$$H=u\sin\theta\cdot\frac{u\sin\theta}{g}-\frac{1}{2}g\left(\frac{u\sin\theta}{g}\right)^2=\frac{u^2\sin^2\theta}{g}-\frac{u^2\sin^2\theta}{2g}=\frac{u^2\sin^2\theta}{2g}.$$

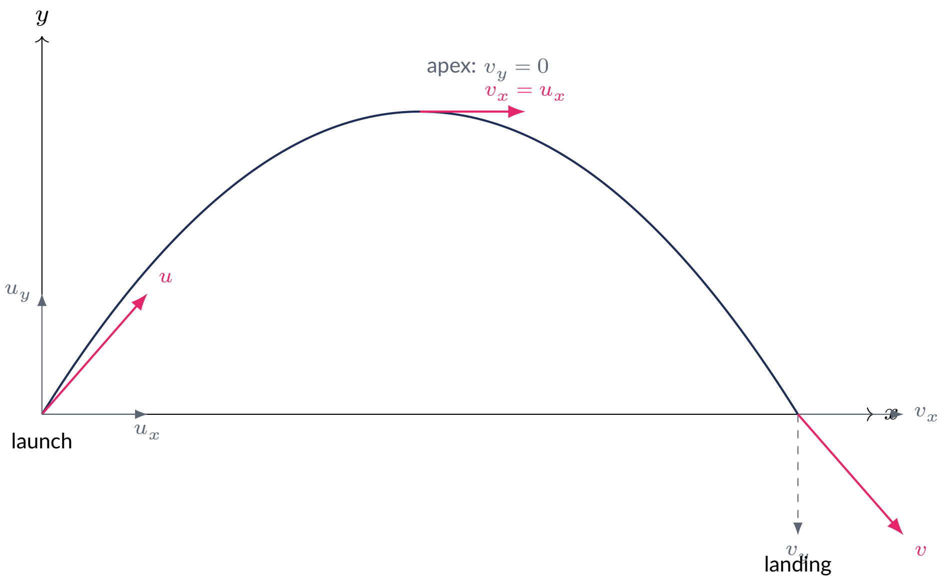
Projectile launched at angle θ from level ground, speed u:

$$u_x=u\cos\theta\ \text{(constant)}\qquad u_y=u\sin\theta\ \text{(initial)}$$

$$T=\frac{2u\sin\theta}{g}\qquad R=\frac{u^2\sin 2\theta}{g}\qquad H=\frac{u^2\sin^2\theta}{2g}$$

Range is maximised (for a given u) at $\theta=45^\circ$, since $\sin 2\theta$ peaks at $2\theta=90^\circ$. Two complementary angles (θ and $90^\circ-\theta$) give the same range but different flight times and heights.

Quick check (complementary angles): for $u=20$ m/s, launching at $\theta=30^\circ$ gives $R=\dfrac{400\sin 60^\circ}{9.81}=\dfrac{400(0.8660)}{9.81}=35.3$ m; launching at $\theta=60^\circ=90^\circ-30^\circ$ gives $R=\dfrac{400\sin 120^\circ}{9.81}=\dfrac{400(0.8660)}{9.81}=35.3$ m as well (since $\sin 120^\circ=\sin 60^\circ$) – identical ranges, even though the 60° launch spends longer in the air and reaches a greater height.



Parabolic trajectory of a projectile launched at angle θ. The horizontal velocity component stays constant throughout the flight; the vertical component decreases to zero at the apex, then grows in magnitude (downward) until landing.

Ví dụ mẫu · Angled projectile launch

A ball is launched at 30 m/s at 40° above the horizontal from level ground. Find the time of flight, the range, and the maximum height.

$\sin 40^\circ=0.6428$, $\cos 40^\circ=0.7660$, $\sin 80^\circ=0.9848$.

$$T = \frac{2u\sin\theta}{g} = \frac{2(30)(0.6428)}{9.81} = \frac{38.57}{9.81} = \boxed{3.93\,\text{s}}.$$
$$R = \frac{u^2\sin 2\theta}{g} = \frac{900(0.9848)}{9.81} = \frac{886.3}{9.81} = \boxed{90.4\,\text{m}}.$$
$$H = \frac{u^2\sin^2\theta}{2g} = \frac{900(0.4132)}{19.62} = \frac{371.9}{19.62} = \boxed{19.0\,\text{m}}.$$

GIẢI THÍCH TIẾNG VIỆT

VN

Với bài toán ném xiên từ mặt đất bằng phẳng (điểm rơi cùng độ cao điểm ném), ba công thức $T = \frac{2u\sin\theta}{g}$, $R = \frac{u^2\sin 2\theta}{g}$, $H = \frac{u^2\sin^2\theta}{2g}$ chỉ đúng khi điểm rơi **cùng độ cao** với điểm phóng. Nếu vật rơi xuống một độ cao khác (ví dụ ném từ mái nhà), phải quay lại phương trình $y(t) = u\sin\theta\, t - \frac{1}{2}gt^2$ và giải cho t ứng với độ cao thực tế – không được áp dụng máy móc công thức T hay R ở trên.

(!) Lỗi thường gặp: Thời gian bay (đối với phóng ngang hoặc phóng xiên rơi cùng độ cao) chỉ phụ thuộc vào **thành phần vận tốc theo phương thẳng đứng** và g – **không** phụ thuộc vào tốc độ ngang u_x. Hai vật được thả từ cùng độ cao, một vật đứng yên và một vật được ném ngang, sẽ chạm đất **cùng lúc** nếu bỏ qua sức cản không khí.

1.1.6 Uniform Circular Motion: Kinematics

An object moving at constant speed around a circle of radius r is in uniform circular motion. Although the speed is constant, the velocity is not, because its direction continuously changes – so the object is accelerating even though $|v|$ never changes.

Period T is the time for one complete revolution. **Frequency** $f = \frac{1}{T}$ is the number of revolutions per unit time (SI unit hertz, Hz). **Angular velocity** ω is the rate of change of angular position, in rad/s: $\omega = \frac{2\pi}{T} = 2\pi f$, since one full revolution sweeps out an angle of 2π radians in time T.

Angular quantities in this course are always expressed in **radians**, not degrees: one full revolution is 2π radians (≈ 6.283 rad), and the simple relation $s = r\theta$ (arc length = radius $\times$ angle) is only valid when θ is measured in radians – this is, in fact, the defining property of the radian as a unit of angle. Calculators must be kept in radian mode throughout circular motion and rotational dynamics work.

The angular displacement θ (in radians) swept out by the radius line is related to the arc length travelled, s, along the circle by $s = r\theta$. Dividing both sides by the time taken gives the linear (tangential) speed in terms of angular velocity: $v = r\omega$ – the arc length swept per second, scaled by the radius. Combined with $\omega = 2\pi/T$, this gives $v = \frac{2\pi r}{T}$ (circumference divided by period).

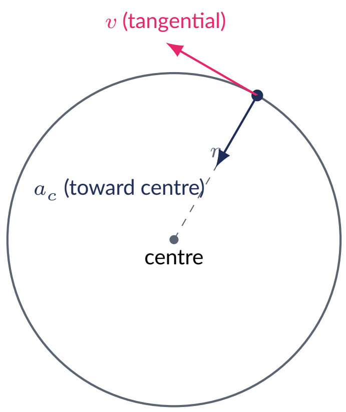
Because the velocity vector's direction is always tangent to the circle while its magnitude stays constant, the velocity vector itself rotates at angular rate ω – and a rotating constant-magnitude vector has an associated **centripetal acceleration** directed toward the centre:

$$a_c = \frac{v^2}{r} = \omega^2 r$$

(the two forms are equivalent since $v = \omega r$). This acceleration exists purely because the direction of v is changing; it says nothing yet about what force causes it – that is the subject of Section A.2.

Uniform circular motion, radius r:

$$f = \frac{1}{T} \qquad \omega = 2\pi f = \frac{2\pi}{T} \qquad v = \omega r = \frac{2\pi r}{T} \qquad a_c = \frac{v^2}{r} = \omega^2 r \text{ (directed toward the centre)}$$



In uniform circular motion the velocity v is always tangent to the circle, while the centripetal acceleration $a_c = v^2/r$ points radially inward, toward the centre. Because a_c is always perpendicular to v, it changes the direction of the velocity but never its magnitude.

VN **GIẢI THÍCH TIẾNG VIỆT**

Trong chuyển động tròn đều, tốc độ không đổi nhưng vector vận tốc thì **luôn đổi hướng**, nên vẫn có gia tốc – gọi là gia tốc hướng tâm $a_c = v^2/r = \omega^2 r$, luôn chỉ vào tâm đường tròn. Đừng nhầm "chuyển động đều" (tốc độ không đổi) với "không có gia tốc" – đây là hai khái niệm khác nhau khi quỹ đạo là đường cong.

Ví dụ mẫu · Centripetal acceleration of a wheel

A point on the rim of a wheel of radius 0.50 m completes one revolution every 2.0 s. Find the frequency, angular velocity, linear speed, and centripetal acceleration.

$f = \frac{1}{T} = \frac{1}{2.0} = \boxed{0.50\,\text{Hz}}$. $\omega = \frac{2\pi}{T} = \frac{2\pi}{2.0} = \boxed{3.14\,\text{rad/s}}$.

$v = \omega r = 3.14(0.50) = \boxed{1.57\,\text{m/s}}$.

$a_c = \frac{v^2}{r} = \frac{1.57^2}{0.50} = \boxed{4.93\,\text{m/s}^2}$ (check: $\omega^2 r = 3.14^2(0.50) = 4.93\,\text{m/s}^2$, consistent).

1.2 A.2 Forces, Momentum and Gravitation

1.2.1 Newton's Laws of Motion and Free-Body Diagrams

Isaac Newton published all three laws of motion together with the law of universal gravitation (Section A.2.5) in his *Philosophiae Naturalis Principia Mathematica* (1687), unifying, for the first time, the motion of falling objects on Earth and the motion of the planets under a single mathematical framework.

Newton's First Law (law of inertia): an object remains at rest, or continues moving with constant velocity in a straight line, unless acted on by a net (unbalanced) external force. A net force is required to *change* an object's velocity, not to sustain it.

Newton's Second Law: the net force on an object equals the rate of change of its momentum; for constant mass this reduces to

$$\vec{F}_{\text{net}} = m\vec{a}.$$

The acceleration produced is proportional to the net force, inversely proportional to the mass, and points in the same direction as the net force.

Newton's Third Law: whenever object A exerts a force on object B, object B exerts an equal-magnitude, oppositely-directed force on object A, along the same line of action. These two forces act on *different* objects, at the same instant, and are always the same type of force – they never cancel each other, because they act on different bodies. Everyday examples: a swimmer pushes water backward, and the water pushes the swimmer forward; a rocket expels exhaust gas downward at high speed, and the gas pushes the rocket upward; a person walking pushes backward on the ground with their foot, and the ground pushes them forward.

A **free-body diagram** isolates a single object and draws every external force acting on it as an arrow from the object, with length roughly proportional to magnitude. Common forces:

- **Weight** $W = mg$, always vertically downward (toward the centre of the planet).
- **Normal force** N, perpendicular to the contact surface, pushing away from it.
- **Tension** T, along a string/rope/cable, pulling away from the object along the line of the string.
- **Static friction** $f_s \le \mu_s N$: opposes the *tendency* of relative sliding between surfaces not yet sliding, adjusting its magnitude up to a maximum of $\mu_s N$.
- **Kinetic friction** $f_k = \mu_k N$: acts once surfaces are sliding, opposing the relative sliding motion, with (essentially) constant magnitude $\mu_k N$ (typically $\mu_k < \mu_s$, so friction "lets go" slightly once motion begins).
- **Hooke's law** (springs): $F = -kx$, where x is the displacement of the spring from its natural length and k is the spring constant (N/m); the minus sign shows the spring force always opposes the displacement – a restoring force.

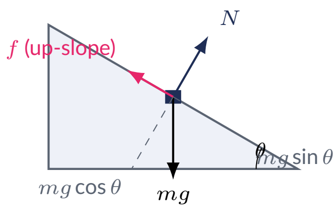
Typical friction coefficients between common material pairs (approximate, for reference):

Surfaces	μ_s	μ_k
Rubber on dry concrete	1.0	0.7
Steel on steel	0.6	0.4
Wood on wood	0.5	0.3
Ice on ice	0.1	0.03

Mass and weight are not the same quantity. Mass m (kg) is an intrinsic property of an object – the amount of matter it contains, and its resistance to acceleration – and does not change with location. Weight $W = mg$ (N) is a *force*, specifically the gravitational pull on that mass, and depends on the local gravitational field strength g; the same object weighs less on the Moon (smaller g) despite having the same mass everywhere.

$$\text{1st law: } \vec{F}_{\text{net}} = 0 \iff \vec{v} = \text{constant} \qquad \text{2nd law: } \vec{F}_{\text{net}} = m\vec{a} \qquad \text{3rd law: } \vec{F}_{A\to B} = -\vec{F}_{B\to A}$$

$$W = mg \qquad f_s \le \mu_s N \qquad f_k = \mu_k N \qquad F_{\text{spring}} = -kx$$



Free-body diagram of a block on a rough incline of angle θ. The weight mg resolves into $mg\sin\theta$ (along the slope) and $mg\cos\theta$ (perpendicular, into the slope). The normal force N balances $mg\cos\theta$; kinetic friction $f = \mu_k N$ acts up the slope when the block slides down.

General strategy for force problems

1. Choose the object of interest and sketch a free-body diagram showing every external force acting on *that object only* (never draw a force an object exerts on something else).
2. Choose a convenient coordinate system – for problems on an incline, it is almost always easiest to align one axis along the slope and the other perpendicular to it.
3. Resolve every force into components along the chosen axes.
4. Apply $\sum F = ma$ separately along each axis (with $a = 0$ along any axis on which the object does not accelerate).

VN **GIẢI THÍCH TIẾNG VIỆT**

Bốn bước làm bài toán lực: (1) chọn vật cần phân tích và vẽ sơ đồ lực tự do; (2) liệt kê mọi lực tác dụng lên vật đó (trọng lực, phản lực, lực căng, ma sát, lực đàn hồi...); (3) chọn hệ trục toạ độ (thường dọc/vuông góc mặt phẳng nghiêng nếu có mặt nghiêng); (4) áp dụng $\sum F = ma$ theo từng trục. Với mặt phẳng nghiêng, luôn phân tích trọng lực mg thành hai thành phần: $mg\sin\theta$ dọc mặt nghiêng và $mg\cos\theta$ vuông góc mặt nghiêng.

(!) Lỗi thường gặp: Trọng lực mg tác dụng lên vật và phản lực pháp tuyến N từ mặt bàn **không phải** là một cặp lực-phản lực theo Định luật 3 Newton, dù chúng thường bằng nhau về độ lớn khi vật đứng yên trên mặt phẳng ngang. Cặp lực-phản lực thật sự của mg là lực hấp dẫn mà **vật** tác dụng ngược lại lên **Trái Đất**; cặp lực-phản lực của N là lực mà vật tác dụng xuống mặt bàn. Hai lực trong một cặp Định luật 3 luôn tác dụng lên **hai vật khác nhau**, không bao giờ triệt tiêu nhau trên cùng một vật.

Ví dụ mẫu · Block on a rough incline

A $5.0\,\text{kg}$ block slides down a 30° incline with $\mu_k = 0.20$. Find the normal force and the block's acceleration.

$N = mg\cos\theta = 5.0(9.81)(0.8660) = \boxed{42.5\,\text{N}}$.
Along the slope: $mg\sin\theta - \mu_k N = ma \implies a = g(\sin\theta - \mu_k\cos\theta) = 9.81(0.500 - 0.20 \times 0.8660)$ $= 9.81(0.500 - 0.1732) = 9.81(0.3268) = \boxed{3.21\,\text{m/s}^2}$ down the slope.

Ví dụ mẫu · Apparent weight in a lift

A person of mass $70\,\text{kg}$ stands on a weighing scale inside a lift (elevator) that is accelerating upward at $2.0\,\text{m/s}^2$. Find the scale reading.

The scale reading equals the normal force N it exerts upward on the person. Applying $F_{\text{net}} = ma$ (up positive): $N - mg = ma \implies N = m(g + a) = 70(9.81 + 2.0) = 70(11.81) = \boxed{827\,\text{N}}$. This exceeds the person's true weight ($mg = 687\,\text{N}$) – the scale reads a larger "apparent weight" whenever the lift accelerates upward; it would read *less* than mg if the lift accelerated downward, and exactly mg if the lift moved at constant velocity (or was stationary).

VN **GIẢI THÍCH TIẾNG VIỆT**

"Trọng lượng biểu kiến" (apparent weight) là số chỉ trên cân, chính là độ lớn phản lực N mà mặt cân tác dụng lên người – **không phải** lúc nào cũng bằng trọng lượng thật mg. Khi thang máy tăng tốc đi lên, $N > mg$ (cảm giác nặng hơn); khi tăng tốc đi xuống, $N < mg$ (cảm giác nhẹ hơn); nếu thang máy rơi tự do, $N = 0$ – trạng thái "không trọng lượng" biểu kiến, dù trọng lực thật vẫn tác dụng bình thường.

Many problems involve two or more objects connected by a light, inextensible string, so the objects share a common acceleration magnitude even though they may move in different directions (e.g. one up, one down, linked over a pulley). The standard technique: write Newton's second law separately for each object, choosing for each object a positive direction that matches the direction it actually accelerates, then combine (typically add) the two equations to eliminate the unknown tension.

Ví dụ mẫu · Atwood machine (connected masses)

Two blocks, $m_1 = 3.0\,\text{kg}$ and $m_2 = 5.0\,\text{kg}$, are connected by a light inextensible string over a light, frictionless pulley. Find the acceleration of the system and the tension in the string.

Since $m_2 > m_1$, block 2 accelerates downward and block 1 accelerates upward, both with the same magnitude a.
Block 2 (down positive): $m_2 g - T = m_2 a$.
Block 1 (up positive): $T - m_1 g = m_1 a$.

Adding the two equations eliminates T: $m_2 g - m_1 g = (m_1 + m_2)a \implies a = \dfrac{(m_2 - m_1)g}{m_1 + m_2} = \dfrac{(5.0 - 3.0)(9.81)}{8.0} = \boxed{2.45\,\text{m/s}^2}$.
Substituting back: $T = m_1(g + a) = 3.0(9.81 + 2.45) = \boxed{36.8\,\text{N}}$ (check: $T = m_2(g - a) = 5.0(9.81 - 2.45) = 36.8\,\text{N}$, consistent).

GIẢI THÍCH TIẾNG VIỆT

VN Với hệ hai vật nối bằng dây qua ròng rọc (Atwood machine), mẹo giải nhanh: viết định luật 2 Newton cho **từng vật riêng biệt**, chọn chiều dương là chiều vật đó **thực sự** chuyển động, rồi **cộng** hai phương trình lại để triệt tiêu lực căng dây T (ẩn số chung). Công thức nhanh cho gia tốc khi $m_2 > m_1$: $a = \dfrac{(m_2 - m_1)g}{m_1 + m_2}$.

1.2.2 Momentum and Impulse

Linear momentum $\vec{p} = m\vec{v}$ is a vector, same direction as velocity, SI unit kg · m/s (equivalently N s). Newton's second law in its most general form is stated in terms of momentum:

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt},$$

which reduces to $\vec{F}_{\text{net}} = m\vec{a}$ whenever mass is constant, since then $\dfrac{d\vec{p}}{dt} = m\dfrac{d\vec{v}}{dt} = m\vec{a}$. The momentum form is more general because it also correctly describes situations of changing mass (e.g. a rocket burning fuel), beyond the reach of a fixed-mass formula.

Impulse $\vec{J}$ is the product of a (possibly time-varying) net force and the time interval over which it acts; for a constant force, $\vec{J} = \vec{F}\,\Delta t$. Rearranging the momentum form of Newton's second law over a finite interval gives the **impulse-momentum theorem**:

$$\vec{J} = \vec{F}_{\text{net}}\,\Delta t = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_i.$$

This is especially useful when a force varies rapidly and unpredictably over a short contact time (a bat hitting a ball, an airbag inflating): the theorem gives the *average* force from the momentum change without needing the detailed force-time curve, and vice versa.

$$\vec{p} = m\vec{v} \qquad \vec{J} = \vec{F}\,\Delta t = \Delta\vec{p} \qquad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}$$

A large momentum change delivered over a *longer* contact time requires a *smaller* average force (and vice versa) – the physics behind airbags, crumple zones, and bending your knees on landing.

The same principle works in reverse when a *larger* force is desired for a given momentum change: a golfer or tennis player "follows through" on a swing, keeping the club or racket in contact with the ball for as long as possible, which – for a required change in the ball's momentum – actually corresponds to *extending* the contact time to maximise the impulse delivered within the swing's available force, rather than minimising it; a boxer, by contrast, tries to "ride" a punch (increasing the time over which their head decelerates it) specifically to reduce the peak force experienced.

GIẢI THÍCH TIẾNG VIỆT

VN Định lý xung lượng - động lượng $J = F\Delta t = \Delta p$ giải thích vì sao kéo dài thời gian va chạm (túi khí, đệm, gập gối khi tiếp đất) làm giảm lực trung bình tác dụng lên cơ thể với cùng một độ biến thiên động lượng Δp. Đây là nguyên lý an toàn cốt lõi trong thiết kế ô tô và các môn thể thao.

Ví dụ mẫu · Impulse from a bouncing ball

A 0.50 kg ball hits a wall at 20 m/s and bounces straight back at 15 m/s. The contact time is 0.010 s. Find the average force exerted by the wall on the ball.

Taking the incoming direction as positive: $\Delta p = m(v_f - v_i) = 0.50(-15 - 20) = 0.50(-35) = -17.5$ kg · m/s.

$F_{\text{avg}} = \dfrac{|\Delta p|}{\Delta t} = \dfrac{17.5}{0.010} = \boxed{1750\text{ N}}$, directed away from the wall.

Impulse is a vector, so when a collision is not head-on, only the velocity component *perpendicular* to the contact surface is affected by an idealised (frictionless, elastic) bounce – the component parallel to the surface is unchanged.

Ví dụ mẫu · Impulse from an angled bounce

A 0.20 kg ball strikes a smooth wall at 10 m/s, at 45° to the wall's normal, and rebounds elastically at the same speed and angle (angle of reflection = angle of incidence). Find the impulse delivered to the ball by the wall.

Only the component of velocity perpendicular to the wall reverses; the component parallel to the wall is unchanged, so the impulse is purely perpendicular to the wall.

Perpendicular component of velocity: $v_\perp = 10\cos 45^\circ = 7.07$ m/s, reversing from $+7.07$ to -7.07 m/s.

$J = \Delta p_\perp = m(v_{f,\perp} - v_{i,\perp}) = 0.20(-7.07 - 7.07) = 0.20(-14.14) = \boxed{-2.83\text{ N}\cdot\text{s}}$, i.e. magnitude 2.83 N · s, directed straight out of the wall (along the normal).

1.2.3 Conservation of Momentum and Collisions

For an isolated system (no net external force), Newton's third law guarantees that the internal forces between colliding objects come in equal-and-opposite pairs acting over the same time interval, so their impulses cancel exactly. The total momentum of the system is therefore unchanged by the collision:

$$\vec{p}_{\text{total, before}} = \vec{p}_{\text{total, after}} \qquad \text{i.e.} \qquad m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \ \ (\text{two-body, 1D}).$$

This law of **conservation of momentum** holds regardless of what kind of collision occurs – momentum conservation does not require kinetic energy to be conserved. Momentum conservation is, in fact, one of the most fundamental principles in all of physics: unlike $F_{\text{net}} = ma$ in its simplest form, it continues to hold even in the relativistic and quantum-mechanical regimes explored later in this course (Section A.5, and in later themes), provided momentum itself is defined appropriately in each setting.

Collisions are classified by what happens to kinetic energy:

- **Elastic collision**: both momentum and total kinetic energy are conserved (approximately true for very hard objects, and essentially exact at the atomic/molecular scale).
- **Inelastic collision**: momentum is conserved but total kinetic energy is not (some KE converts to heat, sound, deformation).
- **Perfectly inelastic collision**: the objects stick together and move with a common final velocity – the case in which the *maximum possible* kinetic energy, consistent with momentum conservation, is lost.

Deriving the elastic-collision formulas (two bodies, 1D, masses m_1, m_2, initial velocities u_1, u_2): conserve momentum and kinetic energy simultaneously,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2, \qquad \tfrac{1}{2}m_1u_1^2 + \tfrac{1}{2}m_2u_2^2 = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2.$$

Solving this pair of simultaneous equations (factor the energy equation as a difference of squares and divide by the momentum equation) yields

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2}u_1 + \frac{2m_2}{m_1 + m_2}u_2, \qquad v_2 = \frac{2m_1}{m_1 + m_2}u_1 + \frac{m_2 - m_1}{m_1 + m_2}u_2.$$

Two useful limiting cases: if $m_1 = m_2$ (equal masses), these formulas reduce to $v_1 = u_2$ and $v_2 = u_1$ – the two objects simply *exchange* velocities. If $m_2 \gg m_1$ (a light object bouncing off a much heavier, effectively immovable one – a ball bouncing off a wall, for instance), $v_1 \to -u_1$ (the light object reverses direction, essentially undisturbed in speed) and $v_2 \to 0$ (the heavy object barely moves).

Momentum conservation (isolated system): $\sum p_{\text{before}} = \sum p_{\text{after}}$.
Elastic collision (1D, two bodies):

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2}u_1 + \frac{2m_2}{m_1 + m_2}u_2 \qquad v_2 = \frac{2m_1}{m_1 + m_2}u_1 + \frac{m_2 - m_1}{m_1 + m_2}u_2$$

Perfectly inelastic collision: $v_{\text{common}} = \dfrac{m_1u_1 + m_2u_2}{m_1 + m_2}$.

VN GIẢI THÍCH TIẾNG VIỆT

Động lượng **luôn** bảo toàn trong hệ kín, bất kể va chạm loại gì. Động năng chỉ bảo toàn trong va chạm đàn hồi. Với va chạm mềm (dính vào nhau), dùng công thức vận tốc chung $v = \dfrac{m_1u_1 + m_2u_2}{m_1 + m_2}$ rồi so sánh động năng trước/sau để tìm phần năng lượng đã chuyển hoá thành nhiệt, âm thanh, biến dạng.

(!) Lỗi thường gặp: Định luật bảo toàn động lượng **luôn** đúng cho hệ kín, nhưng động năng **chỉ** bảo toàn trong va chạm đàn hồi. Học sinh thường mặc định cả hai đại lượng đều bảo toàn trong mọi va chạm – đây là lỗi sai nghiêm trọng. Với va chạm mềm (perfectly inelastic), động năng bị mất là **lớn nhất có thể** trong khi vẫn thoả mãn bảo toàn động lượng.

Momentum conservation also governs the reverse process: an **explosion** or **recoil**, in which a single object initially at rest separates into two (or more) fragments that fly apart. Kinetic energy *increases* in this process (drawn from a store of chemical, elastic, or nuclear energy released in the separation), but momentum is still conserved. Since the total momentum before the explosion is zero, the fragments must fly apart with equal and opposite momenta: $m_1v_1 = -m_2v_2$.

Quick check: a 2.0 kg object at rest splits into two fragments, of mass 0.50 kg and 1.5 kg. If the lighter fragment moves off at 12 m/s, the heavier fragment must move in exactly the opposite direction, at $v = \dfrac{0.50(12)}{1.5} = 4.0$ m/s, so that the total momentum remains zero.

Ví dụ mẫu · Elastic collision of two carts

Cart 1 (2.0 kg, moving at 6.0 m/s) collides elastically with stationary cart 2 (4.0 kg). Find both final velocities and verify momentum and kinetic energy are conserved.

$v_1 = \dfrac{m_1 - m_2}{m_1 + m_2}u_1 = \dfrac{2-4}{6}(6.0) = \boxed{-2.0\,\text{m/s}}$; $\quad v_2 = \dfrac{2m_1}{m_1+m_2}u_1 = \dfrac{4}{6}(6.0) = \boxed{4.0\,\text{m/s}}$.

Momentum: before $= 2.0(6.0) = 12\,\text{kg}\cdot\text{m/s}$; after $= 2.0(-2.0) + 4.0(4.0) = -4 + 16 = 12\,\text{kg}\cdot\text{m/s}$ – conserved.

Kinetic energy: before $= \frac{1}{2}(2.0)(6.0)^2 = 36$ J; after $= \frac{1}{2}(2.0)(2.0)^2 + \frac{1}{2}(4.0)(4.0)^2 = 4 + 32 = 36$ J – also conserved, as required for an elastic collision.

Ví dụ mẫu · Perfectly inelastic collision

The same two carts as above (cart 1: 2.0 kg at 6.0 m/s; cart 2: 4.0 kg at rest) instead collide and stick together. Find their common final velocity and the kinetic energy lost.

$v_f = \dfrac{m_1u_1 + m_2u_2}{m_1+m_2} = \dfrac{12}{6.0} = \boxed{2.0\,\text{m/s}}$.

$KE_{\text{before}} = 36$ J (as above). $KE_{\text{after}} = \frac{1}{2}(6.0)(2.0)^2 = 12$ J.

Kinetic energy lost $= 36 - 12 = \boxed{24\,\text{J}}$ ($66.7\,\%$ of the initial kinetic energy), converted to heat, sound and deformation.

1.2.4 Circular Motion Dynamics

Section A.1 showed that any object moving on a circular path at constant speed has a centripetal acceleration $a_c = v^2/r$ directed toward the centre. By Newton's second law, this acceleration must be produced by a net force of matching magnitude and direction – the **centripetal force**:

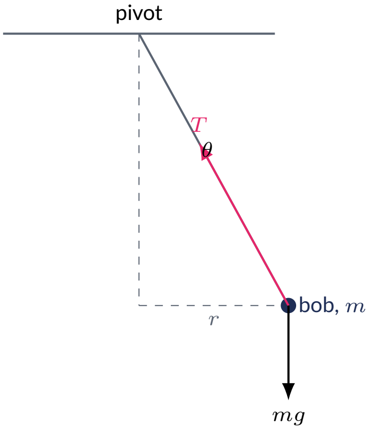
$$F_{\text{net}} = ma_c = \frac{mv^2}{r}.$$

"Centripetal force" is not a new, separate type of force; it is simply the name given to whichever combination of familiar forces (tension, gravity, friction, normal force) happens to supply the net inward force required for circular motion in a given situation. In every circular-motion problem, the first step is to identify *which physical force(s)* play this role:

- **Conical pendulum**: the horizontal component of the string tension supplies the centripetal force, while the vertical component supports the weight.
- **Car on a flat curve**: friction between tyres and road supplies the centripetal force.
- **Banked curve (frictionless)**: the horizontal component of the normal force supplies the centripetal force.
- **Satellite in orbit**: gravity itself supplies the entire centripetal force (Section A.2.5).

$$F_{\text{net}} = \frac{mv^2}{r} = m\omega^2 r \quad \text{(directed toward the centre of the circular path)}$$

This net force is supplied by ordinary forces already present – never add an extra "centrifugal force" arrow to a free-body diagram written in an inertial (non-rotating) frame.



Conical pendulum: the bob moves in a horizontal circle of radius r while the string sweeps out a cone of half-angle θ. The vertical component of tension $T\cos\theta$ supports the weight mg; the horizontal component $T\sin\theta$ supplies the centripetal force.

(!) Lỗi thường gặp: Trong chuyển động tròn không nằm ngang (ví dụ vòng lặp thẳng đứng – loop-the-loop), hướng "vào tâm" **thay đổi liên tục** quanh quỹ đạo: ở đáy vòng lặp, tâm nằm phía trên nên lực hướng tâm hướng lên; ở đỉnh vòng lặp, tâm nằm phía dưới nên lực hướng tâm hướng xuống. Vì vậy, phương trình $F_{\text{net}} = mv^2/r$ phải được viết lại (dấu các lực thành phần) tại **từng vị trí cụ thể** trên quỹ đạo – không dùng chung một phương trình cho mọi điểm.

VN **GIẢI THÍCH TIẾNG VIỆT**

"Lực hướng tâm" không phải một loại lực mới, mà là **tên gọi** cho hợp lực (net force) hướng vào tâm quỹ đạo tròn – hợp lực này luôn do các lực quen thuộc (lực căng dây, trọng lực, ma sát, phản lực) tạo thành, tuỳ từng bài toán cụ thể. Bước đầu tiên khi giải bài toán chuyển động tròn luôn là xác định **lực nào** (hoặc thành phần nào của lực nào) đóng vai trò cung cấp lực hướng tâm.

Ví dụ mẫu · Conical pendulum

A 0.20 kg bob on a string of length 0.50 m moves in a horizontal circle, with the string making a constant angle of 20° with the vertical. Find the tension in the string, the speed of the bob, and the period of the motion.

Vertically: $T\cos\theta = mg \implies T = \dfrac{mg}{\cos\theta} = \dfrac{0.20(9.81)}{\cos 20^\circ} = \dfrac{1.962}{0.9397} = \boxed{2.09\,\text{N}}$.

Radius of the circle: $r = L\sin\theta = 0.50\sin 20^\circ = 0.171$ m.

Horizontally: $T\sin\theta = \dfrac{mv^2}{r} \implies v = \sqrt{rg\tan\theta} = \sqrt{0.171(9.81)(0.3640)} = \sqrt{0.6106} = \boxed{0.781\,\text{m/s}}$.

Period: $T_{\text{period}} = \dfrac{2\pi r}{v} = \dfrac{2\pi(0.171)}{0.781} = \boxed{1.38\,\text{s}}$.

1.2.5 Newton's Law of Gravitation and Orbital Motion

Newton's law of universal gravitation states that every pair of point masses attracts each other with a force

$$F = \frac{Gm_1m_2}{r^2},$$

directed along the line joining them, where $G = 6.67 \times 10^{-11}$ N · m^2/kg^2 is the universal gravitational constant and r is the centre-to-centre separation. The force is always attractive, and by Newton's third law each mass pulls on the other with equal magnitude.

For a uniform sphere of mass M (such as a planet), the gravitational force it exerts on an external mass is the same as if all of M were concentrated at its centre. A mass m at the surface (distance R, the planet's radius, from the centre) therefore experiences $F = \frac{GMm}{R^2}$. Comparing this with the familiar weight expression $F = mg$ gives the **gravitational field strength** at the surface:

$$mg = \frac{GMm}{R^2} \implies g = \frac{GM}{R^2}.$$

Quick check (Mars): $M = 6.42 \times 10^{23}$ kg, $R = 3.39 \times 10^6$ m: $g = \frac{(6.67 \times 10^{-11})(6.42 \times 10^{23})}{(3.39 \times 10^6)^2} = \frac{4.282 \times 10^{13}}{1.149 \times 10^{13}} \approx 3.73$ m/s^2.

Because real planets are not perfectly uniform spheres (Earth, for instance, bulges slightly at the equator and spins on its axis), the measured value of g varies slightly by location – from about 9.78 m/s^2 at the equator to about 9.83 m/s^2 at the poles. Throughout this course, the single standard value $g = 9.81$ m/s^2 is used for all Earth-based calculations unless stated otherwise.

Part of this equator-to-pole variation is purely geometric (Earth's equatorial bulge places the equator's surface slightly farther from the planet's centre than the poles are), but part is dynamical, connecting directly back to Section A.2.4: an object standing at the equator is undergoing circular motion as Earth rotates, and so requires a small net centripetal force directed toward Earth's axis. Part of the true gravitational force is "used up" supplying this centripetal requirement, leaving a smaller *measured* (apparent) weight than at the poles, where an object on the rotation axis needs no centripetal force at all.

For a satellite of mass m in a circular orbit of radius r around a planet of mass M, gravity supplies the entire centripetal force:

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \implies v = \sqrt{\frac{GM}{r}}.$$

Orbital speed depends only on M and r, not on the orbiting mass m at all. Since $v = 2\pi r/T$,

$$\frac{2\pi r}{T} = \sqrt{\frac{GM}{r}} \implies T = 2\pi\sqrt{\frac{r^3}{GM}} \implies T^2 = \frac{4\pi^2}{GM}r^3.$$

Because $4\pi^2/GM$ is the same constant for every satellite orbiting the same central body, this is exactly **Kepler's third law**: $T^2 \propto r^3$.

> **(!) Lỗi thường gặp:** Đừng nhầm lẫn g (chữ thường) với G (chữ hoa). $g \approx 9.81$ m/s^2 là **gia tốc trọng trường tại một vị trí cụ thể** (phụ thuộc vào hành tinh và độ cao), còn $G = 6.67 \times 10^{-11}$ N · m^2/kg^2 là **hằng số hấp dẫn** – một hằng số phổ quát, giống nhau ở mọi nơi trong vũ trụ. Hai đại lượng này có đơn vị hoàn toàn khác nhau và không được hoán đổi cho nhau.

Johannes Kepler first found this relationship empirically around 1619, from decades of planetary position data, long before Newton showed (in 1687) that it follows directly from the inverse-square law of gravitation combined with circular (or, more generally, elliptical) orbital motion.

Newton's law of gravitation has predictive power well beyond describing already-known orbits: in 1846, astronomers Urbain Le Verrier and John Couch Adams independently used small, unexplained deviations in the observed orbit of Uranus to calculate – purely from Newtonian gravitation – the mass and position of an unseen, more distant planet whose gravity was perturbing it. Telescopes were pointed at the predicted location, and Neptune was found almost exactly there, a striking confirmation of the theory's predictive accuracy.

Why astronauts in orbit appear "weightless"

A common misconception is that astronauts aboard the International Space Station (orbiting at an altitude of only a few hundred kilometres) are weightless because "there is no gravity in space". This is false: at that altitude, Earth's gravitational field strength is still about 90% of its surface value. The astronauts and their spacecraft are in fact continuously accelerating toward Earth at essentially this same rate – they are in a state of continuous free fall, orbiting rather than falling straight down only because of their large tangential velocity. Because the astronauts and everything around them (including any object they release) fall together at the same rate, there is no *relative* acceleration between an astronaut and the cabin – producing the sensation, and appearance, of weightlessness, even though real gravity is very much still acting.

$$F = \frac{Gm_1m_2}{r^2} \qquad g = \frac{GM}{R^2}\ (\text{surface})$$

$$v_{\text{orbit}} = \sqrt{\frac{GM}{r}} \qquad T = 2\pi\sqrt{\frac{r^3}{GM}} \qquad T^2 \propto r^3\ (\text{Kepler III})$$

VN **GIẢI THÍCH TIẾNG VIỆT**

Công thức $g = GM/R^2$ cho biết gia tốc trọng trường bề mặt phụ thuộc vào khối lượng **và** bán kính hành tinh, không chỉ khối lượng – đây là lý do các hành tinh khác nhau có g khác Trái Đất dù khối lượng có thể tương đương. Với quỹ đạo tròn, tốc độ quỹ đạo $v = \sqrt{GM/r}$ **không** phụ thuộc khối lượng vệ tinh m – vệ tinh nhẹ hay nặng ở cùng bán kính quỹ đạo đều bay với cùng tốc độ.

Ví dụ mẫu · Satellite in low orbit

A satellite orbits Earth ($M = 5.97 \times 10^{24}$ kg, $R = 6.37 \times 10^6$ m) at an altitude of 300 km. Find its orbital speed and period.

Orbital radius: $r = R + h = 6.37 \times 10^6 + 0.30 \times 10^6 = 6.67 \times 10^6$ m.
$GM = (6.67 \times 10^{-11})(5.97 \times 10^{24}) = 3.982 \times 10^{14}$ m^3/s^2.

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{3.982 \times 10^{14}}{6.67 \times 10^6}} = \sqrt{5.970 \times 10^7} = \boxed{7.73 \times 10^3\ \text{m/s}} \approx 7.73\ \text{km/s}.$$

$$T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{2.967 \times 10^{20}}{3.982 \times 10^{14}}} = 2\pi\sqrt{7.45 \times 10^5} = 2\pi(863) = \boxed{5.43 \times 10^3\ \text{s}} \approx 90.4\ \text{min}.$$

1.3 A.3 Work, Energy and Power

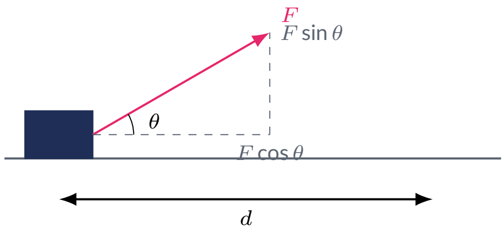
1.3.1 Work Done by a Force

In physics, **work** has a precise, narrower meaning than in everyday language: a force does work on an object only while that object is displaced, and only the component of the force *along* the displacement contributes. For a constant force F acting while the object undergoes displacement d, with θ the angle between the force and displacement directions,

$$W = Fd\cos\theta.$$

Work is a scalar (SI unit joule, $1\,\text{J} = 1\,\text{N}\cdot\text{m}$), and it can be positive (the force has a component along the motion – energy transferred *to* the object), negative (the force opposes the motion – energy transferred *away* from the object), or zero (the force is perpendicular to the motion, $\theta = 90°$).

Two important zero-work cases: the normal force N on an object moving along a surface is always perpendicular to the displacement, so it never does work; likewise *static* friction on an object not sliding relative to a surface does no work on the system as a whole, since there is no relative sliding, hence no dissipation. **Kinetic** friction, by contrast, always opposes the relative sliding motion and so always does *negative* work on the sliding object, converting mechanical energy into heat (and some sound).



A force F applied at angle θ above the direction of displacement d. Only the component $F\cos\theta$, parallel to the displacement, does work: $W = Fd\cos\theta$. The perpendicular component $F\sin\theta$ does no work.

$$W = Fd\cos\theta$$

$\theta = 0°$: $W = Fd$ (maximum, force along the motion). $\theta = 90°$: $W = 0$ (force perpendicular to motion – true for N always, and for the centripetal force in circular motion). $\theta = 180°$: $W = -Fd$ (force directly opposes motion, e.g. kinetic friction).

VN **GIẢI THÍCH TIẾNG VIỆT**

Công chỉ được sinh ra bởi **thành phần lực dọc theo phương dịch chuyển**. Phản lực pháp tuyến N không bao giờ sinh công khi vật trượt dọc bề mặt vì luôn vuông góc với chuyển động; lực căng dây trong chuyển động tròn đều cũng không sinh công vì luôn vuông góc với vận tốc tức thời. Ma sát trượt thì luôn sinh công **âm**, chuyển cơ năng thành nhiệt.

(!) Lỗi thường gặp: Phản lực pháp tuyến N **không bao giờ** sinh công lên vật trượt dọc theo một mặt phẳng, vì N luôn vuông góc với phương chuyển động – kể cả khi mặt phẳng đó nghiêng hay cong. Đừng nhầm điều này với ma sát: ma sát trượt (kinetic friction) luôn sinh công **âm** vì nó luôn ngược chiều chuyển động tương đối.

Ví dụ mẫu · Work done pulling a crate

A crate is pulled $8.0\,\text{m}$ across a floor by a rope with tension $50\,\text{N}$, held at $30°$ above the horizontal. Find the work done by the tension.

$W = Fd\cos\theta = 50(8.0)\cos 30° = 400(0.8660) = \boxed{346\,\text{J}}$.

1.3.2 The Work-Energy Theorem

Kinetic energy $KE = \frac{1}{2}mv^2$ is the energy an object possesses because of its motion. Starting from Newton's second law for a constant net force along the direction of motion, $F_{\text{net}} = ma$, and using

$v^2 = u^2 + 2as$ (so $as = \frac{1}{2}(v^2 - u^2)$):

$$W_{\text{net}} = F_{\text{net}}\, s = mas = \tfrac{1}{2}m(v^2 - u^2) = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \Delta KE.$$

This is the **work-energy theorem**: the net work done by *all* forces acting on an object equals its change in kinetic energy. It holds even when the net force is not constant, and provides an alternative to SUVAT for many force-and-motion problems, especially when the force varies, or when only speeds – not detailed accelerations – are required.

$$W_{\text{net}} = \Delta KE = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

VN **GIẢI THÍCH TIẾNG VIỆT**

Định lý công - động năng cho phép tính tốc độ cuối cùng của vật mà **không cần biết chi tiết** về gia tốc hay thời gian – chỉ cần tổng công của tất cả các lực. Đây là công cụ rất mạnh khi lực thay đổi dọc đường đi (ví dụ lực ma sát trên đoạn đường gồ ghề, hay lực đàn hồi của lò xo).

Ví dụ mẫu · Work-energy theorem

A $3.0\,$kg object moving at $2.0\,$m/s is acted on by a net force of $12\,$N parallel to its motion over a distance of $5.0\,$m. Find its final speed.

$W_{\text{net}} = Fd = 12(5.0) = 60\,$J. $KE_i = \frac{1}{2}(3.0)(2.0)^2 = 6.0\,$J. $KE_f = KE_i + W_{\text{net}} = 6.0 + 60 = 66\,$J.
$v_f = \sqrt{\frac{2KE_f}{m}} = \sqrt{\frac{2(66)}{3.0}} = \sqrt{44} = \boxed{6.63\,\text{m/s}}$.

1.3.3 Potential Energy

Potential energy is stored energy associated with an object's position or configuration, available to be converted into kinetic energy (or other forms) by a **conservative** force – one for which the work done moving between two points does not depend on the path taken (gravity and the ideal spring force are both conservative; kinetic friction is not). For example, the work done by gravity raising an object from the ground to a height h is $-mgh$ regardless of whether the object travels straight up, along a zigzag ramp, or by any other route between the same two heights – only the net change in height matters. Kinetic friction has no such property: sliding an object twice as far (even between the same start and end points, via a longer path) does twice the (negative) work, so friction cannot be assigned a potential energy.

Gravitational potential energy (near a planet's surface, where g is essentially uniform over the height range considered): choosing a reference height where $PE = 0$,

$$PE_{\text{grav}} = mgh,$$

where h is the height above the chosen reference level. Only *changes* in height (hence in PE) are physically meaningful – the reference level is a free choice.

Elastic potential energy, stored in an ideally-Hookean spring displaced a distance x from its natural length: since the spring force grows linearly from 0 to kx as it is stretched or compressed by x, the work done against it (the area under the F–x graph, a triangle of base x and height kx) is

$$PE_{\text{elastic}} = \tfrac{1}{2}kx^2.$$

$$PE_{\text{grav}} = mgh \qquad PE_{\text{elastic}} = \tfrac{1}{2}kx^2$$

Both formulas require a defined reference point ($h = 0$, or $x = 0$ at the spring's natural length).

GIẢI THÍCH TIẾNG VIỆT

VN Thế năng luôn cần một **mốc tham chiếu** ($h = 0$ ở đâu, hoặc $x = 0$ ứng với chiều dài tự nhiên của lò xo). Chỉ có **độ biến thiên** thế năng mới có ý nghĩa vật lý; giá trị tuyệt đối của thế năng phụ thuộc vào việc ta chọn mốc ở đâu và không phải là đại lượng "thật" có thể đo trực tiếp.

Ví dụ mẫu · Spring launching a block

A horizontal spring with $k = 400$ N/m is compressed 0.15 m and used to launch a 0.60 kg block along a frictionless surface. Find the elastic potential energy stored and the block's launch speed.

$PE = \frac{1}{2}kx^2 = \frac{1}{2}(400)(0.15)^2 = \boxed{4.5\text{ J}}$.

All of this converts to kinetic energy: $\frac{1}{2}mv^2 = 4.5 \implies v = \sqrt{\frac{2(4.5)}{0.60}} = \sqrt{15} = \boxed{3.87\text{ m/s}}$.

1.3.4 Conservation of Mechanical Energy

Mechanical energy is the sum $E_{\text{mech}} = KE + PE$. When only conservative forces do work on a system (no friction, no air resistance, no external applied force doing net work), mechanical energy is conserved:

$$KE_i + PE_i = KE_f + PE_f.$$

When non-conservative forces (typically friction) are also present, energy is not lost from the universe – it is converted to other forms (heat, sound) – but mechanical energy specifically decreases. The work-energy theorem still applies in its most general form:

$$KE_i + PE_i + W_{\text{non-conservative}} = KE_f + PE_f,$$

where $W_{\text{non-conservative}}$ is negative when friction does negative work (as it almost always does on the object under study); the mechanical energy "lost" equals the magnitude of the work done against friction.

No friction/resistance: $KE_i + PE_i = KE_f + PE_f$.
With friction: $KE_i + PE_i - |W_{\text{friction}}| = KE_f + PE_f$, where $|W_{\text{friction}}| = f_k \times (\text{distance slid})$.

A single energy-conservation equation can track *several* forms of potential energy at once, provided every force involved is conservative.

Ví dụ mẫu · Gravitational PE converting to elastic PE

A 2.0 kg block starts from rest at the top of a frictionless incline, 1.5 m above a horizontal frictionless surface, and slides down onto a spring ($k = 500$ N/m) waiting at the bottom. Find the maximum compression of the spring.

All of the gravitational PE released converts to elastic PE at maximum compression (the block is momentarily at rest there, so $KE = 0$ at both the start and the point of maximum compression):

$$mgh = \frac{1}{2}kx^2 \implies x = \sqrt{\frac{2mgh}{k}} = \sqrt{\frac{2(2.0)(9.81)(1.5)}{500}} = \sqrt{\frac{58.9}{500}} = \sqrt{0.1177} = \boxed{0.343\text{ m}}.$$

GIẢI THÍCH TIẾNG VIỆT

VN Khi có ma sát, cơ năng **không bảo toàn** – một phần chuyển thành nhiệt. Luôn viết phương trình dạng: (cơ năng đầu) − (công cản của ma sát) = (cơ năng cuối). Công của ma sát $|W_{\text{friction}}| = f_k \times d$ với d là quãng đường **trượt thực tế**, không phải độ dịch chuyển theo phương ngang hay phương thẳng đứng.

Ví dụ mẫu · Sliding down a rough incline

A 4.0 kg block starts from rest at the top of a 5.0 m-long incline at 25° to the horizontal, with $\mu_k = 0.15$. Find its speed at the bottom.

Height dropped: $h = L\sin\theta = 5.0\sin 25^\circ = 5.0(0.4226) = 2.113$ m.
Gravitational PE released: $mgh = 4.0(9.81)(2.113) = 82.9$ J.
Friction force: $f_k = \mu_k mg\cos\theta = 0.15(4.0)(9.81)(0.9063) = 5.33$ N.
Work done against friction: $f_k L = 5.33(5.0) = 26.7$ J.

$$KE_f = 82.9 - 26.7 = 56.2\text{ J} \implies v = \sqrt{\frac{2(56.2)}{4.0}} = \sqrt{28.1} = \boxed{5.30\text{ m/s}}.$$

At the top of a vertical circular loop, both gravity and the track's normal force point toward the centre of the loop (i.e. downward, since the centre lies below the car at that point), so $N + mg = \frac{mv^2}{r}$. The track can only *push*, never pull, so $N \geq 0$; the minimum possible speed at the top – the point at which the track is on the verge of losing contact with the car ($N = 0$) – satisfies $mg = \frac{mv_{\text{min}}^2}{r} \implies v_{\text{min}} = \sqrt{gr}$. Combined with energy conservation between the bottom and top of the loop (height difference $2r$), this sets the minimum speed required at the bottom to complete the loop.

Ví dụ mẫu · Loop-the-loop: minimum speed

A roller-coaster car enters a frictionless vertical circular loop of radius 8.0 m. Find (a) the minimum speed at the top of the loop needed to maintain contact with the track, and (b) the corresponding minimum speed at the bottom of the loop.

(a) $v_{\text{min,top}} = \sqrt{gr} = \sqrt{9.81(8.0)} = \sqrt{78.5} = \boxed{8.86\text{ m/s}}$.
(b) Energy conservation, bottom to top (height gained $= 2r$): $\frac{1}{2}v_{\text{bottom}}^2 = \frac{1}{2}v_{\text{top}}^2 + g(2r) \implies v_{\text{bottom}}^2 = v_{\text{top}}^2 + 4gr = 78.5 + 4(9.81)(8.0) = 78.5 + 313.9 = 392.4$.
$v_{\text{min,bottom}} = \sqrt{392.4} = \boxed{19.8\text{ m/s}}$.

1.3.5 Power and Efficiency

Power is the rate at which work is done or energy is transferred/transformed:

$$P = \frac{W}{t} = \frac{dE}{dt},$$

measured in watts (1 W $= 1$ J/s; a common non-SI unit still seen in vehicle and engine specifications is the horsepower, 1 hp ≈ 746 W). For an object moving at velocity v under an applied force F in the direction of motion,

$$P = \frac{Fd}{t} = Fv,$$

since $d/t = v$. This form is especially convenient for vehicles and motors, where power output at a given instant is simply (driving force) $\times$ (speed).

Efficiency compares useful output to total input, for energy, power, or work (the ratio is the same regardless of which pair is used, provided the same time interval is used):

$$\text{efficiency} = \frac{\text{useful energy output}}{\text{total energy input}} = \frac{P_{\text{useful}}}{P_{\text{input}}},$$

expressed as a decimal fraction (always ≤ 1 for any real process, since some input energy is always dissipated, typically as heat) or as a percentage by multiplying by 100%. Typical real-world efficiencies vary enormously by technology: an incandescent light bulb converts only around 5% of its electrical energy input into visible light (the rest becomes heat), a modern LED bulb reaches roughly 40%, a petrol car engine is typically 25–30% efficient at converting the chemical energy of fuel into useful motion, and a well-designed electric motor can exceed 90%.

$$P = \frac{W}{t} = \frac{\Delta E}{t} \qquad P = Fv \text{ (constant-velocity case)} \qquad \text{efficiency} = \frac{\text{useful output}}{\text{total input}} \ (\times 100\%)$$

VN **GIẢI THÍCH TIẾNG VIỆT**

Công suất là **tốc độ** chuyển hoá năng lượng, không phải bản thân năng lượng – hai máy có thể thực hiện cùng một công nhưng công suất khác nhau nếu thời gian thực hiện khác nhau. Hiệu suất luôn nhỏ hơn hoặc bằng 100% vì luôn có năng lượng hao phí (thường là nhiệt do ma sát/điện trở).

Ví dụ mẫu · Motor lifting a load

A motor lifts a 200 kg load through a height of 10 m in 8.0 s. Find its useful power output; if the electrical power supplied is 3000 W, find its efficiency.

Useful work: $W = mgh = 200(9.81)(10) = 19620$ J.

$P_{\text{useful}} = \frac{19620}{8.0} = \boxed{2453\text{ W}}$ (≈ 2.45 kW).

Efficiency $= \frac{2453}{3000} = 0.818 = \boxed{81.8\%}$.

1.4 A.4 Rigid Body Mechanics (AHL)

This content is assessed at Additional Higher Level (AHL) only.

1.4.1 Torque and Rotational Equilibrium

So far every object has been treated as a point (or as translating without rotating). Real objects have size, and a force applied off-centre can cause rotation. The rotational analogue of force is **torque** (or moment of a force),

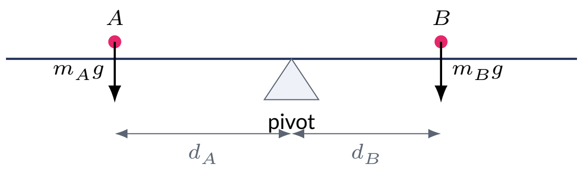
$$\tau = Fr\sin\theta = Fr_{\perp},$$

where r is the distance from the pivot (axis of rotation) to the point where the force is applied, θ is the angle between the force vector and the line from the pivot to the point of application, and $r_{\perp} = r\sin\theta$ is the **perpendicular distance** from the pivot to the line of action of the force (the "lever arm"). Equivalently, $\tau = F_{\perp}r$, using only the component of the force perpendicular to r. Torque is measured in newton-metres (N m) and is taken positive for one sense of rotation (e.g. counter-clockwise) and negative for the other, by convention.

An extended object is in **translational equilibrium** when the net force on it is zero, and in **rotational equilibrium** when the net torque about *any* chosen pivot is zero. An object at rest satisfying both is in **static equilibrium**. A useful fact: if the net torque is zero about one pivot (and the net force is also zero), it is automatically zero about every other pivot as well – so a pivot can be chosen strategically, typically at the point where an unknown force acts, to eliminate that unknown from the torque equation.

$$\tau = Fr\sin\theta = Fr_\perp$$

Equilibrium of a rigid body requires *both*: $\sum \vec{F} = 0$ (translational) and $\sum \tau = 0$ about any pivot (rotational).



A rigid beam balanced on a pivot. For rotational equilibrium, the clockwise and counter-clockwise torques about the pivot must balance: $m_A g\, d_A = m_B g\, d_B$.

GIẢI THÍCH TIẾNG VIỆT VN

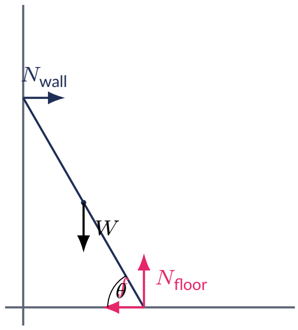
Khi giải bài toán cân bằng vật rắn, luôn chọn trục quay (pivot) đi qua điểm đặt của một lực **chưa biết** – lực đó sẽ có mô-men bằng 0 (vì $r = 0$) và tự động biến mất khỏi phương trình mô-men, giúp bài toán đơn giản hơn nhiều. Nhớ kiểm tra cả hai điều kiện: tổng lực bằng 0 **và** tổng mô-men bằng 0.

Ví dụ mẫu · Balancing a seesaw

On a seesaw pivoted at its centre, child A (mass 30 kg) sits 1.8 m from the pivot. Where must child B (mass 25 kg) sit to balance the seesaw?

Torque balance about the pivot: $m_A g\, d_A = m_B g\, d_B \implies d_B = \dfrac{m_A d_A}{m_B} = \dfrac{30(1.8)}{25} = \dfrac{54}{25} = \boxed{2.16\,\text{m}}$ from the pivot, on the opposite side.

A second classic equilibrium problem combines translational *and* rotational equilibrium at once: a ladder leaning against a wall, where four different forces all act at different points along the ladder.



A ladder leaning against a smooth (frictionless) wall, with its foot on a rough floor. Four forces act: weight W at the midpoint, normal force N_{floor} and friction f at the base, and normal force N_{wall} at the top.

Ví dụ mẫu · Ladder against a smooth wall

A uniform ladder of mass 20 kg and length 4.0 m leans against a smooth (frictionless) vertical wall, its foot on rough ground, making an angle of 60° with the ground. Find the minimum coefficient of static friction at the floor needed to prevent the ladder slipping.

Vertical equilibrium: $N_{\text{floor}} = W = mg = 20(9.81) = 196.2$ N.
Horizontal equilibrium: $f = N_{\text{wall}}$.

Torque about the base (this eliminates N_{floor} and f, both acting there): the weight acts at the midpoint, moment arm $\frac{L}{2}\cos\theta$; the wall's normal force acts at the top, moment arm $L\sin\theta$:

$$N_{\text{wall}}\,L\sin\theta = W\cdot\tfrac{L}{2}\cos\theta \implies N_{\text{wall}} = \frac{W}{2\tan\theta} = \frac{196.2}{2\tan 60^\circ} = \frac{196.2}{2(1.732)} = 56.6\,\text{N}.$$

So $f = 56.6\,$N, and the minimum coefficient of friction is

$$\mu_{s,\min} = \frac{f}{N_{\text{floor}}} = \frac{56.6}{196.2} = \boxed{0.289}.$$

GIẢI THÍCH TIẾNG VIỆT

VN

Bài toán thang dựa vào tường là ví dụ kinh điển cần dùng **cả hai** điều kiện cân bằng: tổng lực bằng 0 theo hai phương, **và** tổng mô-men bằng 0. Mẹo quan trọng: chọn trục quay tại **chân thang** để hai lực chưa biết (N_{floor} và f) tự động biến mất khỏi phương trình mô-men, chỉ còn lại phương trình liên hệ giữa trọng lượng và phản lực tường.

1.4.2 Moment of Inertia

Just as mass measures an object's resistance to a change in linear velocity (inertia), **moment of inertia** I measures its resistance to a change in angular velocity – but unlike mass, I depends not just on how much mass an object has, but on *how that mass is distributed relative to the axis of rotation*. For a single point mass m at distance r from the axis, $I = mr^2$; for an extended rigid body, I is the sum of mr^2 over every mass element. Mass concentrated far from the axis contributes far more to I than the same mass close to the axis (because of the r^2 dependence) – which is why, for example, a hoop has a larger moment of inertia than a solid disk of the same mass and radius.

Standard results (stated here without derivation, since deriving them requires methods beyond the scope of this course):

Shape (axis of rotation)	**Moment of inertia**
Point mass, distance r from axis	$I = mr^2$
Thin rod, about centre (length L)	$I = \frac{1}{12}ML^2$
Thin rod, about one end (length L)	$I = \frac{1}{3}ML^2$
Solid disk / cylinder, central axis (radius R)	$I = \frac{1}{2}MR^2$
Thin hoop / ring, central axis (radius R)	$I = MR^2$
Solid sphere, about a diameter (radius R)	$I = \frac{2}{5}MR^2$

The same object can have a different I about different axes – always state (or check) which axis a given I refers to. Moment of inertia adds for a composite object: $I_{\text{total}} = \sum_i I_i$ (sum of the moments of inertia of its parts, all about the *same* axis).

A large moment of inertia is not always undesirable: flywheels in engines and machinery are deliberately built with mass concentrated near the rim (like a hoop) precisely to maximise I for a given mass, so they store a large amount of rotational kinetic energy ($\frac{1}{2}I\omega^2$) and resist changes in rotation speed, smoothing out fluctuations in power delivery. Conversely, a gymnast or diver curls into a tight ball to *minimise* I (pulling mass close to the rotation axis), allowing rapid spins for a given angular momentum – the same physics as the spinning skater discussed below.

> **VN GIẢI THÍCH TIẾNG VIỆT**
> Mô-men quán tính I đóng vai trò như "khối lượng" trong chuyển động quay – nhưng phụ thuộc vào **cách phân bố khối lượng** quanh trục quay, không chỉ tổng khối lượng. Vật có khối lượng nằm càng xa trục quay thì I càng lớn (do tỉ lệ với r^2), nên vành tròn ($I = MR^2$) có mô-men quán tính lớn hơn đĩa đặc cùng khối lượng và bán kính ($I = \frac{1}{2}MR^2$) – khối lượng của đĩa đặc phân bố gần trục hơn.

1.4.3 Rotational Dynamics: Newton's Second Law, Rotational KE and Angular Momentum

The rotational analogue of $F_{\text{net}} = ma$ is

$$\tau_{\text{net}} = I\alpha,$$

where α is angular acceleration (rad/s^2) – a net torque produces an angular acceleration proportional to the torque and inversely proportional to the moment of inertia, exactly paralleling the linear case.

A rotating object has **rotational kinetic energy**

$$KE_{\text{rot}} = \tfrac{1}{2}I\omega^2,$$

the rotational analogue of $\frac{1}{2}mv^2$.

Angular momentum is the rotational analogue of linear momentum:

$$L = I\omega.$$

In the absence of a net external torque, angular momentum is conserved:

$$L_i = L_f \implies I_i\omega_i = I_f\omega_f.$$

This is a conservation law independent of linear momentum conservation – a system can conserve angular momentum while its linear momentum changes, and vice versa. A classic demonstration: a spinning skater (or a person on a frictionless rotating stool) pulls their arms inward, decreasing I; since no external torque acts about the vertical spin axis (friction/air resistance neglected), L stays constant, so ω must increase.

$$\tau_{\text{net}} = I\alpha \qquad KE_{\text{rot}} = \tfrac{1}{2}I\omega^2 \qquad L = I\omega \qquad (\tau_{\text{net}} = 0 \implies L = \text{constant})$$

Every rotational quantity introduced in this section has a direct translational analogue, and every rotational equation is structurally identical to its translational counterpart with the substitutions $s \to \theta$, $v \to \omega$, $a \to \alpha$, $m \to I$, $F \to \tau$:

Quantity	Translational	Rotational
Displacement	s	θ
Velocity	v	ω
Acceleration	a	α
Inertia	mass m	moment of inertia I
Newton's second law	$F_{\text{net}} = ma$	$\tau_{\text{net}} = I\alpha$
Momentum	$p = mv$	$L = I\omega$
Kinetic energy	$\frac{1}{2}mv^2$	$\frac{1}{2}I\omega^2$

Ví dụ mẫu · Torque accelerating a flywheel

A solid disk ($I = \frac{1}{2}MR^2$), mass 4.0 kg and radius 0.25 m, starts from rest and is subjected to a constant net torque of 2.0 N · m. Find (a) its moment of inertia, (b) its angular acceleration, (c) its angular velocity after 5.0 s, and (d) its rotational kinetic energy at that time.

(a) $I = \frac{1}{2}MR^2 = \frac{1}{2}(4.0)(0.25)^2 = \boxed{0.125\,\text{kg}\cdot\text{m}^2}$.

(b) $\alpha = \frac{\tau_{\text{net}}}{I} = \frac{2.0}{0.125} = \boxed{16.0\,\text{rad/s}^2}$.

(c) $\omega = \alpha t = 16.0(5.0) = \boxed{80\,\text{rad/s}}$.

(d) $KE_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}(0.125)(80)^2 = \boxed{400\,\text{J}}$.

GIẢI THÍCH TIẾNG VIỆT

VN

Bảo toàn động lượng góc $L = I\omega$ giải thích vì sao vận động viên trượt băng quay nhanh hơn khi thu tay vào (I giảm nên ω tăng để giữ L không đổi). Lưu ý: động năng quay **không** bảo toàn trong quá trình này – nó **tăng** lên nhờ công cơ (nội lực) mà vận động viên chủ động thực hiện khi kéo tay vào.

Ví dụ mẫu · Spinning skater

A figure skater spins with arms extended, moment of inertia 5.0 kg · m^2, at 1.5 rad/s. She pulls her arms in, reducing her moment of inertia to 2.0 kg · m^2. Find her new angular velocity and the ratio of her final to initial rotational kinetic energy.

Angular momentum is conserved (no external torque about the spin axis): $I_1\omega_1 = I_2\omega_2 \implies \omega_2 = \frac{5.0(1.5)}{2.0} = \boxed{3.75\,\text{rad/s}}$.

$KE_1 = \frac{1}{2}(5.0)(1.5)^2 = 5.625$ J; $KE_2 = \frac{1}{2}(2.0)(3.75)^2 = 14.06$ J.

Ratio $\frac{KE_2}{KE_1} = \boxed{2.5}$ – the extra kinetic energy comes from the internal (muscular) work she does pulling her arms in.

1.4.4 Rolling Without Slipping

A wheel, ball, or cylinder that rolls without slipping combines translation of its centre of mass with rotation about its own centre, linked by the rolling condition

$$v_{\text{cm}} = \omega r,$$

where v_{cm} is the speed of the centre of mass and r is the object's radius. This expresses that the contact point is instantaneously at rest relative to the ground – precisely the "no slipping" condition.

Because the object both translates and rotates, its total kinetic energy is the sum of translational and rotational parts:

$$KE_{\text{total}} = \tfrac{1}{2}mv_{\text{cm}}^2 + \tfrac{1}{2}I\omega^2.$$

For an object of a given shape (fixed ratio I/mr^2) rolling down a slope from rest through a height h, energy conservation gives a final speed *less* than $\sqrt{2gh}$ (the frictionless-sliding result), because part of the released gravitational PE goes into rotational KE rather than translational KE. Objects with a larger I/mr^2 ratio (mass distributed farther from the axis, e.g. a hoop) move more slowly down a given slope than a more centrally-concentrated shape (e.g. a solid sphere), regardless of mass or radius – both cancel out of the result.

$$v_{\text{cm}} = \omega r \qquad KE_{\text{total}} = \tfrac{1}{2}mv_{\text{cm}}^2 + \tfrac{1}{2}I\omega^2$$

Rolling down height h from rest: $v_{\text{cm}} = \sqrt{\dfrac{2gh}{1 + I/(mr^2)}}$ – always less than the frictionless-sliding speed $\sqrt{2gh}$.

VN **GIẢI THÍCH TIẾNG VIỆT**

Vật lăn không trượt có **hai dạng động năng** cùng lúc: động năng tịnh tiến $\frac{1}{2}mv^2$ và động năng quay $\frac{1}{2}I\omega^2$. Vì vậy, cùng một độ cao h, vật lăn luôn đến chân dốc **chậm hơn** vật trượt không ma sát (không quay) – một phần thế năng "bị" chuyển thành động năng quay thay vì động năng tịnh tiến.

Ví dụ mẫu · Cylinder rolling down a ramp

A solid cylinder ($I = \frac{1}{2}MR^2$), mass 3.0 kg, radius 0.20 m, rolls without slipping from rest down a ramp, dropping through a height of 2.0 m. Find its speed at the bottom, and compare with frictionless sliding.

$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\dfrac{v^2}{R^2} = \frac{1}{2}Mv^2 + \frac{1}{4}Mv^2 = \frac{3}{4}Mv^2.$

$v^2 = \dfrac{4gh}{3} = \dfrac{4(9.81)(2.0)}{3} = \dfrac{78.5}{3} = 26.2 \implies v = \boxed{5.11\,\text{m/s}}.$

Frictionless sliding: $v = \sqrt{2gh} = \sqrt{2(9.81)(2.0)} = \sqrt{39.2} = \boxed{6.26\,\text{m/s}}$ – faster, since none of the released PE is diverted into rotational KE.

1.5 A.5 Galilean and Special Relativity (AHL)

This content is assessed at Additional Higher Level (AHL) only.

1.5.1 Frames of Reference and Galilean Relativity

A **frame of reference** is a coordinate system (with a clock) from which motion is measured. An **inertial frame** is one that is not accelerating – Newton's first law holds exactly within it (a free object moves at constant velocity). Any frame moving at constant velocity relative to an inertial frame is itself inertial; any accelerating (including rotating) frame is non-inertial.

Before the twentieth century, physicists assumed velocities simply add between frames – **Galilean relativity**: if a frame S' moves at constant velocity u relative to frame S (both measuring along the same axis), and an object has velocity v in S, then its velocity in S' is

$$v' = v - u.$$

The central claim of Galilean relativity is that the laws of mechanics take the *same form* in every inertial frame: no mechanical experiment performed entirely within a closed, uniformly-moving laboratory can detect that laboratory's velocity (only its acceleration is detectable, e.g. via a pendulum or accelerometer). This idea – that there is no privileged, "absolute rest" frame – survives essentially unchanged into Einstein's theory; what changes is only the rule for how velocities, times, and lengths transform between frames.

Galilean velocity addition: $v' = v - u$ (frame S' moves at velocity u relative to S). The laws of mechanics are identical in every inertial frame – no experiment performed entirely within a closed, uniformly-moving frame can detect uniform motion.

Quick check: a passenger walks forward at 1.5 m/s relative to a train that itself travels at 20 m/s relative to the ground (same direction). In the ground frame, Galilean addition gives the passenger's speed as $20 + 1.5 = 21.5$ m/s; if the passenger instead walks toward the back of the train at 1.5 m/s, the ground-frame speed is $20 - 1.5 = 18.5$ m/s.

VN **GIẢI THÍCH TIẾNG VIỆT**

Thuyết tương đối Galileo (cổ điển): vận tốc cộng đơn giản theo phép cộng vector thông thường, $v' = v - u$. Không có thí nghiệm cơ học nào thực hiện hoàn toàn bên trong một phòng thí nghiệm chuyển động đều (không gia tốc) có thể phát hiện phòng đó đang chuyển động – chỉ có **gia tốc** mới phát hiện được (ví dụ bằng con lắc hay gia tốc kế), không phải vận tốc không đổi.

1.5.2 Einstein's Postulates

By the late nineteenth century, Maxwell's theory of electromagnetism predicted a fixed speed of light $c = 3.00 \times 10^8$ m/s in vacuum – but the theory gave no indication of which frame this speed was measured relative to, which seemed to conflict with the Galilean picture, in which speeds should simply add. Physicists initially assumed light travelled through a hypothetical medium (the "luminiferous aether") that would define a special, absolute rest frame; the famous Michelson-Morley experiment (1887) attempted to detect Earth's motion through this aether by comparing the speed of light in perpendicular directions, but found no difference whatsoever, however the experiment was oriented or repeated at different times of year – a null result with no satisfactory classical explanation. Einstein resolved this in 1905 by elevating the puzzle into two postulates, taken as the axioms of a new theory: **special relativity** – one of several landmark papers Einstein published that same year (his so-called "miracle year"), alongside work explaining the photoelectric effect and Brownian motion.

Einstein's two postulates

1. **Principle of relativity:** the laws of physics (not just mechanics, but all of physics, including electromagnetism) take the same form in every inertial frame of reference.
2. **Constancy of the speed of light:** the speed of light in vacuum, c, has the same value for every inertial observer, regardless of the motion of the light source or of the observer.

The second postulate is the radical one: it flatly contradicts Galilean velocity addition ($v' = v - u$ would predict different observers measure different speeds for the same light beam). Taking it seriously – rather than dismissing it – forces time and space themselves to behave differently from Galilean expectations, exactly as worked out in the rest of this section.

VN **GIẢI THÍCH TIẾNG VIỆT**

Hai tiên đề của Einstein: (1) mọi định luật vật lý (không chỉ cơ học) đều có cùng dạng trong mọi hệ quy chiếu quán tính; (2) tốc độ ánh sáng trong chân không c là **như nhau** với mọi người quan sát quán tính, bất kể nguồn sáng hay người quan sát đang chuyển động thế nào. Tiên đề thứ hai đi ngược lại trực giác thông thường (cộng vận tốc kiểu Galileo) và là nền tảng dẫn đến giãn thời gian, co độ dài.

1.5.3 Time Dilation and Length Contraction

Define the **Lorentz factor**

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}},$$

where v is the relative speed between two inertial frames. Since $v < c$ always for any massive object, $\gamma \geq 1$ always, with $\gamma = 1$ only when $v = 0$ and $\gamma \to \infty$ as $v \to c$. For everyday speeds $v \ll c$, γ

is indistinguishable from 1, which is why relativistic effects are never noticed in ordinary life – they only become significant when v is an appreciable fraction of c.

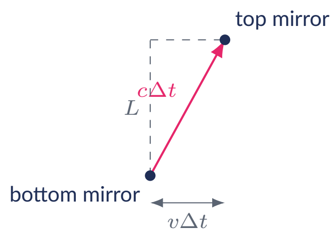
Time dilation: the **proper time** Δt_0 between two events is the time interval measured by a clock present at both events (a clock at rest in the frame where the two events happen at the same place). Any other inertial observer, who sees that clock moving at speed v, measures a *longer* interval between the same two events:

$$\Delta t = \gamma\,\Delta t_0 \qquad (\gamma \geq 1 \implies \Delta t \geq \Delta t_0).$$

"Moving clocks run slow" – more precisely, any observer judges a clock moving relative to them to be ticking slower than their own identical clock.

This result can be derived directly from Einstein's second postulate using a simple thought experiment – the **light clock**. A light clock consists of two mirrors separated by a fixed distance L (measured in the clock's own rest frame), with a light pulse bouncing between them; one "tick" (the pulse travelling from one mirror to the other) takes proper time $\Delta t_0 = L/c$ in the clock's own frame.

Now view the same clock from a frame in which it moves horizontally at speed v. During one tick (duration Δt in this frame), the clock itself shifts horizontally by $v\Delta t$, while the light pulse – travelling at speed c in *every* frame, by the second postulate – must cover the diagonal distance $c\Delta t$, the hypotenuse of a right triangle with legs L and $v\Delta t$:



The light clock, viewed from a frame in which it moves at speed v. The light pulse's diagonal path of length $c\Delta t$ is the hypotenuse of a right triangle with legs L and $v\Delta t$.

By Pythagoras: $(c\Delta t)^2 = L^2 + (v\Delta t)^2$. Substituting $L = c\Delta t_0$:

$$c^2\Delta t^2 = c^2\Delta t_0^2 + v^2\Delta t^2 \implies \Delta t^2(c^2 - v^2) = c^2\Delta t_0^2 \implies \Delta t^2 = \frac{\Delta t_0^2}{1 - v^2/c^2} \implies \Delta t = \gamma\Delta t_0.$$

This is exactly the time dilation result, derived from nothing more than the constancy of c and the Pythagorean theorem.

Length contraction: the **proper length** L_0 of an object is its length measured in the frame in which the object is at rest. An observer who sees that object moving at speed v (along the direction of the length being measured) measures a *shorter* length:

$$L = \frac{L_0}{\gamma} \qquad (\gamma \geq 1 \implies L \leq L_0).$$

Length contraction applies *only* to lengths measured along the direction of relative motion – dimensions perpendicular to the motion are unaffected.

> $$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}\ (\geq 1) \qquad \Delta t = \gamma\Delta t_0 \text{ (time dilation)}$$
>
> $$L = \frac{L_0}{\gamma} \text{ (length contraction, along motion only)}$$
>
> Two clean reference values: $v = 0.60c \implies \gamma = 1.25$; $\quad v = 0.80c \implies \gamma = \frac{5}{3} \approx 1.667$.

VN **GIẢI THÍCH TIẾNG VIỆT**

"Proper time" Δt_0 là thời gian đo bởi đồng hồ **đứng yên** tại vị trí xảy ra hai sự kiện; "proper length" L_0 là chiều dài đo trong hệ quy chiếu mà vật **đứng yên**. Mọi người quan sát khác (thấy đồng hồ/vật đó chuyển động) sẽ đo được thời gian **dài hơn** ($\Delta t = \gamma \Delta t_0$) và chiều dài **ngắn hơn** ($L = L_0/\gamma$). Ghi nhớ hai giá trị đẹp: $v = 0.6c \Rightarrow \gamma = 1.25$; $v = 0.8c \Rightarrow \gamma \approx 1.667$.

(!) Lỗi thường gặp: "Proper time" Δt_0 luôn là thời gian đo bởi **đồng hồ đứng yên so với hai sự kiện** (đồng hồ có mặt tại cả hai sự kiện, cùng một vị trí trong hệ quy chiếu của nó) – đây luôn là khoảng thời gian **ngắn nhất** trong mọi hệ quy chiếu quán tính. Tương tự, "proper length" L_0 là chiều dài đo trong hệ quy chiếu mà vật **đứng yên**, và luôn là chiều dài **dài nhất**. Học sinh thường gán nhầm Δt_0 hoặc L_0 cho hệ quy chiếu "đang chuyển động" thay vì hệ quy chiếu "đứng yên so với vật/sự kiện".

Ví dụ mẫu · Time dilation and length contraction

A spaceship travels at $v = 0.80c$ relative to Earth. Its onboard clock measures a proper time interval of 5.0 s for an onboard process, and its proper length is 90 m.

$$\gamma = \frac{1}{\sqrt{1-0.80^2}} = \frac{1}{\sqrt{0.36}} = \frac{1}{0.60} = \boxed{1.667}.$$

Earth-frame time interval: $\Delta t = \gamma \Delta t_0 = 1.667(5.0) = \boxed{8.33\text{ s}}$.

Earth-frame length: $L = \dfrac{L_0}{\gamma} = \dfrac{90}{1.667} = \boxed{54.0\text{ m}}$.

Ví dụ mẫu · Muon time dilation

Muons produced when cosmic rays strike the upper atmosphere have a proper mean lifetime of $\Delta t_0 = 2.2$ μs (measured in the muon's own rest frame) before decaying. A muon travels toward the ground at $v = 0.80c$. Find (a) the mean lifetime as measured in the Earth's frame, and (b) the average distance it travels (in the Earth's frame) before decaying.

(a) $\gamma = 1.667$ at $v = 0.80c$: $\Delta t = \gamma \Delta t_0 = 1.667(2.2) = \boxed{3.67\text{ μs}}$.
(b) $d = v\Delta t = 0.80(3.00 \times 10^8)(3.67 \times 10^{-6}) = \boxed{880\text{ m}}$.
Using the (incorrect) classical proper lifetime alone, a muon could travel only $v\Delta t_0 = 0.80(3.00 \times 10^8)(2.2 \times 10^{-6}) \approx 528$ m on average – far too short a distance for most muons to reach sea level from the upper atmosphere. The much larger measured muon flux at ground level is direct experimental evidence for time dilation.

Time dilation is not merely a theoretical curiosity confirmed by exotic particles: the satellites of the Global Positioning System (GPS) move fast enough, and their onboard atomic clocks must be corrected precisely enough, that special-relativistic time dilation (along with a separate general-relativistic effect from the weaker gravitational field at orbital altitude, beyond the scope of this course) must be accounted for continuously – without these corrections, GPS position errors would accumulate at several kilometres per day.

1.5.4 Relativity of Simultaneity

A direct consequence of the constancy of c is that **simultaneity is relative**: two events that occur at the same time but at different locations, as judged by one inertial observer, are in general *not* simultaneous as judged by a different inertial observer moving relative to the first. This is not a matter of light-travel-time delays being calculated incorrectly (those can be corrected for by any observer); it is a genuine disagreement about which events happen "at the same time" once light-travel delays have already been properly accounted for by each observer in their own frame.

A standard thought experiment: a light source at the midpoint of a moving train emits a flash in both directions simultaneously. A passenger at the train's centre, at rest relative to the train, judges the light to reach both ends of the train at the same instant (each pulse travels an equal distance in the train's own rest frame). An observer standing on the platform, watching the train go by, sees the rear wall of the train moving *toward* the rearward-travelling pulse and the front wall moving *away* from the forward-travelling pulse; since light travels at the same speed c for this observer too (second postulate), the platform observer judges the light to reach the rear of the train *before* it reaches the front – the two arrivals are not simultaneous in the platform frame, even though they are simultaneous in the train frame.

VN **GIẢI THÍCH TIẾNG VIỆT**

Tính đồng thời (simultaneity) chỉ mang tính **tương đối**: hai sự kiện xảy ra "cùng lúc" trong hệ quy chiếu này có thể **không cùng lúc** trong hệ quy chiếu khác đang chuyển động tương đối so với hệ đầu. Đây không phải do tính sai độ trễ ánh sáng, mà là một hệ quả tất yếu của tiên đề tốc độ ánh sáng không đổi – một trong những kết quả phản trực giác nhất của thuyết tương đối hẹp.

1.5.5 Mass-Energy Equivalence and the Twin Paradox

Special relativity implies that mass itself is a form of energy. The **total energy** of an object of rest mass m moving at speed v is

$$E = \gamma mc^2,$$

which splits naturally into a part present even when the object is at rest, the **rest energy**

$$E_0 = mc^2 \qquad (\text{the } v = 0,\ \gamma = 1 \text{ case}),$$

and the additional energy due to motion, the relativistic **kinetic energy**

$$KE = E - E_0 = (\gamma - 1)mc^2.$$

At everyday speeds ($v \ll c$), a series expansion of γ shows $(\gamma - 1)mc^2 \to \frac{1}{2}mv^2$, so this reduces smoothly to the familiar classical kinetic-energy formula – special relativity does not replace Newtonian mechanics so much as correct it, with corrections that vanish at low speed and grow large only as v approaches c.

This also explains why no object with mass can ever be accelerated to reach c: as $v \to c$, $\gamma \to \infty$, so the kinetic energy $KE = (\gamma - 1)mc^2$ required would itself grow without bound. An infinite amount of energy would be needed to reach the speed of light, so c acts as an absolute speed limit for any object with nonzero rest mass; only massless particles (such as photons) can travel *at* c.

Mass-energy equivalence is not merely a theoretical curiosity: because c^2 is such an enormous number ($\approx 9 \times 10^{16}\ \text{m}^2/\text{s}^2$), even a tiny decrease in the total rest mass of a system releases a very large amount of energy. This is the underlying mechanism of nuclear fission (heavy nuclei splitting) and nuclear fusion (light nuclei combining), in which the total rest mass of the products is measurably less than that of the reactants, with the missing mass released as energy consistent with $E = mc^2$.

$$E = \gamma mc^2 \qquad E_0 = mc^2 \qquad KE = (\gamma - 1)mc^2$$

The twin "paradox"

Twin A stays on Earth (a single inertial frame throughout); twin B travels at high speed to a distant star and returns. Each twin, naively applying "moving clocks run slow" to the other, might expect to find the *other* twin younger on reunion – an apparent contradiction, since they cannot both be right. The resolution is that the situation is *not symmetric*: twin B must decelerate, turn around, and re-accelerate to come home, so twin B does not remain in a single inertial frame for the whole trip, while twin A does. Time dilation as derived above (via a constant γ) strictly applies only to observers who stay in a single inertial frame; twin B's frame-switching breaks the symmetry, and a careful calculation shows unambiguously that twin B returns **younger** than twin A. There is no true logical paradox – only an apparent one that arises from applying a symmetric-looking formula to a genuinely asymmetric physical situation.

VN **GIẢI THÍCH TIẾNG VIỆT**

Nghịch lý song sinh (twin paradox) không phải là một nghịch lý thật sự: tình huống **không đối xứng** vì người anh/em du hành (twin B) phải giảm tốc, quay đầu, rồi tăng tốc trở về – nghĩa là đổi hệ quy chiếu quán tính – trong khi người ở lại Trái Đất (twin A) luôn ở trong **một** hệ quy chiếu quán tính duy nhất. Công thức giãn thời gian đơn giản $\Delta t = \gamma \Delta t_0$ chỉ áp dụng chắc chắn cho người quan sát không đổi hệ quy chiếu (twin A); do đó twin B trở về trẻ hơn twin A một cách không mơ hồ.

Ví dụ mẫu · Rest energy and kinetic energy of an electron

An electron (rest mass 9.11×10^{-31} kg) moves at $v = 0.60c$. Find its rest energy, total energy, and kinetic energy.

$\gamma = 1.25$ at $v = 0.60c$ (since $\sqrt{1 - 0.36} = 0.80$).

$E_0 = mc^2 = 9.11 \times 10^{-31}(3.00 \times 10^8)^2 = 9.11 \times 10^{-31}(9.00 \times 10^{16}) = \boxed{8.20 \times 10^{-14}\text{ J}}$

(this is close to the well-known electron rest energy of ≈ 0.511 MeV, a useful sanity check).

$E = \gamma E_0 = 1.25(8.20 \times 10^{-14}) = \boxed{1.02 \times 10^{-13}\text{ J}}$.

$KE = (\gamma - 1)E_0 = 0.25(8.20 \times 10^{-14}) = \boxed{2.05 \times 10^{-14}\text{ J}}$.

The kinematics, dynamics, and energy relations developed throughout Sections A.1 to A.4 are entirely Newtonian, and remain extremely accurate for essentially all everyday, terrestrial, and even most astronomical applications – projectiles, vehicles, machinery, and planetary orbits alike. Special-relativistic corrections become significant only when speeds approach a substantial fraction of c, a regime essentially never encountered outside particle accelerators, cosmic-ray physics, and a handful of precision technologies such as GPS – which is precisely why Newtonian mechanics, not relativity, remains the working tool for the overwhelming majority of physics and engineering problems.

1.6 Problem Bank

Bài tập luyện tập

A car's velocity-time graph is a straight line from $(0\text{ s}, 0\text{ m/s})$ to $(10\text{ s}, 30\text{ m/s})$. What is the car's displacement during these 10 seconds?

(A) 300 m

(B) 150 m

(C) 45 m

(D) 15 m

Lời giải chi tiết

The graph is a straight line from rest, so displacement equals the area of the triangle under it: $s = \frac{1}{2}(10)(30) = 150$ m. **(B)**.

Bài tập luyện tập

A train starts from rest and accelerates uniformly at $0.80\,\text{m/s}^2$ for $25\,\text{s}$, travels at constant velocity for a further $60\,\text{s}$, and finally decelerates uniformly to rest in $20\,\text{s}$.

(a) Find the maximum velocity reached.

(b) Find the total displacement of the train.

(c) Find the average velocity for the entire journey.

Lời giải chi tiết

(a) $v = u + at = 0 + 0.80(25) = 20\,\text{m/s}$.
(b) Displacement = area under the v–t graph in three phases: $s_1 = \frac{1}{2}(25)(20) = 250\,\text{m}$; $s_2 = (60)(20) = 1200\,\text{m}$; $s_3 = \frac{1}{2}(20)(20) = 200\,\text{m}$. Total $= 250 + 1200 + 200 = \boxed{1650\,\text{m}}$.
(c) Total time $= 25 + 60 + 20 = 105\,\text{s}$. $\bar{v} = \dfrac{1650}{105} = \boxed{15.7\,\text{m/s}}$.

Bài tập luyện tập

A stone is dropped from rest from a height of $45\,\text{m}$. Ignoring air resistance, find its speed just before it hits the ground.

(A) $21.0\,\text{m/s}$

(B) $29.7\,\text{m/s}$

(C) $44.1\,\text{m/s}$

(D) $9.81\,\text{m/s}$

Lời giải chi tiết

$v^2 = u^2 + 2gh = 0 + 2(9.81)(45) = 882.9 \implies v = 29.7\,\text{m/s}$. **(B)**. (Note that this result is independent of the stone's mass – consistent with all objects falling at the same rate when air resistance is neglected.)

Bài tập luyện tập

A ball is thrown vertically upward at $25\,\text{m/s}$ from the edge of a rooftop $20\,\text{m}$ above the ground.

(a) Find the maximum height reached above the rooftop.

(b) Find the time taken to reach that maximum height.

(c) Find the total time from launch until the ball hits the ground.

(d) Find the ball's speed when it hits the ground.

Lời giải chi tiết

Take up as positive, $a = -g$.
(a) $H = \dfrac{u^2}{2g} = \dfrac{25^2}{2(9.81)} = \boxed{31.9\,\text{m}}$ above the rooftop.
(b) $t_{\text{up}} = \dfrac{u}{g} = \dfrac{25}{9.81} = \boxed{2.55\,\text{s}}$.
(c) Taking the rooftop as origin, the ground is at $s = -20\,\text{m}$: $-20 = 25t - 4.905t^2 \implies 4.905t^2 - 25t - 20 = 0$.

$$t = \frac{25 + \sqrt{625 + 4(4.905)(20)}}{2(4.905)} = \frac{25 + \sqrt{1017}}{9.81} = \frac{25 + 31.9}{9.81} = \boxed{5.80\,\text{s}}$$ (rejecting the negative

root).
(d) $v = u - gt = 25 - 9.81(5.80) = \boxed{-31.9\,\text{m/s}}$, i.e. $31.9\,\text{m/s}$ downward.

Bài tập luyện tập

A ball rolls off a horizontal table $1.25\,\text{m}$ high with a horizontal speed of $4.0\,\text{m/s}$. How far from the base of the table does it land?

(A) $1.25\,\text{m}$
(B) $2.0\,\text{m}$
(C) $4.0\,\text{m}$
(D) $5.0\,\text{m}$

Lời giải chi tiết

$t = \sqrt{2h/g} = \sqrt{2(1.25)/9.81} = 0.505\,\text{s}$. Range $= ut = 4.0(0.505) = 2.0\,\text{m}$. **(B).**

Bài tập luyện tập

A javelin is thrown at $22\,\text{m/s}$ at 35° above the horizontal from ground level.
(a) Find the time of flight.
(b) Find the horizontal range.
(c) Find the maximum height reached.
(d) Find the javelin's speed at the highest point of its flight.

Lời giải chi tiết

$u_x = 22\cos 35^\circ = 18.0\,\text{m/s}$, $u_y = 22\sin 35^\circ = 12.6\,\text{m/s}$.
(a) $T = \dfrac{2u\sin\theta}{g} = \dfrac{2(22)(0.5736)}{9.81} = \boxed{2.57\,\text{s}}$.
(b) $R = \dfrac{u^2\sin 2\theta}{g} = \dfrac{484\sin 70^\circ}{9.81} = \dfrac{484(0.9397)}{9.81} = \boxed{46.4\,\text{m}}$.
(c) $H = \dfrac{u^2\sin^2\theta}{2g} = \dfrac{484(0.3290)}{19.62} = \boxed{8.12\,\text{m}}$.
(d) At the highest point the vertical component is zero, so speed $= u_x = \boxed{18.0\,\text{m/s}}$.

Bài tập luyện tập

A laboratory centrifuge spins at 3000 revolutions per minute with the sample held at radius $0.15\,\text{m}$ from the axis.
(a) Find the period and angular velocity of the spin.
(b) Find the linear speed of the sample.
(c) Find the centripetal acceleration of the sample, expressed also as a multiple of g.

Lời giải chi tiết

(a) $f = 3000/60 = 50\,\text{Hz}$, so $T = 1/f = 0.020\,\text{s}$ and $\omega = 2\pi f = \boxed{314\,\text{rad/s}}$.
(b) $v = \omega r = 314(0.15) = \boxed{47.1\,\text{m/s}}$.

(c) $a_c = \omega^2 r = 314^2(0.15) \approx \boxed{1.48 \times 10^4\ \text{m/s}^2}$, i.e. $a_c/g \approx 1.51 \times 10^3$ – about 1500 times g.

Bài tập luyện tập

A 10 kg box on a horizontal floor is pushed with a horizontal force of 40 N. The coefficient of kinetic friction between box and floor is $\mu_k = 0.25$. Find the box's acceleration.

(A) 4.0 m/s^2
(B) 2.5 m/s^2
(C) 1.5 m/s^2
(D) 0.25 m/s^2

Lời giải chi tiết

$N = mg = 10(9.81) = 98.1$ N. $f_k = \mu_k N = 0.25(98.1) = 24.5$ N. Net force $= 40 - 24.5 = 15.5$ N. $a = 15.5/10 = 1.5$ m/s^2. **(C)**.

Bài tập luyện tập

A 8.0 kg block is placed on an incline at 20° to the horizontal. The coefficients of friction between block and incline are $\mu_s = 0.35$ and $\mu_k = 0.25$.

(a) Show that the block does not remain at rest when released.

(b) Find the block's acceleration down the slope.

Lời giải chi tiết

(a) Component of weight along the slope: $mg \sin 20^\circ = 8(9.81)(0.342) = 26.8$ N. Maximum static friction: $\mu_s mg \cos 20^\circ = 0.35(8)(9.81)(0.940) = 25.8$ N. Since 26.8 N $>$ 25.8 N, the driving component exceeds the greatest friction available, so the block slides.
(b) Once sliding: $a = g(\sin 20^\circ - \mu_k \cos 20^\circ) = 9.81(0.342 - 0.25 \times 0.940) = 9.81(0.342 - 0.235) = \boxed{1.05\ \text{m/s}^2}$ down the slope.

Bài tập luyện tập

A 0.15 kg baseball approaches a bat horizontally at 30 m/s and leaves the bat at 40 m/s in the opposite direction. The ball is in contact with the bat for 1.5 ms. Find the average force exerted by the bat on the ball.

(A) 1050 N
(B) 4000 N
(C) 7000 N
(D) 10500 N

Lời giải chi tiết

Taking the incoming direction as positive: $\Delta p = m(v_f - v_i) = 0.15(-40 - 30) = -10.5$ kg · m/s. $F = |\Delta p|/\Delta t = 10.5/0.0015 = 7000$ N. **(C)**.

Bài tập luyện tập

Cart A (3.0 kg, moving at 4.0 m/s) collides with stationary cart B (1.0 kg) on a frictionless track.

(a) If the collision is perfectly elastic, find the velocity of each cart after the collision.

(b) If instead the carts collide and stick together, find their common final velocity and the kinetic energy lost.

Lời giải chi tiết

(a) $v_A = \frac{m_A - m_B}{m_A + m_B} u_A = \frac{2}{4}(4.0) = 2.0\,\text{m/s}$; $v_B = \frac{2m_A}{m_A + m_B} u_A = \frac{6}{4}(4.0) = 6.0\,\text{m/s}$.
Check: momentum before $= 3.0(4.0) = 12\,\text{kg}\cdot\text{m/s}$; after $= 3.0(2.0) + 1.0(6.0) = 12\,\text{kg}\cdot\text{m/s}$, confirming momentum conservation. KE before $= \frac{1}{2}(3.0)(4.0)^2 = 24$ J; after $= \frac{1}{2}(3.0)(2.0)^2 + \frac{1}{2}(1.0)(6.0)^2 = 6 + 18 = 24$ J, confirming KE is also conserved.
(b) $v_f = \frac{m_A u_A + m_B u_B}{m_A + m_B} = \frac{12}{4.0} = \boxed{3.0\,\text{m/s}}$. KE after $= \frac{1}{2}(4.0)(3.0)^2 = 18$ J. KE lost $= 24 - 18 = \boxed{6.0\,\text{J}}$.

Bài tập luyện tập

A 1200 kg car rounds a flat (unbanked) curve of radius 60 m. The coefficient of static friction between tyres and road is $\mu_s = 0.60$. Find the maximum speed at which the car can round the curve without skidding.

(A) 6.0 m/s
(B) 18.8 m/s
(C) 35.3 m/s
(D) 60.0 m/s

Lời giải chi tiết

$\mu_s mg = \frac{mv^2}{r} \implies v = \sqrt{\mu_s g r} = \sqrt{0.60(9.81)(60)} = \sqrt{353} = 18.8\,\text{m/s}$. **(B).**

Bài tập luyện tập

A highway curve of radius r is banked at 15° so that a car travelling at 25 m/s can round it with *no* reliance on friction.

(a) Find the required radius r.

(b) If a car exceeds this design speed on the same curve, state (with a brief reason) which way friction must act to prevent the car sliding.

Lời giải chi tiết

(a) With no friction, resolving forces: $N\cos\theta = mg$ and $N\sin\theta = \frac{mv^2}{r}$. Dividing: $\tan\theta = \frac{v^2}{rg} \implies r = \frac{v^2}{g\tan\theta} = \frac{625}{9.81\tan 15^\circ} = \frac{625}{9.81(0.268)} = \boxed{238\,\text{m}}$.
(b) Above the design speed, the car "wants" to slide up and outward across the bank, so friction acts down the slope (inward and downward along the banked surface) to supply the extra centripetal force needed.

Bài tập luyện tập

Planet X has mass 4.0×10^{24} kg and radius 5.0×10^{6} m.

(a) Find the gravitational field strength at the surface of Planet X.

(b) A satellite orbits Planet X at an altitude of 1.0×10^3 km above the surface. Find its orbital speed and period.

Lời giải chi tiết

(a) $g = \frac{GM}{R^2} = \frac{(6.67 \times 10^{-11})(4.0 \times 10^{24})}{(5.0 \times 10^6)^2} = \frac{2.67 \times 10^{14}}{2.5 \times 10^{13}} = \boxed{10.7\,\text{m/s}^2}$.

(b) $r = R + h = 5.0 \times 10^6 + 1.0 \times 10^6 = 6.0 \times 10^6$ m.

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{2.67 \times 10^{14}}{6.0 \times 10^6}} = \sqrt{4.45 \times 10^7} = \boxed{6.67 \times 10^3\,\text{m/s}}.$$

$$T = \frac{2\pi r}{v} = \frac{2\pi(6.0 \times 10^6)}{6.67 \times 10^3} = \boxed{5650\,\text{s}} \approx 94\,\text{min}.$$

Bài tập luyện tập

Planet 1 orbits a star at distance r_1 with period T_1. Planet 2 orbits the same star at distance $r_2 = 4r_1$. What is planet 2's orbital period, in terms of T_1?

(A) $2T_1$

(B) $4T_1$

(C) $8T_1$

(D) $16T_1$

Lời giải chi tiết

By Kepler's third law, $T^2 \propto r^3$, so $\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{r_2}{r_1}\right)^3 = 4^3 = 64 \implies \frac{T_2}{T_1} = 8$. **(C)**. Note how strongly period grows with orbital radius: quadrupling the distance increases the period eightfold, not merely fourfold.

Bài tập luyện tập

A 5.0 kg box, initially at rest, is pushed 3.0 m across a floor by a horizontal force of 20 N, while friction does -18 J of work on it. Find the box's final speed.

(A) 2.7 m/s

(B) 4.1 m/s

(C) 4.9 m/s

(D) 16.8 m/s

Lời giải chi tiết

$W_{\text{applied}} = Fd = 20(3.0) = 60$ J. $W_{\text{net}} = 60 - 18 = 42\,\text{J} = \Delta KE$. From rest: $v = \sqrt{2(42)/5.0} = \sqrt{16.8} = 4.1$ m/s. **(B)**.

Bài tập luyện tập

A 2.0 kg block starts from rest at the top of a frictionless ramp 3.0 m high. It slides down and then continues across a rough horizontal floor ($\mu_k = 0.40$) before stopping.

(a) Find the block's speed at the bottom of the ramp.

(b) Find the distance the block travels across the rough floor before stopping.

Lời giải chi tiết

(a) $v = \sqrt{2gh} = \sqrt{2(9.81)(3.0)} = \sqrt{58.9} = \boxed{7.67\,\text{m/s}}$.
(b) KE at the bottom $= mgh = 2.0(9.81)(3.0) = 58.9$ J, entirely removed by friction: $f_k d = mgh$, where $f_k = \mu_k mg = 0.40(2.0)(9.81) = 7.85$ N. $d = 58.9/7.85 = \boxed{7.5\,\text{m}}$.

Bài tập luyện tập

A horizontal spring with spring constant $k = 150$ N/m is compressed 0.20 m and used to launch a 0.50 kg ball along a frictionless surface. Find the ball's launch speed.

(A) 1.7 m/s
(B) 2.4 m/s
(C) 3.5 m/s
(D) 6.0 m/s

Lời giải chi tiết

$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 \implies v = x\sqrt{k/m} = 0.20\sqrt{150/0.50} = 0.20\sqrt{300} = 0.20(17.3) = 3.5$ m/s. **(C)**.

Bài tập luyện tập

A vertical spring with $k = 2000$ N/m is compressed 0.12 m by a 0.80 kg ball, then released, launching the ball upward. Neglect the spring's mass and the small height change during release.

(a) Find the elastic potential energy stored in the compressed spring.

(b) Find the additional height the ball rises above the point of release.

Lời giải chi tiết

(a) $PE = \frac{1}{2}kx^2 = \frac{1}{2}(2000)(0.12)^2 = \boxed{14.4\,\text{J}}$.
(b) $mgh = 14.4 \implies h = \dfrac{14.4}{0.80(9.81)} = \dfrac{14.4}{7.85} = \boxed{1.83\,\text{m}}$.

Bài tập luyện tập

An electric pump raises 800 kg of water through a height of 12 m every 40 s. If the electrical power supplied to the pump is 3000 W, find the efficiency of the pump.

(A) 21.5%
(B) 45.0%
(C) 78.5%
(D) 96.0%

Lời giải chi tiết

Useful power $= \dfrac{mgh}{t} = \dfrac{800(9.81)(12)}{40} = \dfrac{94176}{40} = 2354$ W. Efficiency $= 2354/3000 = 0.785 = 78.5\%$. **(C)**.

Bài tập luyện tập

A cyclist and bicycle (total mass 75 kg) travel at a constant 8.0 m/s along a flat road against a total resistive force of 30 N.

(a) Find the mechanical power the cyclist must supply.

(b) If the cyclist's muscles convert metabolic energy into useful mechanical power at 25% efficiency, find the metabolic power input required.

Lời giải chi tiết

(a) At constant velocity, $P = Fv = 30(8.0) = \boxed{240\,\text{W}}$.

(b) $P_{\text{input}} = \dfrac{P_{\text{useful}}}{\text{efficiency}} = \dfrac{240}{0.25} = \boxed{960\,\text{W}}$.

Bài tập luyện tập

A uniform rod of length 2.0 m and weight 40 N is pivoted at one end and held horizontal by a vertical force applied at its far end. Find the magnitude of that force.

(A) 10 N

(B) 20 N

(C) 40 N

(D) 80 N

Lời giải chi tiết

The weight acts at the rod's centre, 1.0 m from the pivot. Torque balance: $F(2.0) = 40(1.0) \implies F = 20$ N. **(B)**.

Bài tập luyện tập

A uniform beam of mass 12 kg and length 5.0 m rests horizontally on two supports, one at each end. A 40 kg person stands 1.5 m from the left end. Find the support force at each end.

Lời giải chi tiết

Beam weight $= 12(9.81) = 117.7$ N at the centre (2.5 m from the left end). Person's weight $= 40(9.81) = 392.4$ N at 1.5 m from the left end.
Torques about the left support: $N_R(5.0) = 117.7(2.5) + 392.4(1.5) = 294.3 + 588.6 = 882.9 \implies N_R = \boxed{176.6\,\text{N}}$.
Force balance: $N_L = 117.7 + 392.4 - 176.6 = \boxed{333.5\,\text{N}}$.

Bài tập luyện tập

A figure skater spins with arms extended, moment of inertia $4.0\,\text{kg}\cdot\text{m}^2$, at angular velocity 2.0 rad/s. She then pulls her arms in, reducing her moment of inertia to $1.6\,\text{kg}\cdot\text{m}^2$.

(a) Find her new angular velocity.

(b) Find the ratio of her final to initial rotational kinetic energy, and explain where the extra energy comes from.

Lời giải chi tiết

(a) $I_1\omega_1 = I_2\omega_2 \implies \omega_2 = \dfrac{4.0(2.0)}{1.6} = \boxed{5.0\,\text{rad/s}}$.

(b) $KE_1 = \frac{1}{2}(4.0)(2.0)^2 = 8.0$ J; $KE_2 = \frac{1}{2}(1.6)(5.0)^2 = 20$ J. Ratio $= \boxed{2.5}$. The extra rotational kinetic energy comes from the internal (muscular) work she does pulling her arms inward against the centripetal requirement – angular momentum is conserved, but kinetic energy is not, since an internal force does positive work on the system.

Bài tập luyện tập

A solid sphere ($I = \frac{2}{5}MR^2$) rolls without slipping down an incline, starting from rest and dropping through a height of 1.5 m. Find its speed at the bottom, and compare it with the speed it would reach sliding down the same height without friction (and hence without rotating).

Lời giải chi tiết

$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\frac{v^2}{R^2} = \frac{1}{2}Mv^2 + \frac{1}{5}Mv^2 = \frac{7}{10}Mv^2.$

$v^2 = \frac{10gh}{7} = \frac{10(9.81)(1.5)}{7} = \frac{147.2}{7} = 21.0 \implies v = \boxed{4.58\,\text{m/s}}.$

Frictionless sliding: $v = \sqrt{2gh} = \sqrt{2(9.81)(1.5)} = \sqrt{29.4} = \boxed{5.43\,\text{m/s}}$ – faster, since none of its PE is diverted into rotational KE.

Bài tập luyện tập

A spacecraft moves at $v = 0.80c$ relative to Earth. A clock on board measures a proper time interval of 3.0 s between two events occurring at the same place on the spacecraft. What time interval do Earth-based observers measure between these same two events?

(A) 1.8 s

(B) 2.4 s

(C) 3.0 s

(D) 5.0 s

Lời giải chi tiết

$\gamma = \frac{1}{\sqrt{1-0.80^2}} = \frac{1}{\sqrt{0.36}} = \frac{1}{0.60} = 1.667.$ $\Delta t = \gamma\Delta t_0 = 1.667(3.0) = 5.0$ s. **(D).**

Bài tập luyện tập

A spaceship has a proper length of 120 m and travels past Earth at $v = 0.60c$.

(a) Find the Lorentz factor γ.

(b) Find the length of the spaceship as measured by an observer on Earth.

(c) Find the time (measured on Earth) for the spaceship to pass a fixed point on Earth.

Lời giải chi tiết

(a) $\gamma = \frac{1}{\sqrt{1-0.60^2}} = \frac{1}{\sqrt{0.64}} = \frac{1}{0.80} = \boxed{1.25}.$

(b) $L = \frac{L_0}{\gamma} = \frac{120}{1.25} = \boxed{96\,\text{m}}.$

(c) $t = \frac{L}{v} = \frac{96}{1.80 \times 10^8} = 5.33 \times 10^{-7}\,\text{s} \approx \boxed{533\,\text{ns}}.$

Bài tập luyện tập

A proton (rest mass 1.67×10^{-27} kg) moves at $v = 0.60c$. Find its kinetic energy.

(A) 1.50×10^{-10} J
(B) 3.76×10^{-11} J
(C) 1.88×10^{-10} J
(D) 7.52×10^{-11} J

Lời giải chi tiết

$\gamma = 1.25$ at $v = 0.60c$. $E_0 = mc^2 = 1.67 \times 10^{-27}(3.00 \times 10^8)^2 = 1.67 \times 10^{-27}(9.00 \times 10^{16}) = 1.503 \times 10^{-10}$ J. $KE = (\gamma - 1)E_0 = 0.25(1.503 \times 10^{-10}) = 3.76 \times 10^{-11}$ J. **(B)**.

Bài tập luyện tập

Twin A stays on Earth. Twin B travels to a star 12 light-years away (as measured in the Earth frame) at $v = 0.80c$, and returns immediately at the same speed.

(a) Find the total time elapsed for twin A, in the Earth frame.

(b) Find the proper time elapsed for twin B (i.e. twin B's own aging) during the trip.

(c) Briefly explain why it is twin B, and not twin A, who ages less – despite each twin seeing the other's clock as "running slow" during the outward and return legs.

Lời giải chi tiết

(a) Total distance travelled (there and back) $= 24$ light-years at $v = 0.80c$: $\Delta t = \dfrac{24\,\text{ly}}{0.80c} = \boxed{30.0\,\text{yr}}$ (using 1 light-year$/c = 1$ year).

(b) $\gamma = 1.667$ at $v = 0.80c$. Twin B's proper time: $\Delta t_0 = \dfrac{\Delta t}{\gamma} = \dfrac{30.0}{1.667} = \boxed{18.0\,\text{yr}}$.

(c) The situation is not symmetric: twin A remains in a single inertial frame for the entire trip, while twin B must decelerate, turn around, and accelerate back at the distant star, switching inertial frames. The simple formula $\Delta t = \gamma \Delta t_0$ applies unambiguously only from the perspective of the twin who stays in one inertial frame throughout (twin A); twin B's frame change breaks the apparent symmetry, so twin B returns having aged only 18.0 years while twin A has aged 30.0 years.

Bài tập luyện tập

Two blocks, $m_1 = 4.0$ kg and $m_2 = 6.0$ kg, are connected by a light inextensible string over a light, frictionless pulley (an Atwood machine). Find the acceleration of the system and the tension in the string.

Lời giải chi tiết

$a = \dfrac{(m_2 - m_1)g}{m_1 + m_2} = \dfrac{(6.0 - 4.0)(9.81)}{10} = \dfrac{19.62}{10} = \boxed{1.96\,\text{m/s}^2}$.

$T = m_1(g + a) = 4.0(9.81 + 1.96) = 4.0(11.77) = \boxed{47.1\,\text{N}}$ (check: $T = m_2(g - a) = 6.0(9.81 - 1.96) = 6.0(7.85) = 47.1$ N, consistent).

Bài tập luyện tập

A roller-coaster car enters a frictionless vertical circular loop of radius $12\,\text{m}$.

(a) Find the minimum speed at the top of the loop needed to maintain contact with the track.

(b) Find the corresponding minimum speed required at the bottom of the loop.

Lời giải chi tiết

(a) $v_{\text{min,top}} = \sqrt{gr} = \sqrt{9.81(12)} = \sqrt{117.7} = \boxed{10.9\,\text{m/s}}$.
(b) $v_{\text{bottom}}^2 = v_{\text{top}}^2 + 4gr = 117.7 + 4(9.81)(12) = 117.7 + 470.9 = 588.6 \implies v_{\text{min,bottom}} = \sqrt{588.6} = \boxed{24.3\,\text{m/s}}$.

Bài tập luyện tập

A $0.010\,\text{kg}$ bullet is fired horizontally into a $2.0\,\text{kg}$ wooden block hanging at rest from a light string. The bullet embeds in the block, and the block+bullet swing upward, rising through a height of $0.15\,\text{m}$.

(a) Find the speed of the block+bullet immediately after the bullet embeds.

(b) Find the initial speed of the bullet.

Lời giải chi tiết

(a) After the collision, only gravity does work as the block swings upward (energy conservation): $\frac{1}{2}(m_{\text{bullet}} + m_{\text{block}})v^2 = (m_{\text{bullet}} + m_{\text{block}})gh \implies v = \sqrt{2gh} = \sqrt{2(9.81)(0.15)} = \sqrt{2.943} = \boxed{1.72\,\text{m/s}}$.
(b) The collision itself is perfectly inelastic and very short, so momentum is conserved during it (gravity's impulse over such a short time is negligible): $m_{\text{bullet}}u = (m_{\text{bullet}} + m_{\text{block}})v \implies u = \dfrac{(2.010)(1.715)}{0.010} = \dfrac{3.448}{0.010} = \boxed{345\,\text{m/s}}$.

Bài tập luyện tập

A $1000\,\text{kg}$ car, starting from rest, accelerates to $20\,\text{m/s}$ in $8.0\,\text{s}$ on a level road, against a constant resistive force of $400\,\text{N}$.

(a) Find the driving force needed from the engine (assumed constant).

(b) Find the distance travelled during this acceleration phase.

(c) Find the work done by the driving force, and the average power output of the engine.

Lời giải chi tiết

(a) $a = \dfrac{v-u}{t} = \dfrac{20}{8.0} = 2.5\,\text{m/s}^2$. $F_{\text{drive}} - f = ma \implies F_{\text{drive}} = ma + f = 1000(2.5) + 400 = 2500 + 400 = \boxed{2900\,\text{N}}$.
(b) $s = \frac{1}{2}(u+v)t = \frac{1}{2}(0+20)(8.0) = \boxed{80\,\text{m}}$.
(c) $W = F_{\text{drive}}\,s = 2900(80) = \boxed{2.32 \times 10^5\,\text{J}}$. $P_{\text{avg}} = \dfrac{W}{t} = \dfrac{232000}{8.0} = \boxed{2.90 \times 10^4\,\text{W}}$ (29 kW).

Bài tập luyện tập

Two trains, each travelling at $25\,\mathrm{m/s}$, approach each other on parallel straight tracks. Find the rate at which the distance between them decreases (their closing speed), as measured by an observer on the ground.

(A) $0\,\mathrm{m/s}$
(B) $25\,\mathrm{m/s}$
(C) $35\,\mathrm{m/s}$
(D) $50\,\mathrm{m/s}$

Lời giải chi tiết

In the ground frame, the trains move toward each other, so their speeds of approach add (Galilean velocity addition applied to two objects moving in opposite directions): closing speed $= 25 + 25 = 50\,\mathrm{m/s}$. **(D)**.

Bài tập luyện tập

A mechanic applies a force of $80\,\mathrm{N}$ to the end of a $0.30\,\mathrm{m}$ wrench handle, at an angle of 70° to the handle (not perpendicular to it). Find the torque produced on the bolt.

(A) $24.0\,\mathrm{N\cdot m}$
(B) $22.6\,\mathrm{N\cdot m}$
(C) $8.2\,\mathrm{N\cdot m}$
(D) $27.4\,\mathrm{N\cdot m}$

Lời giải chi tiết

$\tau = Fr\sin\theta = 80(0.30)\sin 70^\circ = 24(0.9397) = 22.6\,\mathrm{N\cdot m}$. **(B)**.

Bài tập luyện tập

A solid sphere ($I = \frac{2}{5}MR^2$), a solid disk ($I = \frac{1}{2}MR^2$), and a thin hoop ($I = MR^2$) – all with the same mass and radius – are released from rest simultaneously at the top of an identical incline and roll without slipping to the bottom. Which one reaches the bottom first, and why?

(A) The hoop, since it has the largest moment of inertia
(B) The disk, since its moment of inertia is exactly midway between the other two
(C) The sphere, since the smallest fraction of its kinetic energy goes into rotation
(D) All three arrive together, since mass and radius are equal

Lời giải chi tiết

For any rolling shape, $v^2 = \dfrac{2gh}{1 + I/(mr^2)}$: the smaller the ratio $I/(mr^2)$, the larger the final speed. Since $\frac{2}{5} < \frac{1}{2} < 1$, the sphere has the smallest ratio, so the largest fraction of its released gravitational PE becomes translational (rather than rotational) kinetic energy, and it reaches the bottom first, followed by the disk, then the hoop. **(C)**.

Bài tập luyện tập

A $4.0\,\mathrm{kg}$ rifle, initially at rest, fires a $0.020\,\mathrm{kg}$ bullet at $400\,\mathrm{m/s}$. Find the recoil speed of the rifle immediately after firing.

(A) $0.5\,\text{m/s}$
(B) $2.0\,\text{m/s}$
(C) $8.0\,\text{m/s}$
(D) $20\,\text{m/s}$

Lời giải chi tiết

Total momentum is zero both before and after firing (isolated system, initially at rest): $m_{\text{bullet}}v_{\text{bullet}} + m_{\text{rifle}}v_{\text{rifle}} = 0 \implies v_{\text{rifle}} = -\dfrac{0.020(400)}{4.0} = -2.0\,\text{m/s}$, i.e. a recoil speed of $2.0\,\text{m/s}$ in the direction opposite the bullet. **(B)**.

Bài tập luyện tập

A block of mass $m_1 = 4.0\,\text{kg}$ rests on a frictionless incline at $30°$ to the horizontal. It is connected by a light string, over a light frictionless pulley fixed at the top of the incline, to a second block of mass $m_2 = 6.0\,\text{kg}$ hanging vertically off the edge.

(a) Find the acceleration of the system.

(b) Find the tension in the string.

Lời giải chi tiết

The hanging block (m_2) falls while the block on the incline is dragged up the slope; both accelerate at the same magnitude a.
Hanging block (down positive): $m_2 g - T = m_2 a$.
Block on incline (up-slope positive; the incline is frictionless, so only gravity's component along the slope and the tension act along this axis): $T - m_1 g\sin 30° = m_1 a$.
Adding the two equations eliminates T: $m_2 g - m_1 g\sin 30° = (m_1 + m_2)a$.
$a = \dfrac{m_2 g - m_1 g\sin 30°}{m_1 + m_2} = \dfrac{6.0(9.81) - 4.0(9.81)(0.500)}{10} = \dfrac{58.86 - 19.62}{10} = \dfrac{39.24}{10} = \boxed{3.92\,\text{m/s}^2}$.
$T = m_2(g - a) = 6.0(9.81 - 3.92) = 6.0(5.89) = \boxed{35.3\,\text{N}}$ (check: $T = m_1(a + g\sin 30°) = 4.0(3.92 + 4.905) = 4.0(8.825) = 35.3\,\text{N}$, consistent).

Bài tập luyện tập

A force-time graph for a hammer strike on a nail is a triangle: the force rises linearly from 0 to a peak of $200\,\text{N}$ over the first $0.020\,\text{s}$, then falls linearly back to 0 over the next $0.020\,\text{s}$.

(a) Find the impulse delivered, using the area under the graph.

(b) If this impulse acts on a $0.50\,\text{kg}$ nail (initially at rest, free to move), find the nail's speed immediately afterward.

Lời giải chi tiết

(a) The graph is a triangle of base $0.040\,\text{s}$ and height $200\,\text{N}$: $J = \text{area} = \frac{1}{2}(0.040)(200) = \boxed{4.0\,\text{N}\cdot\text{s}}$.

(b) $J = \Delta p = mv \implies v = \dfrac{J}{m} = \dfrac{4.0}{0.50} = \boxed{8.0\,\text{m/s}}$.

Theme B: The Particulate Nature of Matter

Mục tiêu Unit: Mô hình động học hạt của vật chất và sự phân biệt giữa nhiệt độ với nội năng; nhiệt lượng truyền đi, nhiệt dung riêng, nhiệt ẩn riêng và các cách truyền nhiệt (dẫn nhiệt, đối lưu, phát xạ nhiệt); hiệu ứng nhà kính, định luật Stefan–Boltzmann, định luật Wien và mô hình cân bằng năng lượng của Trái Đất; các định luật chất khí (Boyle, Charles, Gay-Lussac), phương trình khí lý tưởng và thuyết động học phân tử chất khí; nhiệt động lực học ở mức AHL: định luật I và định luật II nhiệt động lực học, hiệu suất động cơ nhiệt và hiệu suất Carnot; dòng điện, suất điện động, định luật Ohm, điện trở suất, mạch điện nối tiếp – song song và mạch phân áp.

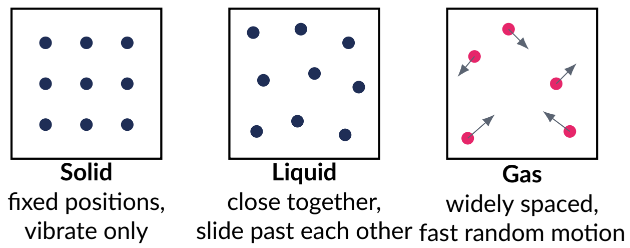
Theme B examines matter not as a continuous substance but as a vast collection of discrete, moving particles, and shows how the statistical behaviour of huge numbers of such particles gives rise to every macroscopic thermal property encountered in everyday life: temperature, pressure, the flow of heat, the radiation of stars and planets, the behaviour of gases, and, through the motion of charge carriers, electric current itself. The same particle model that explains why a metal spoon feels cold to the touch also explains why Venus's atmosphere traps enough heat to melt lead, and why a gas cylinder becomes dangerously hot when compressed rapidly.

2.1 B.1 Thermal Energy Transfers

2.1.1 The Particle Model of Matter

All matter is made of particles (atoms, ions, or molecules) in constant motion. The three familiar states of matter — solid, liquid, gas — differ in how closely the particles are packed, how strongly they interact, and how freely they move.

In a **solid**, particles are packed closely in a roughly fixed arrangement, held by strong intermolecular or interatomic forces; each particle vibrates about a fixed equilibrium position but does not travel through the material. A solid therefore has a fixed shape and a fixed volume. In a **liquid**, particles remain close together (similar average spacing to a solid) but the forces are weaker, so particles can slide past one another; a liquid has a fixed volume but takes the shape of its container. In a **gas**, particles are separated by distances many times their own size, interact only briefly during collisions, and move freely and rapidly in random directions; a gas has neither a fixed shape nor a fixed volume, expanding to fill any container.



Particle spacing, arrangement, and motion in the three common states of matter. Average particle spacing and speed increase from solid to liquid to gas, while the strength of the bonds between particles decreases in the same order.

Property	Solid	Liquid	Gas
Density	high	high (similar to solid)	very low ($\sim$ $1000\times$ smaller)
Compressibility	essentially incompressible	essentially incompressible	highly compressible
Shape	fixed	takes shape of container	fills entire container
Volume	fixed	fixed	not fixed

VN GIẢI THÍCH TIẾNG VIỆT

Ba trạng thái của vật chất khác nhau chủ yếu ở **khoảng cách** giữa các hạt, **lực liên kết** giữa chúng, và **kiểu chuyển động**. Ở thể rắn, hạt xếp sát nhau và chỉ dao động quanh vị trí cố định (lực liên kết mạnh). Ở thể lỏng, hạt vẫn gần nhau nhưng có thể trượt qua nhau (lực yếu hơn). Ở thể khí, hạt cách xa nhau, gần như không tương tác trừ lúc va chạm, và chuyển động hỗn loạn với tốc độ lớn. Đây chính là nền tảng của **thuyết động học phân tử** sẽ dùng xuyên suốt cả chương.

A fourth process, **sublimation** (solid directly to gas, e.g. solid CO2 “dry ice” at room pressure) and its reverse, **deposition** (gas directly to solid, e.g. frost forming from water vapour), occurs when a substance’s solid and gas phases can coexist without an intervening liquid phase. More generally, the temperature at which a substance melts or boils is not fixed once and for all: it depends on the surrounding pressure, which is why water boils below 100°C on a high mountain (lower atmospheric pressure) and why a sealed pressure cooker, which raises the internal pressure, cooks food faster by raising water’s boiling point above 100°C.

2.1.2 Temperature, Internal Energy, and Heat

Three closely related quantities are often confused with one another, so it is essential to define each precisely.

Temperature T is a measure of the *average* translational kinetic energy of the particles in a substance. It does not depend on how much substance is present — a cup of boiling water and a swimming-pool-sized volume of boiling water have the same temperature, because the *average* kinetic energy per particle is the same in both.

Internal energy U is the *total* energy (kinetic *and* potential) of all the particles in a system. The kinetic part comes from particles’ translational, rotational, and vibrational motion; the potential part comes from the intermolecular/interatomic bonds between particles. Because internal energy is a *total*, it does depend on the amount of substance present: the swimming pool of boiling water has vastly more internal energy than the cup, even though both are at the same temperature.

Heat (thermal energy transferred, often written Q) is energy that flows from one body to another *because of a temperature difference* between them. Heat is not a property a body “has”; it is energy *in transit*, exchanged only while a temperature difference exists.

Temperature is most naturally measured on the **Kelvin (absolute) scale**, on which 0 K (**absolute zero**) corresponds to the theoretical state of minimum possible particle kinetic energy, and every physically meaningful temperature is positive. The more familiar **Celsius scale** is offset from the Kelvin scale by a constant, $T(\text{K}) = T(^{\circ}\text{C}) + 273$. Because the two scales differ only by a constant offset, not by a change of scale factor, a *temperature difference* ΔT has the same numerical value in kelvin as in degrees Celsius; but an *absolute* temperature T, as required by the gas laws, the Stefan–Boltzmann law, and $\overline{KE} = \frac{3}{2}kT$ later in this chapter, must always first be converted to kelvin.

Temperature $\propto$ average kinetic energy per particle (an intensive quantity — independent of amount).
Internal energy $U = \sum(\text{kinetic energy} + \text{potential energy})$ of all particles (an extensive quantity — depends on amount).
Heat Q is energy transferred between bodies due to a temperature difference; it flows spontaneously from hotter to colder.

(!) Lỗi thường gặp: "Heat" and "temperature" are not the same thing, and neither is "hot" the same as "a lot of energy." A struck match momentarily reaches a very high *temperature* but contains very little total *internal energy* (few particles); a bathtub of lukewarm water has a much lower temperature but far greater internal energy (vastly more particles). Never write "heat of an object" as if heat were a stored quantity — only Q, the energy *transferred*, is meaningful, and only while a temperature difference drives the transfer.

VN GIẢI THÍCH TIẾNG VIỆT

Nhiệt độ đo động năng **trung bình** của mỗi hạt — không phụ thuộc vào lượng chất. **Nội năng** là tổng động năng cộng thế năng của **toàn bộ** các hạt trong hệ — phụ thuộc vào lượng chất. **Nhiệt lượng** Q là năng lượng được truyền đi do có chênh lệch nhiệt độ, không phải là một đại lượng vật thể "chứa sẵn". Mẹo nhớ: nhiệt độ giống "mức trung bình", nội năng giống "tổng số dư trong tài khoản", còn nhiệt lượng giống "giao dịch chuyển khoản" xảy ra khi có chênh lệch.

2.1.3 Thermal Equilibrium

When two bodies at different temperatures are placed in contact, thermal energy flows spontaneously from the hotter body to the colder body (never the reverse, for a spontaneous process). This transfer continues until both bodies reach the same temperature, at which point they are said to be in **thermal equilibrium**: no further *net* heat flows between them, even though particles continue to exchange energy at the molecular level in both directions.

Two bodies in **thermal equilibrium** have equal temperature and zero net heat flow between them. Left in contact and isolated from their surroundings, any two bodies at different initial temperatures will always evolve toward a single common final (equilibrium) temperature, never away from it.

VN GIẢI THÍCH TIẾNG VIỆT

Cân bằng nhiệt: khi hai vật ở nhiệt độ khác nhau tiếp xúc, nhiệt tự động truyền từ vật nóng sang vật lạnh cho đến khi hai vật có **cùng nhiệt độ** — lúc đó không còn dòng nhiệt **ròng** nào nữa. Đây chính là nguyên lý làm nền cho toàn bộ các bài toán **nhiệt lượng kế (calorimetry)** ở phần sau: nhiệt lượng do vật nóng mất đi đúng bằng nhiệt lượng vật lạnh nhận được.

Thermal equilibrium is the working principle behind every thermometer: a thermometer is left in contact with an object until the two reach a common temperature, at which point the thermometer's own reading (the length of a mercury or alcohol column, the resistance of a sensor, the voltage of a thermocouple) can be taken as equal to the object's temperature. A good thermometer is therefore small compared with the object it measures, so that reaching equilibrium disturbs the object's own temperature as little as possible.

2.1.4 Specific Heat Capacity

When a substance is heated (or cooled) without undergoing a phase change, the temperature change is proportional to the heat added and inversely proportional to the mass present. The constant of proportionality, characteristic of the material, is the **specific heat capacity** c — the energy required to raise the temperature of 1 kg of the substance by 1 K.

$$Q = mc\,\Delta T,$$

where Q is the heat transferred (J), m is the mass (kg), c is the specific heat capacity (J kg^{-1}K^{-1}), and $\Delta T = T_f - T_i$ is the temperature change (K or °C — since it is a *difference*, the size of a kelvin and a degree Celsius are identical, so ΔT may be computed directly in either scale).

Substance	**Specific heat capacity c (J kg^{-1}K^{-1})**
Water (liquid)	4200
Ice	2100
Steam (water vapour)	2010
Aluminium	900
Iron	450
Copper	390
Lead	130

Ví dụ mẫu · Specific heat capacity

Find the heat required to raise the temperature of 3.0 kg of liquid water from 15°C to 75°C. ($c_{water} = 4200$ J/(kg · K).)

$$Q = mc\,\Delta T = (3.0)(4200)(75 - 15) = (3.0)(4200)(60) = 7.56 \times 10^5 \text{ J} = 756 \text{ kJ}.$$

GIẢI THÍCH TIẾNG VIỆT

VN

Công thức $Q = mc\Delta T$ chỉ áp dụng khi vật **không đổi trạng thái** (không nóng chảy, không sôi) — nếu không, nhiệt độ sẽ tăng/giảm liên tục theo Q. Nước có nhiệt dung riêng rất lớn (4200 J kg^{-1}K^{-1}) so với hầu hết kim loại — đây là lý do nước biển/hồ điều hoà khí hậu tốt hơn đất liền: cùng một lượng nhiệt, nước thay đổi nhiệt độ chậm hơn nhiều.

This property has large practical consequences. Metals with a low specific heat capacity (copper, iron, aluminium) heat up and cool down quickly, which is useful in cookware that must respond rapidly to changes in stove temperature. Substances with a high specific heat capacity, above all water, resist temperature change, which is exploited in cooling systems (car radiators, power-station cooling towers) and explains why coastal regions experience milder, more stable temperatures across the seasons than inland regions at the same latitude, since the adjacent ocean absorbs or releases large amounts of thermal energy for only a small change in its own temperature.

2.1.5 Specific Latent Heat

During a **phase change** (melting, freezing, boiling, condensing, sublimating), a substance absorbs or releases thermal energy *without any change in temperature*. All of the energy exchanged goes into breaking (or forming) the intermolecular bonds that hold particles in their arrangement — that is, into changing the *potential* energy component of the internal energy, not the kinetic energy component that temperature measures.

The **specific latent heat of fusion** L_f is the energy per unit mass needed to change a substance between solid and liquid (melting or freezing) at constant temperature. The **specific latent heat of vaporization** L_v is the energy per unit mass needed to change a substance between liquid and gas (boiling/evaporating or condensing) at constant temperature. For a given substance, L_v is always substantially larger than L_f, because vaporization must overcome essentially *all* of the intermolecular attraction (particles separate to gas-like distances), whereas fusion only needs to loosen the rigid fixed arrangement of a solid into the more mobile arrangement of a liquid.

$$Q = mL,$$

where L is the specific latent heat of fusion L_f (melting/freezing) or of vaporization L_v (boiling/condensing), in J kg^{-1}. Unlike $Q = mc\Delta T$, there is no ΔT term — temperature is constant throughout the phase change.

Process (water, at 1 **atm)**	**Specific latent heat (J kg**$^{-1}$**)**
Fusion: ice ↔ liquid water, at $0°$C	$L_f = 3.34 \times 10^5$
Vaporization: liquid water ↔ steam, at $100°$C	$L_v = 2.26 \times 10^6$

(!) Lỗi thường gặp: A very common error is to apply $Q = mc\Delta T$ across a phase change, or to assume temperature keeps rising while ice melts or water boils. Remember: *while* a substance is changing phase, T is exactly constant and $Q = mL$ applies; *before* and *after* the phase change (while it is entirely solid, entirely liquid, or entirely gas), T changes and $Q = mc\Delta T$ applies. A multi-stage problem (e.g. ice below $0°$C turning into steam above $100°$C) must be split into separate stages, each governed by the correct formula.

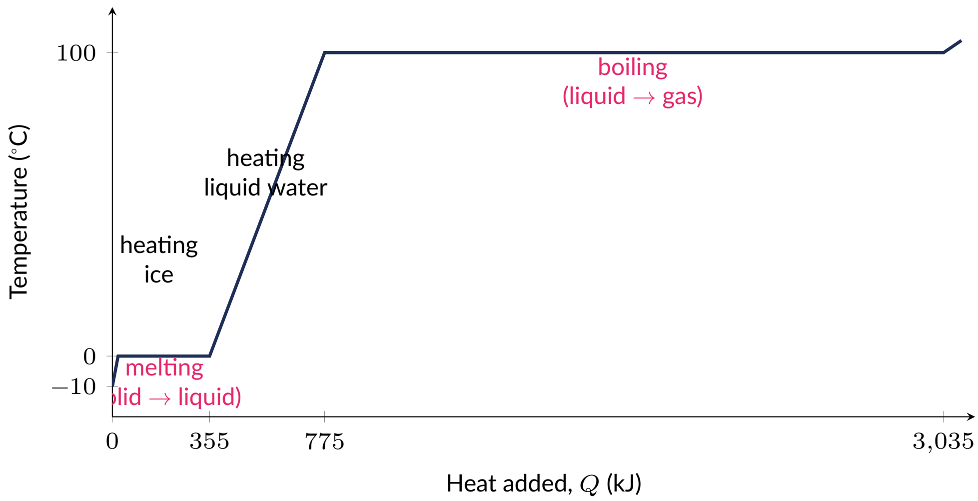
VN **GIẢI THÍCH TIẾNG VIỆT**

Trong quá trình chuyển thể (nóng chảy, hoá hơi...), nhiệt độ **không đổi** dù vẫn có nhiệt lượng truyền vào/ra — toàn bộ năng lượng dùng để phá vỡ (hoặc hình thành) liên kết giữa các hạt, tức là thay đổi **thế năng**, không phải động năng. Vì vậy công thức chuyển sang $Q = mL$ (không có ΔT). L_v luôn lớn hơn L_f nhiều lần vì hoá hơi cần tách hạt ra xa nhau hoàn toàn, còn nóng chảy chỉ cần làm hạt "lỏng lẻo" hơn một chút.

Evaporation provides a familiar cooling mechanism even well below a liquid's boiling point: at any temperature, a small fraction of surface molecules move fast enough to escape into the vapour phase, and these escaping molecules carry away above-average kinetic energy, leaving the remaining liquid with a lower average kinetic energy, and hence a lower temperature. This is the physical basis of sweating as a cooling mechanism, and of evaporative ("swamp") coolers used in hot, dry climates.

2.1.6 The Heating Curve: Warming and Phase Changes

Combining specific heat capacity and specific latent heat produces the characteristic **heating curve** of a substance: a graph of temperature against heat added, showing alternating sloped segments (single-phase warming, governed by $Q = mc\Delta T$) and flat plateaus (phase changes, governed by $Q = mL$, at constant T).



Temperature of 1.0 kg of H2O as heat is added steadily, starting as ice at -10°C. Temperature stays constant on the two plateaus (melting, boiling) — all incoming energy there breaks intermolecular bonds rather than raising kinetic energy.

Ví dụ mẫu · The full heating curve

Find the total heat required to convert 1.0 kg of ice at -10°C into steam at 100°C. Use $c_{ice} = 2100\,\text{J}/(\text{kg}\cdot\text{K})$, $L_f = 3.34\times10^5\ \text{J/kg}$, $c_{water} = 4200\,\text{J}/(\text{kg}\cdot\text{K})$, $L_v = 2.26\times10^6\ \text{J/kg}$.

Stage 1 — warm ice from -10°C to 0°C:

$$Q_1 = mc_{ice}\Delta T = (1.0)(2100)(10) = 2.1\times10^4\,\text{J} = 21\,\text{kJ}.$$

Stage 2 — melt the ice at 0°C:

$$Q_2 = mL_f = (1.0)(3.34\times10^5) = 3.34\times10^5\,\text{J} = 334\,\text{kJ}.$$

Stage 3 — warm liquid water from 0°C to 100°C:

$$Q_3 = mc_{water}\Delta T = (1.0)(4200)(100) = 4.2\times10^5\,\text{J} = 420\,\text{kJ}.$$

Stage 4 — boil the water at 100°C:

$$Q_4 = mL_v = (1.0)(2.26\times10^6) = 2.26\times10^6\,\text{J} = 2260\,\text{kJ}.$$

Total:

$$Q_{total} = Q_1 + Q_2 + Q_3 + Q_4 = 21 + 334 + 420 + 2260 = 3035\,\text{kJ} \approx 3.0\times10^6\,\text{J}.$$

Notice boiling alone (Q_4) consumes nearly three-quarters of the total energy — a direct consequence of $L_v \gg L_f$.

Ví dụ mẫu · Removing heat: freezing and further cooling

Find the heat that must be *removed* to freeze 2.0 kg of liquid water at 0°C into ice at 0°C, and then cool that ice further down to -15°C.

$$Q_{freeze} = mL_f = (2.0)(3.34\times10^5) = 6.68\times10^5\,\text{J}.$$
$$Q_{cool} = mc_{ice}\Delta T = (2.0)(2100)(15) = 6.3\times10^4\,\text{J}.$$

$$Q_{total} = Q_{freeze} + Q_{cool} = 6.68 \times 10^5 + 6.3 \times 10^4 \approx \boxed{7.31 \times 10^5 \text{ J removed}}.$$

Every stage of a heating curve works equally well in reverse: the same $Q = mc\Delta T$ and $Q = mL$ relations apply, only with heat leaving the substance instead of entering it.

VN **GIẢI THÍCH TIẾNG VIỆT**

Đường cong đun nóng (heating curve) là bức tranh tổng hợp: đoạn **dốc** dùng $Q = mc\Delta T$ (nhiệt độ đổi), đoạn **nằm ngang** dùng $Q = mL$ (nhiệt độ không đổi, đang chuyển thể). Bài toán nhiều giai đoạn (đá lạnh → hơi nước) phải **tách riêng từng đoạn** rồi cộng lại — không được gộp chung một công thức.

2.1.7 Calorimetry

When two substances at different temperatures are mixed inside an insulated container (a **calorimeter**, ideally exchanging no energy with the outside world), energy conservation demands that the thermal energy lost by the hotter substance exactly equals the thermal energy gained by the colder substance, until both reach a single common final temperature.

$$Q_{lost\ by\ hot} = Q_{gained\ by\ cold} \quad \Longrightarrow \quad m_{hot}c_{hot}(T_{hot} - T_f) = m_{cold}c_{cold}(T_f - T_{cold}),$$

where T_f is the common final (equilibrium) temperature. If a phase change occurs during the mixing, the corresponding mL term must be added to the appropriate side.

Ví dụ mẫu · Calorimetry

A 0.20 kg block of metal at $150°$C is dropped into 0.50 kg of water at $20°$C inside an insulated calorimeter. The mixture reaches a final temperature of $25°$C. Find the specific heat capacity of the metal. ($c_{water} = 4200$ J/(kg · K).)

Heat gained by the water:

$$Q_{water} = m_w c_w (T_f - T_{w,i}) = (0.50)(4200)(25 - 20) = (0.50)(4200)(5) = 1.05 \times 10^4 \text{ J}.$$

Heat lost by the metal equals heat gained by the water:

$$m_m c_m (T_{m,i} - T_f) = Q_{water} \Rightarrow (0.20)c_m(150 - 25) = 1.05 \times 10^4 \Rightarrow (0.20)c_m(125) = 1.05 \times 10^4.$$

$$25\, c_m = 1.05 \times 10^4 \Rightarrow c_m = 420 \text{ J/(kg} \cdot \text{K)}.$$

Ví dụ mẫu · Mixing three substances

0.30 kg of aluminium ($c = 900$ J/(kg · K)) at $90°$C, 0.50 kg of water ($c = 4200$ J/(kg · K)) at $10°$C, and 0.20 kg of copper ($c = 390$ J/(kg · K)) at $60°$C are combined in an insulated calorimeter. Find the final equilibrium temperature.

Using $\sum m_i c_i (T_f - T_i) = 0$ for all three substances simultaneously:

$$(0.30)(900)(T_f - 90) + (0.50)(4200)(T_f - 10) + (0.20)(390)(T_f - 60) = 0.$$

$$270(T_f - 90) + 2100(T_f - 10) + 78(T_f - 60) = 0.$$

$$(270 + 2100 + 78)T_f = (270)(90) + (2100)(10) + (78)(60) = 24300 + 21000 + 4680 = 49980.$$

$$2448\,T_f = 49980 \Rightarrow T_f \approx \boxed{20.4°\text{C}}.$$

The result lies closest to water's initial temperature, since water contributes by far the largest mc product (2100, versus 270 and 78 for the two metals) and therefore dominates the energy balance.

(!) Lỗi thường gặp: The most common calorimetry error is a sign mistake in ΔT: always subtract so that the hot object's ΔT (initial minus final) and the cold object's ΔT (final minus initial) are both *positive* quantities, then set the two heat magnitudes equal. Writing a single signed equation with both objects' ΔT defined the same way (e.g. both as $T_f - T_i$) and setting the sum to zero, $\sum m_i c_i (T_f - T_{i,i}) = 0$, avoids the sign confusion entirely and generalizes cleanly to three or more substances mixed together.

VN **GIẢI THÍCH TIẾNG VIỆT**

Bài toán nhiệt lượng kế luôn dựa trên **bảo toàn năng lượng**: nhiệt do vật nóng toả ra = nhiệt do vật lạnh thu vào (giả sử không mất nhiệt ra ngoài). Cách an toàn nhất để tránh nhầm dấu: viết $\sum m_i c_i (T_f - T_{i,\text{ban đầu}}) = 0$ cho **tất cả** các vật cùng lúc, rồi giải ra T_f — không cần tự phân biệt vật nào "mất", vật nào "nhận" nhiệt.

Problem-solving strategy: calorimetry with a possible phase change

- Identify every substance present, together with its mass, initial temperature, and initial phase.
- Decide whether the final state might involve a phase change for any substance (e.g. ice that fully or partially melts) — if the problem does not state the final temperature, it may be necessary to check whether a proposed final temperature is physically consistent (e.g. that not *more* ice melts than is actually present).
- Write one energy term per substance and per stage, using $Q = mc\Delta T$ within a single phase and $Q = mL$ for any phase change, then apply conservation of energy: total heat lost equals total heat gained.
- Solve algebraically for the unknown quantity, whether that is a final temperature, an unknown mass, or an unknown specific heat or latent heat.

2.1.8 Modes of Heat Transfer: Conduction, Convection, and Radiation

Thermal energy moves from a hotter region to a colder region by three distinct physical mechanisms.

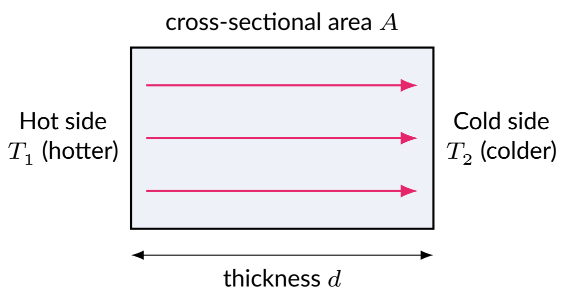
Conduction is the transfer of thermal energy through direct particle-to-particle collisions, with no bulk motion of the material itself. Faster-vibrating (hotter) particles collide with slower neighbours and transfer kinetic energy to them; the disturbance propagates through the material without any particle travelling far from its original position. Conduction is most effective in solids (especially metals, where free electrons carry energy efficiently) and least effective in gases (particles are far apart, so collisions are infrequent).

Convection is the transfer of thermal energy by the bulk movement of a fluid (liquid or gas). When part of a fluid is heated, it expands, becomes less dense, and rises; cooler, denser fluid sinks to take its place, setting up a circulating **convection current** that carries thermal energy along with the moving fluid itself. Convection requires a fluid that is free to move (it does not occur in rigid solids).

Radiation is the transfer of thermal energy by electromagnetic waves, emitted by all objects with temperature above absolute zero. Unlike conduction and convection, radiation requires *no medium at all* — it is the only mechanism by which energy can cross a vacuum, which is how essentially all of the Sun's energy reaches Earth.

> For steady-state conduction through a uniform slab of thickness d and cross-sectional area A, with a temperature difference ΔT across it, the rate of heat conduction (power) is
>
> $$P = \frac{kA\,\Delta T}{d},$$
>
> where k is the **thermal conductivity** of the material ($\mathrm{W\,m^{-1}K^{-1}}$) — a material property describing how readily it conducts heat. P increases with larger k, larger A, or larger ΔT, and decreases with larger thickness d.

In construction and engineering, the insulating property of a layer of material is often quoted as its **thermal resistance** (or R-value), $R \equiv d/k$ — the reciprocal of the quantity appearing in the conduction formula above. A larger R-value means a better insulator, and the R-values of several layers combined in series (e.g. brick, insulation, and interior cladding in a wall) simply add, $R_{total} = R_1 + R_2 + \cdots$, directly analogous to resistors connected in series, met again in Section B.5.4.



Conduction of thermal energy through a slab of thickness d and cross-sectional area A: faster-vibrating particles on the hot side (T_1) collide with and pass energy to slower neighbours, carrying energy down the temperature gradient toward the cold side (T_2).

Mode	Mechanism	Medium needed?	Typical example
Conduction	particle-particle collisions, kinetic energy passed along; no bulk motion	Yes (solid/liquid/gas)	heat travelling along a metal spoon left in a hot pan
Convection	bulk movement of a fluid; warmer, less dense fluid rises, cooler fluid sinks	Yes (fluid only)	convection currents heating a room; boiling water
Radiation	emission/absorption of electromagnetic waves	No — works through vacuum	solar energy travelling to Earth through space

Materials differ enormously in thermal conductivity k: metals, with mobile free electrons available to carry energy, conduct heat far more readily than non-metals, and gases, with widely-spaced, weakly-interacting particles, conduct heat very poorly — which is why trapped, still air layers (in fibreglass insulation, double-glazed windows, or a winter coat) make effective thermal insulators.

Material	Thermal conductivity k ($\mathrm{W\,m^{-1}K^{-1}}$)
Copper	385
Aluminium	205
Glass	0.80
Water (liquid)	0.60
Wood	0.15
Expanded polystyrene (foam)	0.033
Air (still)	0.024

Ví dụ mẫu · Conduction through a window pane

A single-pane glass window has area $1.5\,\mathrm{m^2}$, thickness $4.0\,\mathrm{mm}$, and thermal conductivity $k = 0.80\,\mathrm{W/(m\cdot K)}$. If the inside temperature is $22°\mathrm{C}$ and the outside temperature is $5.0°\mathrm{C}$, find the rate of heat conduction through the pane.

$$P = \frac{kA\Delta T}{d} = \frac{(0.80)(1.5)(22-5.0)}{0.0040} = \frac{(0.80)(1.5)(17)}{0.0040} = \frac{20.4}{0.0040} = 5100\,\mathrm{W}.$$

GIẢI THÍCH TIẾNG VIỆT

VN

Ba cách truyền nhiệt: **dẫn nhiệt** (va chạm giữa các hạt, không cần di chuyển khối chất — xảy ra tốt nhất trong chất rắn/kim loại); **đối lưu** (cả khối chất lỏng/khí di chuyển do chênh lệch mật độ — chỉ xảy ra trong chất lưu); **bức xạ nhiệt** (sóng điện từ, không cần môi trường — truyền được cả trong chân không, ví dụ năng lượng Mặt Trời đến Trái Đất). Công thức $P = kA\Delta T/d$ chỉ áp dụng cho **dẫn nhiệt** qua một tấm phẳng ở trạng thái ổn định.

2.2 B.2 The Greenhouse Effect

2.2.1 Blackbody Radiation and the Stefan–Boltzmann Law

Every object with a temperature above absolute zero emits electromagnetic radiation from its surface, with the total power radiated increasing sharply as temperature rises. An idealized perfect emitter (and perfect absorber) of radiation is called a **blackbody**. The power radiated by a blackbody is described by the **Stefan–Boltzmann law**.

$$P = \sigma A T^4 \quad \text{(perfect blackbody)}, \qquad P = \varepsilon\sigma A T^4 \quad \text{(real object)},$$

where P is the radiated power (W), $\sigma = 5.67\times10^{-8}\ \mathrm{W/(m^2\cdot K^4)}$ is the **Stefan–Boltzmann constant**, A is the surface area ($\mathrm{m^2}$), T is the absolute (kelvin) temperature, and ε is the **emissivity** of the surface ($0 \le \varepsilon \le 1$), which equals 1 for a perfect blackbody and is smaller for surfaces that emit (and absorb) less efficiently than an ideal blackbody. Because power depends on T^4, even a modest change in temperature causes a large change in radiated power.

Emissivity depends strongly on a surface's material and texture: a matte black surface absorbs and emits radiation very efficiently ($\varepsilon \approx 0.95$), while a polished, shiny metal surface reflects most incident radiation and therefore also emits poorly ($\varepsilon \approx 0.05$) — which is why a polished metal thermos flask retains a hot drink's heat far better than a matte black flask would at the same temperature and size.

Ví dụ mẫu · Stefan–Boltzmann law

A small sphere of radius 0.10 m is at a uniform temperature of 500 K. (a) Treating it as a perfect blackbody, find the power it radiates. (b) If its actual emissivity is $\varepsilon = 0.80$, find the actual power radiated.

Surface area of a sphere: $A = 4\pi R^2 = 4\pi(0.10)^2 = 0.1257$ m^2.

(a)

$$P = \sigma A T^4 = (5.67 \times 10^{-8})(0.1257)(500)^4 = (5.67 \times 10^{-8})(0.1257)(6.25 \times 10^{10}) \approx 445 \text{ W}.$$

(b)

$$P_{real} = \varepsilon P = (0.80)(445) \approx 356 \text{ W}.$$

GIẢI THÍCH TIẾNG VIỆT

VN Định luật Stefan–Boltzmann: mọi vật có nhiệt độ trên độ không tuyệt đối đều bức xạ năng lượng, với công suất tỉ lệ với T^4 — tăng nhiệt độ gấp đôi làm công suất bức xạ tăng gấp 16 lần! Vật đen tuyệt đối ($\varepsilon = 1$) là mô hình lý tưởng; vật thật luôn có $\varepsilon < 1$, bức xạ ít hơn vật đen cùng diện tích, cùng nhiệt độ.

Ví dụ mẫu · Comparing radiated power at two temperatures

A metal plate at 600 K and an identical plate (same area, same emissivity) at 300 K both radiate as blackbodies of the same type. Find the ratio of the power radiated by the hotter plate to the power radiated by the cooler plate.

Since $P = \varepsilon\sigma A T^4$ with ε and A identical for both plates, they cancel in the ratio:

$$\frac{P_{600}}{P_{300}} = \left(\frac{T_{600}}{T_{300}}\right)^4 = \left(\frac{600}{300}\right)^4 = 2^4 = \boxed{16}.$$

Doubling the absolute temperature of a radiating object multiplies its radiated power by a factor of 16 — this steep T^4 dependence is why even a modest temperature increase can dramatically increase radiative heat loss (or gain).

2.2.2 Wien's Displacement Law

A blackbody does not radiate all its energy at a single wavelength; it emits a continuous spectrum, with the intensity peaking at a particular wavelength λ_{max} that depends only on the object's absolute temperature. **Wien's displacement law** states that this peak wavelength is inversely proportional to temperature: hotter objects radiate most strongly at *shorter* wavelengths, cooler objects at *longer* wavelengths.

$$\lambda_{max} T = b, \qquad b \approx 2.90 \times 10^{-3} \text{ m} \cdot \text{K} \quad \text{(Wien's constant)}.$$

Ví dụ mẫu · Wien's law for the Sun and Earth

(a) The Sun's surface temperature is about 5800 K. Find its peak emission wavelength. (b) Earth's average surface temperature is about 288 K. Find its peak emission wavelength.

(a)

$$\lambda_{max} = \frac{b}{T} = \frac{2.90 \times 10^{-3}}{5800} = 5.0 \times 10^{-7} \text{ m} = 500 \text{ nm},$$

which lies in the visible spectrum (green-ish light) — consistent with the Sun appearing bright white/yellow to our eyes.

(b)

$$\lambda_{max} = \frac{b}{T} = \frac{2.90 \times 10^{-3}}{288} = 1.0 \times 10^{-5}\,\text{m} = 1.0 \times 10^{4}\,\text{nm} \approx 10\,\mu\text{m},$$

which lies deep in the infrared — far too long a wavelength for human eyes to detect, which is why Earth's own thermal radiation is invisible to us even though it carries real energy.

(!) Lỗi thường gặp: Do not mix up the direction of Wien's law: *higher* temperature means *shorter* peak wavelength (they are inversely proportional, not directly proportional). A very common slip is to say "the Sun is hotter, so it has a longer peak wavelength" — the opposite is true: the Sun (hot, 5800 K) peaks at a short visible wavelength, while Earth (cool, 288 K) peaks at a much longer infrared wavelength. This is exactly why Earth can absorb short-wavelength sunlight but re-emit long-wavelength infrared — the physical basis of the greenhouse effect below.

VN **GIẢI THÍCH TIẾNG VIỆT**

Định luật Wien: $\lambda_{max}T$ = hằng số — vật càng **nóng** thì đỉnh phổ bức xạ càng dịch về bước sóng **ngắn** hơn. Mặt Trời rất nóng (5800 K) nên đỉnh phổ rơi vào vùng **ánh sáng nhìn thấy**; Trái Đất mát hơn nhiều (288 K) nên đỉnh phổ rơi vào vùng **hồng ngoại** (mắt người không thấy được). Sự khác biệt bước sóng này chính là chìa khoá giải thích hiệu ứng nhà kính ở phần B.2.4.

Wien's law has direct practical applications. Astronomers estimate a star's surface temperature purely from the colour (peak wavelength) of the light it emits, without ever needing to visit the star: a blue-white star is hotter than a red star, since blue light has a shorter wavelength than red light. The same principle underlies thermal-imaging cameras, which detect the infrared radiation naturally emitted by objects near room temperature (peaking, by Wien's law, in the infrared, invisible to the naked eye) and convert it into a false-colour visible image that displays temperature differences directly.

2.2.3 Solar Intensity, Albedo, and the Planetary Energy Balance

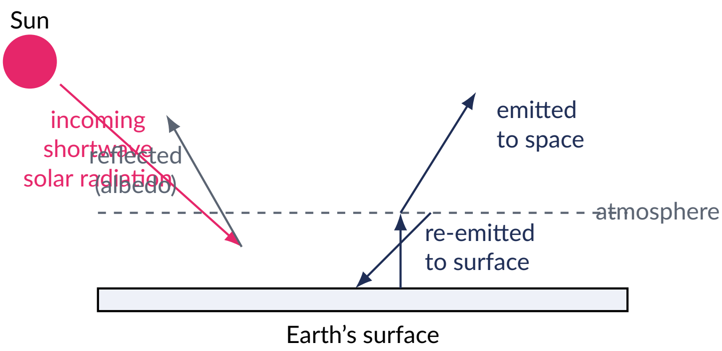
The power per unit area arriving from the Sun at Earth's average distance is called the **solar constant** (or solar intensity), $S \approx 1360\,\text{W/m}^2$. Not all of this incoming radiation is absorbed: a fraction is reflected straight back to space by clouds, ice, and the surface itself. This reflected fraction is the planet's **albedo**, a (a dimensionless number between 0 and 1); Earth's average albedo is about 0.30, meaning about 30% of incoming sunlight is reflected and 70% is absorbed.

To find the planet's equilibrium temperature (assuming, for now, no atmosphere), balance the power *absorbed* against the power *emitted*. Sunlight is essentially parallel rays, so the planet intercepts them over its cross-sectional (disc) area πR^2, where R is the planetary radius; but the planet radiates from its entire spherical surface, area $4\pi R^2$.

Absorbed power: $P_{abs} = (1-a)S\,\pi R^2$. Emitted power (blackbody): $P_{emit} = \sigma\,(4\pi R^2)\,T^4$. At thermal equilibrium, $P_{abs} = P_{emit}$:

$$(1-a)S\pi R^2 = 4\pi R^2\sigma T^4 \quad \Longrightarrow \quad T = \left[\frac{(1-a)S}{4\sigma}\right]^{1/4}.$$

Notice the planetary radius R cancels completely — the no-atmosphere equilibrium temperature depends only on the solar intensity S, the albedo a, and fundamental constants.



Simplified planetary energy-balance diagram. Incoming shortwave solar radiation is partly reflected (albedo) and partly absorbed at the surface. The warmed surface re-radiates longwave infrared; greenhouse gases absorb part of it and re-emit it in all directions, sending some back down and raising the equilibrium surface temperature above the simple no-atmosphere value.

Ví dụ mẫu · Earth's no-atmosphere equilibrium temperature

Using $S = 1360\,\text{W/m}^2$ and albedo $a = 0.30$, estimate Earth's equilibrium temperature if it had no atmosphere.

$$T = \left[\frac{(1-a)S}{4\sigma}\right]^{1/4} = \left[\frac{(0.70)(1360)}{4(5.67\times10^{-8})}\right]^{1/4} = \left[\frac{952}{2.268\times10^{-7}}\right]^{1/4} = \left[4.20\times10^{9}\right]^{1/4}.$$

$$T \approx 255\,\text{K} \approx -18^\circ\text{C}.$$

This is well below Earth's actual observed average surface temperature (about 288 K, 15°C) — the ~ 33 K difference is exactly what the atmosphere's greenhouse effect supplies, as discussed next.

GIẢI THÍCH TIẾNG VIỆT

VN

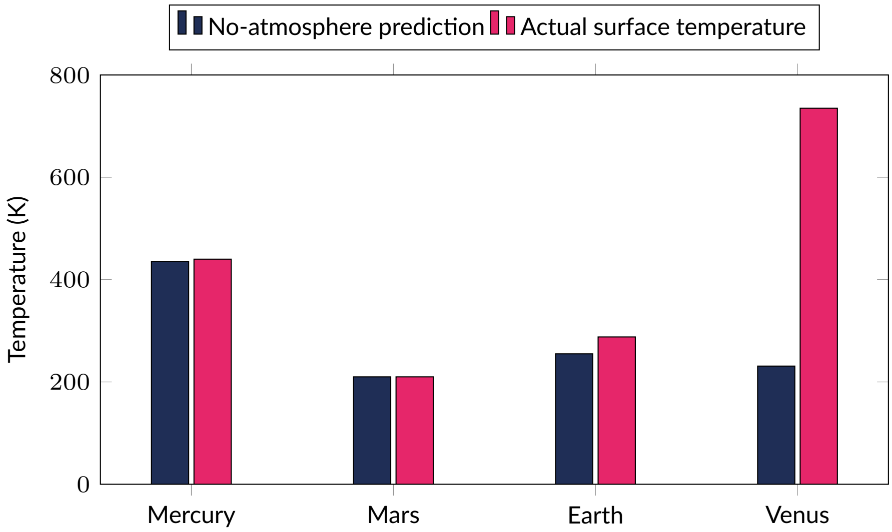
Mô hình cân bằng năng lượng: ở trạng thái cân bằng, **công suất hấp thụ = công suất phát xạ**. Diện tích **hấp thụ** ánh sáng Mặt Trời là tiết diện đĩa πR^2 (vì ánh sáng tới song song), còn diện tích **phát xạ** là toàn bộ mặt cầu $4\pi R^2$ — hệ số 4 trong công thức đến từ tỉ lệ diện tích này. Kết quả tính được cho Trái Đất không khí quyển (≈ 255 K $\approx -18^\circ$C) lạnh hơn nhiều so với thực tế (≈ 288 K $\approx 15^\circ$C) — chênh lệch chính là do hiệu ứng nhà kính.

Ví dụ mẫu · Venus: an extreme real-world greenhouse effect

Venus receives a much higher solar intensity than Earth, $S \approx 2600\,\text{W/m}^2$, but also has a very high albedo, $a \approx 0.75$ (its thick cloud cover reflects most incoming sunlight). Estimate its no-atmosphere equilibrium temperature, and compare it with its actual measured surface temperature of about 735 K.

$$T = \left[\frac{(1-a)S}{4\sigma}\right]^{1/4} = \left[\frac{(0.25)(2600)}{4(5.67\times10^{-8})}\right]^{1/4} = \left[\frac{650}{2.268\times10^{-7}}\right]^{1/4} \approx \boxed{231\,\text{K} \approx -42^\circ\text{C}}.$$

This predicted no-atmosphere temperature is actually *colder* than Earth's (Venus's very high albedo more than compensates for its closeness to the Sun), yet Venus's actual surface temperature is about 735 K (462°C) — roughly 500 K hotter than the simple model predicts. This enormous discrepancy is caused by Venus's atmosphere, which is over 96% CO2 at a surface pressure about 90 times Earth's, producing a runaway greenhouse effect. Venus stands as direct observational evidence that an atmosphere's composition, not merely a planet's distance from its star, ultimately controls its surface temperature.



Predicted no-atmosphere equilibrium temperature versus actual measured surface temperature, for four planets. Mercury and Mars (negligible atmospheres) closely match the simple model; Earth's natural greenhouse effect adds about 33 K; Venus's runaway CO2 greenhouse effect adds roughly 500 K.

2.2.4 The Enhanced Greenhouse Effect and Climate Feedbacks

The simple no-atmosphere model above ignores a crucial detail: Earth's atmosphere is largely transparent to the *incoming* shortwave solar radiation (visible light passes through easily) but strongly *absorbs* some wavelengths of the *outgoing* longwave infrared radiation that the warmed surface emits. Gases capable of this selective absorption — principally carbon dioxide (CO2), methane (CH4), and water vapour (H2O) — are called **greenhouse gases**. Molecules of these gases absorb outgoing infrared photons and then re-emit the energy in *all* directions, including back down toward the surface. This "re-emission downward" delivers extra energy to the surface beyond what direct sunlight alone provides, raising the equilibrium surface temperature above the simple no-atmosphere value calculated previously. This natural warming (the *natural* greenhouse effect) is essential to life on Earth; the **enhanced greenhouse effect** refers to the additional warming caused by human-driven increases in greenhouse gas concentrations, which shifts the whole system toward a new, higher equilibrium temperature.

> Core mechanism: incoming solar radiation is mostly shortwave (peaks in the visible, per Wien's law, since the Sun is hot) and passes through the atmosphere largely unimpeded; outgoing radiation from Earth's surface is mostly longwave infrared (peaks in the IR, since Earth is cool), and greenhouse gases selectively absorb and re-emit much of this outgoing IR, some of it back toward the surface. More greenhouse gas $\Rightarrow$ more absorption/re-emission of outgoing IR $\Rightarrow$ higher equilibrium surface temperature.

A further complication is that the climate system contains **feedback loops** that can amplify (positive feedback) or dampen (negative feedback) an initial warming or cooling. Two important positive feedbacks: the **ice-albedo feedback** — as rising temperatures melt reflective ice/snow, the darker surface exposed underneath (ocean or bare ground) absorbs more sunlight, causing further warming and further melting; and the **water vapour feedback** — warmer air can hold more water vapour (itself a potent greenhouse gas), so initial warming increases atmospheric water vapour, which traps more outgoing IR and causes further warming.

Not all greenhouse gases contribute equally: their warming effect depends on how strongly each gas absorbs infrared radiation and on how long it persists in the atmosphere before being removed by natural processes.

Gas	Principal source	Approx. atmospheric lifetime
Carbon dioxide, CO2	fossil fuel combustion, deforestation	centuries
Methane, CH4	agriculture, livestock, natural gas leaks	~ 12 years
Water vapour, H2O	evaporation, strongly amplified by feedback rather than emitted directly	days

It is worth noting that the everyday glass greenhouse after which the atmospheric effect is named actually traps heat mainly by a *different* mechanism: its glass walls and roof physically block convection, preventing warm air inside from rising away and mixing with cooler air outside, rather than primarily through the selective infrared absorption described above. The atmospheric greenhouse effect is therefore a useful analogy rather than an exact physical parallel to a garden greenhouse.

VN **GIẢI THÍCH TIẾNG VIỆT**

Hiệu ứng nhà kính **tự nhiên** là cần thiết cho sự sống (nếu không có nó, Trái Đất sẽ lạnh hơn thực tế khoảng 33 K). Cơ chế: khí nhà kính (CO2, CH4, hơi nước) hầu như trong suốt với ánh sáng Mặt Trời (bước sóng ngắn) đi vào, nhưng lại hấp thụ mạnh bức xạ hồng ngoại (bước sóng dài) mà Trái Đất phát ra, rồi phát xạ lại một phần xuống mặt đất — làm nhiệt độ cân bằng tăng lên. Hiệu ứng nhà kính **tăng cường** là phần tăng thêm do con người thải thêm khí nhà kính. Các vòng phản hồi (feedback) như băng tan làm giảm suất phản chiếu (albedo), hoặc không khí ấm hơn giữ được nhiều hơi nước hơn, đều khuếch đại xu hướng nóng lên ban đầu.

2.3 B.3 The Gas Laws and Kinetic Theory

2.3.1 Boyle's Law, Charles's Law, and Gay-Lussac's Law

For a fixed mass (fixed number of moles n) of gas, experiments show three simple relationships hold between pressure P, volume V, and absolute temperature T, each valid when the remaining variable is held constant.

Boyle's law: at constant temperature, pressure and volume are inversely proportional. **Charles's law**: at constant pressure, volume is directly proportional to absolute temperature. **Gay-Lussac's law (the pressure law)**: at constant volume, pressure is directly proportional to absolute temperature. All three combine into a single relationship valid for any fixed mass of gas undergoing any change of state.

Each law has a simple qualitative explanation in terms of the kinetic theory of gases, developed fully in Section B.3.3: compressing a gas at constant temperature (Boyle's law) packs the same number of molecules into a smaller volume, so they strike the container walls more frequently, raising pressure; heating a gas at constant pressure (Charles's law) increases the average molecular speed, so the gas must occupy a larger volume for wall collisions to still deliver the same pressure; and heating a gas at constant volume (Gay-Lussac's law) increases molecular speed without allowing extra room, so each wall collision is harder and more frequent, raising pressure directly.

$$P_1V_1 = P_2V_2 \ \ (T, n \text{ const.}), \qquad \frac{V_1}{T_1} = \frac{V_2}{T_2} \ \ (P, n \text{ const.}), \qquad \frac{P_1}{T_1} = \frac{P_2}{T_2} \ \ (V, n \text{ const.}),$$

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \qquad \text{(combined gas law, fixed } n\text{).}$$

Temperature **must** be in kelvin in all of these relationships (they express direct/inverse proportionality with absolute zero as the true origin).

Ví dụ mẫu · The combined gas law

A fixed mass of gas has $P_1 = 1.0 \times 10^5$ Pa, $V_1 = 2.0 \times 10^{-3}$ m^3, $T_1 = 300$ K. It is compressed to $V_2 = 1.2 \times 10^{-3}$ m^3 and heated to $T_2 = 350$ K. Find the new pressure P_2.

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \Rightarrow P_2 = \frac{P_1V_1T_2}{T_1V_2} = \frac{(1.0 \times 10^5)(2.0 \times 10^{-3})(350)}{(300)(1.2 \times 10^{-3})} = \frac{7.0 \times 10^4}{0.36}.$$

$$P_2 \approx 1.94 \times 10^5 \text{ Pa.}$$

(!) Lỗi thường gặp: Always convert Celsius temperatures to kelvin ($T(\text{K}) = T(^\circ\text{C}) + 273$) before using *any* of the gas laws — $P_1V_1 = P_2V_2$-type ratio equations only work with an absolute temperature scale, since they rely on temperature being directly/inversely proportional with a true zero. Plugging Celsius values directly into $V_1/T_1 = V_2/T_2$ (etc.) is one of the most frequent errors in gas-law problems.

VN **GIẢI THÍCH TIẾNG VIỆT**

Ba định luật chất khí: Boyle ($PV =$ const khi T không đổi), Charles ($V/T =$ const khi P không đổi), Gay-Lussac ($P/T =$ const khi V không đổi) — gộp lại thành định luật chất khí tổng quát $P_1V_1/T_1 = P_2V_2/T_2$ cho một lượng khí cố định. **Luôn đổi nhiệt độ sang Kelvin** trước khi thay số — đây là lỗi sai phổ biến nhất trong dạng bài này.

2.3.2 The Ideal Gas Equation

The combined gas law describes how P, V, T change relative to one another for a *fixed* amount of gas; the **ideal gas equation** extends this into a single equation of state valid for *any* amount of an ideal gas, introducing the amount of substance explicitly.

$$PV = nRT \qquad \text{or equivalently} \qquad PV = NkT,$$

where n is the amount of gas in moles, $R = 8.31$ J/(mol · K) is the **universal (molar) gas constant**, N is the number of individual gas particles (molecules), and $k = 1.38 \times 10^{-23}$ J/K is the **Boltzmann constant**. The two forms are linked by

$$N = nN_A, \qquad N_A = 6.02 \times 10^{23} \text{ mol}^{-1} \quad \text{(Avogadro's number),}$$

and indeed $R = N_Ak$ (check: $(6.02 \times 10^{23})(1.38 \times 10^{-23}) \approx 8.31$ J/(mol · K), consistent).

Ví dụ mẫu · The ideal gas equation

A rigid container of volume 0.025 m^3 holds gas at pressure 2.0×10^5 Pa and temperature 300 K. Find (a) the number of moles of gas present, and (b) the number of individual gas molecules.

(a)

$$n = \frac{PV}{RT} = \frac{(2.0 \times 10^5)(0.025)}{(8.31)(300)} = \frac{5000}{2493} \approx 2.0\,\text{mol}.$$

(b)

$$N = nN_A = (2.0)(6.02 \times 10^{23}) \approx 1.2 \times 10^{24}\text{ molecules}.$$

Ví dụ mẫu · Molar mass from density

A gas sample has density $\rho = 1.25\,\text{kg/m}^3$ at pressure 1.01×10^5 Pa and temperature 290 K. Find its molar mass.

Since $n = m/M$, the ideal gas equation $PV = nRT$ becomes $PV = (m/M)RT$, so $PM = (m/V)RT = \rho RT$, giving

$$M = \frac{\rho RT}{P} = \frac{(1.25)(8.31)(290)}{1.01 \times 10^5} = \frac{3012}{1.01 \times 10^5} \approx 0.0298\,\text{kg/mol} = \boxed{29.8\,\text{g/mol}}.$$

This value is very close to the average molar mass of ordinary air (≈ 29 g/mol, reflecting its composition of roughly 78% N2 and 21% O2), so the sample is consistent with being air at everyday conditions.

GIẢI THÍCH TIẾNG VIỆT

VN Phương trình khí lý tưởng $PV = nRT$ (theo số mol) và $PV = NkT$ (theo số hạt) là **hai cách viết của cùng một định luật**, liên hệ qua $N = nN_A$ và $R = N_A k$. Nhớ: n đo bằng mol, N đo bằng số hạt (một số rất lớn, cỡ 10^{23}-10^{24}) — đừng nhầm hai đại lượng này với nhau khi thay vào công thức.

A useful reference point is **standard temperature and pressure (STP)**, conventionally $T = 273$ K (0°C) and $P = 1.00 \times 10^5$ Pa (close to 1 atmosphere). The volume occupied by exactly one mole of *any* ideal gas at STP follows directly from the ideal gas equation:

$$V_m = \frac{nRT}{P} = \frac{(1)(8.31)(273)}{1.00 \times 10^5} = \frac{2268}{1.00 \times 10^5} \approx 2.27 \times 10^{-2}\,\text{m}^3 = 22.7\,\text{L},$$

independent of which gas is being considered — a striking consequence of the ideal gas model, since V_m depends only on the number of particles present, never on their individual size or mass.

2.3.3 Kinetic Theory Assumptions and the Origin of Pressure

The **kinetic theory of gases** models a gas microscopically as a very large number of tiny particles in constant, random motion, and derives the macroscopic gas laws from Newtonian mechanics applied to those particles. The model rests on a set of simplifying assumptions.

Assumptions of the kinetic theory of an ideal gas

- The gas consists of a very large number of identical particles (atoms or molecules), each of negligible volume compared with the volume of the container (treated as point particles).
- Particles move in continuous, random directions with a range of speeds, obeying Newtonian mechanics.
- Collisions between particles, and between particles and the container walls, are perfectly elastic (kinetic energy is conserved in each collision, though it can be redistributed between colliding particles).

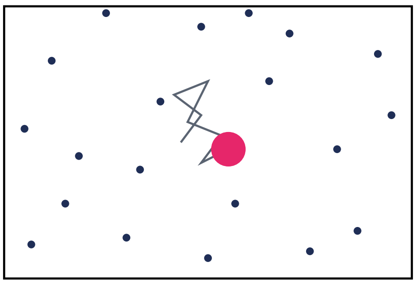
- There are no intermolecular forces between particles except during the brief instant of collision — particles travel in straight lines between collisions.
- The time spent in collisions is negligible compared with the time spent travelling freely between collisions.

On this model, gas **pressure** arises entirely from the enormous number of collisions between gas particles and the walls of their container: each collision delivers a tiny impulse to the wall, and the combined effect of billions of such collisions per second, spread over the wall's area, produces a steady, measurable macroscopic force per unit area.

VN **GIẢI THÍCH TIẾNG VIỆT**

Thuyết động học phân tử dựa trên các giả định: hạt khí là chất điểm, chuyển động hỗn loạn tuân theo cơ học Newton, va chạm đàn hồi hoàn toàn, không có lực tương tác giữa các hạt trừ lúc va chạm, và số hạt rất lớn. **Áp suất khí** sinh ra từ vô số va chạm của các hạt lên thành bình — mỗi va chạm truyền một xung lượng nhỏ, cộng dồn hàng tỉ va chạm mỗi giây tạo thành lực đo được ở cấp độ vĩ mô.

The random collisions at the heart of the kinetic theory also explain two directly observable phenomena: **diffusion**, the gradual mixing of one substance into another (such as the scent of perfume slowly spreading across a still room), and **Brownian motion**, the visible, jittery motion of small pollen grains or smoke particles suspended in a fluid, first explained by Einstein in 1905 as the observable result of asymmetric bombardment by the much smaller, invisibly fast-moving fluid molecules. Both phenomena are direct macroscopic evidence for the reality of the random molecular motion that the kinetic theory assumes.



A schematic of Brownian motion: a comparatively large, visible particle (pink) suspended in a fluid is struck unevenly, from random directions, by vastly more numerous, much smaller, invisibly fast-moving fluid molecules (navy dots), producing the particle's visibly erratic, jittery path (grey).

2.3.4 Average Kinetic Energy and the Microscopic Meaning of Temperature

The kinetic theory provides the microscopic interpretation of temperature promised in Section B.1: temperature is a direct measure of the average translational kinetic energy of the gas particles.

$$\overline{KE} = \tfrac{3}{2}kT,$$

where $\overline{KE}$ is the average translational kinetic energy *per molecule*, k is the Boltzmann constant, and T is the absolute temperature. Average kinetic energy per particle depends *only* on absolute temperature — not on the identity, mass, or pressure of the gas. This is the precise microscopic definition that was described qualitatively in Section B.1.

VN **GIẢI THÍCH TIẾNG VIỆT**

Công thức $\overline{KE} = \frac{3}{2}kT$ là "định nghĩa vi mô" chính xác của nhiệt độ: động năng tịnh tiến trung bình của MỘT hạt khí chỉ phụ thuộc vào nhiệt độ tuyệt đối, không phụ thuộc loại khí, khối lượng phân tử, hay áp suất. Hai chất khí khác nhau ở cùng nhiệt độ có **cùng** động năng trung bình mỗi hạt, dù khối lượng phân tử của chúng khác nhau hoàn toàn.

2.3.5 Root-Mean-Square Speed

Because $\overline{KE} = \frac{1}{2}m\overline{v^2}$ for a particle of mass m (where $\overline{v^2}$ is the mean of the squared speed, averaged over all particles), setting this equal to $\frac{3}{2}kT$ and solving for a characteristic speed gives the **root-mean-square (rms) speed** of the gas particles.

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}kT \quad \Longrightarrow \quad v_{rms} \equiv \sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}},$$

where m is the mass of a single molecule and $M = mN_A$ is the **molar mass** (mass of one mole, in kg mol^{-1} — care is needed to convert from the more commonly tabulated g mol^{-1}). v_{rms} is not the same as the mean (average) speed or the most probable speed of the distribution, though all three are of similar magnitude and all increase with $\sqrt{T}$.

Ví dụ mẫu · Average kinetic energy and rms speed

Find the average translational kinetic energy per molecule and the rms speed of oxygen gas (O2, molar mass $M = 0.032$ kg/mol) at $T = 300$ K.

$$\overline{KE} = \frac{3}{2}kT = \frac{3}{2}(1.38 \times 10^{-23})(300) = 6.21 \times 10^{-21}\text{ J}.$$

$$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(8.31)(300)}{0.032}} = \sqrt{\frac{7479}{0.032}} = \sqrt{2.337 \times 10^5} \approx 483\text{ m/s}.$$

(!) Lỗi thường gặp: The molar mass M in $v_{rms} = \sqrt{3RT/M}$ must be in kg mol^{-1}, not the more familiar g mol^{-1} from chemistry (e.g. O2: 32 g/mol $= 0.032$ kg/mol). Forgetting the factor of 1000 conversion is a very common source of an rms speed answer that is off by a factor of about 32 (i.e. $\sqrt{1000}$).

VN **GIẢI THÍCH TIẾNG VIỆT**

Tốc độ căn quân phương $v_{rms} = \sqrt{3RT/M} = \sqrt{3kT/m}$ suy ra trực tiếp từ $\overline{KE} = \frac{3}{2}kT = \frac{1}{2}m\overline{v^2}$. **Lưu ý đơn vị:** M phải đổi sang kg/mol (chia 1000 từ giá trị g/mol quen thuộc trong Hoá học) trước khi thay vào công thức — đây là lỗi rất hay gặp.

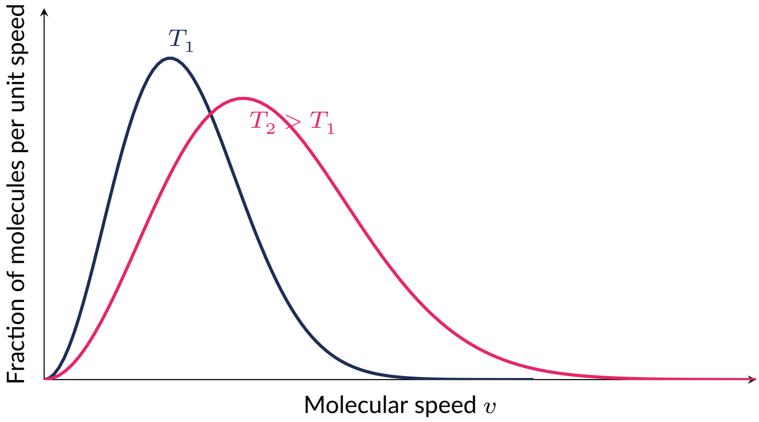
At a fixed temperature, lighter molecules move faster on average than heavier ones, since $v_{rms} \propto 1/\sqrt{M}$.

Gas	Molar mass M (kg/mol)	v_{rms} at 300 K (m/s)
Hydrogen, H2	0.0020	1930
Helium, He	0.0040	1370
Nitrogen, N2	0.028	517
Oxygen, O2	0.032	483
Carbon dioxide, CO2	0.044	412

This large spread in speeds at fixed temperature has real atmospheric consequences: the lightest gases (hydrogen, helium) reach speeds closer to a planet's escape velocity, so over billions of years a planet with weaker gravity or a warmer atmosphere gradually loses its lightest gases to space — which is why Earth's atmosphere today contains negligible free hydrogen and helium despite both being the most abundant elements in the universe.

2.3.6 The Maxwell–Boltzmann Speed Distribution

Not every gas molecule moves at exactly v_{rms}; individual particle speeds are spread over a wide range due to the random nature of collisions. The **Maxwell–Boltzmann distribution** describes the statistical spread of molecular speeds within a gas sample at a given temperature: a skewed curve that rises from zero, reaches a peak (the most probable speed), and falls off gradually toward higher speeds (a long tail, since no upper limit on speed is forbidden, only increasingly improbable).



The Maxwell–Boltzmann speed distribution at two temperatures, $T_2 > T_1$. Raising the temperature shifts the peak toward higher speeds, broadens the curve, and lowers its peak height — the area under each curve (representing the total fraction of molecules, 100%) stays the same.

As temperature increases, the whole curve shifts toward higher speeds (matching the rise in $v_{rms} \propto \sqrt{T}$), and it broadens and flattens, since a wider range of speeds becomes accessible while the total area under the curve (representing all 100% of the molecules) must remain fixed.

GIẢI THÍCH TIẾNG VIỆT

VN Phân bố Maxwell–Boltzmann mô tả sự **phân tán tốc độ** của các phân tử khí — không phải mọi hạt đều chuyển động ở đúng v_{rms}. Khi nhiệt độ tăng: đỉnh phân bố dịch sang tốc độ **cao hơn**, đường cong **loãng và thấp hơn** (vì diện tích dưới đường cong luôn không đổi, bằng 100% số hạt).

2.3.7 Real Gases: Deviations from Ideal Behaviour

The ideal gas equation is an excellent approximation for real gases under most everyday conditions, but it breaks down under **high pressure** and/or **low temperature**. At high pressure, gas particles are forced close together, and the assumption that particle volume is negligible compared with the container volume is no longer accurate — the actual volume available for particles to move in is less than the container volume, so real gases resist compression more than the ideal gas law predicts. At low temperature, particles move more slowly, and the previously-negligible attractive intermolecular forces become significant relative to particles' (now smaller) kinetic energies; these attractive forces pull particles together, reducing the pressure below the ideal-gas prediction, and eventually causing condensation into a liquid — a phase change the ideal gas model, having no interparticle forces at all, cannot describe.

GIẢI THÍCH TIẾNG VIỆT

VN Khí lý tưởng chỉ là mô hình gần đúng: ở **áp suất cao** (hạt bị ép sát, thể tích riêng của hạt không còn bỏ qua được) hoặc **nhiệt độ thấp** (hạt chuyển động chậm, lực hút giữa các phân tử trở nên đáng kể, có thể dẫn đến hoá lỏng), khí thật lệch khỏi phương trình $PV = nRT$.

More sophisticated equations of state, such as the **van der Waals equation**, correct the ideal gas equation by adding a term that accounts for the finite volume of the molecules themselves and a term that accounts for their mutual attraction. Such corrections become significant precisely in the high-pressure, low-temperature regime described above, and vanish (reducing back to $PV = nRT$) in the low-pressure, high-temperature limit, where the kinetic theory's assumptions are best satisfied. Every real gas also has a **critical temperature**, above which no amount of increased pressure can force it to liquefy: beyond this point thermal kinetic energy so thoroughly overwhelms the intermolecular attractive forces that a distinct liquid phase can no longer form, and the liquid and gas phases become indistinguishable.

2.4 B.4 Thermodynamics (AHL)

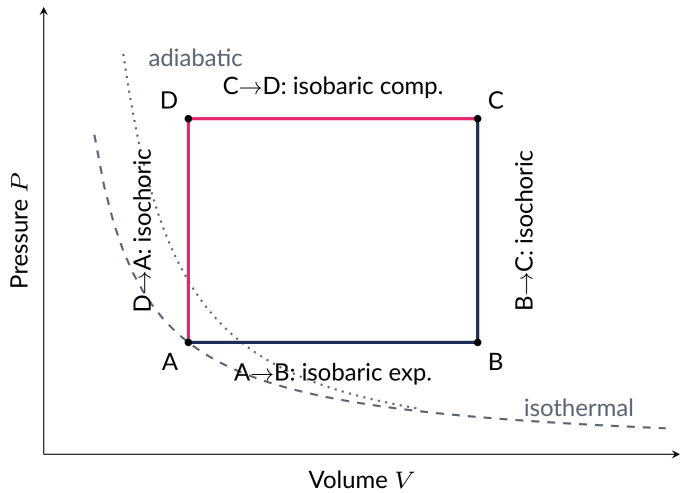
This content is assessed at Additional Higher Level (AHL) only.

2.4.1 State Variables and Thermodynamic Processes

A fixed sample of gas can be fully described, at any instant, by its **state variables**: pressure P, volume V, absolute temperature T, and amount n. A **thermodynamic process** is any change the gas undergoes between two such states, and processes are classified by which variable is held fixed.

Process	Held constant	Key relation
Isobaric	pressure P	$W_{by} = P\Delta V$
Isochoric (isovolumetric)	volume V	$W_{by} = 0,\ \Delta U = Q$
Isothermal	temperature T	$\Delta U = 0$ (ideal gas), $Q = W_{by}$
Adiabatic	no heat exchange	$Q = 0,\ \Delta U = -W_{by}$

On a pressure–volume (P–V) diagram, an **isobaric** process is a horizontal line (constant P) and an **isochoric** process is a vertical line (constant V). An **isothermal** process traces a curve of the form $PV =$ const (a hyperbola, since T and hence PV is fixed); an **adiabatic** process traces a similarly-shaped but *steeper* curve, $PV^\gamma =$ const (with $\gamma > 1$ a constant depending on the gas), because in an adiabatic expansion the gas must also cool (no heat enters to compensate), causing pressure to fall off faster with increasing volume than in the isothermal case.



A rectangular P–V cycle built from isobaric legs (A→B, C→D, horizontal) and isochoric legs (B→C, D→A, vertical). For comparison, a dashed isothermal curve ($PV =$ const, a hyperbola) and a dotted adiabatic curve ($PV^\gamma =$ const, steeper) are sketched at left.

VN **GIẢI THÍCH TIẾNG VIỆT**

Bốn quá trình nhiệt động lực học cơ bản: **đẳng áp** (P không đổi, đường ngang trên đồ thị P–V), **đẳng tích** (V không đổi, đường thẳng đứng), **đẳng nhiệt** (T không đổi, đường hyperbol $PV =$ const), **đoạn nhiệt** (không trao đổi nhiệt, $Q = 0$, đường cong dốc hơn đường đẳng nhiệt vì khí vừa giãn vừa tự lạnh đi).

Real adiabatic processes typically occur when a change happens too quickly for significant heat exchange to take place, rather than because a system is literally, perfectly insulated. The rapid compression of air in a diesel engine cylinder is approximately adiabatic, and the resulting temperature rise (not a spark plug) is what ignites the injected fuel; conversely, the rapid expansion of gas escaping from a punctured tyre or an aerosol can is approximately adiabatic, and the gas noticeably cools as it expands (the can feels cold to the touch), because it does work on the surrounding air while receiving no compensating heat input.

2.4.2 The First Law of Thermodynamics

The **first law of thermodynamics** is a statement of conservation of energy applied to a gas (or any thermodynamic system): the change in a system's internal energy equals the heat added to it plus the work done on it.

$$\Delta U = Q + W,$$

using the convention that Q is the heat added *to* the gas (positive if the gas absorbs heat, negative if it releases heat) and W is the work done *on* the gas (positive if the gas is compressed by an external force, negative if the gas itself does work by expanding). For a process at constant pressure, the work done *by* an expanding or contracting gas is

$$W_{by} = P\Delta V \qquad (W = -W_{by} \text{ in the } \Delta U = Q + W \text{ convention}),$$

and more generally, for *any* process, the work done by a gas equals the area under the curve on a P–V diagram between the initial and final volumes.

Ví dụ mẫu · The first law of thermodynamics

A gas expands at constant pressure $P = 2.0 \times 10^5$ Pa from $V_1 = 1.0 \times 10^{-3}$ m^3 to $V_2 = 2.5 \times 10^{-3}$ m^3, while absorbing $Q = 750$ J of heat. Find the work done by the gas, the work done on the gas, and the change in internal energy.

$$W_{by} = P\Delta V = (2.0 \times 10^5)(2.5 \times 10^{-3} - 1.0 \times 10^{-3}) = (2.0 \times 10^5)(1.5 \times 10^{-3}) = 300 \text{ J}.$$

Work done *on* the gas: $W = -W_{by} = -300$ J (the gas does positive work on its surroundings as it expands).

$$\Delta U = Q + W = 750 + (-300) = 450 \text{ J}.$$

The internal energy of the gas increases by 450 J.

(!) Lỗi thường gặp: The sign convention for W in $\Delta U = Q + W$ trips up many students: here W is work done *on* the gas, so an *expanding* gas (doing positive work *on its surroundings*) corresponds to a *negative* W in this equation, while a *compressed* gas (surroundings doing positive work *on* the gas) corresponds to a *positive* W. Always state explicitly which convention a problem is using, and double-check the sign of W against the physical picture (expansion should reduce ΔU for fixed Q, compression should increase it) before trusting an algebraic answer.

GIẢI THÍCH TIẾNG VIỆT

VN Định luật I nhiệt động lực học $\Delta U = Q + W$ là bảo toàn năng lượng, với quy ước Q = nhiệt **truyền vào** khí, W = công **thực hiện lên** khí. Khí **giãn nở** tự làm công lên môi trường xung quanh ⇒ W (công lên khí) mang dấu **âm**. Khí **bị nén** ⇒ W mang dấu **dương**. Công của khí giãn/nén ở áp suất không đổi: $W_{by} = P\Delta V$; tổng quát hơn, công luôn bằng diện tích dưới đồ thị P–V.

Ví dụ mẫu · An isochoric process

A rigid, sealed container of gas is heated, absorbing $Q = 620\,\text{J}$ of thermal energy. Find the work done on the gas and the resulting change in its internal energy.

Because the container is rigid, $\Delta V = 0$, so $W_{by} = P\Delta V = 0$, and hence $W = 0$ (no work is done in either direction).

$$\Delta U = Q + W = 620 + 0 = \boxed{620\,\text{J}}.$$

For an isochoric process, *all* of the heat added goes directly into increasing internal energy, since no work can be done when the volume cannot change.

2.4.3 The Second Law of Thermodynamics and Entropy

The first law permits energy to flow in either direction between two objects, as long as total energy is conserved — yet in nature, heat is *never* observed to flow spontaneously from a colder object to a hotter one. This directionality is captured by the **second law of thermodynamics**: thermal energy never flows spontaneously from a colder region to a hotter region, and the total **entropy** of an isolated system never decreases over time (it increases for any real, irreversible process, and stays exactly constant only in the idealized limit of a perfectly reversible process).

Entropy S can be thought of as a measure of the disorder, or the number of equivalent microscopic arrangements, available to a system; the second law says that isolated systems evolve, overwhelmingly probably, toward states of higher disorder/entropy, not lower.

For a reversible transfer of heat Q to (or from) a system at constant absolute temperature T, the entropy change is

$$\Delta S = \frac{Q}{T}.$$

$\Delta S > 0$ when heat flows *into* a region (entropy increases); $\Delta S < 0$ when heat flows *out of* a region (entropy decreases there). The *total* entropy change of an isolated system (summed over every part of it) can never be negative.

GIẢI THÍCH TIẾNG VIỆT

VN Định luật II nhiệt động lực học: nhiệt **không tự động** chảy từ vật lạnh sang vật nóng, và entropy (độ hỗn loạn) của một hệ cô lập **không bao giờ giảm**. Công thức $\Delta S = Q/T$ tính độ thay đổi entropy khi có nhiệt lượng Q truyền ở nhiệt độ (gần như) không đổi T — ΔS dương khi hệ **nhận** nhiệt, âm khi hệ **mất** nhiệt.

Ví dụ mẫu · Entropy change during melting

Find the entropy change of $0.50\,\text{kg}$ of ice as it melts completely at $0°\text{C}$ ($273\,\text{K}$). ($L_f = 3.34 \times 10^5$ J/kg.)

$$Q = mL_f = (0.50)(3.34 \times 10^5) = 1.67 \times 10^5\,\text{J}.$$

$$\Delta S = \frac{Q}{T} = \frac{1.67 \times 10^5}{273} \approx \boxed{612\,\text{J/K}}.$$

Entropy increases, consistent with melting being a process that increases molecular disorder: a rigid, ordered crystal lattice becomes a disordered liquid.

2.4.4 Heat Engines and the Carnot Efficiency

A **heat engine** is any device that converts thermal energy into useful mechanical work by operating in a repeating cycle: it absorbs heat Q_{hot} from a hot reservoir, converts part of it into net work W_{net}, and exhausts the remaining energy Q_{cold} to a cold reservoir. Its **efficiency** measures what fraction of the absorbed heat is converted into useful work.

$$\eta = \frac{W_{net}}{Q_{hot}} = 1 - \frac{Q_{cold}}{Q_{hot}}.$$

No real engine can exceed the theoretical maximum set by the **Carnot efficiency**, achieved only by an idealized, perfectly reversible engine operating between two fixed reservoir temperatures T_{hot} and T_{cold} (in kelvin):

$$\eta_{max} = 1 - \frac{T_{cold}}{T_{hot}}.$$

This is an **upper bound**: every real engine, with its unavoidable friction and irreversibilities, has $\eta < \eta_{max}$ for the same two operating temperatures.

Ví dụ mẫu · Carnot efficiency

A heat engine operates between a hot reservoir at 600 K and a cold reservoir at 300 K. (a) Find the maximum possible (Carnot) efficiency. (b) If the engine actually absorbs 1000 J of heat from the hot reservoir per cycle and produces 350 J of net work, find its actual efficiency, and compare it with the Carnot limit.

(a)

$$\eta_{max} = 1 - \frac{T_{cold}}{T_{hot}} = 1 - \frac{300}{600} = 1 - 0.50 = 0.50 = 50\%.$$

(b)

$$\eta_{actual} = \frac{W_{net}}{Q_{hot}} = \frac{350}{1000} = 0.35 = 35\%.$$

Since $35\% < 50\%$, the real engine's efficiency is consistent with the second law: it falls short of, and never exceeds, the Carnot upper bound for the same two reservoir temperatures.

(!) Lỗi thường gặp: Two frequent errors: (1) using Celsius rather than kelvin temperatures in $\eta_{max} = 1 - T_{cold}/T_{hot}$ — since this is a *ratio* of temperatures (not a difference), it is not scale-independent, and only the kelvin scale gives the correct physical result; (2) claiming or calculating a real engine's efficiency *above* the Carnot limit for its operating temperatures — this always signals an arithmetic or conceptual error, since η_{max} is a strict physical upper bound that no real (irreversible) engine can reach or exceed.

Real engines fall well short of the Carnot limit because real processes are never perfectly reversible: friction between moving parts converts some useful energy directly into waste heat, thermal energy leaks out through cylinder walls and pipes before it can be converted to work, and real gases are compressed and expanded too quickly to remain in the idealized equilibrium state that the Carnot analysis assumes. Typical real heat engines — car engines, coal- or gas-fired power-station turbines — achieve only roughly 30% to 45% of their own theoretical Carnot efficiency limit.

GIẢI THÍCH TIẾNG VIỆT

VN Hiệu suất động cơ nhiệt $\eta = W_{net}/Q_{hot} = 1 - Q_{cold}/Q_{hot}$ đo tỉ lệ nhiệt lượng chuyển thành công có ích. Hiệu suất **Carnot** $\eta_{max} = 1 - T_{cold}/T_{hot}$ (nhiệt độ tính bằng Kelvin) là **giới hạn lý thuyết tối đa** mà không động cơ thật nào vượt qua được — luôn kiểm tra $\eta_{actual} \leq \eta_{max}$ để tự kiểm tra đáp số.

2.4.5 Refrigerators and Heat Pumps

A **refrigerator** (or heat pump, or air conditioner) is, conceptually, a heat engine run in reverse: rather than letting heat flow from hot to cold to extract useful work, external work is deliberately *input* to force heat to flow from a cold region to a hot region — exactly the direction that would never happen spontaneously. A refrigerator's compressor does mechanical work W on the working fluid, which absorbs heat Q_{cold} from the cold interior and exhausts a larger amount of heat $Q_{hot} = Q_{cold} + W$ to the warmer room. This does not violate the second law, because the process is not spontaneous: it requires a continuous input of external work, so the refrigerator plus its power supply plus the room it sits in is not simply "heat flowing unaided from cold to hot."

The effectiveness of a refrigerator is measured by its **coefficient of performance**,

$$\text{COP} = \frac{Q_{cold}}{W},$$

the heat removed from the cold interior per unit of work input. Unlike a heat engine's efficiency, COP can exceed 1, since a refrigerator merely relocates existing thermal energy rather than converting it into work. A heat pump used to heat a building is rated similarly, as Q_{hot}/W, and for the same reason a heat pump can deliver more heating energy to a room than the electrical energy it consumes, making it substantially more efficient than direct electrical resistance heating.

Ví dụ mẫu · Heat pump coefficient of performance

A heat pump has a coefficient of performance (for heating) of COP $= 4.0$ and consumes electrical power $W = 500\,\text{W}$. Find the rate at which it delivers heat to the room.

$$\text{COP} = \frac{Q_{hot}}{W} \Rightarrow \frac{Q_{hot}}{t} = \text{COP} \times \frac{W}{t} = (4.0)(500) = \boxed{2000\,\text{W} = 2.0\,\text{kW}}.$$

The heat pump delivers four times as much heating power as the electrical power it consumes — the additional energy is thermal energy extracted from the (colder) outside air or ground, not created from nothing.

GIẢI THÍCH TIẾNG VIỆT

VN Tủ lạnh (hoặc bơm nhiệt) về bản chất là một động cơ nhiệt chạy **ngược**: dùng công từ bên ngoài (máy nén) để ép nhiệt di chuyển từ nơi lạnh sang nơi nóng — điều không thể tự xảy ra. Việc này **không vi phạm** định luật II vì luôn cần công đầu vào liên tục; định luật II chỉ cấm dòng nhiệt tự phát từ lạnh sang nóng khi **không có** công can thiệp từ bên ngoài.

2.5 B.5 Electric Current and Simple Circuits

2.5.1 Electric Current

Electric current I is the rate of flow of electric charge past a point in a circuit.

$$I = \frac{\Delta q}{\Delta t},$$

measured in amperes (A = C/s), where Δq is the charge that flows in time Δt. Conventional current is defined as the direction that positive charge would flow, from higher to lower potential (even in metals, where the actual moving charge carriers are negatively charged electrons drifting the opposite way).

Microscopically, current is carried by a huge number of charge carriers (free electrons in a metal) drifting slowly through the conductor, superimposed on their much faster random thermal motion. If n is the number density of charge carriers (carriers per unit volume), q is the charge per carrier, v_d is their average **drift velocity**, and A is the conductor's cross-sectional area, then the current is

$$I = nqv_dA.$$

Because n is enormous in a typical metal (of order 10^{28}–10^{29} m^{-3}), the drift velocity needed to carry an everyday current is remarkably small — often less than a millimetre per second, even though the electrical signal itself (the establishment of the electric field driving the current) propagates through the circuit at a substantial fraction of the speed of light.

VN **GIẢI THÍCH TIẾNG VIỆT**

Dòng điện $I = \Delta q/\Delta t$ là tốc độ dịch chuyển của điện tích. Ở cấp độ vi mô, $I = nqv_dA$: mật độ hạt tải điện n trong kim loại cực lớn, nên tốc độ trôi (drift velocity) v_d của từng electron thực ra **rất chậm** (thường dưới 1 mm/s) — dù tín hiệu điện truyền đi gần như tức thời trong mạch.

Current that flows steadily in one direction is called **direct current (DC)**, produced by sources such as batteries and cells; current that periodically reverses direction is called **alternating current (AC)**, the form in which electricity is generated and distributed by power grids worldwide. This chapter's circuit analysis — Ohm's law, series/parallel combinations, electrical power — applies directly to DC circuits, and to AC circuits at any single instant, though a full treatment of continuously time-varying AC voltage and current requires additional tools beyond this chapter's scope.

2.5.2 EMF and Potential Difference

The **electromotive force (EMF)** ε of a source (a battery or cell) is the total energy supplied to each unit of charge that passes through the source, converting chemical (or other) energy into electrical energy. The **potential difference (voltage)** V across a component is the energy converted from electrical energy into other forms (heat, light, etc.) per unit charge passing through that component.

$$\varepsilon = \frac{W}{q} \qquad \text{and} \qquad V = \frac{W}{q},$$

where ε (EMF) is the energy supplied per unit charge *by the source*, and V (potential difference) is the energy converted per unit charge *in a component*; both are measured in volts (V = J/C). For an ideal source with no internal resistance, the EMF equals the terminal voltage; for a real source, the two differ, as developed in Section B.5.6.

VN **GIẢI THÍCH TIẾNG VIỆT**

Suất điện động (EMF) ε là năng lượng nguồn cung cấp cho mỗi đơn vị điện tích; **hiệu điện thế (p.d.)** V là năng lượng **chuyển hoá** tại một linh kiện (điện trở, bóng đèn...) cho mỗi đơn vị điện tích. Hai khái niệm cùng đơn vị Volt nhưng khác vai trò: EMF mô tả nguồn **cấp** năng lượng, p.d. mô tả nơi năng lượng **bị tiêu hao/chuyển hoá**.

2.5.3 Ohm's Law and Resistivity

For many conductors, at constant temperature, the current through the conductor is directly proportional to the potential difference across it — this proportionality is **Ohm's law**, and conductors that obey it are called **ohmic**.

$$V = IR,$$

where R is the **resistance** of the conductor (measured in ohms, Ω = V/A), a property that depends on the conductor's material and geometry. For a uniform wire or bar of length L and cross-sectional area A,

$$R = \frac{\rho L}{A},$$

where ρ is the **resistivity** of the material (units $\Omega \cdot$ m) — an intrinsic material property, independent of the object's shape or size, unlike R itself. Resistance increases with length (charge carriers travel further, more collisions) and decreases with cross-sectional area (more parallel "lanes" for current to flow through).

For most metals, resistivity itself increases with temperature (more energetic lattice vibrations scatter drifting electrons more often), which is exactly the microscopic reason a filament lamp's resistance rises as it heats up, discussed quantitatively in Section B.5.8. Some materials, notably semiconductors, show the opposite trend: their resistivity *decreases* sharply with rising temperature, because heating frees additional charge carriers, an effect that outweighs the increased scattering.

Ví dụ mẫu · Resistance from resistivity

A copper wire has length 2.0 m and cross-sectional area 1.0×10^{-6} m^2. Given the resistivity of copper $\rho = 1.68 \times 10^{-8}\ \Omega \cdot$ m, find its resistance.

$$R = \frac{\rho L}{A} = \frac{(1.68 \times 10^{-8})(2.0)}{1.0 \times 10^{-6}} = \frac{3.36 \times 10^{-8}}{1.0 \times 10^{-6}} = 3.4 \times 10^{-2}\ \Omega.$$

Resistivity varies enormously between conductors, semiconductors, and insulators, spanning more than twenty orders of magnitude.

Material	Resistivity ρ ($\Omega \cdot$ m, at room temperature)
Copper	1.68×10^{-8}
Aluminium	2.65×10^{-8}
Nichrome (heating-element alloy)	1.10×10^{-6}
Silicon (pure, a semiconductor)	$\sim 2.3 \times 10^{3}$
Glass (an insulator)	$\sim 10^{10}$–10^{14}

(!) Lỗi thường gặp: Ohm's law, $V = IR$, is not a universal law of nature — it is an experimental description that holds only for *ohmic* conductors held at constant temperature. Many real components (filament lamps, diodes, thermistors) are *non-ohmic*: their resistance $R = V/I$ still makes sense as a ratio at any instant, but it is not a fixed constant, so I is not proportional to V. Always check whether a component is stated (or shown by its I–V graph) to be ohmic before assuming $V \propto I$.

VN GIẢI THÍCH TIẾNG VIỆT

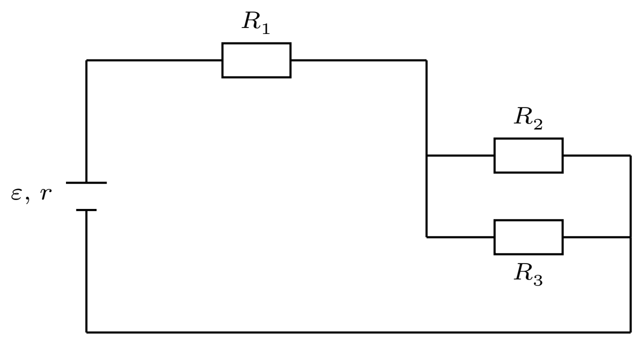
Định luật Ohm $V = IR$ chỉ đúng cho vật liệu **ohmic** ở nhiệt độ không đổi — không phải định luật vật lý phổ quát. Điện trở suất ρ là tính chất **riêng của vật liệu** (không phụ thuộc hình dạng), còn điện trở $R = \rho L/A$ phụ thuộc thêm vào chiều dài và tiết diện của vật thể cụ thể.

2.5.4 Resistors in Series and in Parallel

When resistors are connected **in series** (one after another, forming a single path), the same current flows through each of them (charge has nowhere else to go), and the total voltage across the combination is the sum of the individual voltages. When resistors are connected **in parallel** (side by side, forming separate branches between the same two nodes), the same voltage appears across each of them, and the total current entering the combination is the sum of the individual branch currents.

Both rules follow directly from Ohm's law. In series, $V_{total} = V_1 + V_2 + \cdots = IR_1 + IR_2 + \cdots = I(R_1 + R_2 + \cdots)$, and comparing with $V_{total} = IR_{total}$ gives $R_{total} = R_1 + R_2 + \cdots$. In parallel, $I_{total} = I_1 + I_2 + \cdots = V/R_1 + V/R_2 + \cdots = V(1/R_1 + 1/R_2 + \cdots)$, and comparing with $I_{total} = V/R_{total}$ gives $1/R_{total} = 1/R_1 + 1/R_2 + \cdots$.

Series: $R_{total} = R_1 + R_2 + \cdots$ (same I through each; voltages add).
Parallel: $\dfrac{1}{R_{total}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \cdots$ (same V across each; currents add).
A series combination always has *more* total resistance than its largest individual resistor; a parallel combination always has *less* total resistance than its smallest individual resistor.



A cell of EMF ε and internal resistance r drives current through R_1 in series with the parallel combination of R_2 and R_3. The full current flows through R_1; it splits between R_2 and R_3 in inverse proportion to their resistance, then recombines before returning to the cell.

Ví dụ mẫu · A series–parallel network

In the circuit above, $R_1 = 4.0\,\Omega$, $R_2 = 6.0\,\Omega$, $R_3 = 3.0\,\Omega$, and the supply provides 12 V (assume negligible internal resistance for this calculation). Find the total resistance, the total current supplied, and the current through each of R_2 and R_3.

Parallel combination of R_2, R_3:

$$\frac{1}{R_p} = \frac{1}{6.0} + \frac{1}{3.0} = \frac{1}{6.0} + \frac{2}{6.0} = \frac{3}{6.0} \Rightarrow R_p = 2.0\,\Omega.$$

Total resistance: $R_{total} = R_1 + R_p = 4.0 + 2.0 = 6.0\,\Omega$.
Total current: $I_{total} = V/R_{total} = 12/6.0 = 2.0$ A.
Voltage across the parallel section: $V_p = I_{total}R_p = (2.0)(2.0) = 4.0$ V.

$$I_{R_2} = \frac{V_p}{R_2} = \frac{4.0}{6.0} \approx 0.67\,\text{A}, \qquad I_{R_3} = \frac{V_p}{R_3} = \frac{4.0}{3.0} \approx 1.33\,\text{A}.$$

Check: $I_{R_2} + I_{R_3} = 0.67 + 1.33 = 2.0\,\text{A} = I_{total}$ ✓.

VN **GIẢI THÍCH TIẾNG VIỆT**

Mắc **nối tiếp**: dòng điện qua mọi điện trở **bằng nhau**, hiệu điện thế **cộng lại**, $R_{total} = R_1 + R_2 + \ldots$ luôn **lớn hơn** điện trở thành phần lớn nhất. Mắc **song song**: hiệu điện thế trên mọi nhánh **bằng nhau**, dòng điện **cộng lại**, $1/R_{total} = 1/R_1 + 1/R_2 + \ldots$ nên R_{total} luôn **nhỏ hơn** điện trở thành phần nhỏ nhất. Luôn kiểm tra lại tổng dòng các nhánh song song bằng dòng tổng đi vào.

Problem-solving strategy: series–parallel networks

- Redraw the circuit, working from the branches furthest from the supply inward, replacing each series or parallel group with its single equivalent resistance, until only one resistor remains.
- Apply $V = IR$ to this single equivalent resistance (with the full supply voltage) to find the total current leaving the source.
- Work back outward through the simplified stages: at each series stage the current already found is shared by every resistor in that group; at each parallel stage, the voltage already found is shared by every resistor in that group, and Ohm's law then gives each branch's individual current.

2.5.5 Electrical Power

The rate at which electrical energy is converted to other forms in a circuit component is its **electrical power**, obtained by combining the definitions of current and potential difference with Ohm's law.

$$P = IV = I^2R = \frac{V^2}{R}.$$

All three forms are equivalent for a resistor obeying Ohm's law; which one is most convenient depends on which two of I, V, R are known in a given problem.

Ví dụ mẫu · Electrical power

A lamp is rated "$12\,\text{V}$, $60\,\text{W}$" (its normal operating voltage and power). Find the current it draws and its resistance at this operating point.

$$I = \frac{P}{V} = \frac{60}{12} = 5.0\,\text{A}.$$

$$R = \frac{V}{I} = \frac{12}{5.0} = 2.4\,\Omega \qquad \left(\text{check: } R = \frac{V^2}{P} = \frac{144}{60} = 2.4\,\Omega ✓\right).$$

Excessive current through a component dissipates power as heat at a rate $P = I^2R$; if this heat cannot be removed fast enough, wires and components can overheat and pose a fire hazard. This is why circuits are protected by **fuses** (a thin wire deliberately designed to melt and break the circuit once current exceeds a safe threshold) or **circuit breakers** (an automatic switch that trips under the same condition and can be reset, unlike a fuse).

GIẢI THÍCH TIẾNG VIỆT

VN

Công suất điện $P = IV = I^2R = V^2/R$ là ba cách viết tương đương của cùng một đại lượng — chọn công thức nào tuỳ vào hai đại lượng đã biết trong đề bài. Với thiết bị ghi "định mức" (ví dụ "12 V, 60 W"), có thể suy ra dòng điện và điện trở hoạt động bình thường của nó.

2.5.6 Internal Resistance and Terminal Voltage

A real battery or cell is not a perfect source of EMF: it has some internal resistance r, arising from the resistive materials inside it, which dissipates some energy *within* the source itself whenever current flows. The voltage actually available at the cell's terminals (to drive current through the external circuit) is therefore always somewhat less than the full EMF.

$$V_{terminal} = \varepsilon - Ir,$$

where ε is the EMF, I is the current drawn from the cell, and r is the internal resistance. Equivalently, since the same current flows through the external resistance R and through r in series, $\varepsilon = I(R + r)$. The terminal voltage drops as more current is drawn (larger I means a larger "lost volts" Ir across the internal resistance).

Ví dụ mẫu · Internal resistance

A cell has EMF $\varepsilon = 9.0$ V and internal resistance $r = 0.50\,\Omega$. It is connected to an external resistor $R = 4.0\,\Omega$. Find the current in the circuit and the terminal voltage of the cell.

$$I = \frac{\varepsilon}{R + r} = \frac{9.0}{4.0 + 0.50} = \frac{9.0}{4.5} = 2.0\text{ A}.$$
$$V_{terminal} = \varepsilon - Ir = 9.0 - (2.0)(0.50) = 9.0 - 1.0 = 8.0\text{ V}.$$

Check: $V = IR = (2.0)(4.0) = 8.0$ V — consistent.

(!) Lỗi thường gặp: The EMF of a cell is what a voltmeter reads across its terminals *only when no current is drawn* (an "open-circuit" measurement); as soon as current flows, the terminal voltage is always *less* than the EMF (for a cell delivering current) by exactly Ir. Do not confuse the EMF (a fixed property of the source) with the terminal voltage (which depends on how much current is being drawn at that moment).

GIẢI THÍCH TIẾNG VIỆT

VN

Pin/nguồn thật luôn có **điện trở trong** r, làm hiệu điện thế ở hai cực (terminal voltage) $V = \varepsilon - Ir$ luôn **nhỏ hơn** suất điện động ε khi có dòng điện chạy qua. Dòng càng lớn, "sụt áp trong" Ir càng lớn, V đo được càng thấp. EMF chỉ bằng V đo được khi mạch **hở** (không có dòng điện).

Ví dụ mẫu · Combining internal resistance with a series–parallel network

A cell of EMF 12 V and internal resistance $r = 0.40\,\Omega$ is connected to an external network: $R_1 = 5.0\,\Omega$ in series with the parallel combination of $R_2 = R_3 = 8.0\,\Omega$. Find the current drawn from the cell, the terminal voltage, and the power dissipated in R_1.

Parallel combination: $\frac{1}{R_p} = \frac{1}{8.0} + \frac{1}{8.0} = \frac{2}{8.0} \Rightarrow R_p = 4.0\,\Omega$. External resistance: $R_{ext} =$

$R_1 + R_p = 5.0 + 4.0 = 9.0\,\Omega.$

$$I = \frac{\varepsilon}{R_{ext} + r} = \frac{12}{9.0 + 0.40} = \frac{12}{9.4} \approx 1.28\,\text{A}.$$

$$V_{terminal} = \varepsilon - Ir = 12 - (1.28)(0.40) \approx 11.5\,\text{V}.$$

$$P_{R_1} = I^2 R_1 = (1.28)^2(5.0) \approx \boxed{8.2\,\text{W}}.$$

Problems like this are solved by combining every tool from this section in sequence: reduce the network to a single equivalent resistance, apply the internal-resistance equation to find the current, then apply the power formula to the specific component asked about.

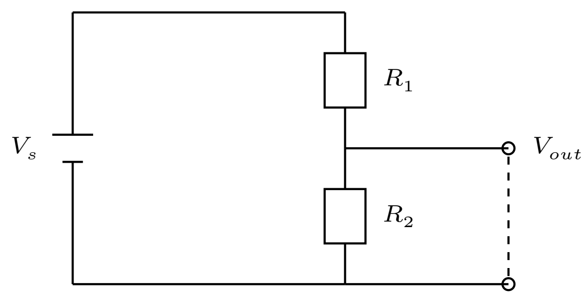
2.5.7 The Potential Divider

A **potential divider** (voltage divider) uses two resistors in series across a supply to produce a smaller, adjustable output voltage, tapped from the junction between the two resistors.

For two resistors R_1, R_2 in series across a supply V_s, with the output taken across R_2:

$$V_{out} = V_s \frac{R_2}{R_1 + R_2}.$$

The same current flows through both resistors (a series circuit), so the supply voltage divides between them in direct proportion to their resistances; changing the ratio $R_1 : R_2$ (e.g. with a variable resistor, or potentiometer) adjusts V_{out} continuously between 0 and V_s.



A potential-divider circuit: resistors R_1, R_2 in series across a supply V_s. The output is tapped between the R_1–R_2 junction and the bottom rail: $V_{out} = V_s\,R_2/(R_1 + R_2)$, adjustable via the resistance ratio.

Ví dụ mẫu · The potential divider

A $6.0\,\text{V}$ supply (negligible internal resistance) is connected across two resistors in series, $R_1 = 2.0\,\text{k}\Omega$ and $R_2 = 4.0\,\text{k}\Omega$, with the output taken across R_2. Find the output voltage.

$$V_{out} = V_s \frac{R_2}{R_1 + R_2} = (6.0)\frac{4.0}{2.0 + 4.0} = (6.0)\frac{4.0}{6.0} = 4.0\,\text{V}.$$

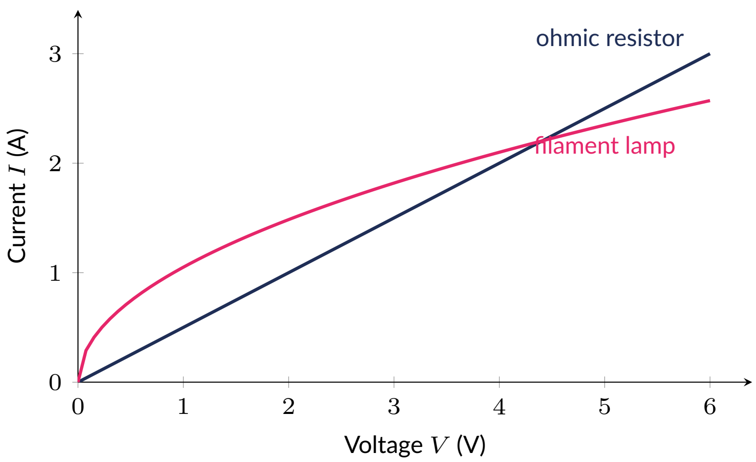
VN **GIẢI THÍCH TIẾNG VIỆT**

Mạch phân áp (potential divider) dùng hai điện trở nối tiếp để "chia" hiệu điện thế nguồn theo đúng tỉ lệ điện trở: $V_{out} = V_s R_2/(R_1 + R_2)$. Đây là cách phổ biến để tạo ra một điện áp đầu ra nhỏ hơn, có thể điều chỉnh được, từ một nguồn cố định.

2.5.8 I–V Characteristic Curves

The relationship between current and voltage for a component is often best displayed as a graph – its I–V **characteristic** – rather than summarized by a single formula, especially for non-ohmic components whose resistance itself changes with operating conditions.

An **ohmic resistor** at constant temperature produces a straight line through the origin on an I–V graph, with slope $1/R$ (constant resistance at every point). A **filament lamp**, by contrast, produces a curve that starts out roughly straight at low voltage/current (where the filament is relatively cool) but bends over and flattens at higher voltage/current, because the increasing current heats the filament, and a hot filament has *higher* resistivity (and hence higher resistance) than a cool one — so equal increases in voltage produce progressively *smaller* increases in current.



I–V characteristics. An ohmic conductor (constant T) gives a straight line through the origin, slope $1/R$. A filament lamp's resistance increases with temperature, so as V rises the curve bends over — its *effective* resistance at any point is V/I (the reciprocal slope of the line from the origin to that point on the curve), not the slope of a tangent line.

Ví dụ mẫu · Reading resistance from an I–V graph

Using the filament-lamp curve above, the graph shows $I \approx 1.5$ A at $V = 2.0$ V, and $I \approx 2.6$ A at $V = 6.0$ V. Find the lamp's resistance at each of these two operating points.

$$R_{(V=2.0)} = \frac{V}{I} = \frac{2.0}{1.5} \approx \boxed{1.3\,\Omega}. \qquad R_{(V=6.0)} = \frac{V}{I} = \frac{6.0}{2.6} \approx \boxed{2.3\,\Omega}.$$

The lamp's resistance nearly doubles between the two operating points, confirming numerically that a filament lamp is strongly non-ohmic: a single resistance value cannot describe it at every voltage.

GIẢI THÍCH TIẾNG VIỆT

VN

Đồ thị I–V: điện trở **thuần ohmic** cho đường **thẳng** qua gốc toạ độ (hệ số góc $= 1/R$ không đổi). Bóng đèn dây tóc cho đường **cong bẹt dần**, vì dòng điện càng lớn làm dây tóc càng nóng, điện trở càng tăng. Với các linh kiện phi tuyến, điện trở tại một điểm bất kỳ trên đồ thị là $R = V/I$ tại điểm đó (không phải độ dốc tiếp tuyến) — phải **đọc từ đồ thị**, không có một công thức duy nhất áp dụng cho mọi điểm.

A third common non-ohmic component is the **semiconductor diode**, which conducts current easily in one direction ("forward bias") but allows almost no current to flow in the reverse direction. Its I–V characteristic shows almost zero current until a small threshold ("turn-on") voltage is reached, after which the current rises very steeply. Because a diode's resistance is different, and strongly voltage-dependent, in each direction, it is a clear example of a component to which $V = IR$ cannot be applied as a fixed proportionality constant, only as an instantaneous ratio at one particular operating point on its graph.

Before turning to practice, the table below collects every core formula of Theme B in one place for quick reference.

B.1 Thermal Energy Transfers	
Specific heat capacity	$Q = mc\Delta T$
Specific latent heat	$Q = mL$ (fusion L_f or vaporization L_v)
Calorimetry (energy conservation)	$Q_{lost\ by\ hot} = Q_{gained\ by\ cold}$
Rate of conduction through a slab	$P = kA\Delta T/d$
B.2 The Greenhouse Effect	
Stefan–Boltzmann law	$P = \sigma AT^4$ (blackbody) or $\varepsilon\sigma AT^4$ (real object)
Wien's displacement law	$\lambda_{max}T = b = 2.90 \times 10^{-3}$ m · K
Planetary equilibrium temperature	$T = [(1-a)S/(4\sigma)]^{1/4}$
B.3 The Gas Laws and Kinetic Theory	
Combined gas law (fixed n)	$P_1V_1/T_1 = P_2V_2/T_2$
Ideal gas equation	$PV = nRT = NkT$, with $N = nN_A$
Average kinetic energy per molecule	$\overline{KE} = \frac{3}{2}kT$
RMS speed	$v_{rms} = \sqrt{3RT/M} = \sqrt{3kT/m}$
B.4 Thermodynamics (AHL)	
First law of thermodynamics	$\Delta U = Q + W$ (with $W_{by} = P\Delta V$ at const. P)
Entropy change (reversible transfer)	$\Delta S = Q/T$
Heat engine efficiency	$\eta = W_{net}/Q_{hot} = 1 - Q_{cold}/Q_{hot}$
Carnot (maximum) efficiency	$\eta_{max} = 1 - T_{cold}/T_{hot}$ (kelvin)
B.5 Electric Current and Simple Circuits	
Current	$I = \Delta q/\Delta t = nqv_dA$
Ohm's law; resistance from resistivity	$V = IR$; $R = \rho L/A$
Series / parallel resistors	$R_{total} = \sum R_i$; $1/R_{total} = \sum 1/R_i$
Electrical power	$P = IV = I^2R = V^2/R$
Terminal voltage of a real cell	$V_{terminal} = \varepsilon - Ir$
Potential divider	$V_{out} = V_sR_2/(R_1+R_2)$

2.6 Problem Bank

Bài 1

Container A holds a small amount of gas at a high temperature; container B holds a much larger amount of the same gas at a slightly lower temperature. Which statement correctly compares their internal energies?

(A) A has greater internal energy, because it has the higher temperature.

(B) B has greater internal energy, because it contains vastly more particles, even though its temperature is slightly lower.

(C) A and B must have equal internal energy, since internal energy is just another name for temperature.

(D) Internal energy cannot be compared without knowing the exact volumes of A and B.

Lời giải chi tiết

Temperature measures *average* kinetic energy per particle, but internal energy is the *total* energy (kinetic + potential) of *all* particles in the system. A system with vastly more particles can have far greater total internal energy even at a somewhat lower temperature, since the huge number of particles outweighs the small difference in average energy per particle. **(B)**.

Bài 2

Which statement about two objects in thermal equilibrium is correct?

(A) They must have the same internal energy.

(B) They must have the same specific heat capacity.

(C) They have the same temperature, and no net heat flows between them.

(D) Thermal equilibrium can only occur between objects of equal mass.

Lời giải chi tiết

Thermal equilibrium is defined by equal temperature and zero *net* heat flow between the two objects – it says nothing about their internal energies, specific heat capacities, or masses, all of which can differ freely. **(C)**.

Bài 3

Find the heat required to raise the temperature of 3.0 kg of aluminium from $15°$C to $75°$C. ($c_{Al} = 900$ J/(kg · K).)

Lời giải chi tiết

$$Q = mc\Delta T = (3.0)(900)(75-15) = (3.0)(900)(60) = \boxed{1.62\times10^5\text{ J} = 162\text{ kJ}}.$$

Bài 4

Find the heat released when 0.30 kg of steam at $100°$C fully condenses to liquid water at $100°$C. ($L_v = 2.26 \times 10^6$ J/kg.)

Lời giải chi tiết

$$Q = mL_v = (0.30)(2.26\times10^6) = \boxed{6.78\times10^5\text{ J} = 678\text{ kJ}}.$$

(Released, not absorbed, since condensation is the reverse of boiling.)

Bài 5

A 0.40 kg block of copper ($c = 390$ J/(kg · K)) at $200°$C is dropped into 0.60 kg of water ($c = 4200$ J/(kg · K)) at $15°$C in an insulated calorimeter. Find the final equilibrium temperature of the mixture.

Lời giải chi tiết

Heat lost by copper = heat gained by water:

$$m_C c_C(200 - T_f) = m_W c_W(T_f - 15) \Rightarrow (0.40)(390)(200 - T_f) = (0.60)(4200)(T_f - 15).$$

$$156(200 - T_f) = 2520(T_f - 15) \Rightarrow 31200 - 156T_f = 2520T_f - 37800.$$

$$69000 = 2676\,T_f \Rightarrow T_f = \boxed{25.8^\circ\text{C}}.$$

Bài 6

A metal rod of cross-sectional area 2.0×10^{-4} m^2, length 0.50 m, and thermal conductivity $k = 200$ W/(m · K) has its ends held at 80°C and 20°C. Find the rate of heat conduction along the rod.

Lời giải chi tiết

$$P = \frac{kA\Delta T}{d} = \frac{(200)(2.0 \times 10^{-4})(80 - 20)}{0.50} = \frac{(200)(2.0 \times 10^{-4})(60)}{0.50} = \frac{2.4}{0.50} = \boxed{4.8\,\text{W}}.$$

Bài 7

A blackbody sphere of radius 0.20 m is at a uniform temperature of 400 K. Find the total power it radiates.

Lời giải chi tiết

$$A = 4\pi R^2 = 4\pi(0.20)^2 = 0.5027\,\text{m}^2.$$

$$P = \sigma A T^4 = (5.67 \times 10^{-8})(0.5027)(400)^4 = (5.67 \times 10^{-8})(0.5027)(2.56 \times 10^{10}).$$

$$P \approx \boxed{730\,\text{W}}.$$

Bài 8

According to Wien's displacement law, as the absolute temperature of a blackbody radiator increases, the wavelength at which it radiates most strongly (λ_{max})...

(A) increases proportionally with T.

(B) decreases, since $\lambda_{max}T$ is a constant.

(C) stays the same regardless of T.

(D) first increases, then decreases.

Lời giải chi tiết

Since $\lambda_{max}T = b$ (a constant), λ_{max} and T are inversely proportional: as T increases, λ_{max} must decrease. **(B)**.

Bài 9

A star has a measured peak emission wavelength of 290 nm. Estimate its surface temperature.

Lời giải chi tiết

$$T = \frac{b}{\lambda_{max}} = \frac{2.90 \times 10^{-3}}{290 \times 10^{-9}} = \frac{2.90 \times 10^{-3}}{2.90 \times 10^{-7}} = \boxed{1.0 \times 10^4\,\text{K}}.$$

Bài 10

A planet receives a solar intensity of $S = 590\,\text{W/m}^2$ and has an albedo of $a = 0.25$. Estimate its no-atmosphere equilibrium temperature.

Lời giải chi tiết

$$T = \left[\frac{(1-a)S}{4\sigma}\right]^{1/4} = \left[\frac{(0.75)(590)}{4(5.67\times10^{-8})}\right]^{1/4} = \left[\frac{442.5}{2.268\times10^{-7}}\right]^{1/4} = \left[1.952\times10^{9}\right]^{1/4}.$$

$$T \approx \boxed{210\,\text{K} \approx -63^\circ\text{C}}.$$

(This is close to the actual average surface temperature of Mars, whose thin atmosphere provides very little additional greenhouse warming.)

Bài 11

Which of the following atmospheric gases is *not* considered a significant contributor to the (natural or enhanced) greenhouse effect?

(A) Carbon dioxide, CO2
(B) Methane, CH4
(C) Water vapour, H2O
(D) Nitrogen, N2

Lời giải chi tiết

Nitrogen makes up about 78% of the atmosphere by volume, but its symmetric diatomic molecule does not absorb infrared radiation the way CO2, CH4, and H2O do, so it plays essentially no role in the greenhouse effect. **(D)**.

Bài 12

A gas at pressure 4.0 atm occupies a volume of 3.0 L. At constant temperature, the pressure is reduced to 1.5 atm. Find the new volume.

Lời giải chi tiết

$$P_1V_1 = P_2V_2 \Rightarrow V_2 = \frac{P_1V_1}{P_2} = \frac{(4.0)(3.0)}{1.5} = \boxed{8.0\,\text{L}}.$$

Bài 13

A gas occupies 2.0 L at 300 K. At constant pressure, find its volume at 450 K.

Lời giải chi tiết

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \Rightarrow V_2 = V_1\frac{T_2}{T_1} = (2.0)\frac{450}{300} = \boxed{3.0\,\text{L}}.$$

Bài 14

A fixed mass of gas has $V_1 = 4.0\times10^{-3}\,\text{m}^3$ at $P_1 = 1.5\times10^{5}$ Pa and $T_1 = 280$ K. It is compressed to $V_2 = 2.0\times10^{-3}\,\text{m}^3$ and heated to $T_2 = 350$ K. Find the new pressure P_2.

Lời giải chi tiết

$$P_2 = \frac{P_1V_1T_2}{T_1V_2} = \frac{(1.5\times10^5)(4.0\times10^{-3})(350)}{(280)(2.0\times10^{-3})} = \frac{2.10\times10^5}{0.56} = \boxed{3.75\times10^5\text{ Pa}}.$$

Bài 15

A tank of volume $0.0500\,\text{m}^3$ holds an ideal gas at pressure 1.2×10^6 Pa and temperature 320 K. Find (a) the number of moles of gas, and (b) the number of gas molecules present.

Lời giải chi tiết

(a)

$$n = \frac{PV}{RT} = \frac{(1.2\times10^6)(0.0500)}{(8.31)(320)} = \frac{60000}{2659.2} \approx \boxed{22.6\,\text{mol}}.$$

(b)

$$N = nN_A = (22.6)(6.02\times10^{23}) \approx \boxed{1.36\times10^{25}\text{ molecules}}.$$

Bài 16

Which of the following is *not* a standard assumption of the kinetic theory of an ideal gas?

(A) The gas consists of a very large number of identical particles in random motion.

(B) Collisions between particles (and with the container walls) are perfectly elastic.

(C) The particles exert significant attractive forces on one another at all separations.

(D) The volume occupied by the particles themselves is negligible compared with the volume of the container.

Lời giải chi tiết

The kinetic theory assumes intermolecular forces are negligible *except* during the brief instant of collision – statement (C), claiming significant attractive forces at all separations, directly contradicts this and is the false assumption. **(C)**.

Bài 17

Find the average translational kinetic energy per molecule and the rms speed of nitrogen gas (N2, molar mass $M = 0.028$ kg/mol) at $T = 350$ K.

Lời giải chi tiết

$$\overline{KE} = \tfrac{3}{2}kT = \tfrac{3}{2}(1.38\times10^{-23})(350) = \boxed{7.25\times10^{-21}\text{ J}}.$$

$$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(8.31)(350)}{0.028}} = \sqrt{\frac{8725.5}{0.028}} = \sqrt{3.116\times10^5} \approx \boxed{558\,\text{m/s}}.$$

Bài 18

At what temperature would the rms speed of hydrogen gas ($M = 0.0020$ kg/mol) equal 1500 m/s?

Lời giải chi tiết

Rearranging $v_{rms}^2 = 3RT/M$:

$$T = \frac{v_{rms}^2 M}{3R} = \frac{(1500)^2(0.0020)}{3(8.31)} = \frac{4500}{24.93} \approx \boxed{180\,\text{K}}.$$

Bài 19

When the temperature of a fixed amount of gas is increased, the Maxwell–Boltzmann speed distribution curve...

(A) shifts toward lower speeds and becomes narrower and taller.

(B) shifts toward higher speeds, becomes broader and flatter, with the total area under the curve unchanged.

(C) keeps exactly the same shape, only its position on the speed axis changes.

(D) develops a second peak at very low speeds.

Lời giải chi tiết

Higher T shifts the distribution toward higher speeds (consistent with $v_{rms} \propto \sqrt{T}$) and spreads it over a wider range, so the peak flattens; since the curve always represents 100% of the molecules, the area underneath stays fixed. **(B)**.

Bài 20

A gas is compressed at constant pressure 3.0×10^5 Pa from 5.0×10^{-3} m^3 to 3.0×10^{-3} m^3, while releasing 200 J of heat to its surroundings. Find the work done *on* the gas and the resulting change in internal energy.

Lời giải chi tiết

$$\Delta V = 3.0 \times 10^{-3} - 5.0 \times 10^{-3} = -2.0 \times 10^{-3}\,\text{m}^3.$$

$$W_{by} = P\Delta V = (3.0 \times 10^5)(-2.0 \times 10^{-3}) = -600\,\text{J} \Rightarrow W_{\text{on gas}} = -W_{by} = \boxed{+600\,\text{J}}.$$

Heat released, so $Q = -200$ J:

$$\Delta U = Q + W = -200 + 600 = \boxed{+400\,\text{J}}\ (\text{internal energy increases}).$$

Bài 21

Which statement correctly describes an adiabatic process undergone by a gas?

(A) The temperature of the gas remains constant throughout.

(B) The pressure of the gas remains constant throughout.

(C) No heat is exchanged between the gas and its surroundings.

(D) The volume of the gas remains constant throughout.

Lời giải chi tiết

By definition, an adiabatic process is one with zero heat exchange, $Q = 0$ – temperature, pressure, and volume are all generally free to change. **(C)**.

Bài 22

A power plant's turbine operates with a hot reservoir at $500\,\text{K}$ and exhausts heat at $320\,\text{K}$. (a) Find the Carnot (maximum) efficiency. (b) If the plant actually runs at 60% of the Carnot efficiency and absorbs 5.0×10^6 J of heat per cycle, find its actual net work output per cycle.

Lời giải chi tiết

(a)

$$\eta_{max} = 1 - \frac{T_{cold}}{T_{hot}} = 1 - \frac{320}{500} = 1 - 0.64 = \boxed{0.36 = 36\%}.$$

(b)

$$\eta_{actual} = (0.60)(0.36) = 0.216 = 21.6\%.$$

$$W = \eta_{actual} Q_{hot} = (0.216)(5.0 \times 10^6) = \boxed{1.08 \times 10^6\,\text{J} = 1.08\,\text{MJ}}.$$

Bài 23

$2000\,\text{J}$ of heat flows irreversibly from a hot reservoir at $400\,\text{K}$ to a cold reservoir at $250\,\text{K}$ (both reservoirs are large enough that their temperatures stay essentially constant). Find the entropy change of each reservoir, and the total entropy change of the universe for this process.

Lời giải chi tiết

Hot reservoir loses heat: $\Delta S_{hot} = -Q/T_{hot} = -2000/400 = -5.0\,\text{J/K}$.
Cold reservoir gains heat: $\Delta S_{cold} = +Q/T_{cold} = +2000/250 = +8.0\,\text{J/K}$.

$$\Delta S_{total} = -5.0 + 8.0 = \boxed{+3.0\,\text{J/K}}.$$

$\Delta S_{total} > 0$, consistent with the second law: entropy increases for this irreversible (spontaneous, hot-to-cold) heat flow.

Bài 24

A refrigerator continuously transfers thermal energy from its cold interior to the warmer room outside. This does not violate the second law of thermodynamics because...

(A) the second law applies only to gases, not to liquids or solids.

(B) heat engines and refrigerators are specifically exempted from the second law.

(C) an external energy input (electrical work on the compressor) drives the process, so heat is not flowing from cold to hot *spontaneously* / unaided.

(D) the refrigerator's interior is not actually colder than the room – only the air near the vents is.

Lời giải chi tiết

The second law forbids only the *spontaneous*, unaided flow of heat from cold to hot. A refrigerator requires a continuous input of external work (via its compressor) to force this heat flow, so it is fully consistent with the second law. **(C)**.

Bài 25

A nichrome wire has resistivity $\rho = 1.10 \times 10^{-6}\ \Omega \cdot$ m, length 1.5 m, and cross-sectional area 5.0×10^{-7} m^2. Find its resistance.

Lời giải chi tiết

$$R = \frac{\rho L}{A} = \frac{(1.10 \times 10^{-6})(1.5)}{5.0 \times 10^{-7}} = \frac{1.65 \times 10^{-6}}{5.0 \times 10^{-7}} = \boxed{3.3\ \Omega}.$$

Bài 26

A circuit consists of $R_1 = 5.0\,\Omega$ in series with the parallel combination of $R_2 = 10\,\Omega$ and $R_3 = 10\,\Omega$, connected to a 15 V supply (negligible internal resistance). Find the total resistance, the total current from the supply, and the current through R_2.

Lời giải chi tiết

$$\frac{1}{R_p} = \frac{1}{10} + \frac{1}{10} = \frac{2}{10} \Rightarrow R_p = 5.0\,\Omega.$$

$$R_{total} = R_1 + R_p = 5.0 + 5.0 = 10\,\Omega.$$

$$I_{total} = \frac{V}{R_{total}} = \frac{15}{10} = \boxed{1.5\,\text{A}}.$$

$$V_p = I_{total} R_p = (1.5)(5.0) = 7.5\,\text{V} \Rightarrow I_{R_2} = \frac{V_p}{R_2} = \frac{7.5}{10} = \boxed{0.75\,\text{A}}.$$

Bài 27

A $100\,\Omega$ resistor carries a current of 0.50 A. Find the power it dissipates.

(A) 12.5 W
(B) 25 W
(C) 50 W
(D) 200 W

Lời giải chi tiết

$$P = I^2 R = (0.50)^2(100) = (0.25)(100) = 25\,\text{W}.$$

(B).

Bài 28

A cell has EMF 12.0 V. When connected to an external resistor $R = 8.0\,\Omega$, the terminal voltage is measured to be 10.4 V. Find the current in the circuit and the internal resistance of the cell.

Lời giải chi tiết

$$I = \frac{V_{terminal}}{R} = \frac{10.4}{8.0} = \boxed{1.3\,\text{A}}.$$

$$V_{terminal} = \varepsilon - Ir \Rightarrow r = \frac{\varepsilon - V_{terminal}}{I} = \frac{12.0 - 10.4}{1.3} = \frac{1.6}{1.3} \approx \boxed{1.2\,\Omega}.$$

Bài 29

A $9.0\,\text{V}$ battery (negligible internal resistance) is connected across a series combination of $R_1 = 1.0\,\text{k}\Omega$ and $R_2 = 2.0\,\text{k}\Omega$. Find the current in the circuit and the output voltage across R_2.

Lời giải chi tiết

$$I = \frac{V}{R_1 + R_2} = \frac{9.0}{1000 + 2000} = \frac{9.0}{3000} = \boxed{3.0 \times 10^{-3}\,\text{A} = 3.0\,\text{mA}}.$$

$$V_{R_2} = IR_2 = (3.0 \times 10^{-3})(2000) = \boxed{6.0\,\text{V}}.$$

Bài 30

As the current through a filament lamp increases from a low starting value, the lamp's resistance...

(A) decreases, because more current means more free charge carriers are available.

(B) stays exactly constant, since resistance is a fixed property of a given lamp.

(C) increases, because the rising current raises the filament's temperature, and resistivity increases with temperature.

(D) drops sharply to zero once the filament becomes white-hot.

Lời giải chi tiết

A filament lamp is non-ohmic: increasing current heats the filament, and for a metal, resistivity (and hence resistance) increases with temperature, so the lamp's resistance rises as the current (and power dissipated) increases – exactly the flattening seen on its I–V graph. **(C)**.

Bài 31

A copper wire of cross-sectional area $2.0 \times 10^{-6}\,\text{m}^2$ carries a current of $3.0\,\text{A}$. The free-electron number density in copper is $n = 8.5 \times 10^{28}\,\text{m}^{-3}$ and the electron charge magnitude is $e = 1.60 \times 10^{-19}\,\text{C}$. Find the drift velocity of the electrons.

Lời giải chi tiết

$$I = nqv_dA \Rightarrow v_d = \frac{I}{neA} = \frac{3.0}{(8.5 \times 10^{28})(1.60 \times 10^{-19})(2.0 \times 10^{-6})}.$$

$$neA = (8.5 \times 10^{28})(1.60 \times 10^{-19}) = 1.36 \times 10^{10}; \qquad (1.36 \times 10^{10})(2.0 \times 10^{-6}) = 2.72 \times 10^4.$$

$$v_d = \frac{3.0}{2.72 \times 10^4} \approx \boxed{1.1 \times 10^{-4}\,\text{m/s}}.$$

This is an extremely slow drift speed – confirming that individual electrons move only a fraction of a millimetre per second even while carrying a substantial current.

Bài 32

A sealed, rigid cylinder of volume $0.0200\,\text{m}^3$ contains $n = 4.0\,\text{mol}$ of an ideal gas at $T = 290\,\text{K}$. The gas is then heated at constant volume until its pressure has doubled. Find (a) the initial pressure, (b) the final temperature, and (c) the average translational kinetic energy per molecule in the final state.

Lời giải chi tiết

(a)

$$P_1 = \frac{nRT_1}{V} = \frac{(4.0)(8.31)(290)}{0.0200} = \frac{9639.6}{0.0200} \approx \boxed{4.82 \times 10^5\ \text{Pa}}.$$

(b) At constant volume (Gay-Lussac's law), P/T is constant, and the pressure doubles:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \Rightarrow T_2 = T_1\frac{P_2}{P_1} = (290)(2) = \boxed{580\ \text{K}}.$$

(c)

$$\overline{KE} = \tfrac{3}{2}kT_2 = \tfrac{3}{2}(1.38 \times 10^{-23})(580) \approx \boxed{1.20 \times 10^{-20}\ \text{J}}.$$

Theme C: Wave Behaviour

Mục tiêu Unit: Chương này trình bày dao động điều hòa đơn giản (SHM) và dao động tắt dần/cưỡng bức; mô hình sóng tổng quát, phương trình sóng, quang phổ điện từ, khúc xạ và phân cực; hiện tượng chồng chập và giao thoa sóng (thí nghiệm hai khe Young, nhiễu xạ, cách tử); sóng dừng và cộng hưởng trên dây đàn hồi và trong cột không khí; và hiệu ứng Doppler đối với âm thanh và ánh sáng. Sau chương này, học sinh cần thiết lập và vận dụng thành thạo các công thức SHM, đọc và phân tích đồ thị sóng, tính toán giao thoa/nhiễu xạ, xác định tần số họa âm của sóng dừng, và áp dụng hiệu ứng Doppler vào các bài toán định lượng.

3.1 Simple Harmonic Motion

Many of the most important periodic phenomena in physics and engineering — vibrating structures, oscillating electrical circuits, atoms bound within molecules, ocean tides driven by resonant coupling to the Moon's gravity, the suspension of a car, the reed of a musical instrument — can be modelled, at least to a good approximation, by the same simple mathematical description developed in this section. Mastering the kinematics and energetics of two canonical examples, the mass-spring system and the simple pendulum, provides a toolkit that generalises directly to every later section of this chapter, because a travelling wave is, at its core, nothing more than a very large number of linked oscillators, each individually performing SHM, passing their motion on to their neighbours with a small time delay.

3.1.1 Defining simple harmonic motion

A great many physical systems — a mass bouncing on a spring, a pendulum swinging through small angles, a guitar string, the current in an oscillating circuit — repeat the same pattern of motion over and over. The simplest and most important of these repeating patterns is **simple harmonic motion** (SHM). SHM is defined by a single condition on the acceleration of the oscillating object.

Khái niệm cốt lõi

A body performs SHM if its acceleration is directly proportional to its displacement from a fixed equilibrium position, and always directed *towards* that equilibrium position (i.e. it always opposes the displacement).

Writing x for the displacement from equilibrium and a for the acceleration, this condition is

$$a = -\omega^2 x,$$

where ω (units rad s^{-1}) is a constant called the **angular frequency** of the oscillation. The minus sign is essential: it guarantees that whenever the object is displaced to positive x, the acceleration pulls it back towards $x = 0$, and vice versa. This is why SHM is also called a **restoring**-force motion.

For a mass m attached to an ideal spring of spring constant k, Hooke's law gives the restoring force $F = -kx$. Applying Newton's second law, $F = ma$, gives

$$ma = -kx \quad \Longrightarrow \quad a = -\frac{k}{m}x.$$

Comparing with $a = -\omega^2 x$ identifies the angular frequency of a mass-spring oscillator as $\omega = \sqrt{k/m}$.

It is worth contrasting SHM with motion that is merely *periodic* without being simple harmonic. A ball bouncing elastically under gravity repeats its motion every bounce, but during each flight its acceleration is the constant $-g$, not proportional to its height above the floor, so the motion is periodic but **not** SHM. By contrast, a mass on a spring, a pendulum swinging through small angles, a vibrating string, and the charge oscillating in an LC circuit are all genuine SHM systems, because in each case a restoring influence grows in direct proportion to the displacement from equilibrium, never in some other way (such as remaining constant, or growing with the square of the displacement).

Defining equation of SHM: $a = -\omega^2 x$.

Mass-spring restoring force: $F = -kx \implies \omega = \sqrt{\dfrac{k}{m}}$.

VN **GIẢI THÍCH TIẾNG VIỆT**

SHM (dao động điều hòa đơn giản) được định nghĩa bằng MỘT điều kiện duy nhất: gia tốc luôn tỉ lệ thuận với độ dời x nhưng ngược chiều (dấu trừ). Nói cách khác, lực luôn "kéo" vật về vị trí cân bằng, và độ lớn của lực kéo càng lớn khi vật càng đi xa vị trí cân bằng. Không phải cứ dao động qua lại là SHM — chỉ khi $a \propto -x$ mới đúng là SHM. Công thức $F = -kx$ (định luật Hooke) là ví dụ kinh điển nhất tạo ra SHM.

3.1.2 The general solution: displacement, velocity, and acceleration

Additional Higher Level (AHL) content: the phase constant ϕ and the full time-dependent solution below.

Solving the differential equation $a = \dfrac{d^2x}{dt^2} = -\omega^2 x$ gives displacement as a function of time. Depending on the initial condition (where the object starts and how it is moving at $t = 0$), the solution takes the form

$$x(t) = x_0 \cos(\omega t + \phi) \qquad \text{or equivalently} \qquad x(t) = x_0 \sin(\omega t + \phi),$$

where x_0 is the **amplitude** (the maximum magnitude of displacement) and ϕ is a **phase constant** fixed by the initial conditions. It is usually most convenient to choose $t = 0$ so that $\phi = 0$: if the object starts at maximum displacement $x = x_0$ with zero velocity, use $x(t) = x_0 \cos(\omega t)$; if it starts at $x = 0$ moving with maximum speed, use $x(t) = x_0 \sin(\omega t)$.

Differentiating $x(t) = x_0 \cos(\omega t + \phi)$ with respect to time gives the velocity,

$$v(t) = \frac{dx}{dt} = -x_0 \omega \sin(\omega t + \phi),$$

and differentiating again gives the acceleration,

$$a(t) = \frac{dv}{dt} = -x_0 \omega^2 \cos(\omega t + \phi) = -\omega^2 x(t),$$

which confirms the original defining relation $a = -\omega^2 x$.

A very useful result eliminates time altogether and relates speed directly to displacement. Using $\sin^2 + \cos^2 = 1$,

$$v^2 = x_0^2 \omega^2 \sin^2(\omega t + \phi) = \omega^2 \left[x_0^2 - x_0^2 \cos^2(\omega t + \phi)\right] = \omega^2 (x_0^2 - x^2),$$

so that

$$v = \pm \omega \sqrt{x_0^2 - x^2}.$$

This shows immediately that speed is greatest when $x = 0$ (passing through equilibrium) and zero when $x = \pm x_0$ (the turning points). Conversely, the magnitude of acceleration $|a| = \omega^2 |x|$ is zero at $x = 0$ and greatest at the extremes $x = \pm x_0$.

Because $a = -\omega^2 x$ is a linear relation, a graph of acceleration a against displacement x for an SHM oscillator is always a straight line through the origin with gradient $-\omega^2$. This is exploited as a standard experimental technique: measuring the acceleration at a series of known displacements and plotting a against x allows ω^2 (and, for a spring, the spring constant $k = m\omega^2$) to be extracted from the gradient of the best-fit line, without ever needing to time a full oscillation directly. The phase constant ϕ, meanwhile, encodes nothing more than the choice of when the stopwatch is started: shifting the origin of time simply shifts ϕ, while the physically meaningful quantities — amplitude, angular frequency, and hence period, frequency, maximum speed, and maximum acceleration — are completely unaffected by that choice.

$$x(t) = x_0 \cos(\omega t + \phi) \qquad v(t) = -x_0\omega \sin(\omega t + \phi) \qquad a(t) = -\omega^2 x(t)$$

$$v = \pm\omega\sqrt{x_0^2 - x^2} \qquad v_{\text{max}} = \omega x_0 \text{ (at } x = 0) \qquad a_{\text{max}} = \omega^2 x_0 \text{ (at } x = \pm x_0)$$

GIẢI THÍCH TIẾNG VIỆT

VN

Công thức $v = \pm\omega\sqrt{x_0^2 - x^2}$ cực kỳ hữu ích vì nó KHÔNG chứa thời gian t — cho phép tìm vận tốc ngay tại một vị trí x bất kỳ mà không cần biết vật đang ở thời điểm nào. Hãy nhớ: tốc độ lớn nhất ($v_{\text{max}} = \omega x_0$) xảy ra khi vật đi NGANG qua vị trí cân bằng ($x = 0$); gia tốc lớn nhất ($a_{\text{max}} = \omega^2 x_0$) xảy ra khi vật ở BIÊN ($x = \pm x_0$), nơi tốc độ bằng không. Hai đại lượng này đạt cực trị tại hai vị trí đối lập nhau.

Ví dụ mẫu · SHM kinematics from displacement

A particle oscillates in SHM with amplitude $x_0 = 0.050$ m and angular frequency $\omega = 4.0$ rad s^{-1}. Find (a) its maximum speed, (b) its maximum acceleration, and (c) its speed when the displacement is $x = 0.030$ m.

(a) $v_{\text{max}} = \omega x_0 = 4.0 \times 0.050 = 0.20$ m s^{-1}.

(b) $a_{\text{max}} = \omega^2 x_0 = (4.0)^2 \times 0.050 = 0.80$ m s^{-2}.

(c) $v = \omega\sqrt{x_0^2 - x^2} = 4.0\sqrt{(0.050)^2 - (0.030)^2} = 4.0\sqrt{0.0025 - 0.0009} = 4.0\sqrt{0.0016} = 4.0(0.040) = 0.16$ m s^{-1}.

Ví dụ mẫu · Finding the phase constant from initial conditions

An oscillator has amplitude $x_0 = 0.12$ m and angular frequency $\omega = 5.0$ rad s^{-1}. At $t = 0$ its displacement is $x = 0.060$ m and it is moving in the positive direction. Using $x(t) = x_0 \cos(\omega t + \phi)$, find the phase constant ϕ.

At $t = 0$: $x_0 \cos\phi = 0.060 \Rightarrow \cos\phi = \dfrac{0.060}{0.12} = 0.500 \Rightarrow \phi = 60^\circ$ or $\phi = -60^\circ$ (i.e. $\pm\frac{\pi}{3}$ rad).

To choose between the two, use the velocity: $v(t) = -x_0\omega\sin(\omega t + \phi)$, so at $t = 0$, $v = -x_0\omega\sin\phi$. Since the particle moves in the *positive* direction, $v > 0$, which requires $\sin\phi < 0$, i.e. ϕ negative.

$$\phi = -60^\circ = -\tfrac{\pi}{3} \text{ rad.}$$

3.1.3 Period and frequency

The **period** T is the time for one complete oscillation, and the **frequency** $f = 1/T$ is the number of oscillations per second. Because $\cos(\omega t + \phi)$ repeats every time ωt increases by 2π,

$$T = \frac{2\pi}{\omega}, \qquad f = \frac{1}{T} = \frac{\omega}{2\pi}.$$

For the **mass-spring system**, substituting $\omega = \sqrt{k/m}$ gives

$$T = 2\pi\sqrt{\frac{m}{k}}.$$

Notice that T does not involve x_0 at all — the period of SHM is completely **independent of amplitude**. A spring oscillating with a large amplitude and one oscillating with a small amplitude, released at the same instant, complete each cycle in exactly the same time (the large-amplitude motion simply moves faster to cover the greater distance).

For a **simple pendulum** of length L swinging through small angles θ, the restoring force along the arc is the tangential component of gravity, $F = -mg\sin\theta$. For small θ (in radians), $\sin\theta \approx \theta = x/L$ where x is the arc-length displacement, so

$$F \approx -mg\frac{x}{L} \quad\Longrightarrow\quad a = -\frac{g}{L}x,$$

which is exactly the SHM form $a = -\omega^2 x$ with $\omega^2 = g/L$. Hence

$$T = 2\pi\sqrt{\frac{L}{g}}.$$

Just as for the spring, this period is independent of amplitude (provided the small-angle approximation holds, typically $\theta \lesssim 10^\circ$) *and* independent of the mass of the bob — only L and g matter.

Squaring the pendulum formula gives $T^2 = \dfrac{4\pi^2}{g}L$, a linear relation between T^2 and L. This underlies a standard laboratory method for determining the local gravitational field strength: measuring the period for several different pendulum lengths and plotting T^2 (vertical axis) against L (horizontal axis) produces a straight line through the origin whose gradient equals $4\pi^2/g$, from which g can be found without needing to measure g by any other means. The same graphical strategy — plot the square of the period against whatever length or mass parameter appears under the square root — works equally well for the mass-spring system, where a graph of T^2 against m has gradient $4\pi^2/k$.

Angular frequency → period: $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{1}{\omega^2}}$.

Mass-spring system: $T = 2\pi\sqrt{\dfrac{m}{k}}$ (independent of amplitude).

Simple pendulum (small angle): $T = 2\pi\sqrt{\dfrac{L}{g}}$ (independent of amplitude *and* mass).

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Đây là hai điểm học sinh IB hay bị mất điểm: (1) chu kỳ SHM KHÔNG phụ thuộc biên độ — dao động biên độ lớn hay nhỏ đều mất cùng một khoảng thời gian cho một chu kỳ; (2) với con lắc đơn, chu kỳ KHÔNG phụ thuộc khối lượng vật nặng — chỉ phụ thuộc chiều dài dây L và gia tốc trọng trường g. Công thức con lắc đơn CHỈ đúng khi góc lệch nhỏ (xấp xỉ $\sin\theta \approx \theta$); với góc lệch lớn, chu kỳ thực tế sẽ lớn hơn giá trị tính từ công thức này.

Ví dụ mẫu · Period of a mass-spring system and a pendulum

(a) A 0.50 kg mass is attached to a spring of constant $k = 200$ N m^{-1}. Find the period and frequency of its oscillation.

(b) A simple pendulum has length $L = 1.00$ m. Find its period on Earth ($g = 9.81$ m s^{-2}).

(a) $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.50}{200}} = 2\pi\sqrt{0.0025} = 2\pi(0.0500) = 0.314$ s.

$$f = \frac{1}{T} = \frac{1}{0.314} = 3.18 \text{ Hz}.$$

(b) $T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{1.00}{9.81}} = 2\pi\sqrt{0.1019} = 2\pi(0.3193) = 2.01$ s.

(!) Lỗi thường gặp: Nhiều học sinh nghĩ rằng biên độ lớn hơn thì "vật đi xa hơn nên mất nhiều thời gian hơn cho một chu kỳ" — điều này SAI đối với SHM lý tưởng. Biên độ lớn hơn cũng đồng nghĩa với tốc độ trung bình lớn hơn, và hai hiệu ứng triệt tiêu lẫn nhau, giữ nguyên T. Đừng nhầm lẫn: biên độ ảnh hưởng đến v_{max} và a_{max}, nhưng KHÔNG ảnh hưởng đến T hay f.

3.1.4 Energy in simple harmonic motion

The quantitative (graphical and algebraic) treatment below is Additional Higher Level (AHL) content; at Standard Level only the qualitative interchange between kinetic and potential energy is required.

As an oscillator moves, energy continuously converts between kinetic energy (KE) and potential energy (PE — elastic potential energy for a spring, gravitational potential energy for a pendulum), while the **total mechanical energy** remains constant (assuming no damping/friction).

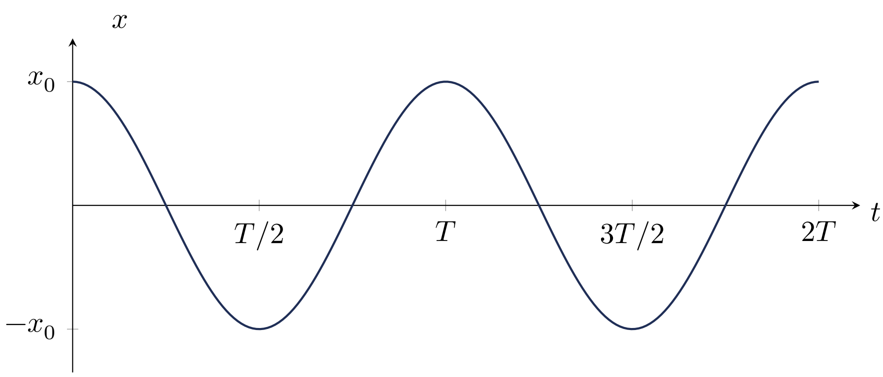
At the extremes $x = \pm x_0$, the object is momentarily at rest, so all the energy is potential: $E = \frac{1}{2}kx_0^2$. At the equilibrium position $x = 0$, all the energy is kinetic and the speed is maximum: $E = \frac{1}{2}mv_{\text{max}}^2$. Since both expressions equal the same constant total energy,

$$E = \frac{1}{2}kx_0^2 = \frac{1}{2}mv_{\text{max}}^2.$$

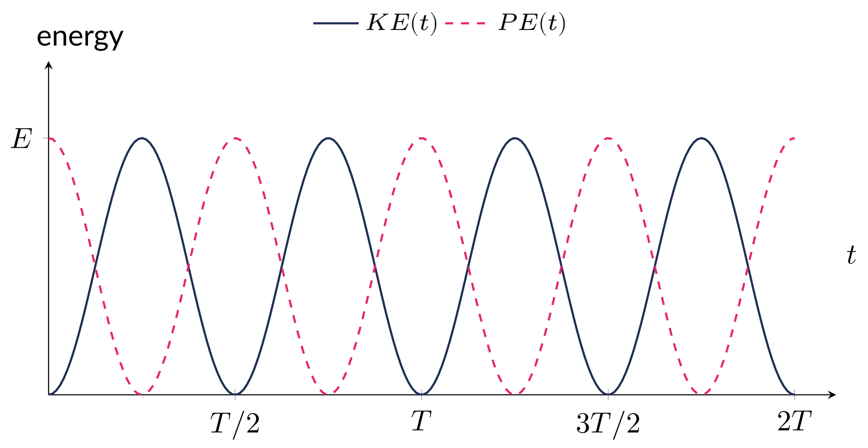
At any intermediate displacement x (with instantaneous speed v), the energy splits between the two forms:

$$KE = \frac{1}{2}mv^2, \qquad PE = \frac{1}{2}kx^2, \qquad KE + PE = E \text{ (constant, at all times)}.$$

A graph of PE against displacement x is an upward-opening parabola ($PE = \frac{1}{2}kx^2$), zero at equilibrium and maximum at the turning points; a graph of KE against x is the complementary downward-opening parabola ($KE = E - \frac{1}{2}kx^2$), maximum at equilibrium and zero at the turning points. At every value of x the two curves add to the same constant total E, so graphically one is simply the vertical mirror image of the other, reflected about the horizontal line $y = E$ that represents the total energy.



Displacement-time graph $x(t) = x_0 \cos(\omega t)$, for an oscillator released from $x = x_0$ at $t = 0$. The pattern repeats every period T.



KE and PE oscillate out of phase and each complete *two* full cycles per period T (since both depend on $\sin^2$ or $\cos^2$); their sum $KE + PE = E$ is constant at every instant.

$E = \frac{1}{2}kx_0^2 = \frac{1}{2}mv_{\text{max}}^2$ (total energy, constant) $KE = \frac{1}{2}mv^2$ $PE = \frac{1}{2}kx^2$ $KE + PE = E$.

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Điểm mấu chốt: tổng năng lượng E không đổi trong suốt quá trình dao động (bỏ qua ma sát) — nó chỉ liên tục "chuyển hóa qua lại" giữa động năng và thế năng. Tại vị trí biên ($x = \pm x_0$): toàn bộ là thế năng, động năng bằng 0. Tại vị trí cân bằng ($x = 0$): toàn bộ là động năng, thế năng bằng 0. Đồ thị KE và PE theo thời gian dao động với tần số GẤP ĐÔI tần số dao động của $x(t)$ — đây là chi tiết hay bị hỏi trong đề trắc nghiệm.

Ví dụ mẫu · Energy conservation in SHM

A 0.20 kg mass oscillates on a spring with $k = 50$ N m^{-1} and amplitude $x_0 = 0.10$ m. Find (a) the total energy, (b) the speed at $x = 0.050$ m, and (c) the maximum speed.

(a) $E = \frac{1}{2}kx_0^2 = \frac{1}{2}(50)(0.10)^2 = \frac{1}{2}(50)(0.010) = 0.25$ J.

(b) $PE = \frac{1}{2}kx^2 = \frac{1}{2}(50)(0.050)^2 = \frac{1}{2}(50)(0.0025) = 0.0625$ J.

$$KE = E - PE = 0.25 - 0.0625 = 0.1875 \text{ J} \implies v = \sqrt{\frac{2\,KE}{m}} = \sqrt{\frac{2(0.1875)}{0.20}} = \sqrt{1.875} = 1.37 \text{ m s}^{-1}.$$

(c) $v_{\text{max}} = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(0.25)}{0.20}} = \sqrt{2.5} = 1.58$ m s^{-1}.

3.1.5 Damped oscillations

Real oscillators always lose energy to resistive forces (friction, air resistance, internal viscous forces), so in practice the amplitude gradually decreases. This is called **damping**. The amount of resistance present determines the qualitative behaviour of the system, and three regimes are recognised.

Damping regime	Behaviour	Typical example
Light (under-) damping	Oscillates with a slowly, exponentially decreasing amplitude; many cycles are completed before the motion dies out.	A pendulum or tuning fork swinging freely in air.
Critical damping	Returns to equilibrium in the **shortest possible time** *without* overshooting or oscillating at all.	Car suspension / shock absorbers; analogue (moving-coil) meter needles.
Heavy (over-) damping	Returns to equilibrium *without* oscillating, but more **slowly** than the critically damped case.	A door-closing mechanism submerged in a very viscous fluid.

On a displacement-time graph, light damping still looks broadly sinusoidal, but each successive peak is smaller than the last: the peaks trace out a smoothly decaying envelope (often very close to exponential decay) rather than staying at constant amplitude, while the time between successive peaks remains very close to (though not identically equal to) the undamped period. Critical and heavy damping, in contrast, produce no peaks at all after release — the displacement-time graph simply approaches zero smoothly, faster in the critically damped case than in the heavily damped case, and in neither case does the system ever cross back through equilibrium more than once.

VN GIẢI THÍCH TIẾNG VIỆT

Ba mức độ tắt dần cần phân biệt rõ: (1) **tắt dần nhẹ** — vẫn dao động qua lại nhiều lần nhưng biên độ giảm dần; (2) **tắt dần tới hạn** — trở về vị trí cân bằng NHANH NHẤT mà KHÔNG dao động (không vượt qua vị trí cân bằng); (3) **tắt dần mạnh (quá tới hạn)** — cũng không dao động, nhưng trở về cân bằng CHẬM hơn tắt dần tới hạn. Mẹo nhớ: giảm xóc ô tô được thiết kế ở chế độ tới hạn — vừa không "nảy" lên xuống, vừa ổn định nhanh nhất.

3.1.6 Forced oscillations and resonance

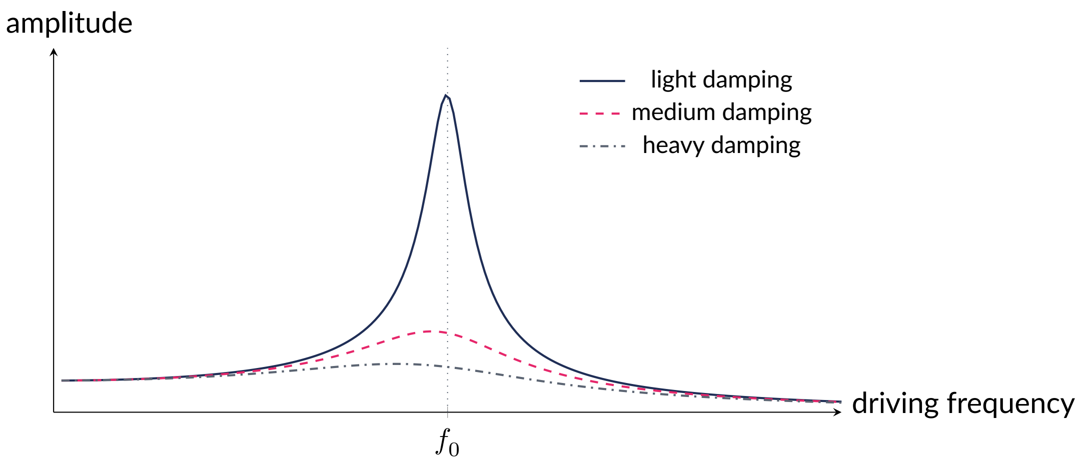
Every oscillating system has its own **natural frequency** f_0 — the frequency at which it oscillates freely once disturbed. If instead an external periodic force continually drives the system at some frequency f_{driver}, the system undergoes **forced oscillation** at the driving frequency.

When the driving frequency is close to the natural frequency, $f_{\text{driver}} \approx f_0$, the amplitude of the forced oscillation becomes very large. This phenomenon is **resonance**. Physically, at resonance the driving force is doing work on the system in step with its natural motion on every cycle, so energy is transferred into the oscillation efficiently rather than being partly "wasted" fighting against the system's own timing.

Everyday and technological examples of resonance include:

- A child on a swing goes highest when pushed at exactly the swing's own natural frequency.
- Bridges and tall buildings can be driven into large, potentially destructive oscillations by wind gusts or earthquake vibrations whose frequency content overlaps the structure's natural frequency (famously the 1940 Tacoma Narrows Bridge collapse).
- A radio tuning circuit is an electrical oscillator whose natural (resonant) frequency is adjusted (e.g. via a variable capacitor) to match the frequency of the desired broadcast station, selectively amplifying that one signal.

Damping reduces the sharpness and height of the resonance peak: a heavily damped system shows only a modest amplitude increase near f_0, spread over a wider range of driving frequencies, while a lightly damped system shows a very tall, narrow resonance peak. A graph of steady-state oscillation amplitude against driving frequency — a **resonance curve** — therefore always peaks at (or extremely close to) the natural frequency f_0; the location of that peak is fixed by the system's own physical properties (mass and stiffness for a mechanical oscillator, inductance and capacitance for an electrical one), while its height and sharpness are controlled entirely by how much damping is present.



Resonance curves for the same oscillator under three different amounts of damping. All three peak at the same natural frequency f_0; lighter damping gives a taller, narrower peak, while heavier damping gives a lower, broader one.

GIẢI THÍCH TIẾNG VIỆT

VN Cộng hưởng (resonance) xảy ra khi tần số của lực cưỡng bức (driving frequency) gần bằng tần số dao động riêng (natural frequency) của hệ — lúc đó biên độ dao động tăng vọt. Đây là lý do vì sao đẩy xích đu đúng nhịp thì xích đu bay rất cao, nhưng đẩy sai nhịp thì gần như không tác dụng. Giảm chấn (damping) càng lớn thì đỉnh cộng hưởng càng thấp và càng "tù" (trải rộng ra nhiều tần số hơn) thay vì nhọn và cao.

(!) Lỗi thường gặp: Lỗi thường gặp: nghĩ rằng cộng hưởng chỉ xảy ra khi tần số cưỡng bức bằng CHÍNH XÁC tần số riêng. Trong thực tế (có giảm chấn), biên độ đạt cực đại tại một tần số *gần* f_0 và vẫn lớn trong một dải tần số hẹp xung quanh f_0 — không phải một điểm duy nhất. Cũng đừng nhầm "tần số riêng" (natural frequency, tính chất nội tại của hệ) với "tần số cưỡng bức" (driving frequency, do ngoại lực áp đặt).

3.2 The Wave Model

Having established how a single point oscillates in SHM, this section turns to what happens when that oscillation is passed from one point of a medium to its immediate neighbour, generating a travelling disturbance — a wave. A common handful of quantities (amplitude, frequency, wavelength, speed, intensity) describe every kind of wave, whether it is a ripple on a pond, a sound wave in air, or an electromagnetic wave crossing empty space, and the relationships between them recur throughout the remainder of this chapter, from interference to the Doppler effect.

3.2.1 Transverse and longitudinal waves

A **wave** is a transfer of energy (and information) through a medium (or through vacuum, for electromagnetic waves) without a net transfer of matter. Each particle of the medium oscillates about a fixed equilibrium position as the wave passes; it is the pattern of oscillation, not the particles themselves, that travels.

Khái niệm cốt lõi

In a **transverse wave**, the oscillation of the medium is perpendicular to the direction the wave travels (e.g. waves on a string, water surface waves, all electromagnetic waves including light). In a **longitudinal wave**, the oscillation is parallel to the direction of travel, producing alternating regions of compression and rarefaction (e.g. sound waves in air, and the P-waves generated by earthquakes).

A **wavefront** is a surface (or, in two dimensions, a line/curve) joining all points of a wave that are in the same phase — for example, all the points currently at a crest. A **ray** is a line drawn perpendicular to the wavefronts, showing the local direction of energy travel. Far from a point source, wavefronts become nearly flat (**plane waves**) and the rays become parallel straight lines. Close to a point source, wavefronts are (segments of) circles in two dimensions or spheres in three dimensions, and the rays point radially outward from the source; this distinction matters later in the chapter, since the inverse-square law for intensity (Section C.2.4) applies specifically to spherical wavefronts spreading from a point source.

A stretched slinky spring can demonstrate both wave types with the same apparatus: shaking one end from side to side sends a transverse wave rippling along it, while sharply pushing and pulling one end along its own length sends a longitudinal wave of compressions and rarefactions travelling down the coils — a useful reminder that the transverse/longitudinal classification describes only the relationship between the oscillation direction and the propagation direction, not some property of the medium itself.

VN **GIẢI THÍCH TIẾNG VIỆT**

Sóng ngang (transverse): phương dao động VUÔNG GÓC với phương truyền sóng — ví dụ ánh sáng, sóng trên dây, sóng mặt nước. Sóng dọc (longitudinal): phương dao động TRÙNG với phương truyền sóng, tạo ra các vùng nén (compression) và giãn (rarefaction) xen kẽ — ví dụ điển hình nhất là âm thanh. Mẹo phân biệt nhanh: âm thanh KHÔNG truyền được trong chân không (cần môi trường vật chất để "nén-giãn"), còn ánh sáng thì truyền được trong chân không vì là sóng điện từ ngang.

3.2.2 The wave equation and wave graphs

For any periodic wave, the speed v, frequency f, and wavelength λ are related by the fundamental **wave equation**:

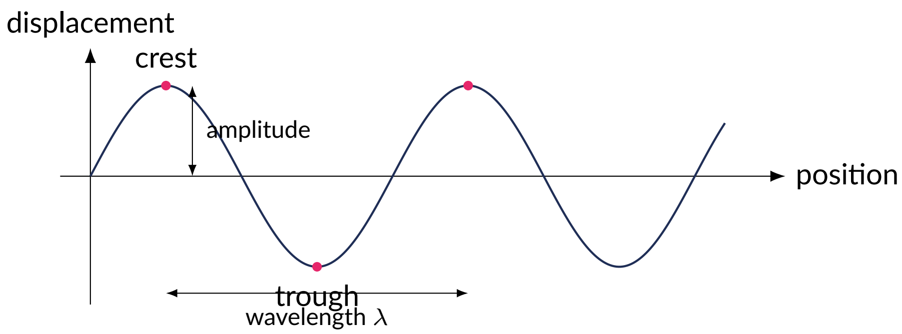
$$v = f\lambda.$$

This simply expresses that in one period $T = 1/f$, the wave pattern advances forward by exactly one wavelength λ, so $v = \lambda/T = f\lambda$.

Two different kinds of graph are used to describe a wave, and it is essential not to confuse them:

- A **displacement-position** graph is a "snapshot" of the whole wave at one instant in time; the horizontal axis is position, and the distance between two successive crests (or any two points a full cycle apart) is the **wavelength** λ.
- A **displacement-time** graph tracks a single point of the medium as time passes; the horizontal axis is time, and the wave's cosine/sine shape has exactly the same form as the SHM displacement-time graph of Section C.1 (each particle of the medium itself performs SHM as the wave passes through it) — the time for one repeat is the **period** T.

The frequency of a wave is fixed by whatever is generating it (the source) and does not change as the wave crosses into a new medium. The wave equation $v = f\lambda$ then shows that if the speed v changes on entering a new medium — which it generally does, since wave speed depends on the mechanical or electromagnetic properties of the medium the wave is travelling through — the wavelength λ must change in direct proportion while f stays fixed. This principle is used again later in the chapter, for refraction (Section C.2.7) and for sound crossing from air into water.



Displacement-position snapshot of a transverse wave: amplitude is the maximum displacement from equilibrium; wavelength is the distance between successive crests.

Wave equation: $v = f\lambda$. Displacement-position graph → shows λ. Displacement-time graph → shows T.

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Công thức $v = f\lambda$ là công thức nền tảng nhất của chương sóng — hầu như bài toán nào cũng dùng đến. Lưu ý phân biệt hai loại đồ thị: đồ thị li độ theo VỊ TRÍ (một khoảnh khắc, cho biết bước sóng λ) và đồ thị li độ theo THỜI GIAN (tại một điểm cố định, cho biết chu kỳ T) — nhìn qua có thể giống hệt nhau về hình dạng (đều là hàm sin/cos) nhưng trục hoành mang ý nghĩa hoàn toàn khác nhau.

Ví dụ mẫu · Applying $v = f\lambda$

A sound wave in air has frequency 256 Hz and travels at 340 m s^{-1}. Find (a) its wavelength in air, and (b) its wavelength after it enters water, where the speed of sound is 1480 m s^{-1} (frequency is unchanged on crossing into a new medium).

(a) $\lambda_{\text{air}} = \dfrac{v}{f} = \dfrac{340}{256} = 1.33$ m.

(b) $\lambda_{\text{water}} = \dfrac{v}{f} = \dfrac{1480}{256} = 5.78$ m.

Ví dụ mẫu · Reading amplitude, wavelength, and period from graphs

A transverse wave travelling on a string is described by two graphs recorded by a student: a displacement-position snapshot shows a maximum displacement of 2.5 cm and successive crests 0.40 m apart; a displacement-time graph, recorded at one fixed point on the string, shows the same point returning to the same displacement and velocity every 0.25 s. Find (a) the amplitude, (b) the wavelength, (c) the frequency, and (d) the wave speed.

(a) The peak height of the displacement-position graph directly gives the amplitude: $x_0 = 2.5$ cm $= 0.025$ m.

(b) The distance between successive crests on the displacement-position graph is the wavelength: $\lambda = 0.40$ m.

(c) The repeat time on the displacement-time graph is the period: $T = 0.25$ s, so $f = \dfrac{1}{T} = \dfrac{1}{0.25} = 4.0$ Hz.

(d) $v = f\lambda = 4.0 \times 0.40 = 0.16$ m s^{-1}.

3.2.3 Phase difference

Two points on the same wave (or two different waves of the same frequency) generally reach their maximum displacement at different times; this timing offset is the **phase difference**. It can be expressed as a fraction of a full cycle, in degrees, or in radians, and it is directly linked to the **path difference** (the difference in distance travelled) between the two points, measured in wavelengths:

$$\text{phase difference} = \frac{\text{path difference}}{\lambda} \times 360^\circ = \frac{\text{path difference}}{\lambda} \times 2\pi \text{ rad}.$$

Two points separated by a path difference of exactly λ (or any whole number of wavelengths) are **in phase** (phase difference 0°, or an integer multiple of 360°); two points separated by $\lambda/2$ (or an odd multiple of it) are **in antiphase** (phase difference 180°).

> phase difference $= \dfrac{\Delta x}{\lambda} \times 360^\circ = \dfrac{\Delta x}{\lambda} \times 2\pi$ rad, where Δx is the path difference.

On superimposed displacement-time graphs, two waves that are in phase rise and fall together, peak for peak; two waves in antiphase have one curve's peaks aligned exactly with the other curve's troughs. Any other phase difference produces peaks offset from each other by the same fraction of the period as the fraction of a wavelength computed above — an equivalence between a fraction of a wavelength (a spatial idea) and a fraction of a period (a temporal idea) that is only possible because every point of the wave oscillates at exactly the same frequency.

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Độ lệch pha (phase difference) chỉ đơn giản là "khoảng cách đường đi" Δx được đổi ra đơn vị "phần trăm của một bước sóng", rồi nhân với 360° (hoặc 2π rad) để ra góc lệch pha. Hai điểm cách nhau đúng 1λ, 2λ, 3λ,...thì luôn CÙNG PHA (dao động y hệt nhau tại mọi thời điểm); cách nhau $\lambda/2$, $3\lambda/2$,...thì luôn NGƯỢC PHA.

Ví dụ mẫu · Phase difference from path difference

Two points on a wave of wavelength 0.80 m are separated (along the direction of travel) by 0.60 m. Find the phase difference between them in (a) fraction of a cycle, (b) degrees, (c) radians.

Fraction of a cycle: $\dfrac{0.60}{0.80} = 0.75$ cycle.

(b) $0.75 \times 360^\circ = 270^\circ$. (c) $0.75 \times 2\pi = 1.5\pi \approx 4.71$ rad.

3.2.4 Wave intensity

The **intensity** I of a wave is the power it delivers per unit area perpendicular to the direction of travel:

$$I = \frac{P}{A}, \qquad [I] = \text{W m}^{-2}.$$

For a source that radiates power P uniformly in all directions (an **isotropic point source**), the power at distance r is spread evenly over the surface of an imaginary sphere of radius r, of area $4\pi r^2$. Hence

$$I = \frac{P}{4\pi r^2} \quad \Longrightarrow \quad I \propto \frac{1}{r^2},$$

the **inverse square law**: doubling the distance from the source reduces the intensity to one quarter, tripling the distance reduces it to one ninth, and so on.

Intensity is also proportional to the **square of the wave's amplitude**, $I \propto A_{\text{amp}}^2$ (this follows from energy considerations: the energy of an oscillating particle is proportional to the square of its amplitude, exactly as $E = \frac{1}{2}kx_0^2$ in Section C.1). Combining the two relations for a point source, since $I \propto 1/r^2$ and $I \propto A_{\text{amp}}^2$, it follows that $A_{\text{amp}} \propto 1/r$.

$$I = \frac{P}{A} \qquad \text{Point source: } I = \frac{P}{4\pi r^2} \implies I \propto \frac{1}{r^2} \qquad I \propto A_{\text{amp}}^2 \implies A_{\text{amp}} \propto \frac{1}{r}.$$

VN **GIẢI THÍCH TIẾNG VIỆT**

Định luật nghịch đảo bình phương (inverse square law) là hệ quả trực tiếp của việc năng lượng lan tỏa đều trên mặt cầu diện tích $4\pi r^2$ — diện tích này tăng theo bình phương khoảng cách, nên cường độ (năng lượng trên một đơn vị diện tích) giảm theo bình phương khoảng cách. Đừng quên MỘT LƯU Ý quan trọng: cường độ tỉ lệ với BÌNH PHƯƠNG biên độ, không phải biên độ — nếu khoảng cách tăng gấp đôi, biên độ chỉ giảm còn một nửa (không phải một phần tư).

Ví dụ mẫu · Inverse square law and amplitude

A point source radiates sound uniformly with power $P = 8.0 \times 10^{-3}$ W. Find (a) the intensity at $r = 4.0$ m, (b) the intensity at $r = 8.0$ m, and (c) the ratio of the wave amplitudes at these two distances.

(a) $I_4 = \frac{P}{4\pi r^2} = \frac{8.0 \times 10^{-3}}{4\pi(4.0)^2} = \frac{8.0 \times 10^{-3}}{201.1} = 3.98 \times 10^{-5}$ W m^{-2}.

(b) $I_8 = \frac{8.0 \times 10^{-3}}{4\pi(8.0)^2} = \frac{8.0 \times 10^{-3}}{804.2} = 9.95 \times 10^{-6}$ W m^{-2}.

(c) Since r doubles, I falls to one quarter (check: $3.98 \times 10^{-5}/9.95 \times 10^{-6} \approx 4.0$); since $I \propto A_{\text{amp}}^2$, the amplitude ratio is $A_4/A_8 = \sqrt{4.0} = 2.0$.

It is worth being careful to distinguish **power** P (measured in watts, a property of the source alone, independent of distance) from **intensity** I (measured in W m^{-2}, which necessarily depends on how far away the receiving area is from the source). A source that sounds louder or looks brighter at a given location is not necessarily more powerful — it may simply be closer.

(!) Lỗi thường gặp: Lỗi thường gặp: khi khoảng cách tăng gấp đôi, học sinh hay tính nhầm cường độ giảm còn một nửa (theo tỉ lệ bậc nhất) thay vì một phần tư (theo bình phương). Luôn nhớ bình phương tỉ số khoảng cách khi dùng $I \propto 1/r^2$.

3.2.5 The electromagnetic spectrum

Electromagnetic (EM) waves are transverse waves consisting of oscillating electric and magnetic fields, requiring no medium; in vacuum, *every* EM wave travels at the same speed, $c = 3.00 \times 10^8$ m s^{-1}, regardless of frequency. The families of EM wave differ only in frequency (and correspondingly, via $v = f\lambda$, in wavelength).

Region	Approximate wavelength range
Radio waves	greater than 1 m (up to km)
Microwaves	1 mm to 1 m
Infrared	700 nm to 1 mm
Visible light	400 nm to 700 nm
Ultraviolet	10 nm to 400 nm
X-rays	0.01 nm to 10 nm
Gamma rays	less than 0.01 nm

Each region of the spectrum has characteristic applications tied to how it interacts with matter. Radio waves carry broadcast and wireless communication signals; microwaves are used in radar and, at a frequency chosen to resonate with water molecules, in microwave ovens; infrared is emitted strongly by warm objects and is used in thermal imaging and remote controls; ultraviolet causes sunburn and is used for sterilisation; X-rays penetrate soft tissue but are absorbed by bone, making them useful for medical imaging; and gamma rays, the most penetrating and highest-energy of all, are used (carefully controlled) in cancer radiotherapy and are emitted during radioactive decay.

> All EM waves travel at $c = 3.00 \times 10^8$ m s^{-1} in vacuum; $c = f\lambda$ links frequency and wavelength for every region of the spectrum.

VN **GIẢI THÍCH TIẾNG VIỆT**

Toàn bộ quang phổ điện từ — từ sóng radio đến tia gamma — đều là CÙNG một loại sóng (sóng điện từ ngang), chỉ khác nhau về tần số/bước sóng, và đều truyền với cùng vận tốc $c = 3.00 \times 10^8$ m/s trong chân không. Thứ tự bước sóng giảm dần (tần số tăng dần) cần thuộc lòng: radio → vi ba (microwave) → hồng ngoại (infrared) → ánh sáng nhìn thấy → tử ngoại (UV) → tia X → tia gamma.

3.2.6 Polarization

Transverse waves can oscillate in any direction perpendicular to their direction of travel. **Unpolarized** light (e.g. from the Sun or an ordinary lamp) contains oscillations spread over all these perpendicular directions simultaneously. **Plane-polarized** light oscillates in only one fixed plane.

A **polarizer** is a filter that transmits only the component of the electric field oscillation parallel to its transmission axis; light emerging from a polarizer is plane-polarized. If plane-polarized light of intensity I_0 then strikes a second polarizer (called an **analyser**) whose transmission axis is at angle θ to the first, **Malus's law** gives the transmitted intensity:

$$I = I_0 \cos^2\theta.$$

When unpolarized light of intensity I_0 first passes through a single polarizer, exactly half the intensity is transmitted (since, averaged over all incoming oscillation directions, $\langle \cos^2\theta \rangle = \frac{1}{2}$), regardless of the polarizer's orientation: $I = I_0/2$.

Because longitudinal waves oscillate only along their direction of travel (there is no perpendicular direction to restrict), they have no analogous "plane" of oscillation to select — **longitudinal waves (such as sound) cannot be polarized.** The very possibility of polarizing a wave is itself evidence that the wave is transverse.

> Unpolarized light through one polarizer: $I = \dfrac{I_0}{2}$. Malus's law (already-polarized light through an analyser at angle θ): $I = I_0\cos^2\theta$. Longitudinal waves cannot be polarized.

VN **GIẢI THÍCH TIẾNG VIỆT**

Hai bước cần phân biệt rõ: (1) ánh sáng KHÔNG phân cực đi qua kính phân cực đầu tiên → cường độ LUÔN giảm còn đúng một nửa ($I_0/2$), bất kể góc của kính; (2) ánh sáng ĐÃ phân cực (sau kính đầu) đi qua kính phân cực thứ hai (analyser) ở góc θ so với kính đầu → dùng định luật Malus $I = I_0\cos^2\theta$. Lỗi rất phổ biến là áp dụng $\cos^2\theta$ ngay từ ánh sáng chưa phân cực ban đầu mà bỏ qua bước chia đôi.

Ví dụ mẫu · Malus's law with two polarizers

Unpolarized light of intensity 80 W m^{-2} passes through a polarizer, then through a second polarizer (analyser) whose axis is at 60° to the first. Find the intensity emerging from the analyser.

After the first polarizer: $I_1 = \frac{I_0}{2} = \frac{80}{2} = 40$ W m^{-2}.

After the analyser (Malus's law): $I_2 = I_1 \cos^2(60^\circ) = 40 \times (0.500)^2 = 40 \times 0.250 = 10$ W m^{-2}.

Polarization has several everyday applications: polarizing sunglasses block much of the strongly, horizontally polarized glare reflected from water or a road surface; LCD screens use polarizing filters together with a liquid-crystal layer to control the brightness transmitted by each pixel; and photoelastic stress analysis passes polarized light through certain transparent plastics under mechanical load, revealing internal stress patterns as coloured fringes. It is also worth noting the special case $\theta = 90^\circ$: since $\cos^2(90^\circ) = 0$, **crossed polarizers** (transmission axes perpendicular to each other) block the beam completely, transmitting no light at all.

(!) Lỗi thường gặp: Lỗi thường gặp: quên hệ số $\frac{1}{2}$ khi ánh sáng ban đầu CHƯA phân cực — chỉ áp dụng $\cos^2\theta$ cho bước đi qua kính phân cực THỨ HAI trở đi, không áp dụng cho bước đầu tiên (từ ánh sáng tự nhiên chưa phân cực).

3.2.7 Refraction and Snell's law

When a wave crosses the boundary between two media in which it travels at different speeds, its direction of travel generally changes — this is **refraction**. The **refractive index** of a medium is defined by

$$n = \frac{c}{v},$$

where v is the speed of light in that medium (so $n \geq 1$ always, with $n = 1$ exactly in vacuum). A medium with a larger n is described as **optically denser**.

Snell's law relates the angles of incidence θ_1 and refraction θ_2 (both measured from the normal to the boundary) to the refractive indices of the two media:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2.$$

Light bends towards the normal when entering a denser medium ($n_2 > n_1$) and away from the normal when entering a less dense medium ($n_2 < n_1$).

If light travels from a denser medium (n_1) towards a less dense medium ($n_2 < n_1$), there is a critical angle of incidence θ_c beyond which no refracted ray exists at all and the light undergoes **total internal reflection**. θ_c is found by setting $\theta_2 = 90^\circ$ in Snell's law:

$$n_1 \sin\theta_c = n_2 \sin 90^\circ = n_2 \quad \Longrightarrow \quad \sin\theta_c = \frac{n_2}{n_1} \quad (n_1 > n_2).$$

Total internal reflection is the principle behind optical fibres, which guide light along their length by repeated total internal reflection at the fibre-cladding boundary.

$n = \frac{c}{v}$ Snell's law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$ Critical angle ($n_1 > n_2$): $\sin\theta_c = \frac{n_2}{n_1}$.

Strictly speaking, the refractive index of a real material depends very slightly on wavelength, a phenomenon called **dispersion**. Violet light is refracted marginally more than red light by ordinary glass, because n is slightly larger at shorter wavelengths. This is why a glass prism spreads white light into a continuous spectrum of colours, and why raindrops split sunlight into a rainbow: each wavelength follows a very slightly different path through refraction (and, for the rainbow, an additional internal reflection inside each droplet).

VN **GIẢI THÍCH TIẾNG VIỆT**

Chiết suất $n = c/v$ càng LỚN thì ánh sáng truyền trong môi trường đó càng CHẬM. Khi ánh sáng đi từ môi trường chiết suất lớn sang môi trường chiết suất nhỏ hơn với góc tới đủ lớn (vượt quá góc tới hạn θ_c), toàn bộ ánh sáng bị phản xạ trở lại — hiện tượng phản xạ toàn phần (total internal reflection) — đây là nguyên lý hoạt động của cáp quang. Lưu ý: phản xạ toàn phần CHỈ xảy ra khi đi từ môi trường chiết suất lớn (n_1) sang môi trường chiết suất nhỏ hơn ($n_2 < n_1$), không xảy ra theo chiều ngược lại.

Ví dụ mẫu · Snell's law and the critical angle

Light travels inside glass of refractive index $n_1 = 1.50$ towards a glass-air boundary ($n_2 = 1.00$). (a) If the angle of incidence inside the glass is $30.0°$, find the angle of refraction in the air. (b) Find the critical angle for total internal reflection at this boundary.

(a) $n_1 \sin\theta_1 = n_2 \sin\theta_2 \Rightarrow 1.50 \sin 30.0° = 1.00 \sin\theta_2$

$$\sin\theta_2 = 1.50 \times 0.500 = 0.750 \implies \theta_2 = 48.6°.$$

(b) $\sin\theta_c = \dfrac{n_2}{n_1} = \dfrac{1.00}{1.50} = 0.667 \implies \theta_c = 41.8°.$

3.3 Superposition and Interference

Because waves add rather than collide, two or more waves can occupy the same region of space simultaneously and combine to produce effects with no counterpart in the motion of particles — most strikingly, regions where waves reinforce each other and regions where they cancel completely. This section develops the conditions under which such interference and diffraction patterns form and remain stable enough to observe and measure, culminating in the two classic quantitative techniques of the topic: the double-slit experiment and the diffraction grating.

3.3.1 The principle of superposition and coherence

Khái niệm cốt lõi

When two or more waves overlap at a point in space, the resultant displacement is the **vector (algebraic) sum** of the individual displacements each wave would produce on its own. This is the **principle of superposition**. After the waves pass through each other, each continues on unaffected — the overlap is purely a temporary combination of displacements.

For a stable, observable interference pattern (a fixed arrangement of reinforcement and cancellation in space) to appear, the sources producing the overlapping waves must be **coherent**: they must have the same frequency and a **constant** (unchanging) phase relationship to one another. Two independent light bulbs, for instance, are not coherent — their relative phase drifts randomly and rapidly, so any interference pattern is smeared out and unobservable. Laser light, or two slits illuminated by a single source, are coherent.

Two coherent point sources emitting into open space — for example, two loudspeakers driven by the same signal generator, or two dippers on a ripple tank driven by the same motor — produce a two-dimensional interference pattern of alternating lines of constructive and destructive interference radiating outward between the sources, each line traced out by the set of points whose path difference from the two sources is constant.

GIẢI THÍCH TIẾNG VIỆT

VN

Nguyên lý chồng chập (superposition) chỉ đơn giản là: khi hai sóng gặp nhau, ly độ tổng hợp tại một điểm bằng TỔNG ĐẠI SỐ (có dấu) của ly độ từng sóng riêng lẻ gây ra. Nhưng để quan sát được một hình ảnh giao thoa ỔN ĐỊNH theo thời gian, hai nguồn sóng bắt buộc phải KẾT HỢP (coherent) — nghĩa là cùng tần số và có độ lệch pha KHÔNG ĐỔI theo thời gian. Đây là lý do thí nghiệm Young dùng ánh sáng từ CÙNG một nguồn chiếu qua hai khe, thay vì dùng hai đèn riêng biệt.

3.3.2 Constructive and destructive interference

When two coherent waves of equal amplitude arrive at a point, the type of interference depends on their **path difference** Δx (the difference in distance each wave has travelled to reach that point):

Constructive interference (waves arrive in phase, resultant amplitude reinforced): $\Delta x = n\lambda$, $n = 0, 1, 2, 3, \ldots$

Destructive interference (waves arrive in antiphase, resultant amplitude cancelled): $\Delta x = \left(n + \frac{1}{2}\right)\lambda$, $n = 0, 1, 2, 3, \ldots$

These conditions strictly assume the two interfering waves have equal (or nearly equal) amplitude, so that destructive interference is close to complete cancellation. If the amplitudes are unequal, the same path-difference conditions still identify the points of maximum and minimum resultant amplitude, but the minima are no longer exactly zero: some residual amplitude, equal to the difference between the two individual amplitudes, remains even at a path difference that satisfies the "destructive" condition.

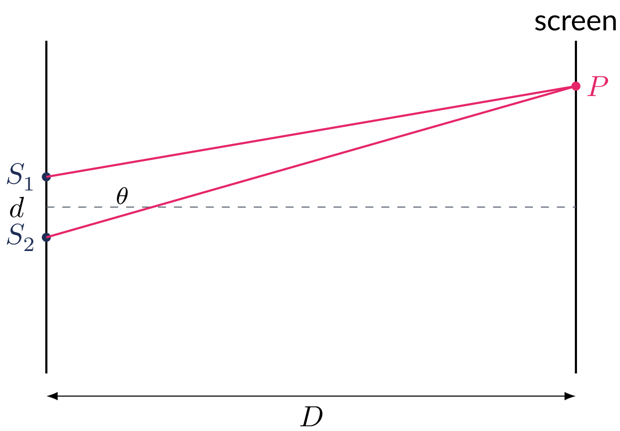
GIẢI THÍCH TIẾNG VIỆT

VN

Điều kiện giao thoa là "ngôn ngữ chung" xuất hiện trong mọi bài toán giao thoa (hai khe Young, cách tử, màng mỏng,...): hiệu đường đi bằng SỐ NGUYÊN LẦN bước sóng ($\Delta x = n\lambda$) → giao thoa TĂNG CƯỜNG (vân sáng); hiệu đường đi bằng SỐ BÁN NGUYÊN LẦN bước sóng ($\Delta x = (n + \frac{1}{2})\lambda$) → giao thoa TRIỆT TIÊU (vân tối).

3.3.3 Young's double-slit experiment

Thomas Young's classic experiment shines coherent light through two narrow, closely-spaced slits S_1 and S_2 (separation d), producing an alternating pattern of bright and dark fringes on a screen a distance D away, where $D \gg d$.



Double-slit geometry: slit separation d, slit-to-screen distance $D \gg d$. The two rays reaching P are essentially parallel, both at angle θ to the central axis, with path difference $S_1P - S_2P \approx d\sin\theta$.

Because $D \gg d$, the two rays travelling to a given point P on the screen are very nearly parallel, both making the same angle θ with the central axis. The path difference between them is then

$$\Delta x = d \sin\theta.$$

Bright fringes (constructive interference) occur where $\Delta x = m\lambda$, i.e. $d \sin\theta_m = m\lambda$ for integer m. For small angles, $\sin\theta \approx \tan\theta \approx \theta$, and the position of the m-th bright fringe on the screen (measured from the centre) is $y_m \approx D\tan\theta_m \approx D\theta_m$. Combining,

$$d \cdot \frac{y_m}{D} = m\lambda \quad \Longrightarrow \quad y_m = \frac{m\lambda D}{d}.$$

The spacing between adjacent bright fringes ($\Delta y = y_{m+1} - y_m$) is therefore

$$\Delta y = \frac{\lambda D}{d}.$$

By the same geometric argument, dark fringes (destructive interference) occur where $\Delta x = \left(m + \frac{1}{2}\right)\lambda$, giving dark-fringe positions $y = \left(m + \frac{1}{2}\right)\frac{\lambda D}{d}$ — exactly midway between each pair of adjacent bright fringes, consistent with the same spacing $\Delta y = \lambda D/d$ applying equally to the gaps between successive dark fringes.

Young's double-slit fringe spacing: $\Delta y = \dfrac{\lambda D}{d}$ (valid for $D \gg d$; d = slit separation, D = slit-to-screen distance).

VN **GIẢI THÍCH TIẾNG VIỆT**

Công thức $\Delta y = \dfrac{\lambda D}{d}$ là công thức trọng tâm của thí nghiệm hai khe Young. Ghi nhớ ý nghĩa từng đại lượng: d càng NHỎ (hai khe gần nhau) thì vân giao thoa càng RỘNG (dễ quan sát hơn); D càng LỚN (màn càng xa) thì vân càng RỘNG. Lỗi tính toán phổ biến nhất là quên đổi đơn vị d (thường cho bằng mm hoặc μm) và D (thường cho bằng m hoặc cm) về CÙNG một đơn vị (mét) trước khi thay vào công thức.

Ví dụ mẫu · Double-slit fringe spacing

In a Young's double-slit experiment, the slit separation is $d = 0.50$ mm, the screen is $D = 2.00$ m away, and the light used has wavelength $\lambda = 600$ nm. Find the fringe spacing.

Converting to metres: $d = 5.0 \times 10^{-4}$ m, $\lambda = 6.00 \times 10^{-7}$ m.

$$\Delta y = \frac{\lambda D}{d} = \frac{(6.00 \times 10^{-7})(2.00)}{5.0 \times 10^{-4}} = \frac{1.20 \times 10^{-6}}{5.0 \times 10^{-4}} = 2.4 \times 10^{-3}\ \text{m} = 2.4\ \text{mm}.$$

(!) Lỗi thường gặp: Lỗi thường gặp: thay trực tiếp d tính bằng mm và D tính bằng m vào cùng một công thức mà KHÔNG đổi đơn vị — dẫn đến kết quả sai lệch 1000 lần. Luôn đổi TẤT CẢ các chiều dài (d, D, λ) về mét trước khi tính Δy.

3.3.4 Diffraction and single-slit diffraction

The qualitative idea of diffraction is Standard Level; the quantitative single-slit minimum condition $a\sin\theta = \lambda$ developed below is Additional Higher Level (AHL) content.

Diffraction is the bending/spreading of waves as they pass around an obstacle or through an aperture (gap). Diffraction is most pronounced — the spreading is greatest — when the size of the aperture or obstacle is comparable to the wavelength of the wave; when the aperture is much larger than λ, the wave passes through with little noticeable spreading (approximately straight-line propagation).

When light of wavelength λ passes through a single slit of width a, it produces a diffraction pattern on a distant screen consisting of a broad, bright **central maximum** flanked by much dimmer secondary maxima, separated by dark minima. The angle to the **first minimum** on either side of the centre satisfies

$$a \sin\theta = \lambda.$$

An important qualitative feature is that the central maximum is **twice as wide** (in angular terms) as any of the secondary (side) maxima, since it extends from $-\theta$ to $+\theta$ where $a \sin\theta = \lambda$, while each side maximum spans only one such angular interval.

This size comparison explains everyday observations. Sound waves, with wavelengths of order 0.1 to 1 m, diffract noticeably around door openings and corners, which is why speech can be heard around a corner even without a direct line of sight to the speaker. Visible light, with wavelength of order 500 nm, diffracts negligibly around everyday-sized objects and apertures, which is why ordinary shadows have sharp edges — noticeable diffraction of light requires an aperture as narrow as a human hair, or narrower.

> Single-slit diffraction, first minimum: $a \sin\theta = \lambda$ (a = slit width). The central maximum is twice as wide (angularly) as each side maximum.

GIẢI THÍCH TIẾNG VIỆT

VN

Nhiễu xạ (diffraction) là hiện tượng sóng "uốn cong" khi đi qua khe hẹp hoặc vòng qua vật cản — càng RÕ RỆT khi kích thước khe/vật cản XẤP XỈ bước sóng. Với nhiễu xạ qua một khe đơn, công thức $a \sin\theta = \lambda$ cho góc tới cực tiểu đầu tiên (KHÔNG phải cực đại) — dễ nhầm với công thức cách tử. Vân sáng trung tâm của nhiễu xạ một khe RỘNG GẤP ĐÔI các vân sáng phụ hai bên.

Ví dụ mẫu · Single-slit diffraction minimum

Light of wavelength 633 nm is incident on a single slit of width 0.015 mm. Find the angle to the first diffraction minimum.

$a = 0.015$ mm $= 1.5 \times 10^{-5}$ m, $\lambda = 6.33 \times 10^{-7}$ m.

$$\sin\theta = \frac{\lambda}{a} = \frac{6.33 \times 10^{-7}}{1.5 \times 10^{-5}} = 0.0422 \implies \theta = 2.42^\circ.$$

3.3.5 The diffraction grating

Additional Higher Level (AHL) content.

A **diffraction grating** is a plate ruled with a very large number of closely and evenly spaced parallel slits. Light passing through many slits interferes constructively, in directions given by

$$d \sin\theta = n\lambda, \qquad n = 0, 1, 2, \dots,$$

where d is the **grating spacing** (the distance between adjacent slits, $d = 1/N$ if the grating is specified by N lines per unit length) and n is the **order** of the maximum. Because so many slits contribute, grating maxima are extremely sharp and bright compared with the broad, low-contrast fringes of a two-slit pattern, making a grating a very precise tool for measuring wavelength.

> Diffraction grating: $d \sin\theta = n\lambda$, where $d = \dfrac{1}{\text{lines per unit length}}$.

The sharpness of grating maxima improves as more slits contribute. With only two slits (Young's experiment) the transition from bright to dark across the pattern is gradual, but with the hundreds or thousands of slits ruled into a typical grating, light from all the slits must line up constructively over only an extremely narrow range of angles before contributions from slits far apart in the grating begin to cancel each other out. The result is a set of maxima that are simultaneously very bright (energy from a huge number of slits reinforcing) and very narrow (precise angular selectivity), which is exactly what makes a grating a suitable tool for precise wavelength measurement, or **spectroscopy**.

VN **GIẢI THÍCH TIẾNG VIỆT**

Công thức cách tử $d \sin\theta = n\lambda$ trông giống công thức nhiễu xạ một khe nhưng KHÔNG PHẢI là một — đây là điều kiện cho các vân SÁNG (cực đại, giao thoa tăng cường của rất nhiều khe), không phải vân tối. Khi đề cho "số vạch trên mm" (lines per mm), nhớ tính $d = 1/N$ và đổi đơn vị cẩn thận (thường N cho theo mm nhưng λ cho theo nm — phải đưa về cùng đơn vị mét).

Ví dụ mẫu · Diffraction grating

A diffraction grating has 300 lines per millimetre. Light of wavelength 589 nm is incident normally on the grating. Find (a) the grating spacing d, (b) the angle of the second-order ($n = 2$) maximum, and (c) the highest order that can be observed.

(a) $d = \dfrac{1}{300\ \text{mm}^{-1}} = 3.33 \times 10^{-3}$ mm $= 3.33 \times 10^{-6}$ m.

(b) $\sin\theta_2 = \dfrac{n\lambda}{d} = \dfrac{2(589 \times 10^{-9})}{3.33 \times 10^{-6}} = \dfrac{1.178 \times 10^{-6}}{3.33 \times 10^{-6}} = 0.354 \Rightarrow \theta_2 = 20.7^\circ$.

(c) The maximum possible order requires $\sin\theta \leq 1$: $n_{\text{max}} \leq \dfrac{d}{\lambda} = \dfrac{3.33 \times 10^{-6}}{589 \times 10^{-9}} = 5.66$, so the highest observable integer order is $n = 5$.

3.3.6 A remark on thin-film interference

Enrichment beyond the core syllabus, included here for completeness.

A closely related phenomenon occurs when light reflects off the top and bottom surfaces of a very thin transparent film (a soap bubble, or a thin layer of oil on water). The two reflected rays have travelled slightly different path lengths (and one may undergo an extra 180° phase shift on reflection, depending on the refractive indices involved), so they interfere. Because the path difference depends on both the film thickness and the wavelength of light, different colours interfere constructively at different thicknesses, producing the shifting rainbow patterns visible on soap films and oil slicks. The same principle, engineered deliberately with a coating of carefully chosen and highly uniform thickness, is used to make **anti-reflective coatings** on camera lenses and eyeglasses: the film thickness is chosen so that the two reflected rays interfere destructively for the wavelengths the eye is most sensitive to, maximising the light that is transmitted through the lens instead of reflected away.

VN **GIẢI THÍCH TIẾNG VIỆT**

Giao thoa màng mỏng (thin-film interference) là lý do tại sao bong bóng xà phòng hay váng dầu trên mặt nước có màu sắc lung linh thay đổi theo góc nhìn — ánh sáng phản xạ từ mặt trên và mặt dưới của lớp màng mỏng giao thoa với nhau, và vì hiệu đường đi phụ thuộc vào cả độ dày màng lẫn bước sóng, mỗi màu sẽ giao thoa tăng cường tại một độ dày khác nhau.

3.4 Standing Waves and Resonance

Superposition applied to two identical waves travelling in opposite directions — rather than two waves travelling side by side towards a distant screen — produces a qualitatively different result: a pattern that oscillates in place without travelling at all. Standing waves and the resonance phenomena associated with them explain how strings, air columns, and countless other bounded systems produce the discrete set of natural frequencies that underlie music, timekeeping, and much of everyday acoustics, and they connect directly back to the resonance idea first introduced for a single oscillator in Section C.1.

3.4.1 Formation of standing waves

A **standing (stationary) wave** forms from the superposition of two identical waves (same frequency, amplitude, and wavelength) travelling in **opposite directions** through the same region — typically an incident wave and its reflection from a boundary. Unlike a travelling wave, a standing wave does not transport energy along its length; instead, energy is exchanged locally between kinetic and potential forms.

Khái niệm cốt lõi

At certain fixed points, called **nodes**, the two component waves are always in antiphase and cancel completely — the displacement there is permanently zero. Midway between adjacent nodes are **antinodes**, where the two waves are always in phase and reinforce — the displacement oscillates with maximum amplitude. Nodes and antinodes stay fixed in space (they do not travel), which is why the pattern is called ”standing.”

Adjacent nodes are separated by half a wavelength, $\lambda/2$; the distance from a node to the next antinode is a quarter wavelength, $\lambda/4$.

Adjacent node-to-node (or antinode-to-antinode) spacing: $\lambda/2$. Node-to-adjacent-antinode spacing: $\lambda/4$.

Because a standing wave is built from two travelling waves of equal amplitude and frequency moving in opposite directions, they carry equal amounts of energy in opposite directions at every point, and the net energy flow along the string or air column averages to exactly zero. Unlike a travelling wave, a standing wave does not transport energy from one end of the medium to the other; it only exchanges energy locally, between kinetic and potential forms, exactly as an SHM oscillator does at each individual point. This is precisely why standing waves are the natural way a musical instrument sustains a steady note: energy fed in continuously by bowing, blowing, or plucking accumulates in the standing-wave pattern rather than radiating straight away along the string or air column.

GIẢI THÍCH TIẾNG VIỆT

VN Sóng dừng hình thành khi HAI sóng giống hệt nhau (cùng tần số, biên độ, bước sóng) truyền ngược chiều nhau và chồng chập — thường gặp nhất là sóng tới và sóng phản xạ từ một đầu cố định/tự do. **Nút sóng** (node): vị trí LUÔN đứng yên (ly độ = 0 mọi lúc). **Bụng sóng** (antinode): vị trí dao động với biên độ CỰC ĐẠI. Khoảng cách nút-nút hoặc bụng-bụng liền kề là $\lambda/2$; khoảng cách nút-bụng liền kề là $\lambda/4$ — đây là hai con số cần thuộc lòng để giải nhanh mọi bài sóng dừng.

3.4.2 Standing waves on a string fixed at both ends

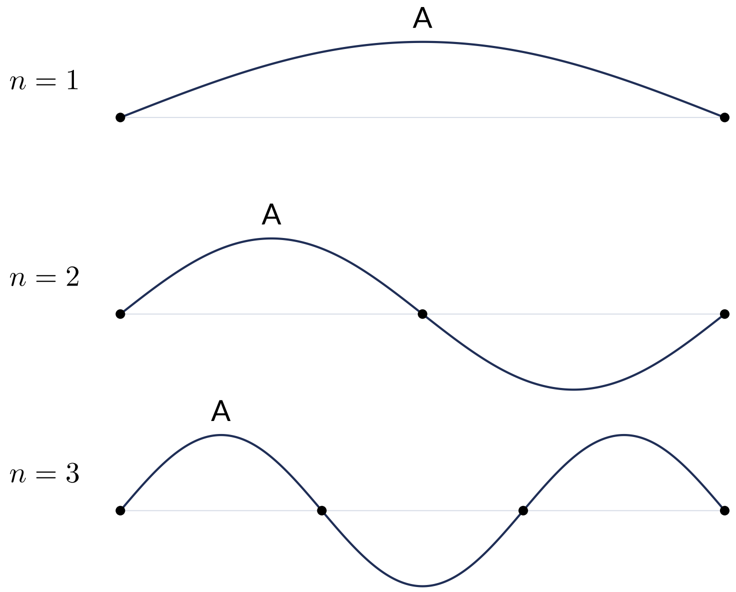
A string of length L, fixed rigidly at both ends, must have a **node at each end** (the string cannot move there). Only certain wavelengths ”fit” this boundary condition — those for which an integer number of half-wavelengths spans the string:

$$L = n \cdot \frac{\lambda_n}{2} \quad \Longrightarrow \quad \lambda_n = \frac{2L}{n}, \qquad n = 1, 2, 3, \ldots$$

Using $v = f\lambda$, the allowed (**harmonic**) frequencies are

$$f_n = \frac{v}{\lambda_n} = \frac{nv}{2L} = nf_1,$$

where $f_1 = v/(2L)$ is the **fundamental frequency** (first harmonic, $n = 1$, a single antinode). $n = 2$ (second harmonic) has two antinodes, $n = 3$ (third harmonic) has three antinodes, and so on — the harmonic number n equals the number of antinodes and also equals $n + 1$, the number of nodes.



The first three harmonics of a string fixed at both ends. Dots mark nodes (fixed points of zero displacement); the curve peaks (labelled A) mark antinodes. The harmonic number n equals the number of antinodes.

> String fixed at both ends: $\lambda_n = \dfrac{2L}{n}$ $\quad f_n = \dfrac{nv}{2L} = nf_1,\ n = 1, 2, 3, \ldots$ (all integer harmonics allowed).

The wave speed v on a string depends on how tightly it is stretched and how massive it is per unit length: increasing the tension increases v and therefore raises every harmonic frequency in proportion — precisely how a guitar or violin string is tuned by turning its tuning pegs. A thicker (more massive per unit length) string under the same tension and length has a lower wave speed and therefore a lower fundamental frequency, which is why the thickest strings on a guitar produce the lowest notes.

GIẢI THÍCH TIẾNG VIỆT

VN

Với dây hai đầu cố định, CẢ hai đầu đều là nút — điều kiện này cho phép TẤT CẢ các họa âm nguyên (n = 1, 2, 3, 4,...), không bỏ sót họa âm nào. Công thức $f_n = nf_1$ có nghĩa là tần số họa âm thứ n luôn gấp n lần tần số cơ bản f_1 — chỉ cần tính f_1 một lần rồi nhân lên.

Ví dụ mẫu · Harmonics on a string

A string of length 1.20 m, fixed at both ends, carries waves at speed 60 m s^{-1}. Find (a) the fundamental frequency, (b) the frequency of the third harmonic, and (c) the wavelength of the second harmonic.

(a) $f_1 = \dfrac{v}{2L} = \dfrac{60}{2(1.20)} = \dfrac{60}{2.40} = 25$ Hz.

(b) $f_3 = 3f_1 = 3(25) = 75$ Hz.

(c) $\lambda_2 = \dfrac{2L}{2} = L = 1.20$ m.

3.4.3 Standing waves in open-open pipes

A pipe open at **both** ends allows air to move freely at each end, so both ends must be displacement **antinodes**. Whenever both boundary conditions are of the **same type** (both nodes, as on a fixed string, or both antinodes, as in an open-open pipe), the allowed wavelengths satisfy the identical mathematical relation $L = n\lambda_n/2$. Consequently, an open-open pipe of length L supports exactly the same harmonic series as a string of the same length:

$$\lambda_n = \frac{2L}{n}, \qquad f_n = \frac{nv}{2L} = nf_1, \qquad n = 1, 2, 3, \ldots$$

where here v is the speed of sound in the air inside the pipe. All integer harmonics (fundamental, second, third, ...) are present.

> Open-open pipe: $f_n = \dfrac{nv}{2L}$, $n = 1, 2, 3, \ldots$ (same harmonic series as a string fixed at both ends).

A flute is the standard real-world example of an (approximately) open-open pipe: both the mouth hole (embouchure) and the far end are open to the atmosphere, so the instrument sounds its full, unbroken series of harmonics — part of the reason a flute has a comparatively bright, harmonically rich tone.

VN **GIẢI THÍCH TIẾNG VIỆT**

Ống hở hai đầu (open-open) có công thức GIỐNG HỆT dây hai đầu cố định — mặc dù bản chất vật lý khác nhau (một bên là nút, một bên là bụng), nhưng vì CẢ HAI đầu đều cùng loại (đều là bụng ở đây), điều kiện toán học ($L = n\lambda/2$) không đổi, nên cho ra đầy đủ tất cả các họa âm nguyên.

Ví dụ mẫu · Open-open pipe harmonics

An organ pipe open at both ends has length 0.85 m and contains air in which the speed of sound is 340 m s^{-1}. Find (a) the fundamental frequency, and (b) the frequency of the third harmonic.

(a) $f_1 = \dfrac{v}{2L} = \dfrac{340}{2(0.85)} = \dfrac{340}{1.70} = 200$ Hz.

(b) $f_3 = 3f_1 = 3(200) = 600$ Hz.

3.4.4 Standing waves in open-closed pipes

A pipe **closed** at one end and **open** at the other has *different* boundary conditions at its two ends. At the closed end, the air is rigidly constrained and cannot oscillate, so it must be a displacement **node**. At the open end, the air is free to move and (to a good approximation) reaches maximum oscillation, so it must be an **antinode**.

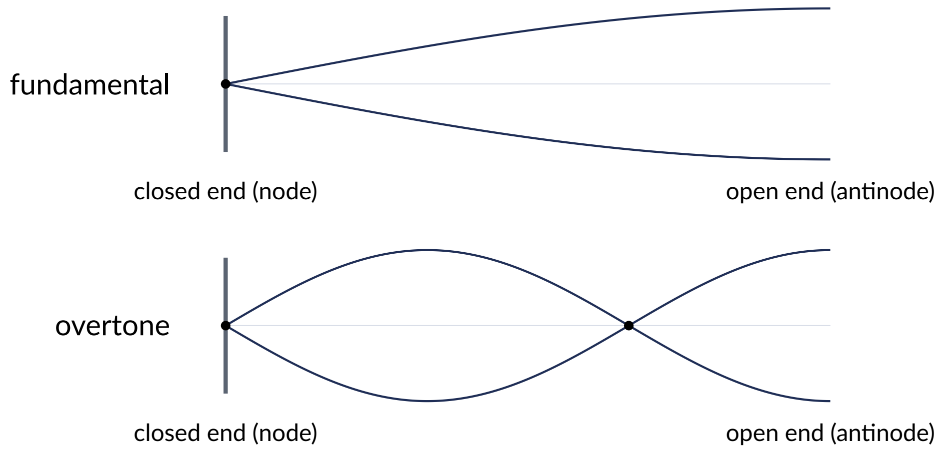
Because the two ends require different types of boundary condition, the allowed patterns are those with a node at one end and an antinode at the other, meaning an **odd** number of quarter-wavelengths must fit into the pipe length L:

$$L = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots \quad \Longrightarrow \quad L = n \cdot \frac{\lambda_n}{4}, \quad n = 1, 3, 5, \ldots \text{ (odd only)}.$$

The allowed frequencies are therefore

$$f_n = \frac{nv}{4L}, \qquad n = 1, 3, 5, \ldots$$

with fundamental $f_1 = v/(4L)$. Even harmonics ($n = 2, 4, 6, \ldots$) simply do not exist for this boundary configuration — the "first overtone" above the fundamental is the *third* harmonic, $f_3 = 3f_1$, not the second.



Closed pipe: fundamental ($L = \lambda/4$, one node, one antinode) and first overtone, the third harmonic ($L = 3\lambda/4$, two nodes, two antinodes).

Open-closed pipe: $f_n = \dfrac{nv}{4L}$, $n = 1, 3, 5, \ldots$ (odd harmonics only). Fundamental: $f_1 = \dfrac{v}{4L}$.

VN **GIẢI THÍCH TIẾNG VIỆT**

Đây là điểm khác biệt QUAN TRỌNG NHẤT giữa ống một đầu kín và các hệ khác: chỉ có các họa âm LẺ ($n = 1, 3, 5, \ldots$) tồn tại, vì một đầu là nút còn đầu kia là bụng — hai điều kiện biên KHÁC LOẠI. "Họa âm bậc hai trên cơ bản" (first overtone) của ống kín thực chất là họa âm bậc BA ($f_3 = 3f_1$), KHÔNG phải bậc hai như dây hay ống hở hai đầu — đây là lỗi học sinh mắc rất thường xuyên.

Ví dụ mẫu · Closed pipe harmonics

A pipe closed at one end has length 0.75 m and contains air in which the speed of sound is 340 m s^{-1}. Find (a) the fundamental frequency and (b) the frequency of the first overtone.

(a) $f_1 = \dfrac{v}{4L} = \dfrac{340}{4(0.75)} = \dfrac{340}{3.00} = 113$ Hz.

(b) The first overtone is the third harmonic: $f_3 = 3f_1 = 3(113) = 340$ Hz.

(!) Lỗi thường gặp: Lỗi thường gặp: áp dụng công thức $f_n = nv/(2L)$ (dùng cho dây hoặc ống hở hai đầu) cho ống MỘT ĐẦU KÍN — dẫn đến bỏ sót yếu tố $4L$ ở mẫu số và tính luôn cả các họa âm chẵn vốn không tồn tại trong ống một đầu kín.

A clarinet behaves acoustically as a closed-open pipe (closed at the reed end, open at the bell), while a flute of similar length behaves as an open-open pipe. Because the closed pipe's fundamental is $f_1 = v/(4L)$ rather than $v/(2L)$, a clarinet sounds roughly an octave lower than a similarly sized flute; and because its even harmonics are missing entirely, it has a characteristically hollow, "woody" tone quality compared with the harmonically fuller flute.

3.4.5 Resonance in standing-wave systems

Every string, air column, or tuning fork has its own set of natural (harmonic) frequencies, exactly as derived above. If such a system is driven — plucked, bowed, blown across, or excited by a nearby vibrating source — at (or very near) one of these natural frequencies, a large-amplitude standing wave builds up: this is the same resonance phenomenon introduced in Section C.1, now applied to an extended (rather than point-like) oscillating system. A struck tuning fork held above a resonance tube, for example, drives the air column strongly only when the tube length matches (a quarter-wavelength multiple of) the fork's frequency; a singer's voice can set a wine glass ringing loudly only at the glass's own natural frequency.

> **GIẢI THÍCH TIẾNG VIỆT**
> VN
> Cộng hưởng trong hệ sóng dừng (dây đàn, cột không khí, âm thoa,...) hoạt động theo đúng nguyên lý đã học ở phần dao động cưỡng bức: khi tần số kích thích TRÙNG (hoặc gần trùng) với một trong các tần số họa âm tự nhiên của hệ, biên độ sóng dừng tăng vọt. Đây là lý do các nhạc cụ dây/hơi tạo ra âm thanh lớn, rõ ràng ở đúng những tần số họa âm đặc trưng của chúng.

The classic laboratory demonstration of this idea is the **resonance tube** experiment: a tuning fork of known frequency is held above a vertical tube partially filled with water, and the water level is lowered gradually until the air column resonates loudly at its fundamental length and, on further lowering, at its next allowed length — allowing the speed of sound to be determined from the measured resonant lengths and the known frequency of the fork.

3.5 The Doppler Effect

The final wave phenomenon considered in this chapter arises whenever a source of waves, an observer, or both are in relative motion: the frequency actually detected differs systematically from the frequency emitted. The same underlying idea — developed first for sound, then adapted for light — has applications ranging from measuring the speed of a passing vehicle to establishing that the universe itself is expanding, and it closes the chapter by tying together the wave equation, frequency, and wavelength introduced all the way back in Section C.2.

3.5.1 Doppler effect for sound

The **Doppler effect** is the change in observed frequency (and wavelength) of a wave caused by relative motion between the source and the observer. When the source and observer are moving *closer together* (whether because the source approaches, the observer approaches, or both), the observed frequency f' is **higher** than the emitted frequency f; when they move *farther apart*, f' is **lower**.

For sound, with v the speed of sound in the medium (taken as 340 m s^{-1} in air unless stated otherwise), v_s the speed of the source, and v_o the speed of the observer, the general Doppler formula is

$$f' = f \cdot \frac{v \pm v_o}{v \mp v_s}.$$

The sign convention: use the **upper** sign ($+v_o$ in the numerator, $-v_s$ in the denominator) for whichever motion brings source and observer *closer together* (raising f'); use the **lower** sign for motion that moves them *farther apart* (lowering f'). Two important special cases are:

Source moving, observer stationary: $$f' = f \cdot \frac{v}{v \mp v_s} \quad \left(\text{i.e. } f' = \frac{fv}{v - v_s} \text{ approaching}\right)$$

Observer moving, source stationary: $$f' = f \cdot \frac{v \pm v_o}{v} \quad \left(\text{i.e. } f' = \frac{f(v + v_o)}{v} \text{ approaching}\right)$$

General: $f' = f \cdot \dfrac{v \pm v_o}{v \mp v_s}$ (top sign → closer together, higher f'; bottom sign → farther apart, lower f').

Source approaching, observer stationary: $f' = \dfrac{fv}{v - v_s}$. Observer approaching, source stationary: $f' = \dfrac{f(v + v_o)}{v}$.

As a source's speed v_s approaches the wave speed v, the denominator $v - v_s$ in $f' = fv/(v - v_s)$ approaches zero and the observed (approaching) frequency rises without bound, because successive wavefronts pile up almost on top of one another just ahead of the source. If the source's speed actually reaches or exceeds the wave speed ($v_s \geq v$), the wavefronts can no longer outrun the source at all: instead they pile up together into a shock front, heard as a **sonic boom** for a supersonic aircraft — a regime the simple Doppler formula above no longer describes.

VN **GIẢI THÍCH TIẾNG VIỆT**

Công thức Doppler tổng quát trông rối vì có cả dấu cộng lẫn dấu trừ, nhưng chỉ cần nhớ MỘT nguyên tắc: nếu chuyển động khiến nguồn và người nghe TIẾN LẠI GẦN nhau, tần số nghe được TĂNG (dùng dấu làm cho f' lớn hơn f); nếu RA XA nhau, tần số GIẢM. Với số hạng vận tốc nguồn ở MẪU SỐ, "tiến lại gần" tương ứng với dấu TRỪ (mẫu số nhỏ hơn → phân số lớn hơn); với số hạng vận tốc người nghe ở TỬ SỐ, "tiến lại gần" tương ứng với dấu CỘNG (tử số lớn hơn).

Ví dụ mẫu · Doppler effect — moving source

A train's horn emits a frequency of 300 Hz. The train approaches a stationary observer at 25 m s^{-1}, then (after passing) recedes at the same speed. Take the speed of sound as 340 m s^{-1}. Find the frequency heard by the observer (a) while the train approaches, and (b) while it recedes.

(a) Approaching (denominator shrinks): $f' = \dfrac{fv}{v - v_s} = \dfrac{300(340)}{340 - 25} = \dfrac{102\,000}{315} = 324$ Hz.

(b) Receding (denominator grows): $f' = \dfrac{fv}{v + v_s} = \dfrac{300(340)}{340 + 25} = \dfrac{102\,000}{365} = 279$ Hz.

3.5.2 Doppler effect for sound: moving observer, and combined motion

The same reasoning applies when the observer, rather than the source, is moving. If the observer moves *towards* a stationary source, more wavefronts are encountered per second than if stationary, raising the observed frequency; moving *away* lowers it. When both source and observer move, the appropriate signs are simply combined in the general formula.

Ví dụ mẫu · Doppler effect — moving observer, and both moving

A stationary source emits a frequency of 1000 Hz; take the speed of sound as 340 m s^{-1}. (a) An observer moves *away* from the source at 15 m s^{-1}. Find the frequency detected. (b) Now suppose instead the source also moves, towards the observer, at 10 m s^{-1}, while the observer moves towards the source at 15 m s^{-1}. Find the frequency detected.

(a) Observer receding: $f' = \dfrac{f(v - v_o)}{v} = \dfrac{1000(340 - 15)}{340} = \dfrac{1000(325)}{340} = 956$ Hz.

(b) Both moving towards each other (numerator up, denominator down):

$$f' = \frac{f(v+v_o)}{v-v_s} = \frac{1000(340+15)}{340-10} = \frac{1000(355)}{330} = 1076 \text{ Hz}.$$

(!) Lỗi thường gặp: Lỗi thường gặp: viết sai dấu trong công thức, ví dụ dùng $f' = f(v+v_s)/v$ khi nguồn đang TIẾN LẠI (đáng lẽ vận tốc nguồn phải nằm ở MẪU SỐ với dấu TRỪ). Cách kiểm tra nhanh: sau khi tính ra f', hãy tự hỏi "nguồn/người nghe đang lại gần hay ra xa?" — nếu lại gần mà $f' < f$ (hoặc ngược lại) thì chắc chắn đã dùng sai dấu.

3.5.3 Doppler effect for light

Electromagnetic waves also exhibit a Doppler shift, but because the speed of light c is a universal constant (not the speed of a mechanical medium), the exact relativistic formula differs from the sound-wave case. However, for relative speeds v much smaller than c, the fractional shift in frequency (and, equivalently, in wavelength) is well approximated by

$$\frac{\Delta f}{f} \approx -\frac{v}{c} \qquad \text{or equivalently} \qquad \frac{\Delta\lambda}{\lambda} \approx \frac{v}{c}.$$

A source **approaching** the observer has its light shifted to **higher** frequency (**shorter** wavelength) — a **blueshift**. A source **receding** has its light shifted to **lower** frequency (**longer** wavelength) — a **redshift**.

$\frac{\Delta\lambda}{\lambda} \approx \frac{v}{c}$ (valid for $v \ll c$). Approaching → blueshift (shorter λ, higher f). Receding → redshift (longer λ, lower f).

Beyond confirming cosmic expansion, precise Doppler measurements of starlight are also used to detect **exoplanets**. A planet's gravity causes its host star to wobble very slightly back and forth, alternately approaching and receding from Earth, which produces a small, periodic Doppler shift in the star's spectrum — the **radial velocity method** — revealing the presence, orbital period, and approximate mass of an otherwise invisible orbiting planet.

VN **GIẢI THÍCH TIẾNG VIỆT**

Hiệu ứng Doppler cho ánh sáng dùng công thức GẦN ĐÚNG $\Delta\lambda/\lambda \approx v/c$, chỉ áp dụng khi tốc độ tương đối v nhỏ hơn nhiều so với c. Nguồn sáng TIẾN LẠI GẦN → bước sóng NGẮN lại, dịch về phía XANH DƯƠNG (blueshift); nguồn sáng RA XA → bước sóng DÀI ra, dịch về phía ĐỎ (redshift). Đây chính là bằng chứng quan trọng cho thấy vũ trụ đang GIÃN NỞ: hầu hết các thiên hà xa xôi đều cho quang phổ bị dịch đỏ.

Ví dụ mẫu · Doppler redshift of a distant galaxy

A hydrogen emission line normally observed in the laboratory at $\lambda = 486.1$ nm is observed in the spectrum of a distant galaxy at 492.0 nm. (a) State whether the galaxy is approaching or receding from Earth. (b) Estimate the galaxy's speed relative to Earth.

(a) The observed wavelength is longer than the laboratory value (492.0 nm $>$ 486.1 nm) — this is a **redshift**, so the galaxy is **receding**.

(b) $\Delta\lambda = 492.0 - 486.1 = 5.9$ nm.

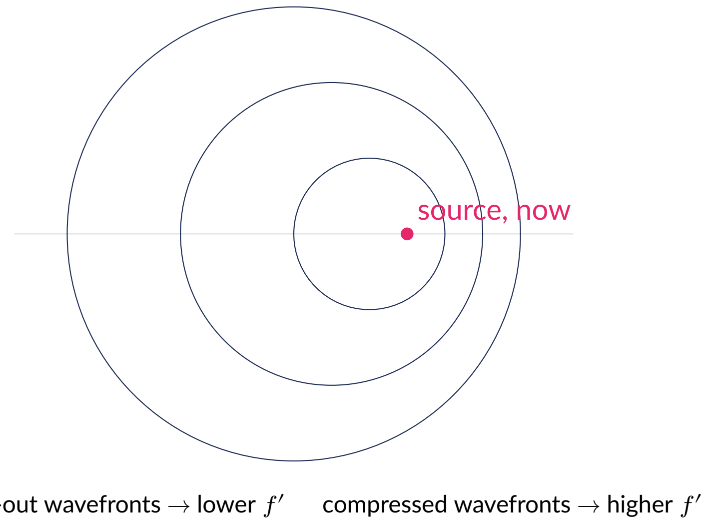
$$\frac{\Delta\lambda}{\lambda} \approx \frac{v}{c} \implies v \approx c \cdot \frac{\Delta\lambda}{\lambda} = (3.00 \times 10^8) \times \frac{5.9}{486.1} = (3.00 \times 10^8)(0.01214) = 3.64 \times 10^6 \text{ m s}^{-1}.$$

3.5.4 Applications of the Doppler effect

The Doppler effect for light provided the observational foundation for one of the most important discoveries in cosmology: nearly every distant galaxy shows a **redshift** in its spectrum, and (per Hubble's law) more distant galaxies are redshifted by proportionally more. This is the principal evidence that the universe is **expanding** — galaxies, on average, are receding from one another, and the farther away a galaxy is, the faster it appears to recede.

Doppler shifts are also exploited in everyday technology:

- **Police (and weather) radar** sends out a radio-frequency pulse and measures the Doppler shift of the reflected signal to determine the speed of a moving vehicle (or the motion of rain/wind within a storm).
- **Medical Doppler ultrasound** directs high-frequency sound waves into the body; reflections from moving red blood cells are Doppler-shifted, and the size of the shift reveals the speed (and direction) of blood flow, used to detect blockages or monitor circulation non-invasively.



Wavefronts emitted at equal time intervals by a source moving to the right. Ahead of the source the wavefronts bunch together (shorter observed wavelength, higher frequency); behind the source they spread apart (longer wavelength, lower frequency).

VN **GIẢI THÍCH TIẾNG VIỆT**

Ứng dụng thực tế của hiệu ứng Doppler rất đa dạng: (1) **dịch đỏ thiên hà** — bằng chứng vũ trụ giãn nở; (2) **radar bắn tốc độ** — đo độ lệch tần số sóng radio phản xạ từ xe đang chạy để suy ra vận tốc; (3) **siêu âm Doppler y khoa** — đo độ lệch tần số sóng siêu âm phản xạ từ hồng cầu đang di chuyển để xác định tốc độ và hướng dòng máu, hỗ trợ chẩn đoán tắc nghẽn mạch máu mà không cần phẫu thuật.

3.6 Problem Bank

Bài tập luyện tập

A particle undergoes SHM with amplitude 0.20 m and period 4.0 s. What is its maximum speed?

(A) 0.10 m s^{-1}
(B) 0.31 m s^{-1}
(C) 0.63 m s^{-1}
(D) 1.57 m s^{-1}

Lời giải chi tiết

$\omega = \frac{2\pi}{T} = \frac{2\pi}{4.0} = 1.571$ rad s^{-1}.

$$v_{\text{max}} = \omega x_0 = 1.571 \times 0.20 = 0.314 \text{ m s}^{-1} \approx 0.31 \text{ m s}^{-1}.$$

(B).

Bài tập luyện tập

A 0.40 kg mass on a spring of constant $k = 160$ N m^{-1} oscillates with amplitude 0.050 m. Find (a) the angular frequency, period, and frequency of the motion; (b) the maximum speed and maximum acceleration; (c) the speed when $x = 0.030$ m.

Lời giải chi tiết

(a) $\omega = \sqrt{k/m} = \sqrt{160/0.40} = \sqrt{400} = 20$ rad s^{-1}. $T = 2\pi/\omega = 2\pi/20 = 0.314$ s. $f = 1/T = 3.18$ Hz.

(b) $v_{\text{max}} = \omega x_0 = 20 \times 0.050 = 1.0$ m s^{-1}. $a_{\text{max}} = \omega^2 x_0 = 400 \times 0.050 = 20$ m s^{-2}.

(c) $v = \omega\sqrt{x_0^2 - x^2} = 20\sqrt{(0.050)^2 - (0.030)^2} = 20\sqrt{0.0025 - 0.0009} = 20\sqrt{0.0016} = 20(0.040) = \boxed{0.80 \text{ m s}^{-1}}$.

Bài tập luyện tập

Which of the following quantities remains constant throughout simple harmonic motion (in the absence of damping)?

(A) Speed
(B) Acceleration
(C) Total mechanical energy
(D) Restoring force

Lời giải chi tiết

Speed, acceleration, and restoring force all vary continuously with displacement; only the total mechanical energy $E = KE + PE$ stays constant at every instant. **(C)**.

Bài tập luyện tập

A simple pendulum has length 0.90 m. (a) Find its period on Earth ($g = 9.81$ m s^{-2}). (b) Find its period on Mars, where $g = 3.71$ m s^{-2}. (c) Find the percentage change in period from Earth to Mars.

Lời giải chi tiết

(a) $T_E = 2\pi\sqrt{L/g} = 2\pi\sqrt{0.90/9.81} = 2\pi\sqrt{0.09174} = 2\pi(0.3029) = 1.90$ s.
(b) $T_M = 2\pi\sqrt{0.90/3.71} = 2\pi\sqrt{0.2426} = 2\pi(0.4925) = 3.09$ s.
(c) Percentage change $= \dfrac{3.09 - 1.90}{1.90} \times 100\% = \boxed{62.6\% \text{ increase}}$.

Bài tập luyện tập

A vehicle's suspension system is designed to return to equilibrium in the shortest possible time *without* oscillating up and down. This describes:

(A) Light damping
(B) Critical damping
(C) Heavy (over-) damping
(D) Resonance

Lời giải chi tiết

By definition, critical damping returns the system to equilibrium fastest while just avoiding any oscillation. **(B)**.

Bài tập luyện tập

A 0.25 kg mass oscillates on a spring with $k = 100$ N m^{-1} and amplitude 0.080 m. Find (a) the total energy of the system; (b) the speed at $x = 0.040$ m; (c) the displacement(s) at which $KE = PE$.

Lời giải chi tiết

(a) $E = \frac{1}{2}kx_0^2 = \frac{1}{2}(100)(0.080)^2 = \frac{1}{2}(100)(0.0064) = 0.32$ J.
(b) $PE = \frac{1}{2}kx^2 = \frac{1}{2}(100)(0.040)^2 = \frac{1}{2}(100)(0.0016) = 0.080$ J. $KE = E - PE = 0.32 - 0.080 = 0.24$ J.

$$v = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2(0.24)}{0.25}} = \sqrt{1.92} = \boxed{1.39 \text{ m s}^{-1}}.$$

(c) $KE = PE$ means each equals $E/2 = 0.16$ J. $PE = \frac{1}{2}kx^2 = 0.16 \Rightarrow x^2 = \dfrac{2(0.16)}{100} = 0.0032 \Rightarrow x = \boxed{\pm 0.057 \text{ m}}$.

Bài tập luyện tập

Soldiers marching in step across a bridge at a certain pace cause the bridge deck to oscillate with dangerously large amplitude. This is best explained by:

(A) Damping
(B) Resonance
(C) Diffraction
(D) Polarization

Lời giải chi tiết

The marching provides a periodic driving force; when its frequency matches (or nearly matches) the bridge's natural frequency, resonance produces a large-amplitude response. **(B)**.

Bài tập luyện tập

Sound is a ________ wave, while light is a ________ wave.

(A) transverse; longitudinal
(B) longitudinal; transverse
(C) longitudinal; longitudinal
(D) transverse; transverse

Lời giải chi tiết

Sound requires compressions/rarefactions parallel to its direction of travel (longitudinal); light is an electromagnetic wave whose field oscillations are perpendicular to its direction of travel (transverse). **(B)**.

Bài tập luyện tập

A sound wave has frequency 256 Hz and speed 340 m s^{-1} in air. (a) Find its wavelength in air. (b) Find its period. (c) Find its wavelength after entering water, where the speed of sound is 1480 m s^{-1} (frequency is unchanged).

Lời giải chi tiết

(a) $\lambda = \dfrac{v}{f} = \dfrac{340}{256} = \boxed{1.33 \text{ m}}$.

(b) $T = \dfrac{1}{f} = \dfrac{1}{256} = \boxed{3.91 \times 10^{-3} \text{ s}}$.

(c) $\lambda_{\text{water}} = \dfrac{1480}{256} = \boxed{5.78 \text{ m}}$.

Bài tập luyện tập

Two points on a transverse wave of wavelength 0.80 m are separated by 0.60 m measured along the direction of travel. Find the phase difference between the two points in (a) fraction of a cycle, (b) degrees, (c) radians.

Lời giải chi tiết

(a) $\dfrac{0.60}{0.80} = 0.75$ cycle.

(b) $0.75 \times 360^\circ = \boxed{270^\circ}$.

(c) $0.75 \times 2\pi = 1.5\pi \approx \boxed{4.71 \text{ rad}}$.

Bài tập luyện tập

A point source has intensity I at distance r. What is its intensity at distance $3r$?

(A) $I/3$
(B) $I/9$
(C) $3I$
(D) $9I$

Lời giải chi tiết

$I \propto 1/r^2$, so tripling r divides intensity by $3^2 = 9$. **(B)**.

Bài tập luyện tập

A loudspeaker radiates sound uniformly in all directions with power 8.0×10^{-3} W. Find (a) the intensity at 4.0 m; (b) the intensity at 8.0 m; (c) the ratio of wave amplitudes at these two distances.

Lời giải chi tiết

(a) $I_4 = \dfrac{P}{4\pi r^2} = \dfrac{8.0 \times 10^{-3}}{4\pi(4.0)^2} = \boxed{3.98 \times 10^{-5} \text{ W m}^{-2}}$.

(b) $I_8 = \dfrac{8.0 \times 10^{-3}}{4\pi(8.0)^2} = \boxed{9.95 \times 10^{-6} \text{ W m}^{-2}}$.

(c) $I_4/I_8 = 4.0$ (distance doubled, intensity quartered); since $I \propto A^2$, amplitude ratio $= \sqrt{4.0} = \boxed{2.0}$.

Bài tập luyện tập

Arrange the following in order of *increasing* wavelength: X-ray, microwave, visible light, gamma ray.

(A) gamma $<$ X-ray $<$ visible $<$ microwave
(B) microwave $<$ visible $<$ X-ray $<$ gamma
(C) gamma $<$ visible $<$ X-ray $<$ microwave
(D) X-ray $<$ gamma $<$ visible $<$ microwave

Lời giải chi tiết

Gamma rays have the shortest wavelength, then X-rays, then visible light, then microwaves (longest of the four). **(A)**.

Bài tập luyện tập

Unpolarized light of intensity 80 W m^{-2} passes through a polarizer, then through an analyser whose transmission axis is at 60° to the first polarizer. Find the intensity emerging from the analyser.

Lời giải chi tiết

After the first (unpolarized → polarized) step: $I_1 = I_0/2 = 80/2 = 40$ W m^{-2}. Malus's law for the analyser: $I_2 = I_1 \cos^2(60^\circ) = 40 \times (0.500)^2 = 40 \times 0.250 = \boxed{10 \text{ W m}^{-2}}$.

Bài tập luyện tập

Light travels from water ($n = 1.33$) into air. (a) If the angle of incidence in the water is 25°, find the angle of refraction in the air. (b) Find the critical angle for total internal reflection at the water-air boundary.

Lời giải chi tiết

(a) $1.33 \sin 25^\circ = 1.00 \sin\theta_2 \Rightarrow \sin\theta_2 = 1.33(0.4226) = 0.5621 \Rightarrow \theta_2 = \boxed{34.2^\circ}$.

(b) $\sin\theta_c = \dfrac{n_2}{n_1} = \dfrac{1.00}{1.33} = 0.7519 \Rightarrow \theta_c = \boxed{48.8^\circ}$.

Bài tập luyện tập

A stable, observable interference pattern requires the two overlapping sources to be:

(A) equal in amplitude
(B) coherent
(C) polarized
(D) longitudinal

Lời giải chi tiết

A constant, unchanging phase relationship (coherence) between the sources is required; without it any interference pattern shifts too rapidly to be observed. Equal amplitude is not required for a pattern to exist (though it does affect how dark the minima are), and neither polarization nor being longitudinal is a requirement for interference at all. **(B)**.

Bài tập luyện tập

Two coherent sources emit waves of wavelength 0.50 m. At a point P the path difference is 1.75 m. (a) State, with reasoning, whether interference at P is constructive or destructive. (b) Find the smallest non-zero path difference that would produce constructive interference.

Lời giải chi tiết

(a) $\frac{1.75}{0.50} = 3.5 = 3 + \frac{1}{2}$, i.e. a half-integer number of wavelengths ⇒ **destructive interference**.

(b) Smallest non-zero constructive path difference is one full wavelength: $\boxed{0.50 \text{ m}}$.

Bài tập luyện tập

In a Young's double-slit setup, the slit separation is 0.25 mm, the screen is 1.50 m away, and the measured fringe spacing is 3.6 mm. Find the wavelength of the light used.

Lời giải chi tiết

$$\Delta y = \frac{\lambda D}{d} \Rightarrow \lambda = \frac{\Delta y\, d}{D} = \frac{(3.6 \times 10^{-3})(0.25 \times 10^{-3})}{1.50} = \frac{9.0 \times 10^{-7}}{1.50} = \boxed{6.0 \times 10^{-7} \text{ m} = 600 \text{ nm}}.$$

Bài tập luyện tập

In Young's double-slit experiment, if the slit separation d is decreased while D and λ stay fixed, the fringe spacing:

(A) decreases
(B) increases
(C) stays the same
(D) becomes zero

Lời giải chi tiết

$\Delta y = \lambda D/d$ is inversely proportional to d, so decreasing d increases Δy — physically, bringing the slits closer together makes the two paths to a given screen point diverge in angle more quickly, so the same path-difference conditions are met at a wider spacing on the screen. **(B)**.

Bài tập luyện tập

Light of wavelength 633 nm is incident on a single slit of width 0.015 mm. Find the angular position of the first diffraction minimum.

Lời giải chi tiết

$\sin\theta = \dfrac{\lambda}{a} = \dfrac{633\times10^{-9}}{1.5\times10^{-5}} = 0.0422 \Rightarrow \theta = \boxed{2.42^\circ}$.

Bài tập luyện tập

A diffraction grating has 300 lines per millimetre. Light of wavelength 589 nm is incident normally. (a) Find the grating spacing d. (b) Find the angle of the second-order maximum. (c) Find the highest order that can be observed.

Lời giải chi tiết

(a) $d = 1/(300\text{ mm}^{-1}) = 3.33\times10^{-3}\text{ mm} = \boxed{3.33\times10^{-6}\text{ m}}$.

(b) $\sin\theta_2 = \dfrac{2\lambda}{d} = \dfrac{2(589\times10^{-9})}{3.33\times10^{-6}} = 0.354 \Rightarrow \theta_2 = \boxed{20.7^\circ}$.

(c) $n_{\text{max}} \le d/\lambda = 3.33\times10^{-6}/589\times10^{-9} = 5.66 \Rightarrow \boxed{n=5}$.

Bài tập luyện tập

Reducing the width of a single slit (with λ fixed) causes the central diffraction maximum to:

(A) narrow
(B) widen
(C) stay the same width
(D) disappear

Lời giải chi tiết

Since $\sin\theta = \lambda/a$, a smaller a gives a larger θ to the first minimum, so the central maximum widens. This is the opposite of intuition based on ray optics: a narrower gap produces a *more* spread-out pattern, not a narrower one, because diffraction becomes more pronounced as the aperture shrinks toward the size of the wavelength. **(B)**.

Bài tập luyện tập

Adjacent nodes on a standing wave are separated by:

(A) $\lambda/4$
(B) $\lambda/2$
(C) λ
(D) 2λ

Lời giải chi tiết

Nodes occur every half wavelength along a standing wave pattern. **(B)**.

Bài tập luyện tập

A string of length 1.20 m, fixed at both ends, carries transverse waves at speed 60 m s^{-1}. Find (a) the fundamental frequency; (b) the frequency of the third harmonic; (c) the wavelength of the second harmonic.

Lời giải chi tiết

(a) $f_1 = \frac{v}{2L} = \frac{60}{2(1.20)} = \boxed{25 \text{ Hz}}$.

(b) $f_3 = 3f_1 = \boxed{75 \text{ Hz}}$.

(c) $\lambda_2 = \frac{2L}{2} = L = \boxed{1.20 \text{ m}}$.

Bài tập luyện tập

An open-open pipe and a closed (one end closed) pipe have the same length. Compared to the open-open pipe's fundamental frequency, the closed pipe's fundamental frequency is:

(A) the same
(B) half
(C) twice
(D) four times

Lời giải chi tiết

Open-open: $f_1 = v/(2L)$. Closed: $f_1 = v/(4L)$, exactly half the open-open value for the same L — a direct consequence of the closed pipe needing only a quarter wavelength (rather than a half wavelength) to fit between its node and antinode. **(B)**.

Bài tập luyện tập

A pipe closed at one end has length 0.75 m; the speed of sound in the air column is 340 m s^{-1}. (a) Find the fundamental frequency. (b) Find the frequency of the first overtone. (c) Explain why even harmonics are absent from this pipe.

Lời giải chi tiết

(a) $f_1 = \frac{v}{4L} = \frac{340}{4(0.75)} = \frac{340}{3.00} = \boxed{113 \text{ Hz}}$.

(b) First overtone is the third harmonic: $f_3 = 3f_1 = \boxed{340 \text{ Hz}}$.

(c) The closed end must always be a displacement node and the open end must always be a displacement antinode. Meeting both conditions simultaneously requires an odd number of quarter-wavelengths to fit the pipe length, so only odd-numbered harmonics ($n = 1, 3, 5, \ldots$) satisfy the boundary conditions; even harmonics cannot place a node at one end and an antinode at the other.

Bài tập luyện tập

A tuning fork of frequency 440 Hz is held above a vertical tube (open at the top, with the water level — and hence the effective closed bottom end — adjustable), speed of sound 340 m s^{-1}. Find (a) the shortest air-column length that produces resonance, and (b) the next-shortest length that also resonates.

Lời giải chi tiết

Resonance in a closed (one-end) air column occurs when $L = \lambda/4, 3\lambda/4, 5\lambda/4, \ldots$ $\lambda = \frac{v}{f} = \frac{340}{440} = 0.7727$ m.

(a) Shortest: $L = \frac{\lambda}{4} = \frac{0.7727}{4} = \boxed{0.193 \text{ m } (19.3 \text{ cm})}$.

(b) Next: $L = \frac{3\lambda}{4} = 3(0.1932) = \boxed{0.580 \text{ m } (58.0 \text{ cm})}$.

Bài tập luyện tập

As an ambulance with siren sounding approaches you, then passes and recedes, the pitch you hear:

(A) increases continuously throughout

(B) is higher while approaching and lower while receding, with an abrupt drop at closest approach

(C) stays constant

(D) is lower while approaching

Lời giải chi tiết

While approaching, the source motion raises the observed frequency above f; the instant it passes, the relative motion switches to receding and the frequency drops to below f — heard as a sudden downward jump in pitch at the moment of closest approach, not a smooth, continuous change throughout. **(B)**.

Bài tập luyện tập

A train's horn emits 300 Hz. The train approaches a stationary observer at 25 m s^{-1}, then recedes at the same speed after passing. Take the speed of sound as 340 m s^{-1}. Find the frequency heard (a) while approaching, (b) while receding.

Lời giải chi tiết

(a) $f' = \frac{fv}{v - v_s} = \frac{300(340)}{340 - 25} = \frac{102\,000}{315} = \boxed{324 \text{ Hz}}$.

(b) $f' = \frac{fv}{v + v_s} = \frac{300(340)}{340 + 25} = \frac{102\,000}{365} = \boxed{279 \text{ Hz}}$.

Bài tập luyện tập

A stationary source emits a frequency of 1000 Hz; speed of sound $= 340$ m s^{-1}. (a) An observer moves away from the source at 15 m s^{-1}: find the frequency detected. (b) If instead the source moves toward the (now also moving) observer at 10 m s^{-1}, while the observer moves toward the source at 15 m s^{-1}, find the frequency detected.

Lời giải chi tiết

(a) $f' = \frac{f(v - v_o)}{v} = \frac{1000(340 - 15)}{340} = \frac{1000(325)}{340} = \boxed{956 \text{ Hz}}$.

(b) $f' = \dfrac{f(v+v_o)}{v-v_s} = \dfrac{1000(340+15)}{340-10} = \dfrac{1000(355)}{330} = \boxed{1076 \text{ Hz}}$.

Bài tập luyện tập

A radar gun emits a signal toward an approaching car and analyses the reflected signal to determine speed. This technology relies fundamentally on:

(A) diffraction
(B) the Doppler effect
(C) total internal reflection
(D) polarization

Lời giải chi tiết

The frequency shift between the emitted and reflected signal (caused by the car's motion) is a Doppler shift, from which the car's speed is calculated. **(B)**.

Bài tập luyện tập

A hydrogen emission line normally at $\lambda = 486.1$ nm in the laboratory is observed at 492.0 nm in a distant galaxy's spectrum. (a) State whether the galaxy is approaching or receding. (b) Estimate the galaxy's speed relative to Earth.

Lời giải chi tiết

(a) Observed wavelength (492.0 nm) is longer than the lab value (486.1 nm) — a redshift, so the galaxy is **receding**.

(b) $\Delta\lambda = 492.0 - 486.1 = 5.9$ nm.

$$v \approx c \cdot \frac{\Delta\lambda}{\lambda} = (3.00 \times 10^8) \times \frac{5.9}{486.1} = \boxed{3.64 \times 10^6 \text{ m s}^{-1}}.$$

Bài tập luyện tập

Medical Doppler ultrasound is used to measure blood-flow speed non-invasively. Explain, in terms of the physics of the Doppler effect, how this measurement works.

Lời giải chi tiết

A transducer emits high-frequency ultrasound into the body. Moving red blood cells act as (effectively) moving reflectors: as the cells move toward or away from the transducer, the reflected ultrasound is Doppler-shifted up or down in frequency relative to the emitted frequency. By measuring the size (and sign) of this frequency shift and using the Doppler relation to solve for the relevant speed, the flow speed and direction of the blood can be determined without any surgical intervention.

Bài tập luyện tập

A galaxy recedes from Earth at $v = 6.0 \times 10^6$ m s^{-1}. A spectral line is emitted at a laboratory wavelength of 434.0 nm. Find the observed wavelength.

Lời giải chi tiết

$$\frac{\Delta\lambda}{\lambda} \approx \frac{v}{c} = \frac{6.0 \times 10^6}{3.00 \times 10^8} = 0.0200.$$

$$\Delta\lambda = 0.0200 \times 434.0 = 8.68 \text{ nm}.$$

Receding ⇒ redshift (longer wavelength): $\lambda_{\text{obs}} = 434.0 + 8.68 = \boxed{442.7 \text{ nm}}$.

Bài tập luyện tập

In an experiment, a student measures the acceleration of an oscillating mass at several known displacements and plots acceleration a (vertical axis) against displacement x (horizontal axis), obtaining a straight line through the origin with gradient -36 s^{-2}. (a) Find the angular frequency ω and the period T. (b) If the oscillating mass is 0.50 kg on a spring, find the spring constant k.

Lời giải chi tiết

(a) Since $a = -\omega^2 x$, the gradient of an a-x graph is $-\omega^2$, so $\omega^2 = 36$ s^{-2} ⇒ $\omega = 6.0$ rad s^{-1}.

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{6.0} = \boxed{1.05 \text{ s}}.$$

(b) $k = m\omega^2 = 0.50 \times 36 = \boxed{18 \text{ N m}^{-1}}$.

Bài tập luyện tập

A student performs a simple pendulum experiment, measuring the period for several different lengths and plotting T^2 (vertical axis, in s^2) against L (horizontal axis, in m). The resulting straight line through the origin has gradient 4.02 s^2 m^{-1}. Use this gradient to determine the local gravitational field strength g.

Lời giải chi tiết

Since $T^2 = \frac{4\pi^2}{g}L$, the gradient of the T^2-L graph equals $4\pi^2/g$.

$$g = \frac{4\pi^2}{\text{gradient}} = \frac{4\pi^2}{4.02} = \frac{39.48}{4.02} = \boxed{9.82 \text{ m s}^{-2}},$$

consistent (within experimental uncertainty) with the accepted value $g = 9.81$ m s^{-2}.

Bài tập luyện tập

A pipe open at both ends has length 0.60 m; the speed of sound in the air inside is 340 m s^{-1}. Find (a) the fundamental frequency, and (b) the frequency of the second harmonic.

Lời giải chi tiết

(a) $f_1 = \frac{v}{2L} = \frac{340}{2(0.60)} = \frac{340}{1.20} = \boxed{283 \text{ Hz}}$.

(b) $f_2 = 2f_1 = 2(283) = \boxed{567\text{ Hz}}$.

Bài tập luyện tập

Which region of the electromagnetic spectrum is primarily responsible for the images captured by a thermal (heat-sensing) imaging camera?

(A) ultraviolet
(B) infrared
(C) X-ray
(D) microwave

Lời giải chi tiết

Warm objects emit strongly in the infrared, which thermal cameras are designed to detect. **(B)**.

Bài tập luyện tập

White light entering a glass prism separates into a spectrum of colours because:

(A) different colours travel at different speeds in vacuum
(B) the refractive index of glass depends very slightly on wavelength
(C) glass absorbs all colours except one at a time
(D) diffraction spreads the colours apart as they enter the glass

Lời giải chi tiết

All colours of light travel at the same speed c in vacuum; inside glass, however, the refractive index n is very slightly wavelength-dependent (dispersion), so each colour refracts by a slightly different amount. **(B)**.

Bài tập luyện tập

Compared with the same oscillating system under light damping, the same system under heavy damping has a resonance curve (amplitude against driving frequency) that is:

(A) taller and narrower
(B) taller and broader
(C) shorter and narrower
(D) shorter and broader

Lời giải chi tiết

Increasing damping reduces the peak amplitude at resonance and spreads the response over a wider range of driving frequencies, producing a shorter, broader curve. **(D)**.

Bài tập luyện tập

Explain, in terms of wavefronts, why an aircraft travelling faster than the speed of sound produces a sonic boom.

Lời giải chi tiết

Each wavefront the aircraft emits spreads outward at the speed of sound v. If the aircraft itself moves slower than v, it never catches up with wavefronts it has already emitted, and they simply bunch closer together ahead of it (an ordinary Doppler compression). If the aircraft moves faster than v, however, it continually outruns its own previously emitted wavefronts; these wavefronts cannot spread out ahead of the aircraft at all and instead pile up together along a cone-shaped shock front trailing the aircraft. When this concentrated shock front passes an observer on the ground, the sudden, large pressure change is heard as a sonic boom.

Bài tập luyện tập

An oscillator undergoing SHM has a displacement-time graph that is a cosine curve of amplitude 5.0 cm and period 2.0 s. Find (a) the maximum speed, and (b) the maximum acceleration of the oscillator.

Lời giải chi tiết

$\omega = \frac{2\pi}{T} = \frac{2\pi}{2.0} = \pi$ rad s^{-1} $= 3.1416$ rad s^{-1}. $x_0 = 5.0$ cm $= 0.050$ m.

(a) $v_{\text{max}} = \omega x_0 = 3.1416 \times 0.050 = \boxed{0.157 \text{ m s}^{-1}}$.

(b) $a_{\text{max}} = \omega^2 x_0 = (3.1416)^2 \times 0.050 = 9.870 \times 0.050 = \boxed{0.493 \text{ m s}^{-2}}$.

Bài tập luyện tập

On graphs of KE and PE plotted against time for an undamped SHM oscillator, which statement is correct?

(A) The two curves are identical at all times

(B) Each curve has the same period as the displacement-time graph $x(t)$

(C) Each curve completes two full cycles for every one cycle of $x(t)$

(D) KE and PE never sum to a constant value

Lời giải chi tiết

Since $KE \propto \sin^2(\omega t)$ and $PE \propto \cos^2(\omega t)$, each curve oscillates at twice the frequency of $x(t)$ (i.e. half the period), completing two full cycles for every one cycle of the displacement. KE and PE are out of phase with each other (not identical) but always sum to the constant total energy E. **(C)**.

Bài tập luyện tập

Compared with a two-slit (Young's) interference pattern produced with the same slit spacing d and wavelength λ, a diffraction grating pattern with many more slits at the same spacing d produces bright fringes that are:

(A) wider and dimmer

(B) narrower and brighter

(C) the same width and brightness

(D) located at completely different angles

Lời giải chi tiết

The maxima occur at the same angles in both cases (since $d \sin\theta = n\lambda$ depends only on d and λ, not on the number of slits), but with many more slits contributing coherently, the grating maxima are far narrower (sharper angular selectivity) and far brighter (more total energy channelled into each maximum) than the broad, low-contrast fringes from just two slits. **(B)**.

Bài tập luyện tập

Light travelling in air ($n = 1.00$) strikes a transparent block at an angle of incidence of $40.0°$; the angle of refraction inside the block is measured as $25.0°$. Find (a) the refractive index of the block, and (b) the speed of light inside the block.

Lời giải chi tiết

(a) $n_1 \sin\theta_1 = n_2 \sin\theta_2 \Rightarrow 1.00 \sin 40.0° = n_2 \sin 25.0°$

$$n_2 = \frac{\sin 40.0°}{\sin 25.0°} = \frac{0.6428}{0.4226} = \boxed{1.52}.$$

(b) $n_2 = \dfrac{c}{v} \Rightarrow v = \dfrac{c}{n_2} = \dfrac{3.00 \times 10^8}{1.52} = \boxed{1.97 \times 10^8 \text{ m s}^{-1}}$.

Theme D: Fields

Mục tiêu Unit: Trường hấp dẫn — cường độ trường, thế năng hấp dẫn, thế hấp dẫn, tốc độ thoát li, năng lượng quỹ đạo (AHL); trường điện — định luật Coulomb, cường độ điện trường, thế điện, tụ điện; trường từ — lực từ lên dây dẫn và hạt mang điện chuyển động; cảm ứng điện từ, định luật Faraday–Lenz, máy biến áp (AHL). Sau chương này, học sinh có thể thiết lập và sử dụng thành thạo mọi công thức trường hấp dẫn, điện và từ, vẽ đúng các đường sức, và giải các bài toán định lượng nhiều bước kết hợp nhiều loại trường.

4.1 D.1 Gravitational Fields

The concept of a **field** is one of the most powerful ideas in physics: rather than asking how one object can exert a force on another across empty space, field theory says that a mass, charge, or current modifies the surrounding space itself, and any second object placed in that modified space simply responds to the local conditions at its own location. Gravitational fields are the first, and conceptually simplest, example of this idea — every other field studied in this chapter (electric, magnetic) is built on exactly the same logic: a source produces a field; a second object placed in that field experiences a force, energy, and can be described using a potential.

4.1.1 The Gravitational Field Concept and Field Strength

A **gravitational field** is a region of space in which any object with mass experiences a gravitational force. Rather than thinking of gravity as a mysterious action at a distance, field theory describes the source mass as modifying the space around it, so that any other mass placed anywhere in that space feels a force. The strength of this effect at any point is described by the **gravitational field strength**, g, defined as the gravitational force experienced per unit mass placed at that point:

$$g = \frac{F}{m}.$$

Field strength is a **vector** quantity: it has both a magnitude (in $\mathrm{N\,kg^{-1}}$, equivalent to $\mathrm{m\,s^{-2}}$) and a direction — always pointing *towards* the source mass, since gravity is always attractive. A small "test mass" m is imagined at the point of interest purely to probe the field; the field itself exists whether or not a test mass is actually there, and its strength does not depend on the size of the test mass used to measure it.

For a point mass M, or for any point outside a spherically symmetric distribution of mass M (a direct consequence of the shell theorem — a uniform sphere attracts external objects exactly as if all its mass were concentrated at its centre), Newton's law of gravitation gives the magnitude of the force on a test mass m at distance r from the centre as

$$F = \frac{GMm}{r^2},$$

where $G = 6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$ is the universal gravitational constant. Combining this with the defining relation $F = mg$:

$$mg = \frac{GMm}{r^2} \quad \Longrightarrow \quad g = \frac{GM}{r^2}.$$

Notice that the test mass m cancels — g depends only on the source mass M and the distance r, exactly as a field quantity should. This is the same relation used throughout earlier mechanics work to write weight as $W = mg$, now generalised to any distance r from any spherical mass, not just at one planet's surface.

Gravitational field strength (force per unit mass), a vector, always directed towards the source mass:

$$g = \frac{F}{m}, \qquad g = \frac{GM}{r^2} \text{ (point mass, or outside a spherically symmetric mass } M\text{).}$$

Units: $\mathrm{N\,kg^{-1}} \equiv \mathrm{m\,s^{-2}}$.

VN **GIẢI THÍCH TIẾNG VIỆT**

Trường hấp dẫn là một vùng không gian mà tại đó một vật có khối lượng sẽ chịu lực hấp dẫn. Cường độ trường g được định nghĩa là lực trên một đơn vị khối lượng ($g = F/m$) — đây là một đại lượng **vector**, luôn hướng **vào** vật nguồn (vì lực hấp dẫn luôn là lực hút). Với khối lượng điểm hoặc bên ngoài một khối cầu phân bố đều, $g = GM/r^2$: chú ý khối lượng thử m đã triệt tiêu — g chỉ phụ thuộc vào khối lượng nguồn M và khoảng cách r, không phụ thuộc vật đặt vào để "đo" trường. Công thức quen thuộc $g = 9.81$ m/s^2 ở bề mặt Trái Đất chỉ là trường hợp riêng của $g = GM/r^2$ khi $r = R_{\text{Trái Đất}}$.

Ví dụ mẫu · Field strength at altitude

Earth has mass $M = 5.97 \times 10^{24}$ kg and mean radius $R = 6.37 \times 10^6$ m. Find the gravitational field strength (a) at Earth's surface, and (b) at the summit of Mount Everest, altitude $h = 8848$ m above sea level.

(a) At the surface, $r = R = 6.37 \times 10^6$ m:

$$g = \frac{GM}{R^2} = \frac{(6.67 \times 10^{-11})(5.97 \times 10^{24})}{(6.37 \times 10^6)^2} = 9.81 \text{ m/s}^2.$$

(b) At the summit, $r = R + h = 6.37 \times 10^6 + 8848 = 6.379 \times 10^6$ m:

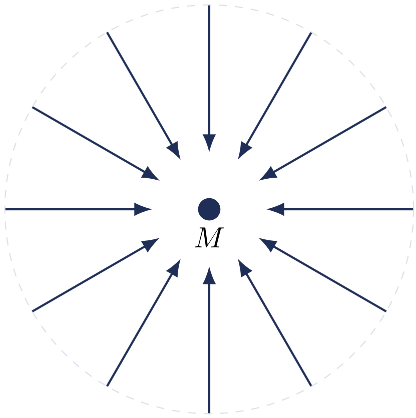
$$g = \frac{GM}{r^2} = \frac{(6.67 \times 10^{-11})(5.97 \times 10^{24})}{(6.379 \times 10^6)^2} = 9.79 \text{ m/s}^2.$$

Even at the top of the highest mountain on Earth, g has fallen by only about $0.2\,\%$ — this is why g can be treated as constant over everyday heights, but the small decrease is real and measurable, and becomes significant for satellites at altitudes of hundreds of kilometres.

4.1.2 Field Lines: Point Mass vs. Near a Planet's Surface

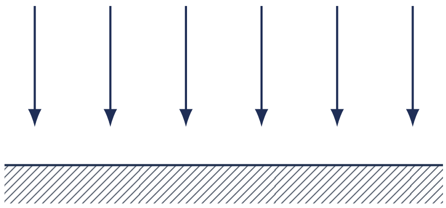
Gravitational fields are visualised using **field lines**: directed lines drawn so that (i) at every point, the tangent to the line gives the direction of g at that point, and (ii) the density of lines (how closely spaced they are) represents the field's magnitude — closely packed lines mean a strong field, widely spaced lines mean a weak one. For an isolated point mass (or any spherical mass, viewed from outside), the field lines are straight, **radial** lines pointing directly inward towards the centre, spaced increasingly far apart as r increases — consistent with $g \propto 1/r^2$ falling off with distance.

This convention — tangent gives direction, spacing gives magnitude — is used consistently for every field studied in this chapter (gravitational, electric, and magnetic). One general consequence follows immediately from the definition: field lines can never cross one another. At any point in space, a field has exactly one direction (the vector sum of all sources acting there); if two field lines crossed, the field would have two different directions at that single point, which is not physically possible.



Field lines around an isolated point mass M: radial, pointing inward, spreading apart with distance (field weakens as $1/r^2$).

Very close to the surface of a large body such as a planet, however, an observer only sees a tiny patch of the overall radial pattern. Over the small range of heights relevant to everyday objects (a building, an aircraft, even a mountain), the field lines are so close to parallel, and so evenly spaced, that the field looks **uniform**: parallel lines pointing straight down, with (to a very good approximation) constant magnitude $g \approx 9.81$ m/s^2.



Near a planet's surface, over a small region, the same radial field looks locally uniform: parallel, equally-spaced field lines, magnitude $g \approx$ constant.

(!) Lỗi thường gặp: Học sinh hay nghĩ "gần bề mặt hành tinh, g không đổi" là một định luật riêng biệt, tách rời khỏi $g = GM/r^2$. Thực ra đây chỉ là một **gần đúng cục bộ**: trường vẫn là trường xuyên tâm $g = GM/r^2$, nhưng vì $h \ll R$ (độ cao khảo sát rất nhỏ so với bán kính hành tinh) nên $r \approx R$ hầu như không đổi, và các đường sức toả ra từ tâm trông gần như song song trên một vùng nhỏ. Nếu độ cao h so sánh được với R (ví dụ vệ tinh ở quỹ đạo), phải dùng công thức đầy đủ $g = GM/r^2$, **không** được dùng $g = 9.81$ m/s^2.

VN GIẢI THÍCH TIẾNG VIỆT

Đường sức trường hấp dẫn: hướng tiếp tuyến cho biết **chiều** của g, mật độ đường cho biết **độ lớn** của g. Với một khối cầu cô lập, đường sức toả ra theo hình xuyên tâm, hướng vào tâm, thưa dần khi ra xa (tương ứng $g \propto 1/r^2$). Trên phạm vi nhỏ gần bề mặt một hành tinh lớn, đoạn cung xuyên tâm đó trông gần như thẳng và song song — đây là lý do vì sao trong đời sống hàng ngày ta luôn coi trọng lực là "thẳng đứng, hướng xuống, độ lớn không đổi".

4.1.3 Combining Gravitational Fields from Multiple Masses

Real problems rarely involve just one isolated source mass. When several masses $M_1, M_2, \ldots$ each produce their own gravitational field at a point P, the **principle of superposition** states that the total field at P is simply the **vector sum** of the individual fields each mass would produce on its own:

$$\vec{g}_{\text{net}} = \vec{g}_1 + \vec{g}_2 + \cdots,$$

with each $\vec{g}_i = GM_i/r_i^2$ directed from P towards M_i. Because field strength is a vector, contributions that point in the same direction add directly, while contributions pointing in opposite directions partially or fully cancel; contributions at an angle to each other must be resolved into components (exactly as with any vector addition in mechanics) and combined component-by-component. This is the field-theory analogue of superposing forces, and it applies equally to any number of source masses.

Superposition of gravitational fields: the net field at a point due to several masses is the vector sum of the fields each mass would produce individually:

$$\vec{g}_{\text{net}} = \sum_i \vec{g}_i, \qquad \vec{g}_i = \frac{GM_i}{r_i^2} \text{ (directed towards } M_i\text{)}.$$

VN **GIẢI THÍCH TIẾNG VIỆT**

Nguyên lý chồng chất (superposition): trường hấp dẫn tổng hợp tại một điểm do nhiều khối lượng gây ra bằng **tổng vector** của từng trường thành phần, $\vec{g}_{\text{net}} = \sum_i \vec{g}_i$, mỗi $\vec{g}_i$ hướng về phía khối lượng M_i tương ứng. Nếu các điểm nguồn và điểm khảo sát nằm trên cùng một đường thẳng, có thể cộng/trừ đại số theo chiều đã quy ước; nếu không thẳng hàng, phải phân tích theo các thành phần vuông góc rồi cộng riêng từng thành phần — hoàn toàn giống cách cộng lực trong cơ học.

Ví dụ mẫu · Superposition of gravitational fields

Two point masses, $M_1 = 4.0 \times 10^5$ kg and $M_2 = 6.0 \times 10^5$ kg, lie on a line, 10.0 m apart. Point P lies on the line between them, 4.0 m from M_1 and 6.0 m from M_2. Find the magnitude and direction of the net gravitational field at P.

The field due to M_1 at P points towards M_1; the field due to M_2 at P points towards M_2 — these are **opposite** directions, since P lies between the two masses.

$$g_1 = \frac{GM_1}{r_1^2} = \frac{(6.67 \times 10^{-11})(4.0 \times 10^5)}{(4.0)^2} = 1.67 \times 10^{-6} \text{ m/s}^2 \text{ (towards } M_1\text{)}.$$

$$g_2 = \frac{GM_2}{r_2^2} = \frac{(6.67 \times 10^{-11})(6.0 \times 10^5)}{(6.0)^2} = 1.11 \times 10^{-6} \text{ m/s}^2 \text{ (towards } M_2\text{)}.$$

Since the two contributions point in opposite directions, they subtract:

$$g_{\text{net}} = g_1 - g_2 = 1.67 \times 10^{-6} - 1.11 \times 10^{-6} = \boxed{5.6 \times 10^{-7} \text{ m/s}^2, \text{ directed towards } M_1}$$

(the larger of the two contributions "wins", since $g_1 > g_2$ here).

4.1.4 Gravitational Potential Energy

Additional Higher Level (AHL) content.

Because the gravitational force is attractive, moving two masses *apart* always requires work to be done against the force, while letting them fall *together* releases energy. To build a consistent energy bookkeeping system that works at any separation (not just near one planet's surface), physicists define gravitational potential energy relative to a reference point at $r = \infty$, where the masses no longer interact at all, and set $E_p = 0$ there.

Consider a mass m a distance r from mass M. Taking the outward radial direction as positive, the gravitational force on m has component $F_r = -\dfrac{GMm}{r^2}$ (negative because the force always points inward, towards M). Potential energy is defined so that the work done by the field as m moves from infinity to r equals $-(E_p(r) - E_p(\infty))$; equivalently,

$$E_p(r) = -\int_\infty^r F_r\,\mathrm{d}r' = -\int_\infty^r \left(-\frac{GMm}{r'^2}\right)\mathrm{d}r' = GMm\int_\infty^r \frac{\mathrm{d}r'}{r'^2} = GMm\left[-\frac{1}{r'}\right]_\infty^r .$$

Evaluating the limits ($1/r' \to 0$ as $r' \to \infty$):

$$E_p(r) = GMm\left(-\frac{1}{r} - 0\right) = -\frac{GMm}{r}.$$

Gravitational potential energy of a mass m a distance r from mass M (taking $E_p = 0$ at $r = \infty$):

$$E_p = -\frac{GMm}{r}.$$

E_p is **always negative** for any finite r — a signature of a **bound** system: energy must be added (work done) to separate the masses to infinity, where $E_p \to 0$ from below. E_p becomes less negative (increases) as r increases, and more negative (decreases) as r decreases.

This general result must connect smoothly to the familiar near-surface formula $E_p = mgh$. Near a planet's surface, write $r = R + h$ with $h \ll R$:

$$E_p(R+h) - E_p(R) = -\frac{GMm}{R+h} - \left(-\frac{GMm}{R}\right) = GMm\left(\frac{1}{R} - \frac{1}{R+h}\right) = GMm\cdot\frac{h}{R(R+h)}.$$

Since $h \ll R$, the denominator $R(R+h) \approx R^2$, giving

$$E_p(R+h) - E_p(R) \approx \frac{GMm}{R^2}h = g\,m\,h,$$

using $g = GM/R^2$ from the previous subsection. This confirms that $E_p = mgh$ is not a separate law but the **local, linear approximation** to $E_p = -GMm/r$, valid only for height changes small compared to the planet's radius, and only for differences in E_p (it uses an arbitrary local zero at the surface, whereas $-GMm/r$ uses a universal zero at infinity).

VN **GIẢI THÍCH TIẾNG VIỆT**

Thế năng hấp dẫn tổng quát là $E_p = -GMm/r$, luôn **âm**, với gốc thế năng bằng 0 đặt tại $r = \infty$ (hai vật ở rất xa nhau, không còn tương tác). Công thức quen thuộc $E_p = mgh$ chỉ là **xấp xỉ tuyến tính cục bộ** của công thức tổng quát, đúng khi độ cao h rất nhỏ so với bán kính hành tinh R — và nó tính **hiệu** thế năng so với một mốc tự chọn ở bề mặt, chứ không phải giá trị tuyệt đối so với vô cực. Lỗi thường gặp: dùng $E_p = mgh$ cho vệ tinh ở quỹ đạo cao — sai, vì khi đó h so sánh được với R, phải dùng $E_p = -GMm/r$.

(!) Lỗi thường gặp: Đừng nhầm dấu: $E_p = -GMm/r$ mang dấu **âm**, và nó **tăng** (bớt âm đi) khi r tăng, chứ không phải giảm. Vật rơi vào gần khối nguồn hơn (r giảm) thì E_p giảm (âm hơn) và động năng tăng — đúng với trực giác vật lý (vật "rơi" giải phóng năng lượng). Học sinh thường quên dấu trừ khi thay số, dẫn đến kết quả dương vô lý cho một hệ liên kết (bound system).

4.1.5 Gravitational Potential and Equipotential Surfaces

Additional Higher Level (AHL) content.

Just as field strength g is force per unit mass, **gravitational potential** V is potential energy per unit mass:

$$V = \frac{E_p}{m} = -\frac{GM}{r}.$$

Unlike g, potential is a **scalar** — it has a magnitude (always negative, in J kg^{-1}) but no direction, which makes it far easier to combine contributions from several masses (simply add the potentials algebraically, whereas field strengths must be added as vectors). Potential is defined at every point in space around M, independent of whether any particular test mass happens to be there.

If a mass m moves between two points where the potential is V_1 and V_2, the change in its potential energy is $\Delta E_p = m(V_2 - V_1) = m\,\Delta V$, and this equals the work that must be done *against* the gravitational field to make that move (by an external agent, at constant speed):

$$W = m\,\Delta V.$$

If $\Delta V > 0$ (moving to a point of higher potential, i.e. further from M), positive external work is required. If $\Delta V < 0$ (moving closer to M), the field itself does positive work, and $W < 0$ for the external agent — equivalently, the object could accelerate under gravity alone, converting potential energy into kinetic energy.

A surface on which every point has the same potential is called an **equipotential surface**. For a spherical mass M, the equipotential surfaces are simply concentric spheres centred on M (since $V = -GM/r$ depends only on r). A key property follows directly from $W = m\Delta V$: because $\Delta V = 0$ for any two points on the *same* equipotential surface, **no net work is done** moving a mass along an equipotential surface. Physically, this is because the gravitational force (and hence g) is always perpendicular to an equipotential surface — motion along the surface has no component in the direction of the force.

> **Gravitational potential** (potential energy per unit mass), a scalar:
>
> $$V = \frac{E_p}{m} = -\frac{GM}{r}, \qquad W = m\,\Delta V \;\; (\text{work to move mass } m \text{ between two points}).$$
>
> **Equipotential surfaces** around a point/spherical mass are concentric spheres; field lines are always perpendicular to them; no work is done moving along one.

> **GIẢI THÍCH TIẾNG VIỆT**
> VN
> Thế hấp dẫn $V = E_p/m = -GM/r$ là một đại lượng **vô hướng** (scalar) — khác với cường độ trường g là vector. Công để di chuyển khối lượng m giữa hai điểm chỉ đơn giản là $W = m\Delta V$. **Mặt đẳng thế** (equipotential surface) là tập hợp các điểm có cùng giá trị V — với khối cầu M, đó là các mặt cầu đồng tâm. Không tốn công di chuyển dọc theo một mặt đẳng thế, vì lực hấp dẫn (và g) luôn vuông góc với mặt đó tại mọi điểm — di chuyển "ngang" không có thành phần nào cùng phương với lực.

Because it is a scalar, gravitational potential from several masses also combines by simple algebraic addition, $V_{\text{net}} = \sum_i V_i = \sum_i(-GM_i/r_i)$, with no need to resolve directions into components — exactly the same simplification demonstrated for electric potential in Section D.2.3, and a useful alternative to vector superposition of $\vec{g}$ whenever only an energy-related quantity (potential, potential energy, or work) is actually required.

4.1.6 Escape Velocity

Additional Higher Level (AHL) content.

Escape velocity is the minimum launch speed an object needs, fired from a planet's surface with no further propulsion, to escape the planet's gravitational pull completely — that is, to reach $r = \infty$ with (at minimum) zero kinetic energy remaining. This is an energy-conservation problem: the total mechanical energy at launch must be at least zero (the value of E_{total} at $r = \infty$ with $v = 0$).

At the surface ($r = R$), with launch speed v_{esc}:

$$E_{\text{total}} = \text{KE} + E_p = \frac{1}{2}mv_{\text{esc}}^2 - \frac{GMm}{R}.$$

Setting this equal to zero (the minimum condition — just barely escaping, arriving at infinity with zero speed left):

$$\frac{1}{2}mv_{\text{esc}}^2 - \frac{GMm}{R} = 0 \implies \frac{1}{2}mv_{\text{esc}}^2 = \frac{GMm}{R}.$$

The mass m of the escaping object cancels (as it must for a field quantity), giving

$$v_{\text{esc}}^2 = \frac{2GM}{R} \implies v_{\text{esc}} = \sqrt{\frac{2GM}{R}}.$$

Escape velocity from the surface of a spherical mass M, radius R:

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}}.$$

Independent of the mass of the escaping object; depends only on the properties of the body being escaped from. Note $v_{\text{esc}} = \sqrt{2}\times$ the speed of a (hypothetical) circular orbit skimming the surface, $v_{\text{orb}} = \sqrt{GM/R}$.

Ví dụ mẫu · Escape velocity of Earth

Find the escape velocity from Earth's surface. ($M = 5.97 \times 10^{24}$ kg, $R = 6.37 \times 10^6$ m.)

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2(6.67 \times 10^{-11})(5.97 \times 10^{24})}{6.37 \times 10^6}} = \sqrt{1.250 \times 10^8} = 1.12 \times 10^4 \text{ m/s}.$$

So $v_{\text{esc}} \approx 11.2$ km/s — roughly 33 times the speed of sound in air. This is the speed a projectile would need at launch (ignoring air resistance) to leave Earth's gravity entirely and never fall back, without any further engine burn.

(!) Lỗi thường gặp: Lỗi thường gặp: quên hệ số 2 trong công thức, viết nhầm $v_{\text{esc}} = \sqrt{GM/R}$ (đây thực ra là công thức tốc độ quỹ đạo tròn sát bề mặt, không phải tốc độ thoát li). Cách nhớ chắc chắn: luôn xuất phát từ bảo toàn năng lượng $\frac{1}{2}mv^2 = GMm/R$, không học thuộc lòng công thức cuối một cách máy móc.

VN **GIẢI THÍCH TIẾNG VIỆT**

Tốc độ thoát li $v_{\text{esc}} = \sqrt{2GM/R}$ là tốc độ **tối thiểu** để một vật phóng đi từ bề mặt (không cần động cơ thêm) thoát khỏi lực hấp dẫn của hành tinh mãi mãi. Xuất phát từ bảo toàn năng lượng: tổng động năng ban đầu vừa đủ bù cho thế năng âm ban đầu, để năng lượng toàn phần bằng 0 — vật "vừa đủ" đến vô cực với vận tốc bằng 0. Khối lượng m của vật thoát li tự triệt tiêu trong phép tính, giống hệt cách m triệt tiêu khi tính g — công thức chỉ phụ thuộc hành tinh xuất phát (M, R), không phụ thuộc vật được phóng.

Field strength and escape velocity both depend only on the mass and radius of the body in question, so they vary considerably across the solar system. The table below applies $g = GM/R^2$ and $v_{\text{esc}} = \sqrt{2GM/R}$ consistently to several bodies, illustrating how both quantities scale with M and R.

Body	Mass (kg)	Surface g (m/s^2)	v_{esc} (km/s)
Mercury	3.30×10^{23}	3.70	4.25
Venus	4.87×10^{24}	8.88	10.4
Earth	5.97×10^{24}	9.81	11.2
Mars	6.42×10^{23}	3.73	5.03
Jupiter	1.90×10^{27}	25.9	60.2
The Moon	7.35×10^{22}	1.62	2.37

Two patterns are worth noting. First, g does not simply track mass: Mars is roughly twice Mercury's mass yet has almost identical surface gravity, because Mars is also a larger, less dense body (larger R partially cancels the larger M in $g = GM/R^2$). Second, v_{esc} grows much faster than g for very massive bodies like Jupiter, since it depends on $\sqrt{M/R}$ rather than M/R^2 — this is why gas giants can retain even the lightest gases (hydrogen, helium) in their atmospheres, while small, low-g bodies like the Moon cannot retain any atmosphere at all over geological time.

It is worth noting that $v_{\text{esc}} = \sqrt{2GM/R}$ is independent of launch *direction* as well as of the escaping object's mass: the derivation used only conservation of energy (total KE and E_p at launch versus at infinity), which does not depend on which direction the object initially travels — firing an object straight up or at any angle requires exactly the same launch speed to escape (though not the same launch speed to just barely *graze* the surface again if it falls short, which does depend on direction). Purely classically, this same escape-velocity formula suggests an extreme limiting case: if a body's mass is compressed into a small enough radius R that v_{esc} would need to exceed the speed of light c, then (in a fully relativistic treatment) not even light itself could escape — a **black hole**. Setting $v_{\text{esc}} = c$ in the (classical, non-relativistic) formula gives an estimate of the critical radius:

$$c = \sqrt{\frac{2GM}{R}} \quad \Longrightarrow \quad R \approx \frac{2GM}{c^2}.$$

For Earth's mass, this radius works out to about $\frac{2(6.67\times10^{-11})(5.97\times10^{24})}{(3.00\times10^{8})^2} \approx 8.9 \times 10^{-3}$ m — under 1 cm: if Earth's entire mass were compressed inside a sphere smaller than this, it would (by this classical estimate) become a black hole. Remarkably, this simple classical calculation gives the same result as the much more sophisticated general-relativistic treatment (the Schwarzschild radius), even though the physical reasoning behind it (Newtonian escape velocity) is not strictly valid at such extreme conditions.

4.1.7 Orbital Energetics (AHL)

This content is assessed at Additional Higher Level (AHL) only.

A satellite of mass m moving in a circular orbit of radius r around a mass M is held in its orbit purely by gravity: the gravitational force supplies exactly the centripetal force required for circular motion. Equating the two:

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \quad \Longrightarrow \quad v^2 = \frac{GM}{r}.$$

The orbital kinetic energy is therefore

$$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{GM}{r}\right) = \frac{GMm}{2r}.$$

The potential energy at radius r is, as before, $E_p = -GMm/r$. The total mechanical energy of the orbiting satellite is the sum:

$$E = \text{KE} + E_p = \frac{GMm}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r}.$$

For a satellite of mass m in a **circular orbit** of radius r around mass M:

$$\text{KE} = \frac{GMm}{2r}, \qquad E_p = -\frac{GMm}{r}, \qquad E_{\text{total}} = \text{KE} + E_p = -\frac{GMm}{2r}.$$

Note the exact relations $E_p = -2\,\text{KE}$ and $E_{\text{total}} = -\text{KE} = \frac{1}{2}E_p$, which hold for *any* circular gravitational orbit.

These fixed ratios between KE, E_p, and E_{total} are a useful shortcut: if a problem gives (or asks for) just one of the three energies for a circular orbit, the other two follow immediately without needing to recompute r. They also give a compact way to describe what happens as a satellite's orbit decays (e.g. due to a trace of atmospheric drag at low altitude): as r slowly decreases, $E_{\text{total}} = -GMm/2r$ becomes *more* negative, so the satellite actually **speeds up** (KE increases) even as it loses total mechanical energy — drag removes energy overall, but the accompanying drop to a smaller, more tightly bound orbit converts more than enough potential energy into kinetic energy to increase the orbital speed. This counterintuitive result is a classic conceptual IB exam question.

The total energy $E = -GMm/2r$ is **negative** for every circular orbit, confirming these are **bound** systems: the satellite cannot drift away to infinity without an external energy input. This connects directly to escape velocity: a trajectory with exactly $E_{\text{total}} = 0$ corresponds to a particle that reaches $r = \infty$ with precisely zero speed left — an unbound, borderline (**parabolic**) trajectory, the same condition used to derive v_{esc}. A trajectory with $E_{\text{total}} > 0$ is also unbound but retains nonzero kinetic energy at infinity — a **hyperbolic** trajectory, as followed by an interstellar probe passing through a solar system without being captured. The three cases are summarised below.

Total energy	Trajectory type	Physical outcome
$E_{\text{total}} < 0$	elliptical / circular	bound orbit, cannot reach $r = \infty$
$E_{\text{total}} = 0$	parabolic	just escapes; $v \to 0$ as $r \to \infty$
$E_{\text{total}} > 0$	hyperbolic	escapes with speed to spare at $r = \infty$

VN **GIẢI THÍCH TIẾNG VIỆT**

Với quỹ đạo tròn quanh khối M: động năng KE $= GMm/2r$ (dương), thế năng $E_p = -GMm/r$ (âm gấp đôi động năng về độ lớn), và tổng năng lượng $E = \text{KE} + E_p = -GMm/2r$ — luôn **âm**, thể hiện đây là hệ **liên kết** (bound). Muốn đưa vệ tinh lên quỹ đạo cao hơn (r tăng), E tăng dần về 0 (bớt âm) — cần cấp thêm năng lượng. Khi $E = 0$: vật vừa đủ thoát ra vô cực với vận tốc bằng 0 (quỹ đạo parabol — biên giới giữa "bị giữ lại" và "thoát li"); khi $E > 0$: quỹ đạo hyperbol, vật thoát ra ngoài với vận tốc dư.

Ví dụ mẫu · Orbital energy and raising an orbit

A satellite of mass $m = 800$ kg orbits Earth ($M = 5.97 \times 10^{24}$ kg) in a circular orbit at altitude 400 km, so $r_1 = R + h = 6.37 \times 10^6 + 4.00 \times 10^5 = 6.77 \times 10^6$ m. (a) Find its kinetic energy, potential energy, and total energy. (b) Find the minimum extra energy needed to send the satellite from this orbit to infinity (i.e. to escape Earth entirely). (c) Find the extra energy needed to raise it instead to a new circular orbit at $r_2 = 2r_1 = 1.354 \times 10^7$ m.

(a)

$$\text{KE} = \frac{GMm}{2r_1} = \frac{(6.67\times10^{-11})(5.97\times10^{24})(800)}{2(6.77\times10^{6})} = 2.35\times10^{10}\ \text{J}.$$

$$E_p = -\frac{GMm}{r_1} = -4.71\times10^{10}\ \text{J}, \qquad E_1 = \text{KE} + E_p = -2.35\times10^{10}\ \text{J}.$$

(b) To escape, the satellite's final total energy must reach (at minimum) $E_\infty = 0$. The energy that must be supplied is

$$\Delta E_{\text{escape}} = E_\infty - E_1 = 0 - (-2.35\times10^{10}) = 2.35\times10^{10}\ \text{J}.$$

(c) At the new radius $r_2 = 1.354\times10^{7}$ m:

$$E_2 = -\frac{GMm}{2r_2} = -1.18\times10^{10}\ \text{J}.$$

$$\Delta E = E_2 - E_1 = (-1.18\times10^{10}) - (-2.35\times10^{10}) = 1.18\times10^{10}\ \text{J}.$$

Doubling the orbital radius requires about 1.18×10^{10} J — roughly half the energy needed to escape entirely, consistent with $E \propto -1/r$: moving from r_1 to $2r_1$ closes half the "energy gap" between the original orbit and full escape ($r \to \infty$).

(!) Lỗi thường gặp: Đừng nhầm: năng lượng **cần cấp thêm** để chuyển quỹ đạo luôn là $\Delta E = E_{\text{sau}} - E_{\text{trước}}$ (năng lượng cuối trừ năng lượng đầu), **không** phải hiệu độ lớn hay lấy trị tuyệt đối tuỳ ý. Vì E càng gần 0 khi r càng lớn, nâng quỹ đạo lên cao hơn luôn cần $\Delta E > 0$ (cấp thêm năng lượng dương), dù E vẫn có thể âm ở quỹ đạo mới.

4.2 D.2 Electric Fields and Capacitance

Electric fields describe how electric charges interact, using precisely the same field-theory framework as gravity in the previous section: a source charge modifies the surrounding space, and any second charge placed there experiences a force. The mathematics is almost identical to the gravitational case — inverse-square force laws, field strength as force per unit "test" quantity, potential energy and potential defined via integrals of force — but with one crucial complication: charge (unlike mass) comes in two signs, so electric forces and fields can be either attractive or repulsive, and electric potential energy can be either positive or negative.

4.2.1 Coulomb's Law

Electric charge is a fundamental property of matter that comes in two kinds, positive and negative. **Coulomb's law** describes the force between two point charges q_1 and q_2 separated by distance r:

$$F = \frac{kq_1q_2}{r^2}, \qquad k = 8.99\times10^{9}\ \text{N m}^2\,\text{C}^{-2}.$$

The constant k is related to the **permittivity of free space**, $\varepsilon_0 = 8.85\times10^{-12}$ C^2 N^{-1} m^{-2}, by $k = 1/(4\pi\varepsilon_0)$; both forms appear on formula sheets and are numerically consistent. The force acts along the straight line joining the two charges: it is **repulsive** when q_1 and q_2 have the same sign, and **attractive** when they have opposite signs — the sign of the product q_1q_2 automatically tells you which.

Coulomb's law has exactly the same mathematical form as Newton's law of gravitation — both are inverse-square laws in the separation r — but the two forces differ in one essential way: gravity is *always* attractive (mass has only one "sign"), whereas the electric force can be either attractive or repulsive depending on the signs of the charges involved. This structural analogy extends through field strength, potential energy, and potential, summarised below.

Quantity	Gravitational	Electric
Force law	$F = \frac{GMm}{r^2}$ (always attractive)	$F = \frac{kq_1q_2}{r^2}$ (attractive or repulsive)
Field strength	$g = \frac{F}{m} = \frac{GM}{r^2}$	$E = \frac{F}{q} = \frac{kq}{r^2}$
Potential energy	$E_p = -\frac{GMm}{r}$ (always negative)	$E_p = \frac{kq_1q_2}{r}$ (sign of q_1q_2)
Potential	$V = -\frac{GM}{r}$ (always negative)	$V = \frac{kq}{r}$ (sign of q)
Work moving through Δ(potential)	$W = m\,\Delta V$	$W = q\,\Delta V$

GIẢI THÍCH TIẾNG VIỆT

VN Định luật Coulomb $F = kq_1q_2/r^2$ có **cùng dạng toán học** với định luật hấp dẫn Newton — đều là định luật nghịch đảo bình phương khoảng cách. Điểm khác biệt cốt lõi: lực hấp dẫn **luôn luôn hút** (vì khối lượng chỉ có một "dấu"), còn lực điện có thể **hút hoặc đẩy** tuỳ theo dấu của q_1 và q_2 — cùng dấu thì đẩy, trái dấu thì hút. Ghi nhớ bảng tương tự (analogy) giữa hai trường giúp nhớ công thức nhanh hơn: chỉ cần thay $G \to k$, $M \to q_1$, $m \to q_2$ — nhưng luôn phải tự kiểm tra dấu và chiều lực riêng cho từng bài toán điện.

Ví dụ mẫu · Coulomb's law

Two point charges, $q_1 = +3.0\ \mu$C and $q_2 = -5.0\ \mu$C, are separated by $r = 0.20$ m in air. Find the magnitude and nature (attractive/repulsive) of the force between them.

$$F = \frac{kq_1q_2}{r^2} = \frac{(8.99\times10^9)(3.0\times10^{-6})(5.0\times10^{-6})}{(0.20)^2} = \frac{0.13485}{0.04} = 3.4\ \text{N}.$$

Since q_1 and q_2 have opposite signs, the force is **attractive**: each charge pulls the other directly along the line joining them, with magnitude $\boxed{F \approx 3.4\ \text{N}}$.

(!) Lỗi thường gặp: Định luật Coulomb chỉ cho **độ lớn** lực khi thay số trực tiếp (dùng trị tuyệt đối của q_1, q_2); chiều lực (hút/đẩy) phải suy luận riêng từ dấu của q_1q_2, không tự động ra từ phép tính. Một lỗi khác: quên đổi đơn vị μC ($= 10^{-6}$ C) hoặc nC ($= 10^{-9}$ C) sang C trước khi thay vào công thức — đây là lỗi số học phổ biến nhất trong các bài Coulomb.

4.2.2 Electric Field Strength and Field Lines

The **electric field strength** E at a point is defined as the electric force experienced per unit *positive* charge placed at that point:

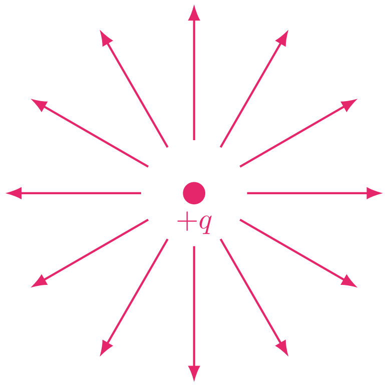
$$E = \frac{F}{q}.$$

Like g, this is a vector field, with direction defined by the direction of the force on a small positive test charge. For a point charge q, combining this definition with Coulomb's law (exactly as was done for g) gives

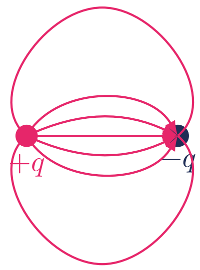
$$E = \frac{kq}{r^2}.$$

If the source charge q is positive, E points radially *outward* (a positive test charge would be pushed away); if q is negative, E points radially *inward* (a positive test charge would be pulled in). The force on any other charge q' placed in the field is then simply $F = q'E$ — repulsive push in the direction of E if $q' > 0$, and in the opposite direction to E if $q' < 0$.

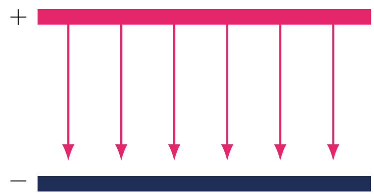
Field-line diagrams for electric fields follow the same rules as for gravity: tangent gives direction, spacing gives relative strength. Three canonical patterns are essential:



(a) single positive point charge



(b) electric dipole ($+q$ and $-q$)



(c) uniform field between charged parallel plates

Field (a) is radial, exactly like a gravitational point-mass field, but pointing *outward* for a positive charge (gravity's field lines always point *inward*, since there is no such thing as "negative mass" to repel). Field (b), the **dipole**, shows lines leaving the positive charge and curving around to terminate on the negative charge — field lines always start on positive charge and end on negative charge. Field (c) shows the region between two oppositely charged parallel plates: away from the edges, the lines are straight, parallel, and evenly spaced, indicating a **uniform** field of constant magnitude and direction, from the positive plate to the negative plate.

Electric field strength (force per unit positive charge), a vector:

$$E = \frac{F}{q}, \qquad E = \frac{kq}{r^2} \text{ (point charge).}$$

Field lines point away from positive charges, into negative charges; line spacing shows relative field strength; between oppositely charged parallel plates (away from the edges) the field is uniform.

VN **GIẢI THÍCH TIẾNG VIỆT**

Cường độ điện trường $E = F/q$ là lực trên một đơn vị điện tích **dương** thử — khác với trường hấp dẫn, E có thể hướng ra ngoài (điện tích dương) hoặc hướng vào trong (điện tích âm). Đường sức điện luôn **xuất phát từ điện tích dương** và **kết thúc ở điện tích âm**. Vùng giữa hai bản tụ tích điện trái dấu (xa mép bản) có điện trường **đều** — các đường sức song song, cách đều nhau — đây là mô hình quan trọng bậc nhất trong các bài tụ điện và hạt mang điện chuyển động giữa hai bản.

Coulomb's law itself was established experimentally in the 1780s using a torsion balance — a lightweight rod suspended by a fine fibre, with a charged sphere at one end deflected by the force from a second fixed charged sphere, the fibre's twist providing a sensitive measure of a very small force. Confirming the inverse-square dependence experimentally (rather than simply assuming it by analogy with gravity) was essential, since even small deviations from an exact inverse-square law would have significant consequences — for instance, the interior of a hollow charged conductor would not be perfectly field-free (see below) if the exponent in Coulomb's law differed even slightly from exactly 2.

One important consequence of these field ideas: inside a solid conductor in electrostatic equilibrium, the electric field is always **zero**. Any excess charge placed on a conductor redistributes itself entirely on the outer surface, arranging itself so that the field inside exactly cancels; if a nonzero field existed inside, free charges (electrons) would continue to accelerate in response to it, contradicting the assumption that the charges have settled into equilibrium. This is the basis of **electrostatic shielding**: a hollow conducting enclosure (a Faraday cage) shields its interior from external electric fields, since the induced surface charges on the conductor cancel the external field everywhere inside — the reason sensitive electronic equipment is often housed in metal enclosures, and why a car (a largely metal shell) offers some protection from external electric fields such as those from lightning.

Ví dụ mẫu · Field of a point charge

A point charge $q = +2.0\ \mu$C sits in isolation. (a) Find the electric field strength at a point 0.50 m away. (b) Find the force on a small charge $q' = +4.0$ nC placed at that point.

(a)

$$E = \frac{kq}{r^2} = \frac{(8.99\times10^9)(2.0\times10^{-6})}{(0.50)^2} = 7.2\times10^4\ \text{N/C},$$

directed radially outward from q (since q is positive).

(b)

$$F = q'E = (4.0\times10^{-9})(7.2\times10^4) = 2.9\times10^{-4}\ \text{N},$$

directed away from q (repulsive, since both charges are positive).

4.2.3 Combining Electric Fields from Multiple Charges

The same superposition principle used for gravitational fields applies to electric fields: the net electric field at a point due to several point charges is the **vector sum** of the fields each charge would produce on its own,

$$\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 + \cdots.$$

The only extra subtlety compared with gravity is that each contribution $\vec{E}_i = kq_i/r_i^2$ points *away* from its source charge if $q_i > 0$, and *towards* it if $q_i < 0$ — the direction of each term depends on the sign of the corresponding charge, not just its position, so the sign must be tracked carefully before adding vectors (unlike gravity, where every contribution always points towards its source).

Superposition of electric fields:

$$\vec{E}_{\text{net}} = \sum_i \vec{E}_i, \qquad \vec{E}_i = \frac{kq_i}{r_i^2}\ \text{(away from } q_i \text{ if } q_i > 0\text{; towards } q_i \text{ if } q_i < 0\text{).}$$

GIẢI THÍCH TIẾNG VIỆT

VN

Nguyên lý chồng chất điện trường giống hệt trường hấp dẫn về mặt toán học (cộng vector), nhưng có một điểm khác biệt quan trọng: chiều của mỗi vector thành phần phụ thuộc vào **dấu** của điện tích nguồn — hướng ra xa nếu nguồn dương, hướng vào nếu nguồn âm — chứ không phải luôn hướng về nguồn như trường hấp dẫn. Học sinh cần vẽ hình rõ ràng, xác định đúng chiều từng vector thành phần trước khi cộng, đặc biệt khi các điện tích không thẳng hàng với điểm khảo sát.

Ví dụ mẫu · Superposition of electric fields

Two positive point charges, $q_1 = +5.0\ \mu$C and $q_2 = +8.0\ \mu$C, lie on a line, 0.50 m apart. Point P lies on the same line, beyond q_2, at a distance of 0.80 m from q_1 and 0.30 m from q_2. Find the net electric field at P.

Since P lies beyond both positive charges on the same side, both fields point in the **same** direction at P (away from both charges, further along the line):

$$E_1 = \frac{kq_1}{r_1^2} = \frac{(8.99 \times 10^9)(5.0 \times 10^{-6})}{(0.80)^2} = 7.0 \times 10^4 \text{ N/C.}$$

$$E_2 = \frac{kq_2}{r_2^2} = \frac{(8.99 \times 10^9)(8.0 \times 10^{-6})}{(0.30)^2} = 8.0 \times 10^5 \text{ N/C.}$$

Since both point the same way, they add directly:

$$E_{\text{net}} = E_1 + E_2 = (7.0 \times 10^4) + (8.0 \times 10^5) = \boxed{8.7 \times 10^5 \text{ N/C}},$$

directed away from both charges, along the line. Notice the closer charge (q_2, only 0.30 m away) dominates the total despite being the smaller charge — the inverse-square dependence on distance matters more here than the size of the charge.

Because potential is a **scalar**, superposing potentials from multiple charges is significantly simpler than superposing fields: there are no directions to resolve into components — every contribution is simply added (or subtracted, if a source charge is negative) as a plain number.

Ví dụ mẫu · Superposition of electric potential

Two point charges, $q_1 = +4.0\ \mu$C and $q_2 = -2.0\ \mu$C, are placed 0.60 m apart. Find the electric potential at the midpoint between them.

The midpoint is $r = 0.30$ m from each charge. Since potential is a scalar, the contributions simply add algebraically (with sign):

$$V = \frac{kq_1}{r} + \frac{kq_2}{r} = \frac{(8.99 \times 10^9)(4.0 \times 10^{-6})}{0.30} + \frac{(8.99 \times 10^9)(-2.0 \times 10^{-6})}{0.30}.$$

$$V = (1.20 \times 10^5) + (-6.0 \times 10^4) = \boxed{6.0 \times 10^4 \text{ V}}.$$

No components or directions were needed anywhere in this calculation — a direct illustration of why potential (energy-based, scalar) calculations are often preferred over field (force-based, vector) calculations when only an energy-related quantity is ultimately required.

4.2.4 Electric Potential Energy and Electric Potential

Additional Higher Level (AHL) content.

By exactly the same reasoning used for gravity — integrating the Coulomb force from infinity to separation r, with $E_p = 0$ at $r = \infty$ — the **electric potential energy** of a pair of point charges q_1 and

q_2 separated by r is

$$E_p = \frac{kq_1q_2}{r}.$$

Unlike the gravitational case, this can be positive *or* negative: $E_p > 0$ for like charges (energy must be supplied to bring them together against their mutual repulsion — pushing them apart releases this energy), and $E_p < 0$ for unlike charges (energy is released bringing them together, since they attract; separating them to infinity requires work).

Electric potential V is potential energy per unit charge — a scalar field, exactly analogous to gravitational potential:

$$V = \frac{E_p}{q}, \qquad V = \frac{kq}{r} \text{ (point charge } q\text{, potential at distance } r).$$

As with gravity, moving a charge q between two points where the potential is V_1 and V_2 changes its potential energy by $\Delta E_p = q(V_2 - V_1) = q\,\Delta V$, and this equals the work done against the field by an external agent:

$$W = q\,\Delta V.$$

Electric potential energy of two point charges, and **electric potential** of a point charge (a scalar):

$$E_p = \frac{kq_1q_2}{r}, \qquad V = \frac{E_p}{q} = \frac{kq}{r}, \qquad W = q\,\Delta V.$$

$E_p > 0$ for like charges, $E_p < 0$ for unlike charges; V takes the sign of the source charge q.

VN **GIẢI THÍCH TIẾNG VIỆT**

Thế năng điện của hai điện tích điểm $E_p = kq_1q_2/r$ có thể **dương** (hai điện tích cùng dấu — cần tốn công đẩy chúng lại gần nhau) hoặc **âm** (trái dấu — chúng tự hút nhau lại, giải phóng năng lượng). Thế điện $V = kq/r$ mang dấu của điện tích nguồn q: dương nếu nguồn dương, âm nếu nguồn âm — khác hẳn thế hấp dẫn $V = -GM/r$ luôn luôn âm. Công thức $W = q\Delta V$ dùng chung logic với trường hấp dẫn: điện tích di chuyển từ nơi thế cao đến nơi thế thấp thì trường thực hiện công dương lên điện tích dương.

Ví dụ mẫu · Electric potential energy of two charges

Two point charges, $q_1 = +4.0\ \mu$C and $q_2 = +6.0\ \mu$C, are held 0.30 m apart. Find the electric potential energy of the system, and state what this value means physically.

$$E_p = \frac{kq_1q_2}{r} = \frac{(8.99 \times 10^9)(4.0 \times 10^{-6})(6.0 \times 10^{-6})}{0.30} = 0.72 \text{ J}.$$

The result is positive because both charges are positive (mutually repulsive): 0.72 J of external work was required to assemble the two charges at this separation, starting from infinite separation. If released, the two charges would fly apart, converting this 0.72 J of potential energy into kinetic energy.

4.2.5 The Uniform Field Between Parallel Plates

Two parallel conducting plates, separated by distance d and connected across a potential difference (voltage) V, produce a field between them that is uniform: constant in both magnitude and direction, from the positive plate to the negative plate (as shown in the field-line figure above), except very close to the edges of the plates.

To find the magnitude of this field, consider moving a positive test charge q directly from the positive plate to the negative plate, a distance d, against the field. The work done is $W = qV$ (definition of potential difference). Because the field is uniform, this is also simply force $\times$ distance: $W = Fd = (qE)d = qEd$. Equating the two expressions for W:

$$qV = qEd \quad \Longrightarrow \quad V = Ed \quad \Longrightarrow \quad E = \frac{V}{d}.$$

Uniform field between parallel plates, separation d, potential difference V:

$$E = \frac{V}{d}.$$

Units: V/m $\equiv$ N/C (these are equivalent units for electric field strength). The field points from the positive plate towards the negative plate.

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Điện trường đều giữa hai bản tụ song song: $E = V/d$, với V là hiệu điện thế giữa hai bản, d là khoảng cách giữa chúng. Đơn vị V/m và N/C là **tương đương** — hãy nhớ cả hai cách viết vì đề thi IB dùng cả hai. Công thức này chỉ áp dụng cho vùng **giữa** hai bản, xa mép bản (nơi trường không còn đều nữa).

Ví dụ mẫu · Field and force between charged plates

Two parallel plates are separated by $d = 2.0$ cm and connected across a potential difference $V = 500$ V. (a) Find the electric field strength between the plates. (b) A proton ($q = +1.60 \times 10^{-19}$ C, $m = 1.67 \times 10^{-27}$ kg) is released from rest next to the positive plate. Find the force on it, and its initial acceleration.

(a)

$$E = \frac{V}{d} = \frac{500}{0.020} = 2.5 \times 10^4 \text{ V/m}.$$

(b)

$$F = qE = (1.60 \times 10^{-19})(2.5 \times 10^4) = 4.0 \times 10^{-15} \text{ N}.$$

$$a = \frac{F}{m} = \frac{4.0 \times 10^{-15}}{1.67 \times 10^{-27}} = 2.4 \times 10^{12} \text{ m/s}^2.$$

The proton accelerates directly towards the negative plate with this (enormous) acceleration — a reminder that even modest laboratory voltages produce huge accelerations on subatomic particles, because their masses are so small.

(!) Lỗi thường gặp: $E = V/d$ chỉ đúng cho trường **đều** (giữa hai bản phẳng song song, xa mép); **không** được áp dụng công thức này cho điện trường của một điện tích điểm — ở đó phải dùng $E = kq/r^2$. Học sinh cũng hay quên đổi cm sang m khi tính d, dẫn đến sai kết quả đúng 2 bậc số mũ (10^2).

The uniform field between charged plates also produces a projectile-motion-like trajectory for a charged particle entering *parallel* to the plates (rather than being released from rest, as in the previous example): the particle's horizontal velocity component stays constant (no force acts horizontally), while the electric force produces constant acceleration perpendicular to the plates — exactly analogous to horizontal projectile motion under gravity, with g replaced by $a = qE/m$.

Ví dụ mẫu · Electron deflection between plates

An electron enters midway between two parallel plates, travelling parallel to them at $v_0 = 2.0 \times 10^7$ m/s. The plates are $L = 2.0$ cm long (in the direction of travel) and separated by $d = 1.0$ cm, with a potential difference of 100 V across them. Find the electron's vertical deflection by the time it exits the plates, and confirm it does not strike either plate.

The field and acceleration (perpendicular to the initial velocity):

$$E = \frac{V}{d} = \frac{100}{0.010} = 1.0{\times}10^4 \text{ V/m}, \qquad a = \frac{eE}{m_e} = \frac{(1.60 \times 10^{-19})(1.0 \times 10^4)}{9.11 \times 10^{-31}} = 1.76{\times}10^{15} \text{ m/s}^2.$$

Time spent between the plates (horizontal motion is unaffected by the field):

$$t = \frac{L}{v_0} = \frac{0.020}{2.0 \times 10^7} = 1.0 \times 10^{-9} \text{ s}.$$

Vertical deflection, exactly as in projectile motion ($y = \frac{1}{2}at^2$, starting with zero vertical velocity):

$$y = \frac{1}{2}at^2 = \frac{1}{2}(1.76 \times 10^{15})(1.0 \times 10^{-9})^2 = \boxed{8.8 \times 10^{-4} \text{ m } (0.88 \text{ mm})}.$$

Since the electron enters midway between the plates, it can deflect up to $d/2 = 5.0$ mm before striking a plate; 0.88 mm is well within this limit, so the electron safely exits the field region, deflected slightly towards the positive plate.

The balance between an electric force and gravity, for a charged object held between parallel plates, is the basis of one of the most famous experiments in physics: Millikan's oil-drop experiment, which first measured the elementary charge e.

Ví dụ mẫu · Balancing electric and gravitational forces

A tiny charged oil drop of mass $m = 1.6 \times 10^{-15}$ kg is held exactly stationary in the air between two horizontal parallel plates separated by $d = 5.0$ mm. The drop carries a charge equal to five elementary charges, $q = 5e = 8.0 \times 10^{-19}$ C. Find the potential difference that must be applied across the plates to hold the drop in equilibrium.

For the drop to be stationary, the upward electric force must exactly balance the downward gravitational force:

$$qE = mg \quad \Longrightarrow \quad E = \frac{mg}{q} = \frac{(1.6 \times 10^{-15})(9.81)}{8.0 \times 10^{-19}} = 1.96 \times 10^4 \text{ V/m}.$$

Using $E = V/d$:

$$V = Ed = (1.96 \times 10^4)(5.0 \times 10^{-3}) = \boxed{98 \text{ V}}.$$

This is exactly the method Millikan used: by varying V until different drops became stationary, and finding that the required charge on every drop was always a whole-number multiple of one smallest value, he demonstrated that electric charge is **quantised** and measured the elementary charge e.

4.2.6 Capacitance and Energy Storage

Additional Higher Level (AHL) content.

A **capacitor** is a device (commonly two conducting plates separated by an insulator) that stores electric charge — and hence electrical energy — when a potential difference is applied across it. Its

capacitance C is defined as the charge stored per unit potential difference:

$$C = \frac{Q}{V}, \qquad \text{unit: farad (F)} = \text{C/V}.$$

Capacitance is a property of the capacitor's geometry, not of Q or V individually — doubling V simply causes Q to double as well, keeping C fixed. For a **parallel-plate capacitor** with plate area A and separation d (with vacuum or air between the plates),

$$C = \frac{\varepsilon_0 A}{d}.$$

Capacitance increases with larger plate area (more room to store charge) and decreases with larger separation (the plates interact less strongly).

To find the energy stored in a charged capacitor, note that as charge is added in small increments dq, the voltage across the capacitor at that moment is $v = q/C$ (it rises as q builds up, following $Q = CV$), so the incremental work is d$W = v\,$dq. This is exactly the area of a thin vertical strip under a graph of voltage v (vertical axis) against accumulated charge q (horizontal axis) — and since $v = q/C$ is a straight line through the origin, the total work to charge the capacitor from 0 to final charge Q (at final voltage V) is the area of the triangle under that line:

$$E_{\text{stored}} = \frac{1}{2}QV.$$

Substituting $Q = CV$ gives two equivalent forms:

$$E_{\text{stored}} = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}.$$

Capacitance: $C = \frac{Q}{V}$ (farads, F); **parallel-plate capacitor:** $C = \frac{\varepsilon_0 A}{d}$.

Energy stored in a charged capacitor:

$$E_{\text{stored}} = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}.$$

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Điện dung $C = Q/V$ là một hằng số phụ thuộc **hình học** của tụ điện (diện tích bản A, khoảng cách d), không phụ thuộc điện tích hay hiệu điện thế đặt vào — $C = \varepsilon_0 A/d$ cho tụ phẳng. Năng lượng tích trữ $E = \frac{1}{2}QV = \frac{1}{2}CV^2 = Q^2/2C$ có hệ số $\frac{1}{2}$ vì hiệu điện thế **tăng dần** trong quá trình nạp (không phải không đổi ngay từ đầu) — đây là diện tích tam giác dưới đồ thị V–Q, **không** phải QV đơn giản (đó là công thức cho một điện tích di chuyển qua hiệu điện thế *không đổi*, khác với việc nạp tụ).

Ví dụ mẫu · Parallel-plate capacitor

A parallel-plate capacitor has plates of area $A = 2.0 \times 10^{-2}$ m² separated by $d = 1.0$ mm (vacuum/air gap), connected to a $V = 12$ V supply. Find (a) its capacitance, (b) the charge stored, (c) the energy stored.

(a)

$$C = \frac{\varepsilon_0 A}{d} = \frac{(8.85 \times 10^{-12})(2.0 \times 10^{-2})}{1.0 \times 10^{-3}} = 1.77 \times 10^{-10}\ \text{F} = 177\ \text{pF}.$$

(b)

$$Q = CV = (1.77 \times 10^{-10})(12) = 2.1 \times 10^{-9}\ \text{C} = 2.1\ \text{nC}.$$

(c)

$$E_{\text{stored}} = \frac{1}{2}CV^2 = \frac{1}{2}(1.77 \times 10^{-10})(12)^2 = 1.3 \times 10^{-8}\ \text{J} = 12.7\ \text{nJ}.$$

4.2.7 Capacitors in Series and Parallel

Additional Higher Level (AHL) content.

Capacitors, like resistors, are frequently connected in combinations, and the equivalent single capacitance of a network follows rules that are (perhaps surprisingly) the **opposite** of the rules for resistors.

For capacitors in **parallel**, each capacitor has the same potential difference V across it (they share the same two connection points), but each stores its own charge $Q_i = C_iV$. The total charge stored is the sum $Q_{\text{total}} = Q_1 + Q_2 + \cdots = (C_1 + C_2 + \cdots)V$, so the equivalent capacitance is

$$C_{\text{parallel}} = C_1 + C_2 + C_3 + \cdots$$

— parallel capacitors simply add, exactly like resistors in *series*. Physically, connecting capacitors in parallel is equivalent to increasing the effective plate area, which is exactly why $C = \varepsilon_0 A/d$ increases with A.

For capacitors in **series**, each capacitor carries the *same* charge Q (charge pushed onto the first plate of the chain must appear, by induction, as an equal and opposite charge at every subsequent junction), but the potential differences across each add up to the total: $V_{\text{total}} = V_1 + V_2 + \cdots = Q(1/C_1 + 1/C_2 + \cdots)$. Since $C_{\text{series}} = Q/V_{\text{total}}$,

$$\frac{1}{C_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

— series capacitors combine like resistors in *parallel*. Physically, connecting capacitors in series is equivalent to increasing the effective plate separation, which is exactly why $C = \varepsilon_0 A/d$ decreases with d. As with resistors in parallel, the equivalent series capacitance is always **smaller** than the smallest individual capacitance in the chain.

Capacitors in parallel: $C_{\text{parallel}} = C_1 + C_2 + \cdots$ (same V across each; charges add).

Capacitors in series: $\frac{1}{C_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$ (same Q through each; voltages add).

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Quy tắc ghép tụ điện **ngược lại** với quy tắc ghép điện trở: tụ điện mắc **song song** thì điện dung **cộng trực tiếp** ($C = C_1 + C_2 + \ldots$, giống điện trở mắc nối tiếp), còn tụ điện mắc **nối tiếp** thì **nghịch đảo cộng** ($1/C = 1/C_1 + 1/C_2 + \ldots$, giống điện trở mắc song song). Mẹo nhớ: mắc song song tụ điện giống như tăng diện tích bản (A tăng $\Rightarrow$ C tăng); mắc nối tiếp giống như tăng khoảng cách giữa bản (d tăng $\Rightarrow C$ giảm).

Ví dụ mẫu · Combining capacitors

Three capacitors, $C_1 = 2.0\ \mu$F, $C_2 = 3.0\ \mu$F, $C_3 = 6.0\ \mu$F, are connected (a) in parallel, (b) in series. Find the equivalent capacitance in each case.

(a) Parallel:

$$C_{\text{parallel}} = C_1 + C_2 + C_3 = 2.0 + 3.0 + 6.0 = \boxed{11.0\ \mu\text{F}}.$$

(b) Series:

$$\frac{1}{C_{\text{series}}} = \frac{1}{2.0} + \frac{1}{3.0} + \frac{1}{6.0} = 0.500 + 0.333 + 0.167 = 1.00\ \mu\text{F}^{-1}.$$

$$C_{\text{series}} = \boxed{1.0\ \mu\text{F}}.$$

As expected, the series combination ($1.0\ \mu$F) is smaller than the smallest individual capacitor ($2.0\ \mu$F), while the parallel combination ($11.0\ \mu$F) is larger than the largest individual capacitor ($6.0\ \mu$F).

(!) Lỗi thường gặp: Học sinh hay áp dụng nhầm công thức điện trở cho tụ điện — nhớ rằng hai công thức **đảo ngược** nhau: song song **cộng trực tiếp** với tụ điện (không phải nghịch đảo như điện trở), nối tiếp thì **nghịch đảo cộng** (không phải cộng trực tiếp như điện trở). Luôn kiểm tra lại bằng trực giác: ghép nối tiếp phải cho điện dung nhỏ hơn mọi tụ thành phần; ghép song song phải cho điện dung lớn hơn mọi tụ thành phần.

4.2.8 RC Circuits: Charging and Discharging

Additional Higher Level (AHL) content.

When a capacitor is charged or discharged through a resistor R, the current does not switch on or off instantaneously — it changes smoothly. As the capacitor charges, the voltage across it rises quickly at first, then more and more slowly as it approaches its final value, following an **exponential approach**: rapid initial change, tailing off asymptotically towards the final voltage but (mathematically) never quite reaching it in finite time. Discharging follows the mirror-image behaviour: an **exponential decay** from the initial voltage down towards zero, again fast at first and progressively slower.

The characteristic timescale of this exponential behaviour is the **time constant**:

$$\tau = RC.$$

A larger resistance R limits the current more strongly, so charge flows more slowly and τ is larger (slower charging/discharging); a larger capacitance C means more charge must flow to reach a given voltage, which also increases τ. After one time constant, the voltage (charging) has reached about $63\,\%$ of its final value; after roughly five time constants, the capacitor is, for practical purposes, fully charged or discharged.

RC time constant: $\tau = RC$ (units: seconds, since □ × F = s). Charging: voltage rises from 0 towards its final value with exponential shape. Discharging: voltage falls from its initial value towards 0 with exponential shape. Larger R or $C \Rightarrow$ slower charging/discharging (larger τ).

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Hằng số thời gian $\tau = RC$ đặc trưng cho tốc độ nạp/xả của tụ điện qua điện trở. Đồ thị điện áp theo thời gian có dạng **hàm mũ**: khi nạp, tăng nhanh lúc đầu rồi chậm dần khi tiến gần giá trị cuối; khi xả, giảm nhanh lúc đầu rồi chậm dần về 0. R hoặc C càng lớn thì τ càng lớn — quá trình nạp/xả càng chậm. IB chỉ yêu cầu hiểu **định tính** dạng đường cong và công thức $\tau = RC$, không cần thuộc công thức hàm mũ đầy đủ.

RC charging and discharging behaviour appears throughout everyday electronics. A camera flash charges a capacitor slowly (large τ, drawing modest current from a battery over a second or two) but discharges it almost instantly through the flash bulb (very small τ, since the bulb has very low resistance) to release a short, intense pulse of light. Timing circuits (for example, the interval between windscreen-wiper sweeps on an "intermittent" setting) often use an RC combination to set a controllable delay, since $\tau = RC$ can be tuned simply by changing R (e.g. with a variable resistor/potentiometer) without needing to change the capacitor.

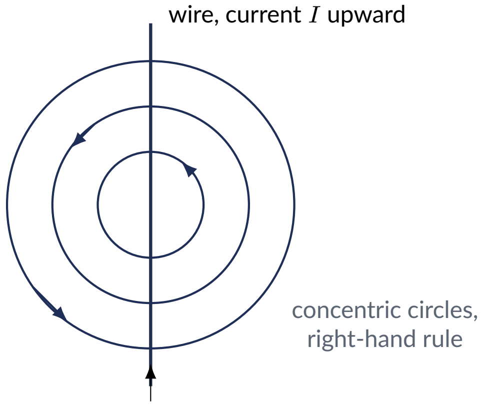
4.3 D.3 Magnetic Fields and Forces on Moving Charges

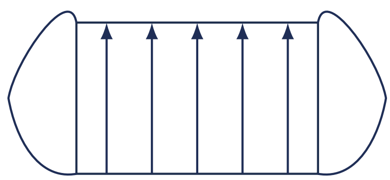
Magnetic fields differ from gravitational and electric fields in an important respect: they are produced not by a static source quantity (mass or charge at rest) but by **moving** charge — electric current. Correspondingly, a magnetic field only exerts a force on charge that is itself moving (or on a current-carrying wire); a stationary charge sitting in a magnetic field feels no magnetic force at all. This section develops the field patterns produced by currents, the force such fields exert on other currents and moving charges, and the resulting circular motion of charged particles — the foundation for later applications such as particle accelerators and mass spectrometers.

4.3.1 Magnetic Field Patterns

Moving electric charge — that is, electric current — produces a **magnetic field**. Unlike gravitational and electric field lines, magnetic field lines never start or end anywhere; they always form closed loops (a direct consequence of there being no known isolated magnetic "charge", or monopole). Three current configurations produce field patterns that recur throughout the course:

- **A long, straight current-carrying wire** produces a field of concentric circles around the wire, in planes perpendicular to the wire. The direction is given by the **right-hand rule**: point the right thumb in the direction of conventional current flow, and the curled fingers show the direction of B circling the wire. Field strength decreases with distance from the wire.
- **A current loop** produces field lines that loop through the centre of the loop (all in the same direction, given by curling the right-hand fingers in the direction of current flow, thumb pointing through the loop) and curve back around the outside — the pattern resembles a single "turn" of a bar magnet.
- **A solenoid** (a tightly wound coil of many loops) produces a field that is strong and very nearly **uniform** inside the coil, parallel to its axis, closely resembling the field of a bar magnet — with field lines emerging from one end (behaving like a north pole) and re-entering the other (behaving like a south pole), then looping back around the outside.





Left: field around a long straight wire (viewed end-on), concentric circles, direction from the right-hand rule. Right: field inside/around a solenoid, approximately uniform inside, like a bar magnet.

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Đường sức từ luôn là các **vòng kín** (không có "điện tích từ" cô lập để đường sức bắt đầu/kết thúc như điện trường). Quanh một dây dẫn thẳng mang dòng điện: đường sức là các vòng tròn đồng tâm, chiều xác định bằng **quy tắc bàn tay phải**: ngón cái chỉ chiều dòng điện, các ngón còn lại cong theo chiều đường sức từ. Bên trong ống dây (solenoid) cuốn nhiều vòng sát nhau, từ trường gần như **đều** và song song trục ống — giống hệt một thanh nam châm.

The solenoid's near-uniform interior field can be understood as the superposition of many individual current loops, stacked side by side: each loop contributes its own field, strongest at its own centre, and when many such loops are packed closely together, the contributions from neighbouring loops overlap and reinforce each other along the central axis, while the field outside the loops largely cancels. This is exactly analogous to the superposition of gravitational or electric fields from many point sources (Sections D.1.3 and D.2.2) — only here the "sources" are current loops rather than point masses or charges. Near the very ends of a real (finite-length) solenoid, this reinforcement is incomplete (there are no neighbouring loops beyond the end to contribute), so the field there is weaker and less uniform than deep inside; the idealised formula $B = \mu_0 n I$ derived in the next section applies strictly only to the interior, far from the ends.

4.3.2 Quantitative Magnetic Field Strength (AHL)

This content is assessed at Additional Higher Level (AHL) only.

The qualitative field patterns of Section D.3.1 can be made quantitative. Each depends on the constant $\mu_0 = 4\pi \times 10^{-7}$ T·m/A, the **permeability of free space** — the magnetic counterpart of ε_0 in electrostatics.

For a long, straight wire carrying current I, the field at perpendicular distance d from the wire has magnitude

$$B = \frac{\mu_0 I}{2\pi d}.$$

The field decreases with distance (inversely, not inverse-square as with point sources) and forms the concentric circles described earlier, with direction from the right-hand rule.

At the **centre** of a single circular current loop of radius R carrying current I, the field magnitude is

$$B = \frac{\mu_0 I}{2R}.$$

For a coil of N tightly-wound loops (all the same radius R, all carrying the same current I), each loop's contribution adds, giving $B = \mu_0 N I/(2R)$ at the centre.

Inside a long **solenoid** with n turns per unit length (i.e. $n = N/\ell$ for a solenoid of N total turns and length ℓ) carrying current I, the field is very nearly uniform and has magnitude

$$B = \mu_0 n I.$$

Notice this does not depend on the solenoid's radius at all — only on the current and the "packing density" of turns along its length; a longer solenoid with the same n and I has the same interior field strength, just over a longer uniform region.

Magnetic field magnitudes ($\mu_0 = 4\pi \times 10^{-7}$ T·m/A):

$$B_{\text{wire}} = \frac{\mu_0 I}{2\pi d}, \qquad B_{\text{loop centre}} = \frac{\mu_0 N I}{2R}, \qquad B_{\text{solenoid (inside)}} = \mu_0 n I \quad (n = N/\ell).$$

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Ba công thức định lượng cho từ trường: dây thẳng dài ($B = \mu_0 I/2\pi d$, giảm theo $1/d$ chứ không phải $1/d^2$ như điện trường điểm), tâm vòng dây tròn ($B = \mu_0 N I/2R$), và bên trong ống dây dài ($B = \mu_0 n I$, với $n = N/\ell$ là số vòng trên một đơn vị chiều dài). Lưu ý: từ trường trong ống dây **không phụ thuộc bán kính** ống — chỉ phụ thuộc mật độ vòng dây n và dòng điện I.

Luôn viết μ_0 **riêng** bên ngoài mọi hay , không gộp chung vào đơn vị.

Ví dụ mẫu · Field of a straight wire and a solenoid

(a) Find the magnetic field strength 4.0 cm from a long straight wire carrying a current of 10.0 A. (b) A solenoid has 800 turns wound uniformly over a length of 40.0 cm and carries a current of 1.5 A. Find the field strength inside it. (c) A single circular loop of radius 0.10 m carries a current of 2.0 A. Find the field strength at its centre.

(a)

$$B = \frac{\mu_0 I}{2\pi d} = \frac{(4\pi \times 10^{-7})(10.0)}{2\pi(0.040)} = \frac{(2 \times 10^{-7})(10.0)}{0.040} = 5.0 \times 10^{-5}\ \text{T}.$$

(b) Turns per unit length: $n = N/\ell = 800/0.400 = 2000$ m^{-1}.

$$B = \mu_0 nI = (4\pi \times 10^{-7})(2000)(1.5) = 3.8 \times 10^{-3}\ \text{T}\ (3.8\ \text{mT}).$$

(c)

$$B = \frac{\mu_0 I}{2R} = \frac{(4\pi \times 10^{-7})(2.0)}{2(0.10)} = 1.3 \times 10^{-5}\ \text{T}.$$

Comparing the three results shows how much more effective a solenoid is at concentrating field strength than either a single wire or a single loop, since it combines the contribution of many loops along its length.

4.3.3 Force on a Current-Carrying Conductor

A current-carrying wire placed in an external magnetic field experiences a force — this is the basic mechanism behind electric motors and loudspeakers. For a straight length L of wire carrying current I, in a uniform field of magnitude B, with the wire making angle θ to the direction of B:

$$F = BIL \sin\theta.$$

The force is **maximum** when the current is perpendicular to the field ($\theta = 90^\circ$, $\sin\theta = 1$), and **zero** when the current is parallel (or antiparallel) to the field ($\theta = 0^\circ$ or 180°, $\sin\theta = 0$) — a current flowing exactly along the field lines experiences no magnetic force at all. The direction of the force is given by the right-hand slap rule (or, equivalently, the vector relation $\vec{F} = I\vec{L} \times \vec{B}$): with the right hand flat, fingers pointing in the direction of the current, then curling towards B, the thumb (or palm, in the "slap" version) points in the direction of the force — the force is always perpendicular to *both* the current direction and B.

Force on a current-carrying conductor in a magnetic field:

$$F = BIL \sin\theta,$$

where θ is the angle between the current direction and B. Maximum at $\theta = 90^\circ$; zero at $\theta = 0^\circ$ or 180°. Direction: right-hand slap rule, always perpendicular to both I and B.

VN **GIẢI THÍCH TIẾNG VIỆT**

Lực từ tác dụng lên dây dẫn mang dòng điện: $F = BIL \sin\theta$, với θ là góc giữa chiều dòng điện và B. Lực **lớn nhất** khi dây vuông góc với từ trường, **bằng 0** khi dây song song với từ trường. Chiều lực xác định bằng quy tắc bàn tay phải (hoặc "quy tắc vỗ tay phải"): các ngón tay chỉ chiều dòng điện, cong về chiều B, ngón cái (hoặc lòng bàn tay) chỉ chiều lực — lực luôn **vuông góc** với cả dòng điện lẫn từ trường.

Ví dụ mẫu · Force on a current-carrying wire

A straight wire of length $L = 0.40$ m carries a current $I = 5.0$ A in a uniform magnetic field of $B = 0.30$ T. Find the force on the wire when (a) the wire is perpendicular to the field, (b) the wire makes an angle of 30° with the field.

(a) $\theta = 90^\circ$:

$$F = BIL \sin 90^\circ = (0.30)(5.0)(0.40)(1) = 0.60 \text{ N}.$$

(b) $\theta = 30^\circ$:

$$F = BIL \sin 30^\circ = (0.30)(5.0)(0.40)(0.5) = 0.30 \text{ N}.$$

Halving $\sin\theta$ (from 1 at 90° to 0.5 at 30°) exactly halves the force, as expected from the direct proportionality to $\sin\theta$.

(!) Lỗi thường gặp: Lỗi thường gặp: quên nhân $\sin\theta$ khi dây dẫn **không** vuông góc với B, mặc định dùng $F = BIL$ cho mọi trường hợp. Chỉ khi $\theta = 90^\circ$ thì $\sin\theta = 1$ và $F = BIL$ mới là công thức đúng và đủ; với góc bất kỳ, luôn phải viết đầy đủ $F = BIL\sin\theta$.

This force on a current-carrying conductor is the working principle of a simple **DC motor**: a rectangular coil carrying current is placed in a magnetic field so that the two sides of the coil parallel to the field's axis each experience a force $F = BIL\sin\theta$, and because current flows in opposite directions along the two opposite sides of the coil, the two forces point in opposite directions — together they form a couple (a torque) that rotates the coil. A split-ring commutator reverses the connection to the current supply every half-turn, so that the current in each side of the coil reverses direction exactly when the torque would otherwise switch from driving the rotation to opposing it, keeping the coil turning continuously in the same sense. The same $F = BIL\sin\theta$ relation, applied continuously in reverse (a coil rotating in a field, rather than being driven by it), is exactly the AC generator of Section D.4.5 — motors and generators are, in this sense, the same device operated in opposite directions.

4.3.4 Force on a Moving Charge and Circular Motion

An individual moving charge, not just a whole current-carrying wire, also experiences a force in a magnetic field. For a charge q moving with speed v at angle θ to a field of magnitude B:

$$F = qvB\sin\theta.$$

This is essentially the same law as for a wire (a current is, after all, many moving charges), with the direction again given by the right-hand rule ($\vec{F} = q\vec{v} \times \vec{B}$, care needed with the sign of q: for a negative charge, the force is in the opposite direction to what the right-hand rule gives for a positive charge moving the same way).

This connection can be made precise. Consider a straight wire segment of length L and cross-sectional area A, containing n charge carriers per unit volume, each of charge q, drifting at speed v perpendicular to a field B. The number of carriers in the segment is $N = nAL$, and each individually feels a force qvB, so the total force on the segment is

$$F_{\text{total}} = N(qvB) = (nAL)(qvB) = (nqvA)LB.$$

The quantity $nqvA$ is exactly the current I carried by the wire (charge per unit time flowing past a cross-section). Substituting $I = nqvA$ recovers

$$F_{\text{total}} = BIL,$$

precisely the formula of Section D.3.3 — confirming that the force on a current-carrying wire is nothing more than the sum of the individual magnetic forces on every moving charge carrier inside it.

The most important special case is $\theta = 90^\circ$ (velocity perpendicular to B), giving $F = qvB$. Because $\vec{F}$ is always perpendicular to $\vec{v}$ in this case, the magnetic force can never do work on the charge (work requires a force component along the direction of motion) — it changes the *direction* of the velocity but never its *magnitude* (speed). A force of constant magnitude, always perpendicular to the velocity, is precisely the condition for **uniform circular motion**.

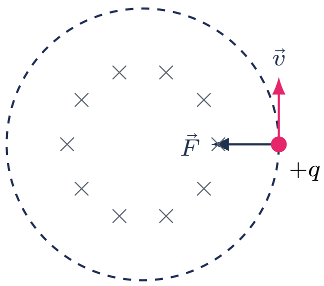
Equating the magnetic force to the centripetal force requirement $F = mv^2/r$:

$$qvB = \frac{mv^2}{r} \implies r = \frac{mv}{qB}.$$

The period of this circular motion is $T = \dfrac{2\pi r}{v}$; substituting $r = mv/(qB)$:

$$T = \frac{2\pi}{v} \cdot \frac{mv}{qB} = \frac{2\pi m}{qB}.$$

Notice that v cancels completely — the period (and hence frequency) of circular motion in a magnetic field is **independent of speed**: faster particles simply move in proportionally larger circles ($r \propto v$), taking exactly the same time to complete one revolution as slower particles of the same charge-to-mass ratio in the same field.



$\vec{B}$ into the page ($\times$); particle follows a circle of radius $r = mv/(qB)$

Force on a moving charge: $F = qvB\sin\theta$; when $\vec{v} \perp \vec{B}$ ($\theta = 90^\circ$), $F = qvB$, always perpendicular to $\vec{v}$, doing no work ⇒ uniform circular motion.

$$r = \frac{mv}{qB} \text{ (radius)}, \qquad T = \frac{2\pi m}{qB} \text{ (period, independent of speed)}.$$

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Khi $\vec{v} \perp \vec{B}$, lực từ $F = qvB$ luôn vuông góc với vận tốc, nên **không sinh công** — chỉ đổi hướng chuyển động, không đổi độ lớn vận tốc, kết quả là hạt chuyển động tròn đều. Bán kính quỹ đạo $r = mv/(qB)$ tỉ lệ thuận với v, nhưng chu kỳ $T = 2\pi m/(qB)$ **không phụ thuộc vào tốc độ** v — đây là kết quả rất hay bị hỏi trong đề thi: hạt nhanh hơn đi vòng tròn lớn hơn nhưng mất đúng một khoảng thời gian như hạt chậm hơn (cùng q, m, B).

Ví dụ mẫu · Electron circling in a magnetic field

An electron ($m = 9.11 \times 10^{-31}$ kg, $q = 1.60 \times 10^{-19}$ C) moves at $v = 2.0 \times 10^6$ m/s perpendicular to a uniform magnetic field of $B = 0.050$ T. Find the radius and period of its circular path.

$$r = \frac{mv}{qB} = \frac{(9.11 \times 10^{-31})(2.0 \times 10^6)}{(1.60 \times 10^{-19})(0.050)} = 2.3 \times 10^{-4} \text{ m } (0.23 \text{ mm}).$$

$$T = \frac{2\pi m}{qB} = \frac{2\pi(9.11 \times 10^{-31})}{(1.60 \times 10^{-19})(0.050)} = 7.2 \times 10^{-10}\ \text{s}.$$

The independence of the period $T = 2\pi m/(qB)$ from speed has a direct practical application: a **cyclotron** accelerates charged particles by repeatedly applying a fixed alternating electric field across a gap, timed at the constant frequency $f_c = 1/T = qB/(2\pi m)$ — the **cyclotron frequency** — while a magnetic field confines the particles to an expanding spiral path. Because the frequency needed to stay synchronised does not change as the particle speeds up (only the radius grows), a single fixed driving frequency can accelerate particles through many revolutions. The same physics explains why charged cosmic-ray particles entering Earth's magnetic field near the poles are guided into tight spirals along the field lines, producing the aurora borealis and aurora australis as they collide with atmospheric molecules.

4.3.5 The Velocity Selector

When a charged particle passes through a region containing *both* an electric field E and a magnetic field B, arranged at right angles to each other and to the particle's velocity, the electric force qE and the magnetic force qvB act in *opposite* directions. For most particles, these two forces do not cancel and the particle is deflected. But for a particle moving at exactly the speed at which the two forces balance,

$$qE = qvB \quad \Longrightarrow \quad v = \frac{E}{B},$$

the net force is zero, and the particle travels through the region **undeflected**, in a straight line. Particles moving faster than $v = E/B$ feel a net magnetic force (which grows with v) and are deflected one way; particles moving slower feel a net electric force and are deflected the other way. A narrow slit placed at the far end of the apparatus then allows only particles with speed very close to $v = E/B$ through — hence the name **velocity selector**.

This principle underlies instruments such as the **mass spectrometer**, where a velocity selector first filters ions to a single known speed, after which the ions enter a region of pure magnetic field and follow a circular arc of radius $r = mv/(qB)$; since v, q, and B are then all known, measuring r allows the mass m of each ion to be determined — the basis of isotope separation and precise mass measurement (a qualitative extension beyond the core velocity-selector idea).

> **Velocity selector:** crossed electric and magnetic fields let only particles of one specific speed pass through undeflected:
> $$qE = qvB \quad \Longrightarrow \quad v = \frac{E}{B}.$$

VN **GIẢI THÍCH TIẾNG VIỆT**

Bộ lọc vận tốc (velocity selector) dùng điện trường E và từ trường B vuông góc với nhau (và vuông góc với vận tốc hạt) sao cho lực điện qE và lực từ qvB **cân bằng nhau**: $qE = qvB \Rightarrow v = E/B$. Chỉ hạt có đúng tốc độ này mới đi thẳng qua khe hẹp mà không bị lệch — hạt nhanh hơn hoặc chậm hơn sẽ bị lệch theo hai hướng khác nhau. Ứng dụng: máy khối phổ (mass spectrometer) dùng nguyên lý này để lọc tốc độ trước khi đo bán kính quỹ đạo trong từ trường, từ đó suy ra khối lượng ion.

Ví dụ mẫu · Velocity selector

A velocity selector has a magnetic field of $B = 0.15$ T and an electric field of $E = 3.0 \times 10^4$ N/C, arranged perpendicular to each other. Find the speed of the (singly charged) particles that pass through undeflected.

$$v = \frac{E}{B} = \frac{3.0 \times 10^4}{0.15} = 2.0 \times 10^5 \text{ m/s}.$$

Note that the charge q cancelled out of the condition $qE = qvB$ — the selected speed depends only on the field strengths E and B, not on the magnitude (or even the sign, since both forces flip together) of the charge, so the device selects by speed regardless of the particle's charge.

(!) Lỗi thường gặp: Trong bộ lọc vận tốc, đừng nhầm điều kiện cân bằng $qE = qvB$ với công thức bán kính $r = mv/(qB)$ của giai đoạn sau (trong khối phổ kế) — đây là hai giai đoạn riêng biệt của thiết bị, dùng hai phương trình khác nhau. Ở giai đoạn chọn vận tốc, **chỉ có** E và B liên quan (không cần biết m); ở giai đoạn đo bán kính, **chỉ có** từ trường B tác dụng (không còn điện trường).

As an extension, once ions of known speed emerge from a velocity selector, a full mass spectrometer sends them into a second region containing only a magnetic field B_2, where they follow a circular arc of radius $r = mv/(qB_2)$ (Section D.3.3). Rearranging for the unknown mass,

$$m = \frac{qB_2 r}{v}.$$

Since q, B_2, and v (from the velocity selector) are all known, measuring r — typically by detecting where the ion strikes a photographic plate or detector — directly yields the ion's mass.

Ví dụ mẫu · Mass spectrometer

Singly charged ions ($q = +1.60 \times 10^{-19}$ C) emerge from a velocity selector at $v = 2.0 \times 10^5$ m/s (as in the worked example above) and enter a second magnetic field region of $B_2 = 0.40$ T, where they follow a circular arc of radius $r = 0.15$ m. Calculate the mass of the ion, in kilograms and in atomic mass units ($1 \text{ u} = 1.66 \times 10^{-27}$ kg).

$$m = \frac{qB_2 r}{v} = \frac{(1.60 \times 10^{-19})(0.40)(0.15)}{2.0 \times 10^5} = \boxed{4.8 \times 10^{-26} \text{ kg}}.$$

$$\frac{m}{1.66 \times 10^{-27} \text{ kg}} \approx \boxed{29 \text{ u}},$$

consistent with a light-to-medium mass ion (for example, a singly ionised silicon atom, mass number ≈ 29).

4.4 D.4 Electromagnetic Induction (AHL)

This content is assessed at Additional Higher Level (AHL) only.

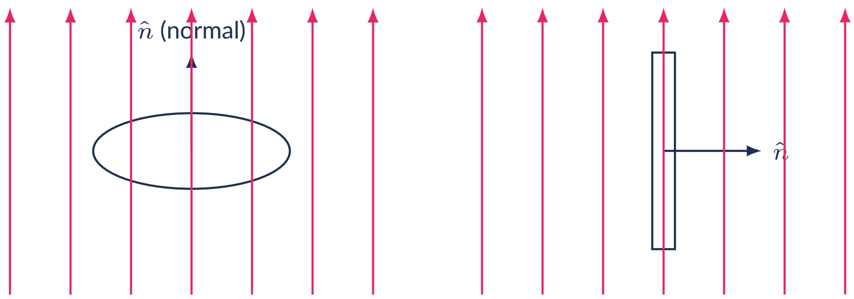
Section D.3 established that a moving charge (current) produces a magnetic field. This section explores the reverse effect: a **changing** magnetic field, or a conductor moving through a magnetic field, produces (induces) an EMF and, in a closed circuit, a current. This phenomenon — electromagnetic induction — is the working principle behind almost every large-scale electricity generator on Earth, as well as transformers, induction cooktops, and many sensors, and it is quantified by two linked laws: Faraday's law (magnitude) and Lenz's law (direction).

4.4.1 Magnetic Flux

Magnetic flux Φ through a flat area A measures how much magnetic field "passes through" that area, taking into account its orientation relative to the field:

$$\Phi = BA\cos\theta,$$

where θ is the angle between the magnetic field $\vec{B}$ and the **normal** (perpendicular) to the area A — not the angle between B and the plane of the area itself. Flux is maximum ($\Phi = BA$) when the area is oriented face-on to the field ($\theta = 0^\circ$, normal parallel to B), and zero when the area is edge-on to the field ($\theta = 90^\circ$, the plane of the loop contains $\vec{B}$, so no field lines pass through it at all). The unit of flux is the **weber** (Wb), where 1 Wb $= 1$ T m^2.



(a) $\theta = 0^\circ$: loop face-on, $\Phi = BA$ (maximum) (b) $\theta = 90^\circ$: loop edge-on, $\Phi = 0$

A coil's flux depends on its orientation relative to B: maximum face-on (a), zero edge-on (b) — this is exactly the geometry exploited by a rotating AC generator coil.

Magnetic flux through area A:

$$\Phi = BA\cos\theta,$$

θ measured between $\vec{B}$ and the **normal** to the area. Maximum ($\Phi = BA$) at $\theta = 0^\circ$; zero at $\theta = 90^\circ$. Unit: weber, 1 Wb $= 1$ T m^2.

VN **GIẢI THÍCH TIẾNG VIỆT**

Từ thông $\Phi = BA\cos\theta$ đo lượng "đường sức từ xuyên qua" một diện tích A, với θ là góc giữa $\vec{B}$ và **pháp tuyến** của mặt (không phải góc giữa B và mặt phẳng!). Từ thông lớn nhất khi mặt vuông góc với đường sức ($\theta = 0^\circ$, pháp tuyến song song B), bằng 0 khi mặt song song với đường sức ($\theta = 90^\circ$). Đây là đại lượng trung tâm của toàn bộ hiện tượng cảm ứng điện từ — mọi suất điện động cảm ứng đều xuất phát từ sự **thay đổi** của Φ.

(!) Lỗi thường gặp: Lỗi thường gặp nhất trong tính từ thông: nhầm góc θ trong $\Phi = BA\cos\theta$ là góc giữa $\vec{B}$ và **mặt phẳng** của khung dây, thay vì góc giữa $\vec{B}$ và **pháp tuyến** (đường vuông góc với mặt phẳng). Hai góc này là **phụ nhau** ($90^\circ - \theta$) — nếu đề cho góc giữa B và mặt phẳng khung dây, phải đổi sang góc với pháp tuyến trước khi dùng công thức, hoặc dùng sin thay vì cos một cách cẩn thận.

4.4.2 Faraday's Law of Electromagnetic Induction

Faraday's law states that an electromotive force (EMF) is induced in a circuit whenever the magnetic flux through it changes, and gives the magnitude of that EMF in terms of the *rate* of change of flux:

$$\varepsilon = -N\frac{d\Phi}{dt},$$

where N is the number of turns in the coil (each turn contributes its own induced EMF, and since the turns are in series, the total EMF is N times that of a single loop). Crucially, flux can change for *any* reason — a changing field strength B (e.g. a magnet moving closer or an AC current in a nearby coil), a changing area A (e.g. a loop being stretched or a rod sliding to change the enclosed area), or a changing orientation θ (e.g. a coil rotating in a fixed field) — and in every case Faraday's law applies unchanged: it is the change in Φ that matters, not the mechanism producing that change.

Faraday's law:

$$\varepsilon = -N\frac{\mathrm{d}\Phi}{\mathrm{d}t}.$$

An EMF is induced whenever flux through a circuit changes — from a changing B, changing A, or changing orientation θ (or any combination).

GIẢI THÍCH TIẾNG VIỆT

VN Định luật Faraday: suất điện động cảm ứng tỉ lệ với **tốc độ thay đổi** từ thông, $\varepsilon = -N\,d\Phi/dt$, với N là số vòng dây. Điều quan trọng: từ thông có thể thay đổi vì **bất kỳ lý do gì** — B đổi (nam châm lại gần/ra xa), A đổi (mạch thay đổi diện tích), hoặc θ đổi (cuộn dây quay) — định luật Faraday áp dụng như nhau cho cả ba trường hợp, chỉ cần tính đúng $d\Phi/dt$.

For a steady (constant) rate of flux change, $\mathrm{d}\Phi/\mathrm{d}t$ can simply be replaced by $\Delta\Phi/\Delta t$. For example, a flat coil of $N = 100$ turns, each of area 0.030 m^2, sits with its plane perpendicular to a magnetic field that increases steadily from 0.20 T to 0.50 T over 2.0 s. The flux through each turn changes from $\Phi_i = BA = (0.20)(0.030) = 6.0 \times 10^{-3}$ Wb to $\Phi_f = (0.50)(0.030) = 1.5 \times 10^{-2}$ Wb, so

$$|\varepsilon| = N\frac{\Delta\Phi}{\Delta t} = 100 \times \frac{(1.5 \times 10^{-2}) - (6.0 \times 10^{-3})}{2.0} = 0.45 \text{ V}.$$

This same pattern — a steady $\Delta\Phi/\Delta t$ over a stated time interval — is the most common way Faraday's law is tested numerically.

4.4.3 Lenz's Law

Faraday's law gives the *magnitude* of the induced EMF; **Lenz's law** gives its *direction* (equivalently, the direction of any induced current that flows as a result): the induced current always flows in the direction that **opposes** the change in flux that produced it. This is the origin of the minus sign in Faraday's law, and it is not an arbitrary rule — it is a direct consequence of **conservation of energy**. If the induced current instead *reinforced* the change in flux, the effect would build on itself with no energy input, which is impossible; opposing the change means work must be done (e.g. to push a magnet into a coil against the "resistance" the coil's induced field provides), and that work is exactly what supplies the electrical energy delivered by the induced current.

Ví dụ mẫu · Direction of an induced current (Lenz's law)

The north pole of a bar magnet is pushed towards one end of a stationary coil of wire connected to form a closed loop. Determine the direction of the induced current in the coil, viewed from the approaching magnet.

As the magnet approaches, the flux through the coil (in the direction from the magnet towards the coil) is **increasing**. By Lenz's law, the induced current must oppose this increase — it must create its own magnetic field, at the near end of the coil, pointing back *towards* the approaching magnet (i.e., the coil's near face must behave like a **north pole**, since like poles repel and this opposes the magnet's approach). Applying the right-hand rule in reverse (curl the fingers in the direction that makes the thumb point back towards the incoming magnet), the induced current flows **counterclockwise** as viewed from the magnet's side. If the magnet is instead withdrawn, the flux decreases, and by the same reasoning the coil's near face must now become a south pole (to attract the departing magnet, opposing its retreat) — so the current reverses to flow **clockwise**.

VN **GIẢI THÍCH TIẾNG VIỆT**

Định luật Lenz: dòng điện cảm ứng luôn có chiều **chống lại** sự thay đổi từ thông đã sinh ra nó — đây là hệ quả trực tiếp của **định luật bảo toàn năng lượng** (nếu dòng cảm ứng lại "tiếp sức" cho sự thay đổi, năng lượng sẽ tự sinh ra vô hạn, vô lý). Mẹo làm bài: khi nam châm **tiến lại gần** cuộn dây, mặt cuộn dây gần nam châm phải trở thành cực **cùng dấu** với cực đang tiến tới (để đẩy lại); khi nam châm **rời xa**, mặt đó phải trở thành cực **trái dấu** (để hút lại, chống sự rời đi).

(!) Lỗi thường gặp: Lỗi thường gặp: xác định chiều dòng cảm ứng theo hướng "tăng cường" từ thông thay vì "chống lại" — đây là lỗi vi phạm bảo toàn năng lượng và luôn sai. Luôn tự hỏi: "từ thông đang tăng hay giảm?", rồi suy ra cực từ cần thiết để **chống lại** xu hướng đó, cuối cùng mới dùng quy tắc bàn tay phải để suy ra chiều dòng điện.

Lenz's law also explains **eddy current braking**, used in some train and roller-coaster braking systems: as a solid conducting metal plate (or wheel) moves through a region of magnetic field, the changing flux through different parts of the metal induces circulating eddy currents within it, in loops that — by Lenz's law — oppose the plate's motion. This induced opposing force converts the plate's kinetic energy directly into heat (resistive dissipation of the eddy currents) with no mechanical contact at all, giving a smooth, wear-free braking force that grows automatically with speed (faster motion means a faster-changing flux, which means larger induced currents and a stronger retarding force).

4.4.4 Motional EMF

A straight conducting rod of length L, moving with speed v perpendicular to a uniform magnetic field B (with the rod itself perpendicular to both v and B), develops an EMF between its ends. Two equivalent arguments derive the same result.

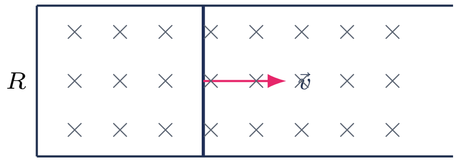
Force-on-charges argument: each free charge q inside the moving rod is itself moving with velocity v through the field B, so it experiences a magnetic force $F = qvB$ along the length of the rod, pushing positive charge to one end and leaving the other end negative. This charge separation continues until the resulting electric field inside the rod exactly balances the magnetic force (equilibrium), at which point $qE_{\text{rod}} = qvB$, so $E_{\text{rod}} = vB$. Since the potential difference across a uniform field is $V = E_{\text{rod}}L$ (from $E = V/d$, Section D.2.4), the EMF of the rod is

$$\varepsilon = vBL.$$

Flux-rule argument: suppose the rod slides along two parallel conducting rails a distance L apart, closing a circuit. In time Δt, the rod sweeps out an additional area $\Delta A = L(v\Delta t)$, so the flux enclosed by the circuit changes by $\Delta\Phi = B\,\Delta A = BLv\,\Delta t$. By Faraday's law (with $N = 1$),

$$\varepsilon = \frac{\Delta\Phi}{\Delta t} = \frac{BLv\,\Delta t}{\Delta t} = BLv,$$

the same result.



$\vec{B}$ into the page; rod slides on rails at speed v

A conducting rod sliding on parallel rails through a uniform field generates a motional EMF $\varepsilon = BLv$, driving a current through resistor R closing the circuit.

Motional EMF of a straight rod, length L, moving at speed v perpendicular to field B (rod perpendicular to both):

$$\varepsilon = BLv.$$

VN **GIẢI THÍCH TIẾNG VIỆT**

Suất điện động cảm ứng do thanh dẫn chuyển động: $\varepsilon = BLv$, có thể suy ra bằng hai cách — (1) lực từ đẩy điện tích tự do trong thanh đến khi cân bằng với điện trường nội tại, hoặc (2) tốc độ thay đổi diện tích quét được nhân với B (quy tắc từ thông của Faraday). Cả hai cách đều cho cùng kết quả — đây là minh hoạ đẹp cho thấy định luật Faraday và lực Lorentz thực chất là hai cách nhìn của cùng một hiện tượng vật lý.

Ví dụ mẫu · Motional EMF on rails

A conducting rod of length $L = 0.50$ m slides along horizontal rails at constant speed $v = 4.0$ m/s, perpendicular to a uniform magnetic field $B = 0.20$ T. The rails are connected by a resistor $R = 2.0\ \Omega$. Find (a) the induced EMF, (b) the induced current.

(a)

$$\varepsilon = BLv = (0.20)(0.50)(4.0) = 0.40 \text{ V}.$$

(b)

$$I = \frac{\varepsilon}{R} = \frac{0.40}{2.0} = 0.20 \text{ A}.$$

As a check on energy conservation: the electrical power dissipated in the resistor is $P_{\text{elec}} = I^2R = (0.20)^2(2.0) = 0.080$ W. This must equal the mechanical power an external agent supplies to keep the rod moving at constant velocity against the retarding magnetic force $F = BIL = (0.20)(0.20)(0.50) = 0.020$ N, since $P_{\text{mech}} = Fv = (0.020)(4.0) = 0.080$ W — the two match exactly, confirming that the work done pushing the rod is converted, via induction, directly into electrical energy dissipated in R.

4.4.5 AC Generation

A coil of N turns, each of area A, rotating with constant angular velocity ω inside a uniform magnetic field B, is the basic model of an **AC generator**. As the coil rotates, the angle between the field and the coil's normal changes continuously as $\theta = \omega t$ (starting face-on to the field at $t = 0$), so the flux through the coil varies as

$$\Phi(t) = BA\cos(\omega t).$$

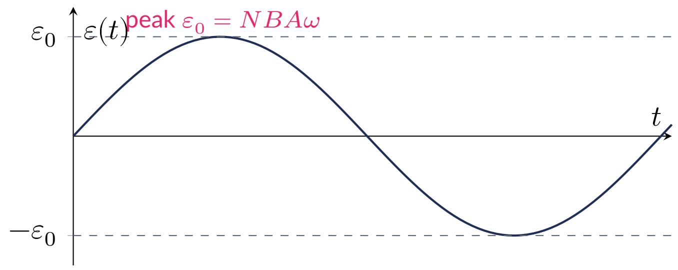
Applying Faraday's law, the induced EMF is (minus) the rate of change of this flux, multiplied by N:

$$\varepsilon(t) = -N\frac{\mathrm{d}\Phi}{\mathrm{d}t} = NBA\omega\sin(\omega t).$$

The EMF therefore oscillates sinusoidally with time, exactly the alternating-current waveform used throughout electrical power systems. Its **peak (maximum) value** occurs when $\sin(\omega t) = 1$:

$$\varepsilon_0 = NBA\omega.$$

This makes physical sense: the EMF is zero when the coil is face-on to the field ($\theta = 0$, flux at its own maximum but momentarily *not changing*, since $\cos\theta$ is at a turning point there) and maximum when the coil is edge-on ($\theta = 90^\circ$, flux passing through zero but changing fastest at that instant).



The generator EMF $\varepsilon(t) = \varepsilon_0 \sin(\omega t)$ oscillates sinusoidally, reaching peak value $\varepsilon_0 = NBA\omega$ twice per rotation.

AC generator (coil of N turns, area A, rotating at angular velocity ω in field B):

$$\varepsilon(t) = NBA\omega \sin(\omega t), \qquad \varepsilon_0 = NBA\omega \ \text{(peak EMF)}.$$

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Máy phát điện xoay chiều: cuộn dây N vòng, diện tích A, quay đều với tốc độ góc ω trong từ trường B tạo ra suất điện động dao động hình sin $\varepsilon(t) = NBA\omega \sin(\omega t)$, với biên độ (giá trị đỉnh) $\varepsilon_0 = NBA\omega$. Lưu ý nghịch lý thú vị: suất điện động bằng 0 khi từ thông **lớn nhất** (cuộn dây đối diện trực tiếp với từ trường — lúc đó Φ đang ở đỉnh, tạm thời không đổi), và suất điện động lớn nhất khi từ thông **bằng 0** (cuộn dây song song với từ trường, Φ đang đổi nhanh nhất).

For example, a generator coil with $N = 200$ turns, area $A = 0.015$ m^2, rotating in a field $B = 0.10$ T at a frequency of $f = 50$ Hz (so $\omega = 2\pi f = 314$ rad/s) produces a peak EMF of $\varepsilon_0 = NBA\omega = (200)(0.10)(0.015)(314) \approx 94$ V — this is exactly how mains AC electricity is generated at power stations, just scaled up enormously in size and field strength.

4.4.6 Transformers

A **transformer** uses electromagnetic induction to change (step up or step down) an AC voltage. It consists of two coils — a **primary** coil (of N_p turns, connected to the input voltage V_p) and a **secondary** coil (of N_s turns, delivering the output voltage V_s) — wound around a common iron core, which concentrates the magnetic flux from the primary and channels nearly all of it through the secondary. Because the same changing flux $\Phi(t)$ passes through every turn of both coils, Faraday's law applied to each coil gives $V_p = N_p\, \mathrm{d}\Phi/\mathrm{d}t$ and $V_s = N_s\, \mathrm{d}\Phi/\mathrm{d}t$ (in magnitude); dividing these two relations, the common $\mathrm{d}\Phi/\mathrm{d}t$ cancels, giving the ideal transformer equation:

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}.$$

A transformer with more turns on the secondary than the primary ($N_s > N_p$) is a **step-up** transformer ($V_s > V_p$); with fewer turns on the secondary ($N_s < N_p$), it is a **step-down** transformer ($V_s < V_p$).

For an **ideal** (lossless) transformer, energy is conserved: the electrical power delivered to the primary equals the power delivered by the secondary,

$$I_p V_p = I_s V_s \quad \Longrightarrow \quad \frac{I_s}{I_p} = \frac{N_p}{N_s}.$$

Note the current ratio is the *inverse* of the turns/voltage ratio: a step-up transformer (higher V_s) necessarily delivers a *lower* I_s for the same power, and vice versa for step-down.

Ideal transformer:

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}, \qquad I_pV_p = I_sV_s \text{ (power in = power out)}.$$

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VN

Máy biến áp lý tưởng: $V_s/V_p = N_s/N_p$ (tỉ số điện áp bằng tỉ số số vòng dây), và vì công suất bảo toàn ($I_pV_p = I_sV_s$), tỉ số dòng điện **ngược lại** với tỉ số điện áp: $I_s/I_p = N_p/N_s$. Đây là lỗi học sinh hay nhầm nhất — tăng áp (step-up) thì **giảm** dòng, hạ áp (step-down) thì **tăng** dòng, để công suất không đổi.

Ví dụ mẫu · Step-down transformer

A transformer connected to the $V_p = 230$ V mains has $N_p = 1000$ turns on the primary and $N_s = 50$ turns on the secondary. A device connected to the secondary draws a current $I_s = 4.0$ A. Find (a) the secondary (output) voltage, (b) the power delivered, (c) the primary current.

(a)

$$V_s = V_p \cdot \frac{N_s}{N_p} = 230 \times \frac{50}{1000} = 11.5 \text{ V}.$$

(b)

$$P = I_sV_s = (4.0)(11.5) = 46 \text{ W}.$$

(c) From $I_pV_p = P$:

$$I_p = \frac{P}{V_p} = \frac{46}{230} = 0.20 \text{ A}.$$

(Check via turns ratio: $I_p = I_s \cdot N_s/N_p = 4.0 \times 0.05 = 0.20$ A — consistent.)

Real (non-ideal) transformers lose a small fraction of the input power to resistive heating and other effects, so their **efficiency** is defined as

$$\text{efficiency} = \frac{P_{\text{output}}}{P_{\text{input}}} \times 100\,\%.$$

For example, if a transformer with 1000 W supplied to its primary delivers only 920 W from its secondary, its efficiency is $920/1000 \times 100\,\% = 92\,\%$; the remaining 80 W is lost to eddy currents, resistive heating in the windings, and hysteresis in the core, discussed below. Well-designed power transformers typically achieve efficiencies above $95\,\%$, but $100\,\%$ (the ideal case assumed throughout this section unless stated otherwise) is never quite reached in practice.

Real transformers are not perfectly lossless. One important source of energy loss is **eddy currents**: the changing magnetic flux induces circulating currents not only in the intended secondary coil but also within the bulk iron core itself (the core is a conductor). These eddy currents dissipate energy as heat (I^2R losses within the core), reducing efficiency and potentially overheating the transformer. The standard mitigation is to build the core from thin, electrically insulated sheets (**laminations**) rather than a single solid block — this breaks up the conducting paths available to eddy currents, greatly reducing them, while barely affecting the core's ability to carry the useful magnetic flux.

Finally, transformers work **only** with alternating current, never direct current: the entire mechanism depends on a continuously *changing* flux to induce an EMF in the secondary coil (Faraday's law requires $d\Phi/dt \neq 0$). A steady DC current in the primary produces a constant flux, so once any initial transient has died away, $d\Phi/dt = 0$ and no EMF is induced in the secondary at all.

This is precisely why electrical power is generated, transmitted, and distributed as AC rather than DC. Power lost to resistive heating in a transmission cable is $P_{\text{loss}} = I^2 R_{\text{cable}}$, so for a fixed amount of power $P = IV$ to be delivered, minimising the current I (by transmitting at very high voltage V) minimises the loss. A step-up transformer at the power station raises the voltage to hundreds of kilovolts for transmission (correspondingly lowering the current), and step-down transformers at local substations and near consumers reduce it again to safe, usable levels. Because transformers require a changing flux, only AC allows this repeated, efficient stepping of voltage up for transmission and back down for use — a major reason AC (rather than DC) became the standard for electricity grids.

This transmission-efficiency argument was the central technical point in the late-19th-century "War of Currents" between proponents of AC (championed by Tesla and Westinghouse, exploiting exactly the transformer principle developed in this section) and DC systems (championed by Edison, whose early DC grids could not step voltage up or down at all, and so suffered large transmission losses over any significant distance). AC's ability to be transformed efficiently is the direct physical reason it prevailed as the basis of essentially all large-scale electricity grids in use today.

> **(!) Lỗi thường gặp:** Học sinh hay quên: máy biến áp **chỉ hoạt động với dòng xoay chiều (AC)**, vì cơ chế hoạt động dựa vào từ thông **biến thiên**. Nếu cấp dòng một chiều (DC) ổn định vào cuộn sơ cấp, từ thông sẽ không đổi sau giai đoạn quá độ ban đầu, $d\Phi/dt = 0$, và không có suất điện động cảm ứng nào ở cuộn thứ cấp — máy biến áp DC không tồn tại theo nghĩa này.

Real transformers and generators also suffer other unwanted energy losses beyond eddy currents: resistive heating in the copper windings (I^2R losses, reduced by using thick, low-resistance wire), and hysteresis losses from repeatedly magnetising and demagnetising the core material as the current alternates (reduced by choosing "soft" magnetic materials that are easy to magnetise and demagnetise). None of these change the ideal relations $V_s/V_p = N_s/N_p$ derived above, which remain the correct starting point for any transformer calculation unless a problem explicitly states an efficiency below $100\,\%$.

4.4.7 Bringing the Three Fields Together

Having now studied gravitational, electric, and magnetic fields in turn, it is worth stepping back to compare them directly, since IB exam questions frequently test whether a student can correctly select *which* field law applies to a given situation.

Field	Source	Acts on	Key feature
Gravitational	mass M (at rest or moving)	any mass m	always attractive; $g = GM/r^2$, $E_p = -GMm/r$
Electric	charge q (at rest or moving)	any charge q'	attractive or repulsive; $E = kq/r^2$, $E_p = kq_1q_2/r$
Magnetic	*moving* charge / current	only *moving* charge or current	force $\perp$ velocity; does no work; produces circular motion when $v \perp B$

A single moving charged particle can, in general, experience gravitational, electric, and magnetic forces simultaneously — a charged dust grain falling through Earth's atmosphere, for example, feels gravity (constant, downward), any local electric field (from atmospheric charge separation), and the Lorentz magnetic force if it has any horizontal velocity component through Earth's magnetic field. Exam questions often combine two of these three fields in one scenario (as in the velocity selector of Section D.3.4, which pairs electric and magnetic forces); always identify each force separately, using the correct law for its type, before combining them as vectors.

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Ba loại trường trong chương này — hấp dẫn, điện, từ — đều dùng chung một khung lý thuyết (cường độ trường, thế năng, thế), nhưng có những khác biệt cốt lõi cần phân biệt rõ: trường hấp dẫn luôn hút; trường điện có thể hút hoặc đẩy tuỳ dấu điện tích; còn trường từ chỉ tác dụng lực lên điện tích **đang chuyển động** (hoặc dòng điện), và lực đó luôn vuông góc với vận tốc nên **không sinh công**. Khi một bài toán kết hợp nhiều loại trường (ví dụ bộ lọc vận tốc — điện và từ), hãy xác định riêng từng lực rồi mới cộng vector.

Strategy: approaching a Fields problem

Before starting any calculation, quickly identify: (1) **which field(s)** are involved — gravitational, electric, magnetic, or a combination; (2) whether the source is a **point** object (use the inverse-square laws $g = GM/r^2$, $E = kq/r^2$) or a **uniform-field** configuration such as parallel plates or deep inside a solenoid (use $E = V/d$, $B = \mu_0 nI$); (3) whether the question asks for a **force**, a **field strength**, an **energy**, or a **potential** — these are related but different quantities, with different units; (4) whether **multiple sources** are present, requiring vector superposition; and (5) for circular-motion and induction problems, whether the question wants the radius/period (kinematics of the motion) or the EMF/force (dynamics). Sketching a quick diagram with field directions marked (using the right-hand rule where relevant) before writing any equation prevents most sign and direction errors.

4.5 Problem Bank

Problem D.1

Which formula correctly gives the gravitational field strength g at distance r from a point mass M?

(A) $g = GMm/r$

(B) $g = GM/r^2$

(C) $g = GM^2/r$

(D) $g = Gm/r^2$

Lời giải chi tiết

Field strength is force per unit mass: $g = F/m = (GMm/r^2)/m = GM/r^2$, independent of the test mass m. **(B).**

Problem D.2

Mars has mass $M = 6.42 \times 10^{23}$ kg and radius $R = 3.39 \times 10^6$ m. Calculate the gravitational field strength at its surface.

Lời giải chi tiết

$$g = \frac{GM}{R^2} = \frac{(6.67 \times 10^{-11})(6.42 \times 10^{23})}{(3.39 \times 10^6)^2} = \boxed{3.73\ \text{m/s}^2}.$$

Problem D.3

A spacecraft of mass 500 kg moves from $r_1 = 8.0 \times 10^6$ m to $r_2 = 1.2 \times 10^7$ m from the centre of a planet of mass $M = 6.0 \times 10^{24}$ kg. Calculate the change in the spacecraft's gravitational potential energy, and state whether energy must be supplied to the spacecraft or is released by it.

Lời giải chi tiết

$$E_{p1} = -\frac{GMm}{r_1} = -2.50 \times 10^{10}\ \text{J}, \qquad E_{p2} = -\frac{GMm}{r_2} = -1.67 \times 10^{10}\ \text{J}.$$

$$\Delta E_p = E_{p2} - E_{p1} = \boxed{+8.3 \times 10^9\ \text{J}}.$$

Since $\Delta E_p > 0$, energy must be **supplied** to the spacecraft (external work is done against gravity) to move it further away.

Problem D.4

A satellite moves at constant speed around Earth in a circular orbit of fixed radius r. How much net work does the gravitational force do on the satellite over one complete orbit?

(A) Positive, equal to its kinetic energy
(B) Negative, equal to its potential energy
(C) Zero
(D) It depends on the orbital speed

Lời giải chi tiết

A circular orbit of fixed r lies entirely on a single gravitational equipotential surface (a sphere centred on Earth). No net work is done moving along an equipotential surface, since the gravitational force is always perpendicular to the satellite's velocity at every instant (it points radially inward, purely centripetal). **(C)**.

Problem D.5

The Moon has mass $M = 7.35 \times 10^{22}$ kg and radius $R = 1.74 \times 10^6$ m. Calculate the escape velocity from the Moon's surface.

Lời giải chi tiết

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2(6.67 \times 10^{-11})(7.35 \times 10^{22})}{1.74 \times 10^6}} = \boxed{2.37 \times 10^3\ \text{m/s}\ (2.37\ \text{km/s})}.$$

This is roughly a fifth of Earth's escape velocity (11.2 km/s) — consistent with the Moon's much smaller mass and radius, which is why the Moon cannot retain a substantial atmosphere.

Problem D.6 (AHL)

A satellite of mass 1200 kg orbits Earth ($M = 5.97 \times 10^{24}$ kg) in a circular orbit of radius $r_1 = 7.00 \times 10^6$ m. (a) Calculate its kinetic energy, potential energy, and total mechanical energy. (b) The satellite is later moved into a new circular orbit at $r_2 = 1.40 \times 10^7$ m (double the original radius). Calculate the energy that must be supplied to achieve this.

Lời giải chi tiết

(a)

$$\text{KE} = \frac{GMm}{2r_1} = 3.41 \times 10^{10}\ \text{J}, \qquad E_p = -\frac{GMm}{r_1} = -6.83 \times 10^{10}\ \text{J}.$$

(b)

$$E_1 = \text{KE} + E_p = \boxed{-3.41 \times 10^{10}\ \text{J}}.$$

$$E_2 = -\frac{GMm}{2r_2} = -1.71 \times 10^{10}\ \text{J}.$$

$$\Delta E = E_2 - E_1 = \boxed{+1.71 \times 10^{10}\ \text{J}}\ \text{must be supplied.}$$

Problem D.7

If the distance between two point charges is tripled (with both charges unchanged), by what factor does the Coulomb force between them change?

(A) It becomes $1/3$ of the original

(B) It becomes $1/9$ of the original

(C) It becomes $3\times$ the original

(D) It becomes $9\times$ the original

Lời giải chi tiết

$F \propto 1/r^2$. Tripling r gives $F \to F/3^2 = F/9$. **(B)**.

Problem D.8

Two point charges, $q_1 = +2.0\ \mu$C and $q_2 = +8.0\ \mu$C, are separated by 0.15 m. Calculate the electric force between them and state whether it is attractive or repulsive.

Lời giải chi tiết

$$F = \frac{kq_1q_2}{r^2} = \frac{(8.99 \times 10^9)(2.0 \times 10^{-6})(8.0 \times 10^{-6})}{(0.15)^2} = \boxed{6.4\ \text{N}}.$$

Both charges are positive, so the force is **repulsive**.

Problem D.9

A point charge $q = +5.0\ \mu$C is isolated in space. Calculate (a) the electric field strength at a point 0.40 m from the charge, (b) the force this field would exert on a charge of $+2.0\ \mu$C placed at that point.

Lời giải chi tiết

(a)

$$E = \frac{kq}{r^2} = \frac{(8.99 \times 10^9)(5.0 \times 10^{-6})}{(0.40)^2} = \boxed{2.8 \times 10^5\ \text{N/C}}.$$

(b)

$$F = q'E = (2.0 \times 10^{-6})(2.8 \times 10^5) = \boxed{0.56\ \text{N}}.$$

Problem D.10

Which of the following statements about electric field lines is correct?

(A) Field lines can cross each other where two fields overlap

(B) Field lines always point from low potential to high potential

(C) Field lines originate on positive charge and terminate on negative charge

(D) The electric field is strongest where field lines are most widely spaced

Lời giải chi tiết

Field lines start on positive charge and end on negative charge; they never cross (the field has one direction at each point); closely-spaced (not widely-spaced) lines indicate a stronger field. **(C)**.

Problem D.11

An electron is placed between two parallel plates separated by $d = 1.5$ cm, across which a potential difference of 300 V is applied. Calculate (a) the electric field strength between the plates, (b) the force on the electron, (c) its initial acceleration ($m_e = 9.11 \times 10^{-31}$ kg, $e = 1.60 \times 10^{-19}$ C).

Lời giải chi tiết

(a)

$$E = \frac{V}{d} = \frac{300}{0.015} = \boxed{2.0 \times 10^4 \text{ V/m}}.$$

(b)

$$F = eE = (1.60 \times 10^{-19})(2.0 \times 10^4) = \boxed{3.2 \times 10^{-15} \text{ N}}.$$

(c)

$$a = \frac{F}{m_e} = \frac{3.2 \times 10^{-15}}{9.11 \times 10^{-31}} = \boxed{3.5 \times 10^{15} \text{ m/s}^2}.$$

Problem D.12

A charge of $+3.0$ μC is moved from a point where the electric potential is 200 V to a point where it is 50 V. Calculate the work done by an external force in moving the charge between these points, and comment on the sign of your answer.

Lời giải chi tiết

$$W = q\,\Delta V = q(V_f - V_i) = (3.0 \times 10^{-6})(50 - 200) = \boxed{-4.5 \times 10^{-4} \text{ J}}.$$

The negative sign means no external work is actually required: the electric field itself does $+4.5 \times 10^{-4}$ J of work on the positive charge as it moves naturally from the higher potential (200 V) to the lower potential (50 V).

Problem D.13

A parallel-plate capacitor has plate area 0.050 m^2 and plate separation 2.0 mm. Calculate (a) its capacitance, (b) the charge stored when it is connected to a 9.0 V battery.

Lời giải chi tiết

(a)

$$C = \frac{\varepsilon_0 A}{d} = \frac{(8.85 \times 10^{-12})(0.050)}{2.0 \times 10^{-3}} = \boxed{2.2 \times 10^{-10} \text{ F } (221 \text{ pF})}.$$

(b)

$$Q = CV = (2.2 \times 10^{-10})(9.0) = \boxed{2.0 \times 10^{-9} \text{ C } (2.0 \text{ nC})}.$$

Problem D.14

A $4.0\ \mu$F capacitor is charged to a potential difference of 12 V. Calculate the energy stored in the capacitor.

Lời giải chi tiết

$$E_{\text{stored}} = \frac{1}{2}CV^2 = \frac{1}{2}(4.0 \times 10^{-6})(12)^2 = \boxed{2.9 \times 10^{-4} \text{ J}}.$$

Problem D.15

A capacitor is charged through a resistor in series. Which change would *increase* the time constant of the charging circuit?

(A) Decreasing R

(B) Decreasing C

(C) Increasing R

(D) Removing the resistor entirely

Lời giải chi tiết

$\tau = RC$: increasing either R or C increases τ. Of the options given, only increasing R does this. **(C)**.

Problem D.16

A long, straight wire carries a current directly out of the page, towards the reader. What is the direction of the magnetic field at a point directly above the wire (in the plane of the page)?

(A) To the left

(B) To the right

(C) Directly away from the wire

(D) Directly towards the wire

Lời giải chi tiết

By the right-hand rule, with the thumb pointing out of the page (direction of current), the fingers curl counterclockwise as seen by the reader. At the point directly above the wire, the counterclockwise tangent direction points to the left. **(A)**.

Problem D.17

A wire of length 0.25 m carrying a current of 8.0 A is placed perpendicular to a uniform magnetic field of 0.60 T. Calculate the force on the wire.

Lời giải chi tiết

$$F = BIL\sin\theta = (0.60)(8.0)(0.25)\sin 90^\circ = \boxed{1.2\text{ N}}.$$

Problem D.18

A current-carrying wire is oriented exactly parallel to an external magnetic field. What is the magnetic force on the wire?

(A) $F = BIL$, the maximum possible value

(B) $F = 0$

(C) $F = BIL/2$

(D) It depends on the wire's resistance

Lời giải chi tiết

$F = BIL\sin\theta$ with $\theta = 0^\circ$ (parallel), so $\sin\theta = 0$ and $F = 0$. **(B)**.

Problem D.19

A charged particle, with charge of magnitude equal to the elementary charge, moves at 3.5×10^5 m/s perpendicular to a magnetic field of 0.80 T. Calculate the magnitude of the magnetic force on the particle.

Lời giải chi tiết

$$F = qvB = (1.60\times10^{-19})(3.5\times10^5)(0.80) = \boxed{4.5\times10^{-14}\text{ N}}.$$

Problem D.20

A proton ($m = 1.67\times10^{-27}$ kg, $q = 1.60\times10^{-19}$ C) moves at 1.5×10^6 m/s perpendicular to a uniform magnetic field of 0.25 T. Calculate (a) the radius of its circular path, (b) the period of its motion.

Lời giải chi tiết

(a)

$$r = \frac{mv}{qB} = \frac{(1.67\times10^{-27})(1.5\times10^6)}{(1.60\times10^{-19})(0.25)} = \boxed{6.3\times10^{-2}\text{ m }(6.3\text{ cm})}.$$

(b)

$$T = \frac{2\pi m}{qB} = \frac{2\pi(1.67\times10^{-27})}{(1.60\times10^{-19})(0.25)} = \boxed{2.6\times10^{-7}\text{ s}}.$$

Problem D.21

A proton and an alpha particle (charge $+2e$, mass approximately 4 times the proton's mass) move at the same speed, perpendicular to the same magnetic field. How does the radius of the alpha particle's circular path compare with that of the proton?

(A) Same radius

(B) Half the radius

(C) Twice the radius

(D) Four times the radius

Lời giải chi tiết

$r = mv/(qB)$. Ratio: $\frac{r_\alpha}{r_p} = \frac{m_\alpha/q_\alpha}{m_p/q_p} = \frac{4m_p/2e}{m_p/e} = \frac{2m_p/e}{m_p/e} = 2$. **(C)**.

Problem D.22

In a velocity selector, the electric field strength is 5.0×10^4 N/C. Only particles moving at 2.5×10^5 m/s pass through undeflected. Calculate the required magnetic field strength.

Lời giải chi tiết

$$v = \frac{E}{B} \implies B = \frac{E}{v} = \frac{5.0 \times 10^4}{2.5 \times 10^5} = \boxed{0.20\ \text{T}}.$$

Problem D.23

In a velocity selector, the electric force on a charged particle points in a fixed direction independent of speed, while the magnetic force points in the opposite direction with magnitude proportional to speed. A particle enters moving *faster* than the selected speed $v = E/B$. In which direction is it deflected?

(A) In the direction of the electric force, since it always dominates

(B) In the direction of the magnetic force, since qvB now exceeds qE

(C) It continues straight, since only slow particles deflect

(D) Perpendicular to both forces

Lời giải chi tiết

The magnetic force qvB grows with speed while the electric force qE stays fixed; for $v > E/B$, $qvB > qE$, so the net force — and hence the deflection — is in the direction of the magnetic force. **(B)**.

Problem D.24

A proton enters a region of uniform magnetic field $B = 0.20$ T, moving at 4.0×10^6 m/s perpendicular to the field. (a) Calculate the radius of its circular path. (b) Calculate the period of its motion. (c) If the proton is replaced by an electron with the same speed in the same field, state and briefly explain how the radius of the path changes.

Lời giải chi tiết

(a)

$$r = \frac{mv}{qB} = \frac{(1.67 \times 10^{-27})(4.0 \times 10^6)}{(1.60 \times 10^{-19})(0.20)} = \boxed{0.21\ \text{m}}.$$

(b)

$$T = \frac{2\pi m}{qB} = \frac{2\pi(1.67 \times 10^{-27})}{(1.60 \times 10^{-19})(0.20)} = \boxed{3.3 \times 10^{-7}\ \text{s}}.$$

(c) Since $r = mv/(qB)$ is directly proportional to mass (with $|q|$, v, B unchanged), and the electron's mass is about $1836\times$ smaller than the proton's, the electron's radius would be about $1836\times$ **smaller** than the proton's — a far tighter circle (curving the opposite way, since the electron's charge is negative).

Problem D.25 (AHL)

A circular coil of area 0.020 m^2 is placed in a uniform magnetic field of 0.40 T, oriented so that the normal to the coil makes an angle of 60° with the field. Calculate the magnetic flux through the coil.

Lời giải chi tiết

$$\Phi = BA\cos\theta = (0.40)(0.020)\cos 60^\circ = \boxed{4.0 \times 10^{-3}\ \text{Wb}}.$$

Problem D.26 (AHL)

The south pole of a bar magnet is withdrawn (moved away) from a coil of wire connected to form a closed loop. Which statement correctly describes the induced current, as determined by Lenz's law?

(A) The induced current creates a north pole at the near face of the coil, to attract the departing south pole

(B) The induced current creates a south pole at the near face of the coil, to repel the departing south pole

(C) No current is induced, since the magnet is moving away rather than towards the coil

(D) The current direction cannot be determined without knowing the coil's resistance

Lời giải chi tiết

The flux from the departing south pole is decreasing. By Lenz's law, the induced current opposes this decrease by trying to keep the magnet in place — the near face of the coil becomes a north pole, which *attracts* the departing south pole (opposite poles attract), opposing its retreat. **(A)**.

Problem D.27 (AHL)

A coil of 250 turns experiences a change in magnetic flux (through each turn) from 2.0×10^{-4} Wb to 6.0×10^{-4} Wb over a time interval of 0.050 s. Calculate the magnitude of the induced EMF.

Lời giải chi tiết

$$|\varepsilon| = N\left|\frac{\Delta\Phi}{\Delta t}\right| = 250 \times \frac{(6.0-2.0)\times 10^{-4}}{0.050} = \boxed{2.0\ \text{V}}.$$

Problem D.28 (AHL)

A conducting rod of length 0.80 m slides along horizontal rails at constant velocity 6.0 m/s, perpendicular to a uniform magnetic field of 0.35 T. The rails are connected by a resistor of $5.0\ \Omega$. Calculate (a) the induced EMF, (b) the current in the circuit, (c) the force that must be applied to the rod to keep it moving at constant velocity.

Lời giải chi tiết

(a)

$$\varepsilon = BLv = (0.35)(0.80)(6.0) = \boxed{1.7\text{ V}}.$$

(b)

$$I = \frac{\varepsilon}{R} = \frac{1.68}{5.0} = \boxed{0.34\text{ A}}.$$

(c) Since the rod moves at constant velocity, the applied force must exactly balance the retarding magnetic force on the current-carrying rod:

$$F = BIL = (0.35)(0.336)(0.80) = \boxed{0.094\text{ N}}.$$

Problem D.29 (AHL)

A generator coil has 150 turns, each of area 0.025 m^2, and rotates in a uniform magnetic field of 0.080 T at a frequency of 60 Hz. Calculate the peak EMF produced.

Lời giải chi tiết

$$\omega = 2\pi f = 2\pi(60) = 377\text{ rad/s}.$$

$$\varepsilon_0 = NBA\omega = (150)(0.080)(0.025)(377) = \boxed{1.1\times 10^2\text{ V }(113\text{ V})}.$$

Problem D.30 (AHL)

A step-up transformer has 80 turns on the primary and 800 turns on the secondary. The primary is connected to a 120 V AC supply and draws a current of 15.0 A. Assuming an ideal (lossless) transformer, calculate (a) the secondary voltage, (b) the secondary current.

Lời giải chi tiết

(a)

$$V_s = V_p\cdot\frac{N_s}{N_p} = 120\times\frac{800}{80} = \boxed{1200\text{ V}}.$$

(b) From power conservation, $I_pV_p = I_sV_s$:

$$I_s = I_p\cdot\frac{N_p}{N_s} = 15.0\times\frac{80}{800} = \boxed{1.5\text{ A}}.$$

Problem D.31 (AHL)

(a) A long, straight wire carries a current of 15.0 A. Calculate the magnetic field strength at a perpendicular distance of 0.12 m from the wire. (b) A solenoid has 1200 turns wound uniformly over a length of 0.60 m and carries a current of 2.0 A. Calculate the magnetic field strength inside the solenoid.

Lời giải chi tiết

(a)

$$B = \frac{\mu_0 I}{2\pi d} = \frac{(4\pi\times 10^{-7})(15.0)}{2\pi(0.12)} = \boxed{2.5\times 10^{-5}\text{ T}}.$$

(b) Turns per unit length: $n = N/\ell = 1200/0.60 = 2000$ m^{-1}.

$$B = \mu_0 nI = (4\pi \times 10^{-7})(2000)(2.0) = \boxed{5.0 \times 10^{-3}\text{ T } (5.0\text{ mT})}.$$

Note that the solenoid, despite carrying less current than the straight wire, produces a field roughly 200 times stronger — concentrating many turns close together is far more effective than a single long wire at building up field strength.

Problem D.32

Two point charges, $q_1 = +6.0\ \mu$C and $q_2 = -6.0\ \mu$C, are fixed 0.20 m apart. Point P lies on the perpendicular bisector of the line joining them, 0.15 m from the midpoint. Calculate the magnitude of the net electric field at P.

Lời giải chi tiết

By symmetry, P is equidistant from both charges:

$$r = \sqrt{(0.10)^2 + (0.15)^2} = \sqrt{0.0325} = 0.180\text{ m}.$$

Each charge produces a field of the same magnitude at P:

$$E_1 = E_2 = \frac{kq}{r^2} = \frac{(8.99 \times 10^9)(6.0 \times 10^{-6})}{0.0325} = 1.66 \times 10^6\text{ N/C}.$$

The field from $+q_1$ points away from q_1; the field from $-q_2$ points towards q_2. Resolving each into components (parallel and perpendicular to the line joining the charges), the components perpendicular to that line (i.e. along PM, M = midpoint) are equal and opposite and **cancel exactly** by symmetry, while the components parallel to the line joining the charges point in the *same* direction and add. Using $\cos\phi = 0.10/0.180 = 0.555$ for the angle ϕ between each field vector and the line joining the charges:

$$E_{\text{net}} = 2E_1 \cos\phi = 2(1.66 \times 10^6)(0.555) = \boxed{1.8 \times 10^6\text{ N/C}},$$

directed parallel to the line joining the charges, from the positive charge's side towards the negative charge's side.

Theme E: Nuclear and Quantum Physics

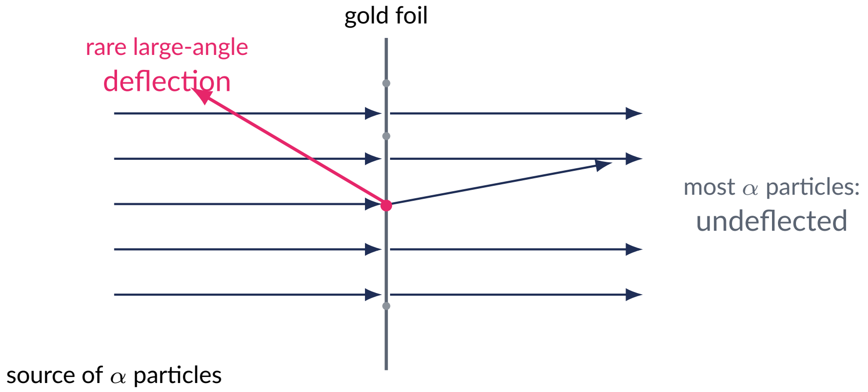
Mục tiêu Unit: Mô hình nguyên tử và quang phổ vạch; vật lý lượng tử (AHL) — hiệu ứng quang điện, lưỡng tính sóng–hạt, nguyên lý bất định Heisenberg; phân rã phóng xạ và định luật phân rã theo hàm mũ; phân hạch hạt nhân, năng lượng liên kết và lò phản ứng hạt nhân; phản ứng nhiệt hạch, sự hình thành và tiến hoá của các ngôi sao, và mô hình chuẩn của vật lý hạt.

5.1 E.1 Structure of the Atom

5.1.1 Early Atomic Models and the Rutherford Scattering Experiment

By the early twentieth century, J. J. Thomson's discovery of the electron led to the **plum-pudding model**: an atom was imagined as a diffuse sphere of positive charge, roughly the size of the whole atom, with negatively charged electrons embedded in it like plums in a pudding. This model predicted that the positive charge — and therefore the mass — of an atom was spread out fairly uniformly.

In 1909, Hans Geiger and Ernest Marsden, working under Ernest Rutherford, fired a narrow beam of fast, positively charged **alpha particles** (α, essentially bare helium nuclei) at an extremely thin sheet of gold foil (only a few hundred atoms thick) and recorded where the particles landed on a surrounding fluorescent screen.



Rutherford's gold-foil experiment: a beam of α particles is fired at a thin gold foil. Most pass straight through; a few deflect through small angles; a very small fraction bounce back through angles greater than 90°.

The observations, and their interpretation, were:

- Most α particles passed straight through the foil with little or no deflection — most of the atom's volume must be empty space.
- A small number deflected through moderate angles — they passed relatively close to a concentration of positive charge.
- A very small fraction (roughly 1 in 8000) bounced almost straight back, through angles greater than 90° — an outcome the plum-pudding model could not explain at all, since a diffuse charge distribution could never exert a large enough force on a fast, massive α particle to reverse its path.

Rutherford concluded that an atom must contain a **tiny, extremely dense, positively charged nucleus** at its centre, carrying essentially all of the atom's mass, surrounded by mostly empty space through which much lighter electrons orbit. Only a head-on (or near head-on) approach to this concentrated charge could produce the strong electrostatic repulsion needed to reverse an α particle's path. This is the **nuclear model** of the atom.

Khái niệm cốt lõi

The Rutherford scattering experiment is a classic example of using a scattering probe to infer the internal structure of something too small to see directly: the pattern of deflections (mostly none, occasionally large) reveals that the atom's positive charge and mass are concentrated in a volume many thousands of times smaller than the atom itself.

VN **GIẢI THÍCH TIẾNG VIỆT**

Thí nghiệm tán xạ hạt α của Rutherford (bắn hạt α vào lá vàng mỏng) là bằng chứng thực nghiệm quan trọng nhất cho mô hình hạt nhân nguyên tử. Điểm mấu chốt cần nhớ: **đa số hạt α xuyên thẳng qua** (nguyên tử chủ yếu là khoảng không rỗng), nhưng **một số rất ít bật ngược trở lại** — điều mô hình "mứt nho" (plum-pudding, điện tích dương dàn trải đều) không thể giải thích được. Từ đó suy ra: điện tích dương và gần như toàn bộ khối lượng nguyên tử tập trung ở một **hạt nhân rất nhỏ và rất đặc** ở tâm.

5.1.2 Estimating Nuclear Size: Closest-Approach Distance and Nuclear Radius

Additional Higher Level (AHL) content.

The scattering data from the Rutherford experiment can be pushed further than a purely qualitative conclusion: it can be used to place a quantitative bound on the size of the nucleus itself. Consider an α particle fired directly, head-on, at a nucleus. As it approaches, the repulsive Coulomb force continuously decelerates it, converting kinetic energy into electric potential energy, until — at some **closest-approach distance** d — the α particle is momentarily at rest and all of its initial kinetic energy has become electric potential energy, before it is repelled straight back out along its incoming path.

Equating the α particle's initial kinetic energy to the Coulomb potential energy at closest approach (using Coulomb's constant $k = 8.99 \times 10^9$ N · m^2/C^2, with the α particle carrying charge $+2e$ and the nucleus $+Ze$):

$$KE_\alpha = \frac{k(2e)(Ze)}{d} \qquad \Rightarrow \qquad d = \frac{2kZe^2}{KE_\alpha}.$$

Because this calculation assumes only the (long-range) Coulomb force acts throughout the approach, the true nuclear radius R must satisfy $R \leq d$: if the α particle actually reached the edge of the nucleus, the additional attractive strong nuclear force would alter the trajectory and the purely-Coulomb prediction would break down. So d, calculated this way, is only an *upper bound* on the nuclear radius.

Ví dụ mẫu · Closest-approach distance for a gold nucleus

An α particle with kinetic energy 5.0 MeV is fired head-on at a gold nucleus ($Z = 79$). Calculate the closest-approach distance.

$$KE_\alpha = 5.0 \times 10^6 \times 1.60 \times 10^{-19} = 8.00 \times 10^{-13} \text{ J}.$$

$$d = \frac{2kZe^2}{KE_\alpha} = \frac{2(8.99\times10^9)(79)(1.60\times10^{-19})^2}{8.00\times10^{-13}} = \frac{2(8.99\times10^9)(79)(2.56\times10^{-38})}{8.00\times10^{-13}}.$$

$$d = \frac{3.64\times10^{-26}}{8.00\times10^{-13}} = 4.5\times10^{-14}\text{ m} = 45\text{ fm}.$$

This is roughly six times larger than the actual radius of a gold nucleus (≈ 7 fm, from the formula below) — confirming that 5.0 MeV α particles never actually reach the nucleus; they are turned back purely by the long-range Coulomb force well before the short-range strong force could ever come into play. This is exactly how the very short range of the strong nuclear force was first inferred: pure Rutherford (Coulomb) scattering, with no deviations, is observed right down to gaps of a few tens of femtometres.

Independent measurements (for instance, using high-energy electron diffraction, which *can* probe distances smaller than typical α-particle closest-approach distances) show that nuclear radius follows a simple empirical pattern.

$$R = R_0A^{1/3}, \qquad R_0 \approx 1.2\text{ fm} = 1.2\times10^{-15}\text{ m}.$$

Since nuclear volume $\propto R^3 \propto A$, and nuclear mass is also (very nearly) proportional to A, nuclear **density** $\rho = \frac{\text{mass}}{\text{volume}}$ comes out approximately the *same* for every nuclide, regardless of size — as if every nucleus were built from incompressible nucleons packed together at one universal density.

Ví dụ mẫu · Radius and density of a uranium-238 nucleus

Estimate the radius and the density of a uranium-238 nucleus ($A = 238$, mass ≈ 238 u).

Radius:

$$R = R_0A^{1/3} = (1.2\times10^{-15})(238)^{1/3} = (1.2\times10^{-15})(6.20) = 7.44\times10^{-15}\text{ m}.$$

Volume:

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(7.44\times10^{-15})^3 = 1.73\times10^{-42}\text{ m}^3.$$

Mass:

$$m = 238\times1.66\times10^{-27} = 3.95\times10^{-25}\text{ kg}.$$

Density:

$$\rho = \frac{m}{V} = \frac{3.95\times10^{-25}}{1.73\times10^{-42}} = 2.3\times10^{17}\text{ kg/m}^3.$$

This is an extraordinary density — comparable to the density inside a neutron star (E.5) — and it is essentially the same value obtained for *any* nuclide, light or heavy, confirming $R \propto A^{1/3}$. It also puts the earlier Rutherford scattering result in perspective: virtually all of an atom's mass is packed into a volume that occupies only about one part in 10^{15} of the atom's total volume, the rest being the near-empty space occupied by the diffuse electron cloud.

(!) Lỗi thường gặp: Khoảng cách tiến gần nhất d tính từ bảo toàn năng lượng Coulomb chỉ là **giới hạn trên** của bán kính hạt nhân ($R \le d$), không phải giá trị chính xác của R — vì phép tính giả định lực Coulomb tác dụng suốt quá trình, trong khi trên thực tế nếu hạt α tới đủ gần, lực hạt nhân mạnh (hút, tầm ngắn) sẽ can thiệp. Ngoài ra, khi tính $R = R_0A^{1/3}$, nhớ lấy **căn bậc**

ba của A (không phải bình phương) — đây là lỗi bấm máy tính rất hay gặp.

VN **GIẢI THÍCH TIẾNG VIỆT**

Từ bảo toàn năng lượng (động năng ban đầu của hạt α chuyển hết thành thế năng Coulomb tại khoảng cách tiến gần nhất d), ta ước lượng được **giới hạn trên** của bán kính hạt nhân: $d = 2kZe^2/KE_\alpha$. Đo đạc độc lập (VD nhiễu xạ electron năng lượng cao) cho công thức bán kính hạt nhân thực nghiệm $R = R_0A^{1/3}$ với $R_0 \approx 1.2$ fm — vì thể tích tỉ lệ với A giống như khối lượng, **mật độ hạt nhân gần như không đổi** với mọi nguyên tố, cỡ 10^{17} kg/m^3 — một con số khổng lồ, cho thấy nguyên tử chủ yếu là khoảng không rỗng.

5.1.3 Atomic Notation, Nuclides, and Isotopes

A nucleus is described completely (in terms of its composition) by two integers:

- the **atomic number** Z — the number of protons in the nucleus (equal to the number of electrons in a neutral atom, and what determines the chemical element);
- the **mass number** A — the total number of nucleons (protons *plus* neutrons) in the nucleus.

The **neutron number** is then $N = A - Z$. A specific combination of Z and N is called a **nuclide**, and is written in nuclide notation as

$$^{A}_{Z}X,$$

where X is the chemical symbol of the element (which is itself already determined by Z, so the subscript Z is sometimes omitted).

$$A = Z + N, \qquad N = A - Z.$$

Isotopes of an element share the same Z (same number of protons, hence the same chemical identity) but differ in N (and therefore in A and in mass). Isotopes of the same element have essentially identical chemical behaviour but different nuclear stability, mass, and (if unstable) decay properties.

Ví dụ mẫu · Nuclide notation and isotopes

Carbon has three naturally occurring isotopes with mass numbers 12, 13, and 14. Carbon's atomic number is $Z = 6$. Write the nuclide notation for each, and state the number of neutrons in each.

For all three, $Z = 6$ (that is what makes them carbon). Using $N = A - Z$:
$^{12}_{6}$C: $N = 12 - 6 = 6$ neutrons.
$^{13}_{6}$C: $N = 13 - 6 = 7$ neutrons.
$^{14}_{6}$C: $N = 14 - 6 = 8$ neutrons (this is the unstable isotope used in radiocarbon dating, discussed in E.3).
All three have identical chemistry (six protons ⇒ six electrons in the neutral atom ⇒ identical bonding behaviour), but different masses and different nuclear stability.

Ví dụ mẫu · Relative atomic mass from isotopic abundance

Naturally occurring chlorine consists of two stable isotopes: chlorine-35 (mass 34.97 u, natural abundance 75.8%) and chlorine-37 (mass 36.97 u, natural abundance 24.2%). Calculate the relative atomic mass of chlorine, as it would appear on a periodic table.

The relative atomic mass is the abundance-weighted average of the isotopic masses:

$$\bar{m} = (0.758)(34.97) + (0.242)(36.97) = 26.51 + 8.95 = 35.45 \text{ u}.$$

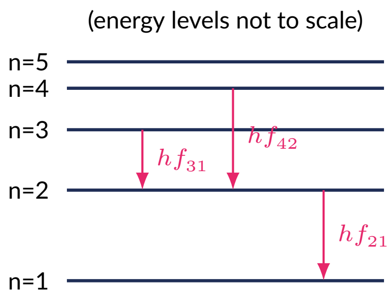
This matches the standard periodic-table value for chlorine (35.45 u) — a value that does not correspond to any single, actual chlorine nucleus, but to the statistical mixture of isotopes found in nature.

Isotopic masses and relative abundances of this kind are measured using a **mass spectrometer**: a sample is first ionized (electrons removed to create positive ions), then the ions are accelerated through a fixed potential difference and passed through a magnetic field. Because the magnetic force provides the centripetal force for circular motion, ions follow a circular path whose radius depends on their mass-to-charge ratio ($r = mv/(qB)$, larger mass giving a larger radius for otherwise identical ions) — so ions of different isotopes, having been accelerated to different speeds by the same potential difference (since $qV = \frac{1}{2}mv^2$) but carrying the same charge, separate onto different circular paths and can be detected and counted individually, revealing both the precise isotopic masses and their relative abundances in the original sample.

(!) Lỗi thường gặp: Đừng nhầm lẫn số khối A (tổng số proton + neutron, gần bằng khối lượng nguyên tử tính theo u) với số hiệu nguyên tử Z (chỉ số proton, quyết định nguyên tố hoá học). Khi cân bằng một phương trình hạt nhân, luôn kiểm tra **riêng biệt** tổng A hai vế và tổng Z hai vế — chúng phải bằng nhau độc lập với nhau, không được cộng gộp.

5.1.4 Emission and Absorption Line Spectra

When a gas at low pressure is excited (for example, by an electric discharge) and the emitted light is passed through a spectrometer, the result is not a continuous rainbow but a series of sharp, discrete bright lines at specific wavelengths — an **emission line spectrum**. Every element produces its own unique pattern of lines, acting as a kind of fingerprint. Conversely, if white light (a continuous spectrum) is passed through a cool sample of the same gas, dark lines appear at exactly the same wavelengths — an **absorption line spectrum**.



A simplified atomic energy-level diagram. An electron dropping from a higher level to a lower one emits a single photon whose energy exactly equals the gap between the two levels, $E_{photon} = hf = E_i - E_f$.

This pattern is explained by assuming that electrons in an atom can only occupy certain **discrete, quantized energy levels** $E_1, E_2, E_3, \ldots$ — never any energy in between. When an electron falls from a higher level E_i to a lower level E_f, it emits a single photon carrying away exactly the energy difference; when an atom absorbs a photon, that photon must supply exactly the energy needed to lift an electron from one allowed level to another (or the photon is not absorbed at all).

$$E_{photon} = hf = \frac{hc}{\lambda} = E_i - E_f \qquad \text{(emission, } E_i > E_f\text{)}.$$

Because the set of allowed levels $\{E_n\}$ is different for every element, the set of possible transition energies — and therefore the set of spectral-line wavelengths — is unique to each element. A photon is absorbed only if its energy exactly matches some allowed transition energy of the absorbing atom; otherwise it passes straight through.

This principle underlies essentially all of astrophysics: starlight, having passed on its way to us through the cooler outer layers of a star's own atmosphere, arrives with a pattern of dark absorption lines superimposed on its continuous spectrum. Matching the wavelengths of these lines against the known laboratory spectra of different elements reveals exactly which elements are present in a star's atmosphere — this is, in fact, the only practical way of determining what distant stars are made of. The same spectral lines also reveal a star's motion: if a star is moving away from (or toward) Earth, every line in its spectrum is shifted very slightly toward longer (or shorter) wavelengths by the Doppler effect, so precisely measuring this shift, relative to the known laboratory wavelength of a recognized spectral line, gives the star's velocity along the line of sight.

VN **GIẢI THÍCH TIẾNG VIỆT**

Quang phổ vạch (phát xạ hoặc hấp thụ) là bằng chứng trực tiếp cho thấy **electron trong nguyên tử chỉ tồn tại ở các mức năng lượng rời rạc**, không liên tục. Khi electron nhảy từ mức cao xuống mức thấp, nó phát ra đúng **một** photon có năng lượng bằng *chính xác* hiệu hai mức ($E = hf = E_i - E_f$) — không nhiều hơn, không ít hơn. Vì mỗi nguyên tố có một bộ mức năng lượng riêng, mỗi nguyên tố cho một "vân tay quang phổ" duy nhất — đây là nguyên lý dùng để nhận diện thành phần hoá học của các ngôi sao xa xôi.

5.1.5 The Bohr Model of Hydrogen

Niels Bohr proposed a specific quantitative model for the hydrogen atom (one proton, one electron) in which the electron's allowed energy levels are given by a simple formula, labelled by a positive integer $n = 1, 2, 3, \ldots$ called the **principal quantum number**.

$$E_n = -\frac{13.6 \text{ eV}}{n^2}, \qquad n = 1, 2, 3, \ldots$$

$n = 1$ is the **ground state** (lowest, most negative energy, $E_1 = -13.6$ eV); higher n are **excited states**, with energies approaching 0 as $n \to \infty$. The negative sign reflects that the electron is *bound* to the nucleus: energy must be supplied to move it to a higher level or to remove it entirely. The **ionization energy** from level n is the energy required to remove the electron from level n out to $n = \infty$ (where $E_\infty = 0$), i.e. $IE_n = -E_n = \frac{13.6 \text{ eV}}{n^2}$; for the ground state, $IE_1 = 13.6$ eV.

Ví dụ mẫu · The first line of the Balmer series

Calculate the energy and the wavelength of the photon emitted when a hydrogen atom's electron falls from $n = 3$ to $n = 2$ (the first, longest-wavelength line of the Balmer series, historically labelled Hα). State which region of the spectrum this lies in.

Step 1 — level energies.

$$E_3 = -\frac{13.6}{3^2} = -\frac{13.6}{9} = -1.511 \text{ eV}, \qquad E_2 = -\frac{13.6}{2^2} = -\frac{13.6}{4} = -3.400 \text{ eV}.$$

Step 2 — photon energy.

$$E_{photon} = E_3 - E_2 = -1.511 - (-3.400) = 1.889 \text{ eV}.$$

Step 3 — convert to joules and find wavelength.

$$E_{photon} = 1.889 \times 1.60 \times 10^{-19} = 3.022 \times 10^{-19} \text{ J}.$$

$$\lambda = \frac{hc}{E_{photon}} = \frac{(6.63 \times 10^{-34})(3.00 \times 10^{8})}{3.022 \times 10^{-19}} = \frac{1.989 \times 10^{-25}}{3.022 \times 10^{-19}} = 6.58 \times 10^{-7} \text{ m} = 658 \text{ nm}.$$

This wavelength lies in the **red** part of the visible spectrum — this is indeed the well-known red Hα line seen in hydrogen emission spectra (and in the light from many astronomical nebulae).

Ví dụ mẫu · Ionization energy from an excited state

A hydrogen atom is already in its first excited state ($n = 2$). Find the energy and wavelength of the photon that would just barely ionize it from this state.

The electron must be lifted from $n = 2$ to $n = \infty$ ($E_\infty = 0$):

$$E = 0 - E_2 = 0 - (-3.400) = 3.400 \text{ eV} = 3.400 \times 1.60 \times 10^{-19} = 5.44 \times 10^{-19} \text{ J}.$$

$$\lambda = \frac{hc}{E} = \frac{1.989 \times 10^{-25}}{5.44 \times 10^{-19}} = 3.66 \times 10^{-7} \text{ m} = 366 \text{ nm}.$$

This photon (energy 3.40 eV, wavelength ≈ 366 nm) lies in the near-ultraviolet — any photon of *at least* this energy (i.e. wavelength 366 nm or shorter) can ionize a hydrogen atom already sitting in $n = 2$, since above the ionization threshold the electron is free and can carry away any excess energy as kinetic energy.

Combining the level formula with $E_{photon} = E_i - E_f$ gives a general expression for the energy of any hydrogen transition, and the full set of hydrogen spectral lines is traditionally grouped into named **series**, according to the lower level n_f the electron lands on:

$$E_{photon} = 13.6 \text{ eV} \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right), \qquad n_i > n_f.$$

Series	Lower level n_f	Spectral region	Example line
Lyman	1	ultraviolet	$n = 2 \to 1$: 10.2 eV
Balmer	2	visible	$n = 3 \to 2$: 1.89 eV (Hα, 658 nm)
Paschen	3	infrared	$n = 4 \to 3$: 0.66 eV

Only the Balmer series lies (partly) in the visible range — this is precisely why it was the first series to be discovered and catalogued (in 1885, decades before the Bohr model existed to explain it), while the Lyman series (higher energy, shorter wavelength) and the Paschen series (lower energy, longer wavelength) both lie outside human vision.

(!) Lỗi thường gặp: Năng lượng mức E_n luôn **âm** (electron bị liên kết), nên khi tính năng lượng photon phát xạ/hấp thụ, hãy lấy *hiệu* $E_i - E_f$ một cách cẩn thận theo đúng dấu — cách an toàn nhất là luôn lấy **giá trị tuyệt đối** của hiệu hai mức làm năng lượng photon, vì photon luôn mang năng lượng dương. Một lỗi phổ biến khác: nhầm n (số lượng tử chính, số nguyên) với N (số neutron) — hai ký hiệu hoàn toàn khác nhau xuất hiện trong cùng chương này.

GIẢI THÍCH TIẾNG VIỆT

VN

Mô hình Bohr cho nguyên tử hydro: $E_n = -13.6$ eV$/n^2$. Trạng thái cơ bản $n = 1$ có năng lượng thấp nhất (âm nhất); năng lượng ion hoá từ mức n là $-E_n$ (năng lượng cần để đưa electron ra $n = \infty$, nơi $E = 0$). Khi electron chuyển mức, năng lượng photon phát ra/hấp thụ đúng bằng hiệu hai mức — dùng công thức này để tính bước sóng vạch quang phổ ($E = hc/\lambda$). Dãy Balmer (electron rơi về $n = 2$) cho các vạch trong vùng ánh sáng nhìn thấy; dãy Lyman (rơi về $n = 1$) nằm trong vùng tử ngoại.

5.1.6 Bohr's Postulate: Quantization of Angular Momentum

Additional Higher Level (AHL) content.

Bohr's energy formula was not simply fitted to match the observed hydrogen spectrum — it followed from a single new postulate added to an otherwise classical picture of an electron orbiting the nucleus. Classically, an orbiting electron continuously accelerates (its velocity direction is always changing), and an accelerating charge should continuously radiate electromagnetic energy; a classical electron should spiral into the nucleus within a fraction of a second, and stable atoms should not exist at all. Bohr's resolution was to postulate that an electron can occupy only those circular orbits for which its orbital angular momentum equals a whole-number multiple of a fixed unit, $h/(2\pi)$:

Bohr's quantization postulate:

$$m_e v r = n\frac{h}{2\pi}, \qquad n = 1, 2, 3, \ldots$$

While in one of these allowed ("stationary") orbits, the electron does *not* radiate energy, regardless of classical expectations — radiation is emitted or absorbed only during a transition between two allowed orbits. Combining this postulate with Newton's second law applied to circular motion (the Coulomb attraction supplying exactly the centripetal force) allows both the allowed orbital radii and the allowed energies $E_n = -13.6$ eV$/n^2$ to be derived from first principles, rather than simply read off the spectrum.

A satisfying physical justification for *why* angular momentum should be quantized in exactly this way came only a decade later, from de Broglie's hypothesis (E.2): if the orbiting electron is pictured as a standing wave wrapped around the nucleus, then a stable orbit requires the orbit's circumference to contain a whole number of de Broglie wavelengths $\lambda = h/(m_e v)$, i.e. $2\pi r = n\lambda$. Substituting $\lambda = h/(m_e v)$ immediately reproduces Bohr's postulate, $m_e v r = nh/(2\pi)$ — showing that Bohr's ad-hoc quantization rule is really just the wave nature of the electron expressing itself as a standing-wave condition.

Ví dụ mẫu · Checking the standing-wave picture for the $n = 1$ orbit

The radius of the $n = 1$ (ground-state) orbit in the Bohr model of hydrogen is $r_1 = 5.29 \times 10^{-11}$ m (the "Bohr radius"), and the electron's kinetic energy in this orbit is 13.6 eV. Find the electron's de Broglie wavelength in this orbit, and compare it with the orbit's circumference.

Speed from kinetic energy:

$$KE = \tfrac{1}{2} m_e v^2 \;\Rightarrow\; v = \sqrt{\frac{2KE}{m_e}} = \sqrt{\frac{2(13.6 \times 1.60 \times 10^{-19})}{9.11 \times 10^{-31}}} = \sqrt{4.78 \times 10^{12}} = 2.19 \times 10^6 \text{ m/s}.$$

De Broglie wavelength:

$$\lambda = \frac{h}{m_e v} = \frac{6.63 \times 10^{-34}}{(9.11 \times 10^{-31})(2.19 \times 10^{6})} = 3.33 \times 10^{-10} \text{ m.}$$

Orbit circumference:

$$2\pi r_1 = 2\pi(5.29 \times 10^{-11}) = 3.32 \times 10^{-10} \text{ m.}$$

The de Broglie wavelength and the orbit circumference agree to within rounding error — exactly one full electron wavelength fits around the $n = 1$ orbit, confirming the standing-wave picture.

VN **GIẢI THÍCH TIẾNG VIỆT**

Giả thuyết Bohr: mô-men động lượng của electron trong quỹ đạo chỉ nhận các giá trị rời rạc $m_e vr = nh/(2\pi)$ — đây chính là điều kiện khiến năng lượng bị lượng tử hoá, chứ không phải một giả định tuỳ tiện. De Broglie (mục E.2) giải thích được **vì sao** lại như vậy: nếu xem electron là sóng đứng quấn quanh hạt nhân, chu vi quỹ đạo phải chứa đúng một số nguyên lần bước sóng de Broglie ($2\pi r = n\lambda$) — thay $\lambda = h/(m_e v)$ vào cho đúng ngay điều kiện của Bohr. Đây là một ví dụ đẹp cho thấy hai ý tưởng tưởng như độc lập (Bohr và de Broglie) thực ra là một.

5.2 E.2 Quantum Physics (AHL)

This content is assessed at Additional Higher Level (AHL) only.

5.2.1 The Photoelectric Effect

When light of sufficiently high frequency shines on a clean metal surface, electrons — called **photoelectrons** — are ejected from the surface, essentially instantaneously. Careful experiments reveal a set of observations that classical wave theory (in which light's energy is spread continuously across its wavefront, and a wave of any frequency should eventually deliver enough energy given enough time or intensity) simply cannot explain:

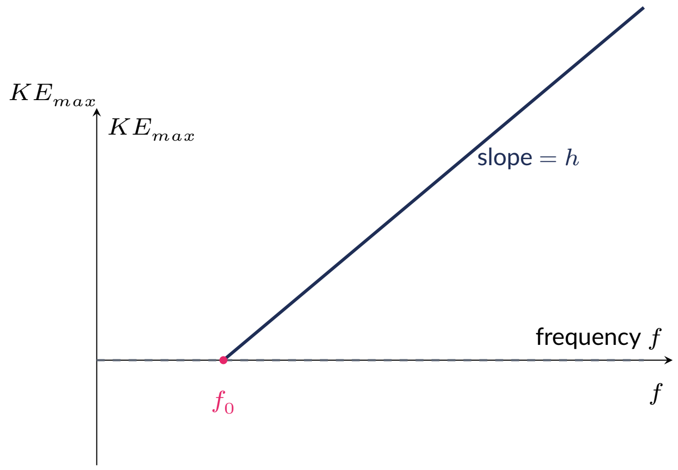
- Emission occurs only above a certain minimum **threshold (cutoff) frequency** f_0, which depends on the metal; below f_0, *no* electrons are emitted, no matter how intense the light or how long it shines.
- Above f_0, emission begins with no measurable delay, even for extremely low intensities.
- Increasing the **intensity** of light above f_0 increases the *number* of photoelectrons emitted per second (and hence the photoelectric current), but does *not* change their maximum kinetic energy.
- Increasing the **frequency** of the light above f_0 increases the maximum kinetic energy of the photoelectrons.

Einstein explained these results (1905) by proposing that light itself is quantized into discrete packets of energy called **photons**, each carrying energy $E = hf$, and that a single photon interacts with a single electron, transferring all of its energy to that one electron in one indivisible event. A minimum energy, the **work function** Φ, is needed to free even the most loosely bound electron from the metal surface; $\Phi = hf_0$. Any photon energy in excess of Φ becomes kinetic energy of the ejected electron. Electrons emitted from deeper below the surface lose some energy escaping, so there is a *maximum* photoelectron kinetic energy, achieved by electrons that were free at the surface itself.

Einstein's photoelectric equation:

$$hf = \Phi + KE_{max}, \qquad \Phi = hf_0, \qquad KE_{max} = eV_s,$$

where V_s is the **stopping potential** — the minimum reverse (retarding) voltage that must be applied to just stop the most energetic photoelectrons from reaching a collector, reducing the photocurrent to zero.



KE_{max} versus frequency f is a straight line: $KE_{max} = hf - \Phi$. The gradient is Planck's constant h (independent of the metal); the horizontal intercept is the threshold frequency $f_0 = \Phi/h$; the vertical intercept (extrapolated) is $-\Phi$.

Ví dụ mẫu · Work function and stopping voltage of sodium

The threshold wavelength for photoemission from a sodium surface is $\lambda_0 = 540$ nm. (a) Find the work function of sodium, in eV. (b) Light of wavelength 400 nm is then shone on the same surface; find the maximum kinetic energy of the emitted photoelectrons and the stopping voltage needed to stop them.

(a) Threshold frequency: $f_0 = \dfrac{c}{\lambda_0} = \dfrac{3.00 \times 10^8}{540 \times 10^{-9}} = 5.556 \times 10^{14}$ Hz.

$$\Phi = hf_0 = (6.63 \times 10^{-34})(5.556 \times 10^{14}) = 3.68 \times 10^{-19}\ \text{J} = \frac{3.68 \times 10^{-19}}{1.60 \times 10^{-19}} = 2.30\ \text{eV}.$$

(b) Photon energy at 400 nm:

$$E = \frac{hc}{\lambda} = \frac{1.989 \times 10^{-25}}{400 \times 10^{-9}} = 4.97 \times 10^{-19}\ \text{J} = 3.11\ \text{eV}.$$

$$KE_{max} = E - \Phi = 3.11 - 2.30 = 0.81\ \text{eV} \qquad \Rightarrow \qquad V_s = \frac{KE_{max}}{e} = 0.81\ \text{V}.$$

(Since KE_{max} in eV is numerically equal to V_s in volts, this last step needs no unit conversion.)

(!) Lỗi thường gặp: Sai lầm kinh điển: cho rằng tăng **cường độ** (intensity) ánh sáng sẽ làm tăng KE_{max} của quang electron. Thực tế, cường độ chỉ quyết định **số lượng** photon đến bề mặt mỗi giây (do đó quyết định số electron bật ra mỗi giây, tức dòng quang điện), còn năng lượng của **mỗi** photon — và do đó KE_{max} — chỉ phụ thuộc vào **tần số** f, theo $hf = \Phi + KE_{max}$. Đây chính là điểm mà thuyết sóng cổ điển thất bại và thuyết photon của Einstein giải thích thành công.

VN **GIẢI THÍCH TIẾNG VIỆT**

Hiệu ứng quang điện: ánh sáng có tần số đủ lớn (trên tần số ngưỡng f_0) chiếu vào bề mặt kim loại làm bật electron ra ngay lập tức. Einstein giải thích bằng mô hình hạt photon: mỗi photon mang năng lượng $E = hf$ và truyền **toàn bộ** năng lượng đó cho **một** electron duy nhất. Công thoát $\Phi = hf_0$ là năng lượng tối thiểu để giải phóng electron khỏi bề mặt; phần năng lượng dư trở thành động năng cực đại $KE_{max} = hf - \Phi = eV_s$. Đồ thị KE_{max} theo f là đường thẳng có độ dốc h — đây là cách thực nghiệm xác định hằng số Planck.

5.2.2 Wave–Particle Duality and the de Broglie Hypothesis

Light shows unmistakably **wave** behaviour in phenomena such as interference and diffraction (constructive/destructive superposition depends on path-length differences measured in wavelengths), yet the photoelectric effect shows unmistakably **particle** behaviour (energy delivered in discrete, indivisible quanta, one photon per electron). Light is neither purely a wave nor purely a stream of classical particles: it exhibits **wave–particle duality**, with the wave description governing propagation and interference, and the particle (photon) description governing energy exchange with matter.

In 1924, Louis de Broglie proposed that this duality is not unique to light: *every* moving particle of matter — electrons, protons, even macroscopic objects — has an associated wavelength.

De Broglie wavelength:

$$\lambda = \frac{h}{p} = \frac{h}{mv}.$$

This wavelength is the same h that governs photon energy, now linking a particle's *momentum* to a wavelength. For everyday (macroscopic) objects, λ is absurdly small and no wave effects are ever observed; for electrons and other subatomic particles accelerated through typical laboratory voltages, λ can be comparable to atomic spacings, so wave effects (such as diffraction) become observable. This was confirmed experimentally by Davisson and Germer (1927), who showed that a beam of electrons fired at a crystal produces a diffraction pattern exactly as a wave of wavelength $\lambda = h/p$ would.

Ví dụ mẫu · De Broglie wavelength of an accelerated electron

An electron, initially at rest, is accelerated through a potential difference of 100 V. Find its de Broglie wavelength.

Step 1 — kinetic energy gained.

$$KE = eV = (1.60 \times 10^{-19})(100) = 1.60 \times 10^{-17} \text{ J}.$$

Step 2 — momentum from $KE = \dfrac{p^2}{2m}$.

$$p = \sqrt{2m_e KE} = \sqrt{2(9.11 \times 10^{-31})(1.60 \times 10^{-17})} = \sqrt{2.92 \times 10^{-47}} = 5.40 \times 10^{-24} \text{ kg} \cdot \text{m/s}.$$

Step 3 — de Broglie wavelength.

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{5.40 \times 10^{-24}} = 1.23 \times 10^{-10} \text{ m} = 0.123 \text{ nm}.$$

This is comparable to the spacing between atomic planes in a crystal (~ 0.1–0.3 nm), which is exactly why fast electrons produce measurable diffraction patterns when fired at crystalline solids.

VN **GIẢI THÍCH TIẾNG VIỆT**

Lưỡng tính sóng–hạt: ánh sáng vừa có tính sóng (giao thoa, nhiễu xạ) vừa có tính hạt (hiệu ứng quang điện). De Broglie mở rộng ý tưởng này cho **mọi** hạt vật chất chuyển động: bước sóng $\lambda = h/p = h/(mv)$. Với vật vĩ mô (bóng, ô tô...), λ nhỏ đến mức không thể quan sát hiệu ứng sóng; với electron được gia tốc qua điện thế cỡ hàng chục–hàng trăm volt, λ đủ lớn (cỡ 0.1 nm) để gây nhiễu xạ khi gặp mạng tinh thể — đây chính là bằng chứng thực nghiệm (Davisson–Germer) xác nhận giả thuyết de Broglie.

The single most striking demonstration of wave–particle duality is the **double-slit experiment** performed with particles (electrons, or even large molecules) sent through the apparatus *one at a time*. Each individual particle is detected on the far screen as a single, localized point of impact — unmistakably particle-like behaviour, with no sign of the particle "splitting" between the two slits. Yet after a great many individual particles have been detected, the accumulated pattern of impact points builds up the same bright-and-dark interference fringe pattern produced by a wave passing through both slits simultaneously. Neither a purely classical particle picture (which predicts no interference at all) nor a purely classical wave picture (which cannot explain the single, sharp detection points) can account for both observations together — only the quantum-mechanical description, in which each particle's probability of arriving at a given point is governed by a wave, reproduces the full pattern.

Khái niệm cốt lõi

The double-slit experiment, run one particle at a time, cannot be resolved by saying "each particle is really a wave" or "each particle is really a tiny ball" — it requires both descriptions together: the particle is detected as a localized point (particle behaviour), but the statistical distribution of many such detection points builds up an interference pattern governed by a wave (wave behaviour). This is the essence of wave–particle duality at its sharpest.

5.2.3 Heisenberg's Uncertainty Principle

Classically, it is assumed that a particle's position and momentum can, in principle, both be measured to arbitrary precision simultaneously. Quantum mechanics shows this assumption to be false: there is a fundamental, unavoidable trade-off between how precisely position and momentum can *both* be known at once — not because of clumsy measuring instruments, but as a basic property of nature arising from wave–particle duality itself (a wave packet localized to a small region of space necessarily contains a spread of wavelengths, and hence of momenta).

Heisenberg's uncertainty principle (position–momentum form, IB data-booklet form):

$$\Delta x\,\Delta p \geq \frac{h}{4\pi}.$$

An analogous relationship links energy and time:

$$\Delta E\,\Delta t \geq \frac{h}{4\pi}.$$

The energy–time form implies that a quantum state that exists for only a short time Δt (such as a short-lived excited nuclear state, or a virtual particle in an intermediate step of an interaction) necessarily has an uncertain, "smeared-out" energy ΔE — observed experimentally as a finite **natural linewidth** in the photon energy emitted when such a state decays, rather than one perfectly sharp energy.

Ví dụ mẫu · Minimum momentum uncertainty of a confined electron

An electron is confined within a region of size $\Delta x = 1.0 \times 10^{-10}$ m (roughly the size of an atom). Estimate the minimum possible uncertainty in its momentum, and in its speed.

$$\Delta p \geq \frac{h}{4\pi\,\Delta x} = \frac{6.63 \times 10^{-34}}{4\pi(1.0 \times 10^{-10})} = \frac{6.63 \times 10^{-34}}{1.257 \times 10^{-9}} = 5.28 \times 10^{-25}\ \text{kg} \cdot \text{m/s}.$$

$$\Delta v \geq \frac{\Delta p}{m_e} = \frac{5.28 \times 10^{-25}}{9.11 \times 10^{-31}} = 5.8 \times 10^{5}\ \text{m/s}.$$

Even in principle, an electron confined to atomic dimensions must have an uncertainty in speed of hundreds of kilometres per second — this is why electrons in atoms cannot be thought of as sitting still at a definite point, and why atoms do not collapse: confining the electron further (smaller Δx) would only *increase* Δp, and hence its kinetic energy.

Ví dụ mẫu · Estimating the range of the weak interaction

In particle physics (E.5), forces are carried by exchanged "virtual" particles that briefly violate energy conservation by an amount ΔE roughly equal to their rest mass energy, for a time Δt consistent with the energy–time uncertainty relation; the maximum distance such a particle can travel, $R \approx c\,\Delta t$, gives a rough estimate of the force's range. The W boson, which mediates the weak interaction, has a rest mass energy of about 80 GeV. Estimate the range of the weak interaction.

$$\Delta E = 80 \times 10^{9} \times 1.60 \times 10^{-19} = 1.28 \times 10^{-8}\ \text{J}.$$

$$\Delta t \approx \frac{h}{4\pi\,\Delta E} = \frac{6.63 \times 10^{-34}}{4\pi(1.28 \times 10^{-8})} = 4.12 \times 10^{-27}\ \text{s}.$$

$$R \approx c\,\Delta t = (3.00 \times 10^{8})(4.12 \times 10^{-27}) = 1.2 \times 10^{-18}\ \text{m}.$$

This matches the experimentally observed range of the weak interaction — vastly smaller than the size of a nucleus — and explains, from the uncertainty principle alone, why the weak force is so short-ranged: its force carriers are very massive. (By contrast, the massless photon that carries the electromagnetic force has $\Delta E \to 0$, allowing $\Delta t \to \infty$ and hence an unlimited range.)

(!) Lỗi thường gặp: Nguyên lý bất định Heisenberg **không phải** là giới hạn do dụng cụ đo còn thô sơ, và cũng không phải nói rằng "hành động đo làm nhiễu loạn hệ". Đây là một **tính chất nội tại của tự nhiên** ở cấp độ lượng tử: một hạt không thể đồng thời có vị trí và động lượng xác định tuyệt đối, bất kể ta đo bằng thiết bị lý tưởng đến đâu. Ngoài ra, $\Delta x\,\Delta p \geq h/(4\pi)$ chỉ cho **giá trị nhỏ nhất có thể** của tích hai độ bất định — không phải giá trị chính xác.

VN **GIẢI THÍCH TIẾNG VIỆT**

Nguyên lý bất định: $\Delta x \cdot \Delta p \geq h/(4\pi)$ — không thể đồng thời biết chính xác tuyệt đối cả vị trí lẫn động lượng của một hạt. Dạng năng lượng–thời gian $\Delta E \cdot \Delta t \geq h/(4\pi)$ giải thích vì sao trạng thái kích thích có thời gian sống ngắn thì "bề rộng" năng lượng (và do đó bề rộng vạch quang phổ phát ra) lớn hơn — trạng thái sống càng ngắn, năng lượng càng "mờ".

5.2.4 The Particle in a Box and Quantum Tunnelling

Quantization of energy is not a special rule invented only for atoms — it is a general consequence of confining a wave to a finite region. Consider a particle confined to a one-dimensional "box" of fixed width L (an idealized model, but a useful one). Just as a wave on a string fixed at both ends can only support standing waves whose wavelength fits an integer number of half-wavelengths into the string's length, a particle confined to a box can only exist in states whose associated de Broglie wavelength fits a standing-wave pattern within the box. Since $\lambda = h/p$ links wavelength to momentum, and kinetic energy depends on momentum, only certain discrete momenta — and therefore only certain discrete energies — are allowed. This is, at heart, exactly why atoms (in which an electron is confined by the attractive pull of the nucleus, rather than by rigid walls) possess discrete energy levels rather than a continuum: confinement itself forces quantization.

> For a particle of mass m confined to an idealized one-dimensional box of width L with impenetrable walls, requiring a whole number of half-wavelengths to fit exactly within the box ($L = n\lambda/2, n = 1, 2, 3, \ldots$) and combining this with $\lambda = h/p$ and $E = p^2/(2m)$ gives the allowed energy levels:
>
> $$E_n = \frac{n^2 h^2}{8mL^2}.$$
>
> Energy increases with the square of n (levels spread further apart at higher n) and decreases sharply as the confining region L grows — a particle confined to a very small region necessarily has a very large minimum (zero-point) energy, exactly the same physical content as the Heisenberg uncertainty principle applied to a confined particle.

Ví dụ mẫu · Ground-state energy of an electron confined to atomic dimensions

Estimate the ground-state ($n = 1$) energy of an electron confined to a one-dimensional box of width $L = 1.0 \times 10^{-10}$ m, roughly the size of an atom.

$$E_1 = \frac{(1)^2 h^2}{8m_e L^2} = \frac{(6.63 \times 10^{-34})^2}{8(9.11 \times 10^{-31})(1.0 \times 10^{-10})^2} = \frac{4.40 \times 10^{-67}}{7.29 \times 10^{-50}} = 6.03 \times 10^{-18} \text{ J}.$$

$$E_1 = \frac{6.03 \times 10^{-18}}{1.60 \times 10^{-19}} = 37.7 \text{ eV}.$$

This simple, idealized model gives a confinement energy of the same order of magnitude (tens of eV) as the real ground-state energy scale of hydrogen (13.6 eV) — a useful check that confinement to atomic-sized regions naturally produces energies in the observed atomic range, even though the particle-in-a-box model does not use the true (Coulomb) potential energy of a real atom.

A related and important quantum phenomenon is **quantum tunnelling**: because a confined particle's position is described by a wave function that does not drop instantly to zero at a barrier, there is a small but nonzero probability of finding the particle *on the far side* of a potential-energy barrier that it does not have enough energy to classically climb over. The particle appears to "tunnel through" the barrier rather than going over it. This is not merely a theoretical curiosity: in E.3 it explains **alpha decay** — an α particle confined inside a heavy nucleus is held in by an attractive nuclear-force barrier that classically it does not have enough energy to escape, yet it has a small but nonzero probability, per unit time, of tunnelling through and escaping, which is exactly why radioactive decay is described statistically (as a probability per unit time) rather than as a fixed, predictable moment.

GIẢI THÍCH TIẾNG VIỆT

VN Mô hình "hạt trong hộp": khi một hạt bị giam trong một vùng không gian hữu hạn, sóng de

Broglie của nó chỉ có thể tồn tại ở những "sóng dừng" nhất định (giống sóng trên dây đàn hai đầu cố định) — do đó năng lượng bị **lượng tử hoá**. Đây chính là lý do sâu xa vì sao nguyên tử có các mức năng lượng rời rạc: electron bị giam bởi lực hút hạt nhân. **Xuyên hầm lượng tử** (quantum tunnelling): hạt có xác suất khác không xuất hiện ở phía bên kia của một "rào thế" mà theo vật lý cổ điển nó không đủ năng lượng để vượt qua — đây là cơ chế giải thích phân rã alpha (mục E.3).

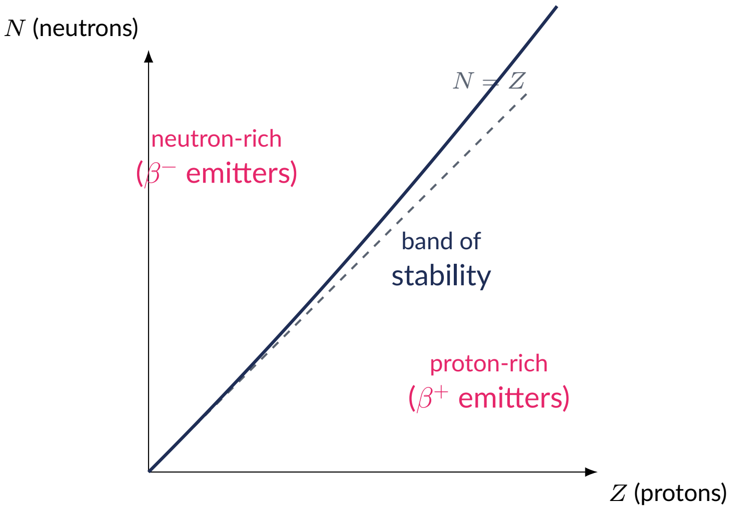
5.3 E.3 Radioactive Decay

5.3.1 Nuclear Stability and the Band of Stability

Not every combination of protons and neutrons forms a stable nucleus. Plotting neutron number N against proton number Z for all known stable nuclides reveals a narrow **band (or line) of stability**:

- For light nuclei ($Z \lesssim 20$), stable nuclides have roughly **equal** numbers of protons and neutrons ($N \approx Z$).
- For heavier nuclei, stable nuclides need progressively **more** neutrons than protons ($N > Z$, with the ratio N/Z rising to about 1.5 near $Z = 82$, lead). This is because as more protons are packed into a nucleus, the (long-range) electrostatic repulsion between them grows rapidly, while the (short-range) strong nuclear force that binds nucleons together only acts between close neighbours; extra neutrons add extra strong-force attraction without adding extra electrostatic repulsion, helping to hold the nucleus together.
- Beyond $Z = 82$ (lead), no combination of N and Z is completely stable — every nuclide with $Z > 82$ is radioactive, though some (such as uranium and thorium) decay so slowly that they persist since the Earth's formation.

A nuclide lying off the band of stability is **radioactive**: it undergoes spontaneous **radioactive decay**, transforming into a different, more stable nuclide (sometimes after a chain of several successive decays) by emitting particles and/or energy.



Sketch of the band of stability on an N-vs-Z plot. Light stable nuclides cluster near $N = Z$; heavy stable nuclides need $N > Z$. Nuclides above the band are neutron-rich (tend to β^- decay); nuclides below are proton-rich (tend to β^+ decay or electron capture).

Within the band of stability, nuclei with certain particular numbers of protons or neutrons — the **magic numbers** $2, 8, 20, 28, 50, 82, 126$ — are noticeably *more* stable (more tightly bound, more abundant in nature, and with a larger number of stable isotopes/isotones) than their neighbours. This mirrors the extra chemical stability of the noble gases, whose electron counts $(2, 10, 18, 36, \dots)$

correspond to fully closed electron shells: in the nuclear **shell model**, protons and neutrons independently occupy their own quantized energy levels within the nucleus (analogous to electron shells in the atom), and a magic number corresponds to a completely filled nuclear shell. Nuclei that are magic in *both* proton number and neutron number simultaneously (such as helium-4 with $Z = N = 2$, or lead-208 with $Z = 82$, $N = 126$) are called **doubly magic** and are exceptionally stable — consistent with the unusually high binding energy per nucleon already calculated for helium-4 earlier in this chapter.

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"Dải bền vững" (band of stability) trên biểu đồ N theo Z: hạt nhân nhẹ bền khi $N \approx Z$; hạt nhân nặng cần $N > Z$ nhiều hơn để bù lại lực đẩy tĩnh điện ngày càng lớn giữa các proton (lực hạt nhân mạnh chỉ tác dụng ở tầm gần, còn lực đẩy Coulomb tác dụng ở tầm xa). Hạt nhân nằm **ngoài** dải này không bền — nó sẽ phân rã phóng xạ để "di chuyển" dần về phía dải bền vững. Các "số ma thuật" $(2, 8, 20, 28, 50, 82, 126)$ proton hoặc neutron tương ứng với lớp vỏ hạt nhân lấp đầy (giống khí hiếm bền về mặt hoá học) — hạt nhân "magic kép" như He-4 hay Pb-208 đặc biệt bền vững.

5.3.2 Types of Radioactive Decay

There are three principal types of radioactive decay, each governed by the conservation of **nucleon number** A, **charge (proton number)** Z, and **mass–energy**.

Alpha (α) decay: the nucleus emits an alpha particle — a helium-4 nucleus, ${}^4_2$He (2 protons + 2 neutrons), tightly bound and therefore relatively easy for a heavy nucleus to eject as a single unit. The parent's mass number falls by 4 and atomic number falls by 2:

$${}^A_Z X \longrightarrow {}^{A-4}_{Z-2}Y + {}^4_2\text{He}.$$

Alpha particles are heavy, slow-moving, and doubly charged, so they interact strongly with matter and are the *least* penetrating: they are stopped by a single sheet of paper or a few centimetres of air, and cannot even penetrate the outer dead layer of human skin (though alpha emitters are dangerous if inhaled or ingested).

Beta-minus (β^-) decay: inside a neutron-rich nucleus, a neutron converts into a proton, emitting a fast electron (the "beta particle") and an electron antineutrino $\bar{\nu}_e$ (required to conserve energy, momentum, and a quantity called lepton number). The atomic number increases by 1; the mass number is unchanged (a neutron simply becomes a proton, not lost from the nucleus):

$${}^A_Z X \longrightarrow {}^{A}_{Z+1}Y + {}^0_{-1}e + \bar{\nu}_e, \qquad (n \to p + e^- + \bar{\nu}_e \text{ at the nucleon level}).$$

Beta particles are much lighter and faster than alpha particles and interact less strongly with matter; they penetrate paper and skin but are stopped by a few millimetres of aluminium.

Beta-plus (β^+) decay: inside a proton-rich nucleus, a proton converts into a neutron, emitting a positron (e^+, the electron's antiparticle) and an electron neutrino ν_e. The atomic number decreases by 1; the mass number is unchanged:

$${}^A_Z X \longrightarrow {}^{A}_{Z-1}Y + {}^0_{+1}e + \nu_e, \qquad (p \to n + e^+ + \nu_e \text{ at the nucleon level}).$$

(β^+ decay occurs only for proton-rich nuclides, and only when energetically allowed; it competes with a related process, electron capture, not required in detail here.)

Gamma (γ) decay: after an α or β decay (or other nuclear process), the daughter nucleus is often left in an excited nuclear energy state. It then drops to a lower nuclear energy state by emitting a high-energy photon, a **gamma ray**. Neither A nor Z changes:

$${}^A_Z X^* \longrightarrow {}^A_Z X + \gamma.$$

Gamma rays are uncharged, massless photons, and are by far the most penetrating: several centimetres of lead (or a metre or more of concrete) are needed to substantially reduce their intensity.

Type	Emitted	Change in (A, Z)	Stopped by
α	$^{4}_{2}$He nucleus	$A-4,\ Z-2$	paper / few cm of air
β^-	electron + $\bar{\nu}_e$	A unchanged, $Z+1$	few mm of aluminium
β^+	positron + ν_e	A unchanged, $Z-1$	few mm of aluminium
γ	high-energy photon	A, Z unchanged	cm of lead / m of concrete

Ví dụ mẫu · Writing and checking nuclear decay equations

Write balanced nuclear equations for: (a) the alpha decay of polonium-210, (b) the beta-minus decay of potassium-40, (c) the beta-plus decay of carbon-11. In each case verify that A and Z are conserved.

(a) Po has $Z = 84$. Alpha decay: $A \to A - 4 = 206$, $Z \to Z - 2 = 82$ (lead):

$$^{210}_{84}\text{Po} \longrightarrow {}^{206}_{82}\text{Pb} + {}^{4}_{2}\text{He}.$$

Check: A: $210 = 206 + 4$ ✓. Z: $84 = 82 + 2$ ✓.

(b) K has $Z = 19$. Beta-minus decay: A unchanged $= 40$, $Z \to Z + 1 = 20$ (calcium):

$$^{40}_{19}\text{K} \longrightarrow {}^{40}_{20}\text{Ca} + {}^{0}_{-1}e + \bar{\nu}_e.$$

Check: A: $40 = 40 + 0 + 0$ ✓. Z: $19 = 20 + (-1) + 0$ ✓.

(c) C has $Z = 6$. Beta-plus decay: A unchanged $= 11$, $Z \to Z - 1 = 5$ (boron):

$$^{11}_{6}\text{C} \longrightarrow {}^{11}_{5}\text{B} + {}^{0}_{+1}e + \nu_e.$$

Check: A: $11 = 11 + 0 + 0$ ✓. Z: $6 = 5 + 1 + 0$ ✓.

(!) Lỗi thường gặp: Trong phân rã β^- và β^+, đừng quên hạt **(phản) neutrino** — dù gần như không khối lượng và cực khó phát hiện, nó vẫn bắt buộc phải xuất hiện trong phương trình để bảo toàn năng lượng, động lượng, và số lepton. Một lỗi khác thường gặp: nhầm chiều thay đổi Z giữa β^- (Z **tăng** — neutron thành proton) và β^+ (Z **giảm** — proton thành neutron); hãy nhớ tên gọi "β^-/β^+" chỉ điện tích của hạt **phát ra**, không phải chiều thay đổi của Z.

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Ba loại phân rã chính: **Alpha** (phát hạt nhân He-4, A giảm 4, Z giảm 2, đâm xuyên yếu nhất); **Beta trừ** (neutron → proton $+e^- + \bar{\nu}_e$, Z tăng 1, A không đổi); **Beta cộng** (proton → neutron $+e^+ + \nu_e$, Z giảm 1, A không đổi); **Gamma** (hạt nhân kích thích phát photon năng lượng cao, A, Z không đổi). Luôn bảo toàn số khối A và điện tích Z ở hai vế phương trình.

5.3.3 Energy Sharing in Alpha Decay

Alpha decay is a two-body process: a parent nucleus, essentially at rest, splits into exactly two products — the daughter nucleus and the α particle. Momentum conservation then requires these two products to recoil in exactly opposite directions with equal-magnitude momenta, $p_\alpha = p_D$. Combined with conservation of energy (the total decay energy Q released is shared between the two kinetic energies), this two-body kinematics completely fixes exactly how much kinetic energy each product receives — every α particle emitted in a given decay carries the *same*, single, discrete kinetic energy. This is why measured α-particle energies from a given source form sharp, discrete lines (useful for identifying the emitting isotope), unlike the beta particles discussed next.

Using $p_\alpha = p_D$ and $KE = \dfrac{p^2}{2m}$:

$$\frac{KE_\alpha}{KE_D} = \frac{m_D}{m_\alpha} \quad \Rightarrow \quad KE_\alpha = Q \cdot \frac{m_D}{m_D + m_\alpha}, \qquad KE_D = Q \cdot \frac{m_\alpha}{m_D + m_\alpha}.$$

Because the daughter nucleus is always far more massive than the α particle, it carries away only a small fraction of Q as recoil kinetic energy; the α particle carries away most of it.

Ví dụ mẫu · Sharing the decay energy between alpha particle and daughter nucleus

Radium-226 decays to radon-222 by alpha emission, releasing a total decay energy $Q = 4.87$ MeV. Using mass numbers as a proxy for mass ratios ($m_D \approx 222$, $m_\alpha \approx 4$, in atomic mass units), find the kinetic energy of the emitted α particle and of the recoiling radon nucleus.

$$KE_\alpha = Q \cdot \frac{m_D}{m_D + m_\alpha} = 4.87 \times \frac{222}{226} = 4.87 \times 0.9823 = 4.78 \text{ MeV}.$$
$$KE_D = Q - KE_\alpha = 4.87 - 4.78 = 0.09 \text{ MeV}.$$

Consistent with the general pattern: the light α particle carries away about 98% of the released energy, while the much heavier daughter nucleus recoils with only a small fraction.

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Phân rã alpha là quá trình **hai vật**: hạt nhân mẹ (đứng yên) tách thành hạt nhân con và hạt α, bay ra ngược chiều nhau với động lượng bằng nhau về độ lớn (bảo toàn động lượng). Điều này khiến động năng phân bổ theo tỉ lệ cố định $KE_\alpha/KE_D = m_D/m_\alpha$ — hạt α nhẹ hơn nhiều nên mang phần lớn năng lượng toả ra, và mọi hạt α từ cùng một phân rã đều có **cùng một mức năng lượng duy nhất** (phổ năng lượng rời rạc).

5.3.4 The Continuous Beta Spectrum and the Neutrino

Beta decay tells a very different story. If β^- decay were also a simple two-body process (nucleus → nucleus + electron), the same momentum-conservation argument used for alpha decay would predict a single, discrete beta-particle energy. But careful measurements in the early twentieth century showed that beta particles from a given source are emitted with a *continuous* range of kinetic energies, from zero up to some well-defined maximum — not a single discrete value. Taken at face value, this appeared to violate conservation of energy, and briefly caused real concern among physicists about whether energy conservation held exactly at the nuclear scale.

In 1930, Wolfgang Pauli proposed a resolution that preserved the conservation laws: beta decay is not a two-body process but a **three-body** process, with the energy shared among the nucleus, the electron, *and* a third particle — extremely light, electrically neutral, and interacting only extremely weakly with matter — later named the **neutrino** (Fermi) and now understood, in β^- decay, to be an antineutrino $\bar{\nu}_e$. With three products sharing a fixed total energy, the energy carried by any *one* of them (such as the beta particle actually detected) can take a continuous range of values, exactly matching what was observed. The neutrino itself proved so weakly interacting that it was not directly detected until 1956 — over a quarter of a century after Pauli's proposal — since the overwhelming majority of neutrinos produced anywhere pass straight through the entire Earth without a single interaction.

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Phổ năng lượng của hạt β (khác hẳn hạt α) là **liên tục** chứ không rời rạc — vì phân rã β thực chất là quá trình **ba vật** (hạt nhân con + electron + neutrino/phản neutrino), năng lượng toả ra được chia theo tỉ lệ khác nhau giữa ba hạt trong mỗi lần phân rã. Đây chính là lý do Pauli đề xuất sự tồn tại của neutrino năm 1930 — để cứu định luật bảo toàn năng lượng — dù hạt này mãi đến năm 1956 mới được phát hiện trực tiếp, vì nó tương tác cực yếu với vật chất.

5.3.5 The Radioactive Decay Law

The decay-constant/exponential treatment below is Additional Higher Level (AHL) content; at Standard Level, half-life is used only via graphs or successive halvings.

Radioactive decay is a random process at the level of an individual nucleus — it is impossible to predict exactly when any one nucleus will decay — but for a large sample, the *rate* of decay is entirely predictable: at any instant, the number of decays per unit time is proportional to the number of undecayed nuclei present, $-\frac{dN}{dt} = \lambda N$, where λ is the **decay constant** (probability of decay per nucleus per unit time, units s^{-1}). Solving this differential equation gives exponential decay.

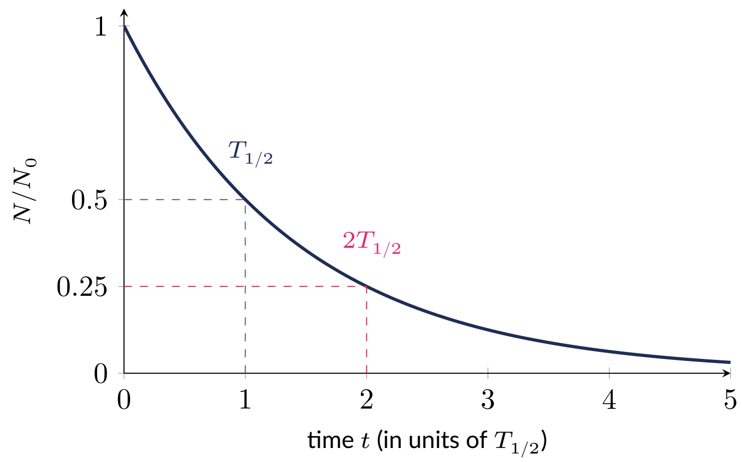
$$N(t) = N_0 e^{-\lambda t}, \qquad A(t) = \lambda N(t) = A_0 e^{-\lambda t},$$

where N_0 is the initial number of undecayed nuclei and $A = \lambda N = -\frac{dN}{dt}$ is the **activity** (number of decays per second), measured in **becquerels** (Bq); 1 Bq $= 1$ decay/s. Both N and A decay with the same exponential form and the same decay constant λ.

The **half-life** $T_{1/2}$ is the time for the number of undecayed nuclei (or the activity) to fall to half its initial value. Setting $N = N_0/2$ at $t = T_{1/2}$:

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}} \Rightarrow \frac{1}{2} = e^{-\lambda T_{1/2}} \Rightarrow \ln 2 = \lambda T_{1/2} \Rightarrow \boxed{T_{1/2} = \frac{\ln 2}{\lambda}}.$$

Every successive half-life halves whatever quantity remained; after n half-lives, $N = N_0(1/2)^n$, regardless of how large or small N_0 was to begin with.



The exponential decay curve $N/N_0 = (1/2)^{t/T_{1/2}} = e^{-\lambda t}$: each successive half-life halves the remaining quantity, regardless of the starting amount.

Ví dụ mẫu · Activity after a given time

A radioactive sample has an initial activity of 1.20×10^6 Bq and a half-life of 6.0 hours. Find (a) its decay constant, (b) its activity after 15.0 hours.

(a)

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.6931}{6.0} = 0.1155\ \text{hr}^{-1}.$$

(b) 15.0 hours corresponds to $15.0/6.0 = 2.5$ half-lives, so

$$A = A_0\left(\frac{1}{2}\right)^{2.5} = (1.20\times 10^6)(0.1768) = 2.12\times 10^5\ \text{Bq}.$$

(Check via the exponential form: $A = A_0e^{-\lambda t} = (1.20 \times 10^6)e^{-(0.1155)(15.0)} = (1.20 \times 10^6)e^{-1.733} = (1.20\times 10^6)(0.1768) = 2.12\times 10^5$ Bq — the same answer, as it must be.)

Ví dụ mẫu · Finding the half-life from activity data

The activity of a radioactive source falls from 360 Bq to 45 Bq over 24 minutes. Find the decay constant and the half-life.

$$A = A_0e^{-\lambda t} \Rightarrow \frac{45}{360} = e^{-\lambda(24)} \Rightarrow 0.125 = e^{-24\lambda}.$$

$$\ln(0.125) = -24\lambda \Rightarrow -2.079 = -24\lambda \Rightarrow \lambda = 0.0866\ \text{min}^{-1}.$$

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.6931}{0.0866} = 8.0\ \text{minutes}.$$

(Sanity check: $360 \to 180 \to 90 \to 45$ Bq is exactly three halvings, i.e. $3T_{1/2} = 24$ min $\Rightarrow T_{1/2} = 8.0$ min — matching the calculation exactly.)

(!) Lỗi thường gặp: λ và $T_{1/2}$ phải dùng **cùng đơn vị thời gian nhất quán** trong suốt bài toán (giây, phút, giờ, năm...) — trộn lẫn đơn vị là lỗi tính toán phổ biến nhất trong dạng bài này. Ngoài ra, $N(t) = N_0e^{-\lambda t}$ và $A(t) = A_0e^{-\lambda t}$ có **cùng** hằng số phân rã λ và cùng chu kỳ bán rã $T_{1/2}$ — không được nhầm hai đại lượng N (số hạt nhân) và A (số phân rã/giây) với nhau, dù cùng dạng hàm mũ.

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Định luật phân rã phóng xạ: $N(t) = N_0e^{-\lambda t}$, hoạt độ $A = \lambda N = A_0e^{-\lambda t}$ (đơn vị becquerel, Bq = số phân rã/giây). Chu kỳ bán rã $T_{1/2} = \ln 2/\lambda$ là thời gian để N (hoặc A) giảm còn một nửa — sau n chu kỳ bán rã, lượng còn lại là $N_0(1/2)^n$, bất kể lượng ban đầu là bao nhiêu. Mẹo giải nhanh: nếu thời gian đã cho là bội số nguyên của $T_{1/2}$, có thể giải bằng cách "chia đôi liên tiếp" thay vì dùng hàm mũ đầy đủ.

5.3.6 Decay Chains

A single decay does not always produce a stable nuclide. Very often, the daughter nucleus is itself radioactive, and decays again — and again — through a whole sequence of successive decays, called a **decay chain** (or decay series), until a stable nuclide is finally reached. The most famous natural example is the uranium-238 decay series, which begins with α decay and proceeds through more than a dozen further α and β^- decays before finally terminating at the stable isotope lead-206.

Step	Nuclide	Half-life	Decay mode
1	$^{238}_{92}$U	4.5×10^9 yr	α
2	$^{234}_{90}$Th	24.1 days	β^-
3	$^{234}_{91}$Pa	1.2 min	β^-
⋮	⋮	⋮	⋮ (further α, β^- steps)
last	$^{206}_{82}$Pb	stable	—

Because each step of a decay chain has a wildly different half-life — from billions of years down to seconds — the chain as a whole is dominated by whichever step is slowest. In a long-undisturbed sample, the chain settles into **secular equilibrium**: every intermediate nuclide in the chain is produced (by the decay of the previous member) at essentially the same rate as it decays away, so the *activity* contributed by every member of the chain becomes equal, even though each has a wildly different half-life and a wildly different (much smaller) quantity present at any instant. This is also why old rocks containing uranium always contain small but measurable traces of every intermediate decay product, including short-lived ones that are constantly being replenished by the slow decay of the uranium further up the chain.

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VN Chuỗi phân rã: hạt nhân con sinh ra sau một lần phân rã có thể vẫn không bền và tiếp tục phân rã, tạo thành một chuỗi nhiều bước cho đến khi đạt hạt nhân bền (VD chuỗi U-238 kết thúc ở Pb-206 bền, qua hơn chục bước phân rã α/β^-). Vì bước chậm nhất trong chuỗi (thường là bước đầu tiên) quyết định tốc độ toàn chuỗi, một mẫu đá cổ chứa uranium luôn có một lượng nhỏ, ổn định của **mọi** sản phẩm trung gian — kể cả những đồng vị có chu kỳ bán rã rất ngắn — vì chúng liên tục được "bù đắp" bởi bước phân rã chậm phía trước (trạng thái cân bằng thế kỷ, secular equilibrium).

5.3.7 Background Radiation, Safety, and Uses of Radioisotopes

Everyone on Earth is continuously exposed to low levels of **background radiation** from natural sources, including cosmic rays from space, radioactive isotopes in rocks and soil (notably **radon gas**, a dense radioactive gas that can accumulate in poorly ventilated basements and is a significant contributor to natural background dose), naturally occurring radioisotopes in food and the human body itself (e.g. potassium-40), and additional exposure from medical procedures (X-rays, nuclear medicine).

Radiation protection for anyone working with radioactive sources rests on three simple, complementary principles:

- **Time** — minimize the time spent near a source, since total dose accumulates with exposure time.
- **Distance** — maximize distance from the source; intensity falls off rapidly with distance (following an inverse-square law for a point source), so even a modest increase in distance substantially reduces dose.
- **Shielding** — place appropriate absorbing material (paper/light plastic for α; aluminium for β; thick lead or concrete for γ) between the source and the body.

Radiation exposure is quantified using two related but distinct quantities. **Absorbed dose** $D = \frac{\text{energy absorbed}}{\text{mass}}$ is measured in **gray** (Gy), where 1 Gy $= 1$ J/kg. **Equivalent dose** $H = D \times w_R$ is measured in **sievert** (Sv), where w_R is a radiation weighting factor that accounts for the fact that different types of radiation cause different amounts of biological damage *per unit of absorbed energy*: α particles, which deposit their energy extremely densely along a very short track, are far more biologically damaging per gray than β particles or γ rays of the same absorbed dose (hence $w_R \approx 20$ for α, versus $w_R \approx 1$ for β and γ).

Radioisotopes also have widespread beneficial uses: in **medicine**, as tracers for diagnostic imaging (e.g. technetium-99m) and as sources for targeted radiotherapy of tumours; in **archaeology and geology**, radiocarbon (carbon-14) dating exploits the known, constant decay rate of a naturally occurring isotope to estimate the age of once-living material; and in **industry**, radioactive sources are used for thickness/density gauging (measuring how much radiation is absorbed by a moving sheet of material) and for non-destructive testing of welds and castings.

Ví dụ mẫu · Radiocarbon dating

The decay constant of carbon-14 is $\lambda = 1.21 \times 10^{-4}$ yr^{-1}. A wooden artifact is found to have a carbon-14 activity equal to 25% of that found in a living tree of the same species. Estimate the age of the artifact.

$25\% = (1/2)^2$, i.e. two half-lives have elapsed. First find the half-life:

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.6931}{1.21 \times 10^{-4}} = 5730 \text{ years.}$$

$$\text{age} = 2 \times T_{1/2} = 2 \times 5730 = 1.15 \times 10^4 \text{ years.}$$

Ví dụ mẫu · Medical tracer decay

A patient is injected with a radioactive tracer of activity 200 MBq for a diagnostic imaging scan; the tracer isotope has a half-life of 6.0 hours. Estimate the activity remaining in the patient's body after 24 hours (ignoring biological elimination, i.e. considering only physical radioactive decay).

24 hours $= 24/6.0 = 4$ half-lives:

$$A = A_0 \left(\frac{1}{2}\right)^4 = \frac{200}{16} = 12.5 \text{ MBq.}$$

Short-lived isotopes such as this one (comparable to technetium-99m, $T_{1/2} \approx 6.0$ hours, widely used in nuclear medicine imaging) are deliberately chosen so that the radioactivity delivers a clear diagnostic signal quickly, then decays away to a small fraction of its initial value within about a day, minimizing the patient's total radiation dose.

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Bức xạ nền tự nhiên đến từ tia vũ trụ, khí radon trong đất đá, và các đồng vị phóng xạ tự nhiên trong cơ thể/thực phẩm. Ba nguyên tắc an toàn bức xạ: giảm **thời gian** tiếp xúc, tăng **khoảng cách**, dùng **lá chắn** phù hợp. Liều hấp thụ (Gray, Gy $=$ J/kg) đo năng lượng hấp thụ trên khối lượng; liều tương đương (Sievert, Sv) nhân thêm hệ số trọng số bức xạ w_R vì hạt α gây hại sinh học nhiều hơn hẳn β/γ với cùng liều hấp thụ. Ứng dụng hữu ích: chẩn đoán/điều trị y khoa, xác định niên đại bằng carbon phóng xạ (^{14}C, dựa vào $T_{1/2} = 5730$ năm), đo độ dày/mật độ vật liệu công nghiệp.

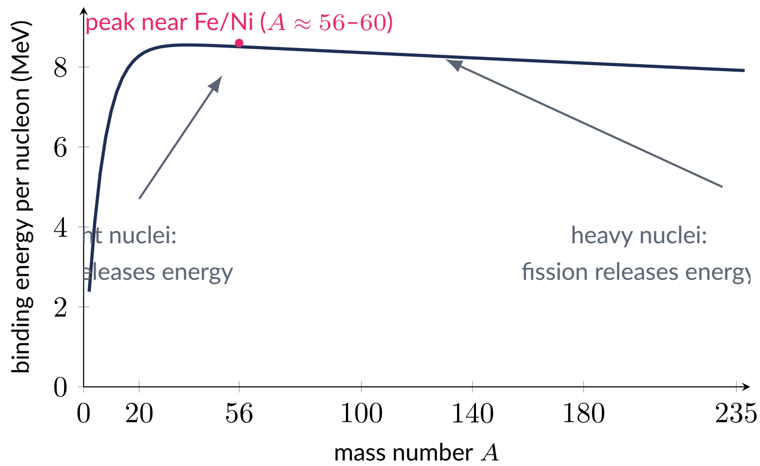
5.4 E.4 Nuclear Fission

5.4.1 Mass–Energy Equivalence and Binding Energy

Careful mass measurements reveal that the mass of any nucleus is always slightly *less* than the sum of the masses of its separate, individual protons and neutrons. This missing mass, the **mass defect** Δm, has literally been converted into the energy that binds the nucleus together, via Einstein's mass–energy equivalence:

$$\Delta m = [Zm_p + Nm_n] - M_{nucleus}, \qquad E_{binding} = \Delta m\, c^2.$$

Because nuclear mass differences are tiny, it is far more convenient to work in **atomic mass units** (u), using $1\ \text{u} = 931.5$ MeV/c^2: then $E_{binding}\,(\text{MeV}) = \Delta m\,(\text{u}) \times 931.5$. The **binding energy** $E_{binding}$ is the energy that would be required to completely separate a nucleus into its individual, free nucleons (equivalently, the energy released when the nucleus is assembled from separate nucleons). The **binding energy per nucleon**, $E_{binding}/A$, measures how tightly, on average, each nucleon is held — it is the standard measure of nuclear stability, allowing fair comparison between nuclei of very different sizes.



Binding energy per nucleon versus mass number A: it rises steeply for the lightest nuclides, peaks near $A \approx 56$–60 (iron/nickel — the most tightly bound nuclides), then decreases slowly toward the heaviest nuclides. Both fission of heavy nuclei and fusion of light nuclei move products *up* toward the peak, releasing energy.

Because the curve rises steeply for light nuclei and peaks around iron/nickel before slowly declining, any nuclear process that moves nucleons *toward* that peak — splitting a heavy nucleus into two medium-mass fragments (**fission**), or combining light nuclei into a heavier one (**fusion**) — produces daughter/product nuclei that are, on average, more tightly bound per nucleon than the starting material. The energy corresponding to this increase in binding energy per nucleon is released, chiefly as kinetic energy of the products.

Ví dụ mẫu · Binding energy per nucleon of the alpha particle

The nucleus of helium-4 (2 protons, 2 neutrons) has a measured mass of 4.0015 u. Using $m_p = 1.67 \times 10^{-27}$ kg and $m_n = 1.675 \times 10^{-27}$ kg (with $1\ \text{u} = 1.66 \times 10^{-27}$ kg), calculate the mass defect, the total binding energy, and the binding energy per nucleon.

Convert nucleon masses to u:

$$m_p = \frac{1.67 \times 10^{-27}}{1.66 \times 10^{-27}} = 1.0060 \text{ u}, \qquad m_n = \frac{1.675 \times 10^{-27}}{1.66 \times 10^{-27}} = 1.0090 \text{ u}.$$

Mass of separate nucleons:

$$2m_p + 2m_n = 2(1.0060) + 2(1.0090) = 2.0120 + 2.0181 = 4.0301 \text{ u}.$$

Mass defect:

$$\Delta m = 4.0301 - 4.0015 = 0.0286 \text{ u}.$$

Binding energy:

$$E_{binding} = 0.0286 \times 931.5 = 26.6 \text{ MeV}.$$

Binding energy per nucleon:

$$\frac{E_{binding}}{A} = \frac{26.6}{4} = 6.66 \text{ MeV/nucleon}.$$

This unusually large binding energy per nucleon (for such a light nuclide) is why the alpha particle is an especially stable, tightly bound configuration — and exactly why heavy nuclei so often decay by ejecting a whole, ready-made helium-4 nucleus rather than individual nucleons.

Nuclide	Mass number A	Binding energy/nucleon	Position on the curve
Helium-4	4	≈ 6.7 MeV	rising steeply (light side)
Carbon-12	12	≈ 7.2 MeV	still rising
Iron-56	56	≈ 8.8 MeV	at the peak (most stable)
Uranium-235	235	≈ 7.6 MeV	declining slowly (heavy side)

This numerical progression is exactly the shape of the binding-energy-per-nucleon curve sketched earlier: a fast rise for the lightest nuclides, a broad, gentle peak around iron, and a slow decline out toward uranium — with iron-56 sitting closest to the summit of nuclear stability of any naturally occurring nuclide.

(!) Lỗi thường gặp: $E_{binding} = \Delta m\, c^2$ dùng **độ hụt khối** Δm — hiệu giữa tổng khối lượng các nucleon riêng lẻ và khối lượng hạt nhân thật sự — chứ **không phải** khối lượng hạt nhân. Luôn tính $\Delta m =$ (tổng khối lượng nucleon rời) $-$ (khối lượng hạt nhân), kết quả phải **dương**. Một lỗi khác: so sánh trực tiếp *tổng* năng lượng liên kết giữa hai hạt nhân khác kích thước để đánh giá độ bền — phải so sánh năng lượng liên kết **trên mỗi nucleon** mới có ý nghĩa.

VN GIẢI THÍCH TIẾNG VIỆT

Khối lượng hạt nhân luôn nhỏ hơn tổng khối lượng các proton và neutron rời rạc tạo thành nó — phần khối lượng "biến mất" (Δm, độ hụt khối) chính là năng lượng liên kết hạt nhân theo $E = \Delta mc^2$ (dùng $1\text{u} = 931.5$ MeV$/c^2$ cho tiện). Năng lượng liên kết **trên mỗi nucleon** là thước đo độ bền hạt nhân; đồ thị năng lượng liên kết/nucleon theo A đạt đỉnh ở sắt/niken ($A \approx 56$) — đây là lý do cả **phân hạch** (hạt nhân nặng tách ra) và **nhiệt hạch** (hạt nhân nhẹ kết hợp) đều toả năng lượng: cả hai đều đưa sản phẩm "tiến gần hơn" tới đỉnh đồ thị.

5.4.2 The Fission Process

Nuclear fission is the splitting of a heavy nucleus into two lighter "daughter" nuclei. In **induced fission**, a heavy nuclide such as uranium-235 absorbs a slow (thermal) neutron, forming a highly excited, unstable compound nucleus (uranium-236) that almost immediately splits into two medium-mass fragments, releasing two or three additional free neutrons and a large amount of energy:

$$^{1}_{0}n + ^{235}_{92}\text{U} \longrightarrow ^{236}_{92}\text{U}^{*} \longrightarrow ^{A_1}_{Z_1}X + ^{A_2}_{Z_2}Y + k\,^{1}_{0}n \quad (k = 2 \text{ or } 3, \text{typically}).$$

Because the two daughter fragments sit much higher on the binding-energy-per-nucleon curve than the original uranium nucleus (uranium is far down the slowly-declining right-hand side of the curve, while medium-mass fragments sit much closer to the peak), the reaction releases a very large amount of energy — overwhelmingly as kinetic energy of the fast-moving fragments and neutrons, which is subsequently converted to heat. In practice, the two daughter fragments are almost never of equal mass: the fission of U-235 overwhelmingly favours *asymmetric* splits, typically one fragment with mass number around 90–100 and the other around 130–145 (as in the worked examples below), rather than two roughly equal halves.

Ví dụ mẫu · Energy released in a fission reaction

In one possible fission pathway, $^{1}_{0}n + ^{235}_{92}\text{U} \rightarrow ^{141}_{56}\text{Ba} + ^{92}_{36}\text{Kr} + 3\,^{1}_{0}n$. Given the masses $m(\text{U-235}) = 235.0439$ u, $m(\text{n}) = 1.0090$ u, $m(\text{Ba-141}) = 140.9139$ u, $m(\text{Kr-92}) = 91.9066$ u, calculate the energy released.

Total mass before:

$$235.0439 + 1.0090 = 236.0529 \text{ u}.$$

Total mass after:

$$140.9139 + 91.9066 + 3(1.0090) = 140.9139 + 91.9066 + 3.0270 = 235.8475 \text{ u}.$$

Mass defect and energy released:

$$\Delta m = 236.0529 - 235.8475 = 0.2054 \text{ u} \qquad \Rightarrow \qquad E = 0.2054 \times 931.5 = 191 \text{ MeV}.$$

This is consistent with the typically quoted value of roughly 200 MeV released per U-235 fission event.

Ví dụ mẫu · Fission rate in a power reactor

A nuclear power reactor generates 3000 MW of thermal power from the fission of U-235, with an average energy release of 200 MeV per fission event. Calculate the number of fission events occurring each second.

Energy per fission, in joules:

$$E_{fission} = 200 \times 10^{6} \times 1.60 \times 10^{-19} = 3.20 \times 10^{-11} \text{ J}.$$

Rate of fission events:

$$\text{rate} = \frac{P}{E_{fission}} = \frac{3.000 \times 10^{9}}{3.20 \times 10^{-11}} = 9.38 \times 10^{19} \text{ fissions/s}.$$

Ví dụ mẫu · Energy density: fission versus chemical combustion

Estimate the energy released per kilogram of U-235 undergoing fission (taking 200 MeV released per fission event), and compare it with the energy released per kilogram of coal burned in an ordinary chemical combustion process (about 3×10^7 J/kg).

Number of U-235 nuclei per kilogram:

$$N = \frac{1 \text{ kg}}{(235)(1.66 \times 10^{-27} \text{ kg})} = \frac{1}{3.90 \times 10^{-25}} = 2.56 \times 10^{24} \text{ nuclei/kg.}$$

Energy released per kilogram:

$$E = N \times (200 \text{ MeV}) = (2.56 \times 10^{24})(3.20 \times 10^{-11} \text{ J}) = 8.2 \times 10^{13} \text{ J/kg.}$$

Ratio to coal:

$$\frac{8.2 \times 10^{13}}{3 \times 10^7} \approx 2.7 \times 10^6.$$

Fission of U-235 releases roughly 2.7 *million times* more energy per kilogram of fuel than burning coal — the fundamental reason nuclear fuel is so energy-dense compared with any chemical fuel: nuclear reactions release binding energy from the strong force, many orders of magnitude stronger than the electromagnetic bonds broken and formed in chemical combustion.

GIẢI THÍCH TIẾNG VIỆT

VN

Phân hạch: hạt nhân nặng (như U-235) hấp thụ một neutron chậm, trở nên không bền và tách thành hai hạt nhân con nhẹ hơn cùng vài neutron tự do, giải phóng năng lượng lớn — vì các hạt nhân con nằm **cao hơn** trên đồ thị năng lượng liên kết/nucleon so với hạt nhân mẹ ban đầu. Cách tính năng lượng toả ra: lấy **độ hụt khối** (khối lượng trước trừ khối lượng sau phản ứng) nhân với 931.5 MeV/u. Về khối lượng, năng lượng toả ra từ phân hạch lớn hơn đốt cháy hoá học hàng **triệu lần** — vì phản ứng hạt nhân giải phóng năng lượng liên kết từ lực hạt nhân mạnh, mạnh hơn rất nhiều so với liên kết hoá học (lực điện từ).

5.4.3 Chain Reactions and Critical Mass

Each fission event releases two or three free neutrons, and if at least one of these goes on to trigger a further fission event in a neighbouring nucleus, the process can become self-sustaining: a **chain reaction**. Whether a chain reaction grows, dies out, or remains steady depends on how many of the released neutrons, on average, go on to cause a further fission — described by the multiplication factor k (average number of fissions directly caused by each fission event):

- **Subcritical** ($k < 1$): on average, fewer than one further fission is caused by each fission event; the reaction rate decreases and the chain reaction dies out.
- **Critical** ($k = 1$): each fission event causes, on average, exactly one further fission; the reaction proceeds at a steady, self-sustaining rate — the operating condition of a controlled nuclear reactor.
- **Supercritical** ($k > 1$): each fission event causes, on average, more than one further fission; the reaction rate grows rapidly (the condition deliberately engineered, uncontrolled, in a fission weapon).

The **critical mass** is the minimum mass of fissile material needed for a self-sustaining chain reaction: too small a sample and too many neutrons escape from the surface without striking another nucleus, so the reaction cannot sustain itself regardless of purity.

In a supercritical assembly, the number of fission events grows geometrically from one neutron generation to the next:

$$N_{\text{generation } n} = N_0 k^n,$$

where N_0 is the number of fission events in the first generation and $k > 1$ is the multiplication factor. This is the same mathematical form as radioactive growth/decay (E.3), except that here the "generations" (each lasting a tiny fraction of a second) play the role of elapsed time, and k plays the role of a growth, rather than decay, factor.

Ví dụ mẫu · Exponential growth in a supercritical chain reaction

In a supercritical assembly with multiplication factor $k = 1.02$, a single initial fission event triggers a chain reaction. Estimate the number of fission events occurring in the 100^{th} neutron generation.

$$N_{100} = N_0 k^{100} = (1)(1.02)^{100}.$$

$$(1.02)^{100} = e^{100 \ln(1.02)} = e^{100(0.0198)} = e^{1.98} \approx 7.2.$$

So only about 7 fission events would be occurring in the 100^{th} generation for this only-just-supercritical assembly ($k = 1.02$) — illustrating that even a multiplication factor only slightly above 1 eventually produces runaway growth, but the growth only becomes dramatic after very many generations; assemblies used in practice for rapid energy release operate with k well above 1, producing far faster growth.

The critical mass of a bare (unshielded, unreflected) sphere of fissile material depends strongly on which isotope is used, since different isotopes have different probabilities of undergoing fission when struck by a neutron of a given energy: a bare sphere of weapons-grade uranium-235 has a critical mass of roughly 52 kg, while a bare sphere of plutonium-239 requires only about 10 kg — plutonium-239 is considerably more fissile, and this critical mass can be reduced further still by surrounding the core with a neutron-reflecting shell that scatters escaping neutrons back into the fissile material.

VN **GIẢI THÍCH TIẾNG VIỆT**

Phản ứng dây chuyền: mỗi lần phân hạch giải phóng vài neutron, có thể kích hoạt phân hạch tiếp theo. Hệ số nhân neutron k: $k < 1$ là **dưới hạn** (phản ứng tắt dần), $k = 1$ là **tới hạn** (phản ứng duy trì ổn định — trạng thái vận hành của lò phản ứng), $k > 1$ là **trên hạn** (phản ứng tăng vọt, mất kiểm soát). **Khối lượng tới hạn** là khối lượng tối thiểu để duy trì phản ứng dây chuyền tự lực — mẫu quá nhỏ thì neutron thoát ra ngoài quá nhiều trước khi kịp gây phân hạch tiếp.

5.4.4 Nuclear Fission Reactors

A nuclear fission (power) reactor is engineered to sustain a fission chain reaction at a steady, controlled rate ($k = 1$) and to extract the resulting energy as useful heat. Its essential components are:

- **Fuel** — typically enriched uranium (with an increased proportion of the fissile isotope U-235 relative to natural uranium), providing the fissile nuclei.
- **Moderator** — a material of low atomic mass (commonly water or graphite) that slows down (thermalizes) the fast neutrons released by fission through repeated elastic collisions, since slow neutrons are far more likely to be captured by a U-235 nucleus and induce further fission than fast ones.
- **Control rods** — made of a strongly neutron-absorbing material (commonly boron or cadmium),

inserted or withdrawn to absorb a controllable fraction of the free neutrons, keeping k at exactly 1 during normal operation, and driven fully in to shut the reactor down ($k \ll 1$) in an emergency.

- **Coolant** — a fluid (often water, sometimes a gas or liquid metal) circulated through the reactor core to remove the heat generated by fission, both preventing the core from overheating and carrying that heat away to generate steam that drives turbines to produce electricity.

Spent fuel removed from a reactor still contains a mixture of highly radioactive fission products (many with short-to-medium half-lives, decaying away over years to centuries) and small quantities of long-lived heavier isotopes formed by neutron capture. Because of the long-lived component, spent fuel requires secure, shielded storage — following exactly the same time/distance/shielding principles introduced earlier in E.3 — for periods that can extend to many thousands of years, an important practical constraint on the deployment of fission power that has no equivalent in fossil-fuel combustion.

VN **GIẢI THÍCH TIẾNG VIỆT**

Bốn thành phần chính của lò phản ứng phân hạch: **nhiên liệu** (uranium làm giàu, cung cấp hạt nhân phân hạch); **chất làm chậm** (nước hoặc than chì, làm chậm neutron nhanh để tăng khả năng gây phân hạch tiếp theo); **thanh điều khiển** (bo hoặc cadmium, hấp thụ neutron để giữ $k = 1$ hoặc dừng lò khẩn cấp); **chất làm mát** (tải nhiệt ra khỏi lò để sinh hơi chạy turbine phát điện, đồng thời tránh lò quá nhiệt).

5.5 E.5 Fusion, Stars and the Standard Model

5.5.1 The Fusion Process

Nuclear fusion is the combination of two light nuclei into a single heavier nucleus. Because the product nucleus sits higher on the binding-energy-per-nucleon curve than the light reactant nuclei (see the figure in E.4), fusion — like fission — releases energy, and per unit mass of fuel it releases substantially *more* energy than fission, since the rise of the binding-energy curve is much steeper on the light-nuclei side than its slow decline on the heavy-nuclei side.

Achieving fusion is nonetheless difficult, because two positively charged nuclei strongly repel one another electrostatically (Coulomb repulsion) as they approach. For the short-range strong nuclear force to take over and bind them together, the nuclei must first be brought extremely close (of order a few femtometres) despite this repulsion. This requires:

- extremely **high temperature** (of order 10^7–10^8 K), so that nuclei have enough random thermal kinetic energy to approach one another closely despite their mutual repulsion;

- extremely **high density/pressure**, so that collisions between nuclei of sufficient energy occur frequently enough to sustain a useful reaction rate.

At such temperatures, matter exists as a **plasma** — a gas of freely moving, unbound ions and electrons — rather than as ordinary atoms.

Ví dụ mẫu · Energy released in deuterium–tritium fusion

The reaction ${}^2_1\text{H} + {}^3_1\text{H} \to {}^4_2\text{He} + {}^1_0n$ is the easiest fusion reaction to initiate in the laboratory. Given $m(\text{H-2}) = 2.0141$ u, $m(\text{H-3}) = 3.0160$ u, $m(\text{He-4}) = 4.0015$ u, $m(\text{n}) = 1.0090$ u, calculate the energy released.

Mass before:

$$2.0141 + 3.0160 = 5.0301 \text{ u}.$$

Mass after:

$$4.0015 + 1.0090 = 5.0105 \text{ u}.$$

Mass defect and energy released:

$$\Delta m = 5.0301 - 5.0105 = 0.0196 \text{ u} \qquad \Rightarrow \qquad E = 0.0196 \times 931.5 = 18.3 \text{ MeV}.$$

Although the energy released per reaction is far smaller than in a single fission event, deuterium–tritium fuel has a much larger energy release *per unit mass of fuel*, which is why fusion is pursued as a future high-density power source.

(!) Lỗi thường gặp: Đừng nhầm điều kiện cho phản ứng nhiệt hạch: nhiệt độ cực cao **không phải** để "phá vỡ" hạt nhân, mà để cung cấp đủ động năng cho các hạt nhân **vượt qua lực đẩy tĩnh điện Coulomb** giữa chúng và tiến đủ gần để lực hạt nhân mạnh (tầm ngắn) phát huy tác dụng. Cũng cần mật độ đủ cao để tần suất va chạm đủ lớn — chỉ nhiệt độ cao thôi (mà mật độ thấp) là không đủ để duy trì phản ứng.

No solid material container could ever physically touch a plasma at 10^7–10^8 K without instantly vaporizing, so a working fusion reactor must confine the plasma *without* solid walls. Two engineering approaches are pursued: **magnetic confinement**, in which strong magnetic fields (arranged, for example, in the toroidal "tokamak" design) exploit the fact that charged particles in a plasma spiral along field lines, holding the plasma away from the walls of the vacuum vessel; and **inertial confinement**, in which a small fuel pellet is compressed and heated almost instantaneously (for example, by an array of powerful lasers striking it from all directions), so that fusion occurs before the pellet has time to fly apart. Unlike a fission reactor, a fusion reactor carries no risk of an uncontrolled chain reaction — if confinement is lost for even a moment, the plasma cools and the reaction simply stops — but achieving a net energy *gain* (more energy released by fusion than is consumed maintaining the plasma) has proved to be a formidable long-term engineering challenge.

GIẢI THÍCH TIẾNG VIỆT

VN Nhiệt hạch: hai hạt nhân nhẹ kết hợp thành một hạt nhân nặng hơn, toả năng lượng (vì sản phẩm nằm cao hơn trên đồ thị năng lượng liên kết/nucleon). Cần nhiệt độ cực cao ($\sim 10^7$–10^8K) và mật độ cao để hạt nhân đủ động năng vượt lực đẩy Coulomb và tiến đủ gần để lực hạt nhân mạnh tác dụng. Ở nhiệt độ này vật chất tồn tại ở dạng **plasma**, phải được giam giữ bằng từ trường mạnh (kiểu tokamak) hoặc bằng nén quán tính (laser) chứ không thể dùng bình chứa vật liệu thông thường nào chịu nổi. Khác với phân hạch, lò nhiệt hạch **không có nguy cơ phản ứng dây chuyền mất kiểm soát** — mất giam giữ là phản ứng tắt ngay.

5.5.2 Fusion in Stars: the Proton–Proton Chain and the CNO Cycle

Stars are powered by nuclear fusion in their cores. In stars with core temperatures similar to or somewhat above the Sun's, the dominant fusion pathway is the **proton–proton (p–p) chain**: through a short sequence of intermediate steps, four hydrogen-1 nuclei (protons) are ultimately converted into one helium-4 nucleus, releasing two positrons, two neutrinos, and a large amount of energy:

$$4\,{}^{1}_{1}\text{H} \longrightarrow {}^{4}_{2}\text{He} + 2\,{}^{0}_{+1}e + 2\nu_e + \text{energy}.$$

In stars considerably hotter and more massive than the Sun, an alternative pathway, the **CNO cycle** (carbon–nitrogen–oxygen cycle), dominates: carbon-12 acts as a catalyst, repeatedly capturing protons and undergoing a cycle of transformations that is regenerated at the end, with the net effect — again — of converting four protons into one helium-4 nucleus and releasing energy. Both pathways achieve the same overall transformation and net energy release; which one dominates in a given star depends primarily on core temperature (the CNO cycle's reaction rate rises much more steeply with temperature, so it dominates only in hotter, more massive stars).

Ví dụ mẫu · Net energy release of the proton–proton chain

Estimate the total energy released when four protons fuse (via the proton–proton chain) into one helium-4 nucleus, neglecting the small rest mass of the emitted positrons and treating the neutrinos as effectively massless. Use $m_p = 1.67 \times 10^{-27}$ kg $= 1.0060$ u and $m(\text{He-4 nucleus}) = 4.0015$ u.

$$4m_p = 4(1.0060) = 4.0241 \text{ u.}$$

$$\Delta m \approx 4.0241 - 4.0015 = 0.0226 \text{ u} \qquad \Rightarrow \qquad E \approx 0.0226 \times 931.5 = 21.1 \text{ MeV.}$$

(A more complete calculation, including the rest mass of the two emitted positrons and their subsequent annihilation, gives a slightly larger commonly quoted total of around 26–27 MeV; the estimate above captures the dominant nuclear contribution.) This single reaction sequence, repeated an enormous number of times per second, is what powers the Sun.

Ví dụ mẫu · The Sun's rate of mass loss

The Sun radiates energy at a rate (its luminosity) of $L = 3.83 \times 10^{26}$ W. Using mass–energy equivalence, estimate the rate at which the Sun loses mass as a result of the nuclear fusion powering this output.

Since $E = mc^2$, the rate of energy loss $L = \frac{dE}{dt}$ corresponds to a rate of mass loss $\frac{dm}{dt} = \frac{L}{c^2}$:

$$\frac{dm}{dt} = \frac{3.83 \times 10^{26}}{(3.00 \times 10^8)^2} = \frac{3.83 \times 10^{26}}{9.00 \times 10^{16}} = 4.26 \times 10^9 \text{ kg/s.}$$

The Sun converts roughly 4.3 billion kilograms of matter into energy every second (a little over four million tonnes per second) — an enormous rate in absolute terms, yet utterly insignificant compared with the Sun's total mass ($\approx 2 \times 10^{30}$ kg), which is why the Sun can continue shining at essentially this same rate for billions of years.

GIẢI THÍCH TIẾNG VIỆT

VN Chuỗi phản ứng proton–proton (chiếm ưu thế trong các sao như Mặt Trời): 4 hạt nhân hydro (proton) kết hợp thành 1 hạt nhân heli-4, phát ra 2 positron, 2 neutrino và năng lượng. Chu trình CNO (chiếm ưu thế ở các sao nóng hơn, khối lượng lớn hơn) dùng carbon làm chất xúc tác nhưng cho **cùng kết quả tổng thể**: 4 proton → 1 heli-4. Sao càng nóng thì chu trình CNO càng chiếm ưu thế so với chuỗi p-p.

5.5.3 Stellar Nucleosynthesis

As a star exhausts the hydrogen fuel in its core, it can — if massive enough — begin fusing progressively heavier elements in succession (helium into carbon and oxygen, and, in the most massive stars, on through neon, oxygen, silicon...), each stage releasing energy because the products still sit higher on the binding-energy-per-nucleon curve than the reactants. This chain of successive fusion stages can build up elements as heavy as **iron**, but stops there: because iron sits at (or extremely close to) the peak of the binding-energy-per-nucleon curve, fusing iron into anything heavier would require an *input* of energy rather than releasing it, so fusion simply cannot proceed further inside an ordinary stellar core.

Elements heavier than iron, observed throughout the universe, are therefore built up by other processes — principally rapid neutron-capture nucleosynthesis occurring during the extreme conditions of a **supernova** explosion (the violent death of a very massive star), where an intense flux of free neutrons allows heavy nuclei to capture neutrons far faster than those nuclei can beta-decay away the resulting instability, building up progressively heavier nuclides that are then blasted out into space, seeding future generations of stars and planets (including, ultimately, the heavy elements that make up the Earth itself).

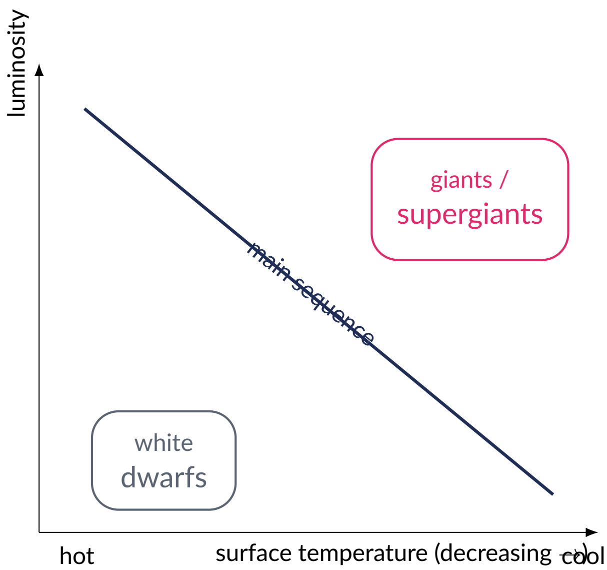
GIẢI THÍCH TIẾNG VIỆT

VN

Tổng hợp hạt nhân trong sao: các giai đoạn nhiệt hạch liên tiếp trong lõi sao đủ nặng có thể tạo ra các nguyên tố nặng dần, tối đa đến **sắt** (Fe) — vì sắt gần đỉnh đồ thị năng lượng liên kết/nucleon, nhiệt hạch từ sắt trở đi sẽ **cần** năng lượng thay vì toả ra. Các nguyên tố nặng hơn sắt được tạo ra qua quá trình bắt neutron nhanh trong vụ nổ **siêu tân tinh** (supernova) — đây là nguồn gốc của hầu hết các nguyên tố nặng trong vũ trụ, kể cả trên Trái Đất.

5.5.4 The Hertzsprung–Russell Diagram and Stellar Evolution

The **Hertzsprung–Russell (HR) diagram** plots stellar **luminosity** (total power radiated, often shown on a logarithmic scale, sometimes expressed as absolute magnitude) against **surface temperature** (with temperature conventionally plotted *decreasing* to the right, and often related to a star's colour/spectral class). Most stars, for most of their lives, lie along a diagonal band called the **main sequence**, running from hot, luminous, blue-white stars at the upper left to cool, faint, red stars at the lower right; a star on the main sequence is fusing hydrogen into helium in its core.



A simplified Hertzsprung–Russell diagram. The main sequence runs from hot/luminous (upper left) to cool/faint (lower right). Giants and supergiants (upper right) are cool but highly luminous because they are enormous; white dwarfs (lower left) are hot but faint because they are tiny.

Two other regions are prominent: **giants and supergiants**, in the upper right, are relatively cool at the surface (hence red/orange in colour) yet extremely luminous, because they are enormous — an immense surface area radiates a great total power even at a modest temperature per unit area. **White dwarfs**, in the lower left, are hot at the surface yet very faint, because they are extremely small (roughly Earth-sized) — a hot but tiny surface radiates relatively little total power.

A star's position and future path on the HR diagram is governed by its mass. A star of mass similar to the Sun spends most of its life on the main sequence fusing hydrogen; once core hydrogen is exhausted, it expands and cools at the surface while its core contracts and heats, moving it off the main sequence and up into the red-giant region as it fuses helium (and, briefly, some heavier elements); eventually it sheds its outer layers and the exposed core settles as a small, hot, dense **white dwarf**, which then simply cools over a very long time with no further fusion. A much more massive star evolves faster and more dramatically: after passing through a supergiant phase and building up a layered structure of successively heavier fused elements up to iron in its core, it can no longer generate energy by fusion and collapses catastrophically, producing a **supernova** explosion that leaves behind an extremely dense **neutron star** or, for the most massive stars, a **black hole**.

> The eventual fate of a collapsed stellar core depends on its mass. A white dwarf is held up against further gravitational collapse not by fusion but by **electron degeneracy pressure**, a purely quantum-mechanical effect (ultimately a consequence of the Pauli exclusion principle, which forbids identical electrons from occupying the same quantum state); this support fails if the core's mass exceeds the **Chandrasekhar limit**, about 1.4 solar masses. A more massive collapsed core instead becomes a **neutron star**, supported by the analogous neutron degeneracy pressure, up to a further limit of roughly 2–3 solar masses — a neutron star packs more mass than the Sun into a sphere only a few kilometres across, reaching densities comparable to (and even exceeding) the nuclear density calculated in E.1. Beyond this limit, no known force can halt the collapse, and a **black hole** results.

> **(!) Lỗi thường gặp:** Trục nhiệt độ trên biểu đồ H–R **giảm dần sang phải** — ngược với quy ước thông thường của trục số. Vì vậy sao **nóng nhất** nằm ở **bên trái**, không phải bên phải. Đây là lỗi đọc đồ thị rất thường gặp — hãy luôn kiểm tra chiều mũi tên/nhãn trục trước khi kết luận sao nào nóng hơn.

> VN **GIẢI THÍCH TIẾNG VIỆT**
> Biểu đồ Hertzsprung–Russell: trục tung là độ trưng (luminosity), trục hoành là nhiệt độ bề mặt nhưng **giảm dần sang phải**. Dải chính (main sequence, chéo từ trên-trái xuống dưới-phải) là nơi hầu hết sao (kể cả Mặt Trời) trải qua phần lớn "cuộc đời" khi đang nhiệt hạch hydro thành heli. Sao khổng lồ đỏ (giants/supergiants) ở góc trên-phải: lạnh nhưng rất sáng vì kích thước cực lớn. Sao lùn trắng (white dwarfs) ở góc dưới-trái: nóng nhưng mờ vì kích thước rất nhỏ. Tiến hoá sao: Mặt Trời sẽ thành sao khổng lồ đỏ rồi sao lùn trắng; sao rất nặng sẽ nổ siêu tân tinh, để lại sao neutron hoặc hố đen.

5.5.5 The Standard Model of Particle Physics

The **Standard Model** is the current, well-tested theoretical framework describing the fundamental building blocks of matter and the interactions between them.

Matter particles fall into two families of fundamental (indivisible, as far as is currently known) particles, both organized into three "generations":

- **Quarks** — six flavours: up, down, charm, strange, top, bottom (each also having a corresponding antiquark). Quarks carry fractional electric charge ($+2/3\,e$ for up-type: up, charm, top; $-1/3\,e$ for down-type: down, strange, bottom) and are never observed in isolation; they are always confined inside composite particles called **hadrons**.
- **Leptons** — the electron, muon, and tau, each with charge $-e$, together with a corresponding neutral, nearly massless neutrino for each (ν_e, ν_μ, ν_τ); each also has a corresponding antiparticle.

Generation	Quarks	Charge (units of e)	Leptons
1st	up (u), down (d)	$+\frac{2}{3}, -\frac{1}{3}$	electron (e^-), ν_e
2nd	charm (c), strange (s)	$+\frac{2}{3}, -\frac{1}{3}$	muon (μ^-), ν_μ
3rd	top (t), bottom (b)	$+\frac{2}{3}, -\frac{1}{3}$	tau (τ^-), ν_τ

Ordinary, everyday matter is built almost entirely from just the first generation (up and down quarks inside protons and neutrons, and electrons); the second- and third-generation particles are far more massive, unstable, and are seen only in high-energy cosmic-ray collisions or particle-accelerator experiments, decaying almost immediately into first-generation particles.

Quarks combine to form two classes of hadrons:

- **Baryons** — three quarks bound together. The proton is uud (charge $+\frac{2}{3}+\frac{2}{3}-\frac{1}{3}=+1$, in units of e); the neutron is udd (charge $+\frac{2}{3}-\frac{1}{3}-\frac{1}{3}=0$).
- **Mesons** — a quark and an antiquark bound together.

The four fundamental interactions, their approximate relative strengths, and their force-carrying gauge bosons:

- **Strong** (relative strength ~ 1, the largest): carried by the **gluon**; binds quarks together inside hadrons, and (as a residual effect) binds protons and neutrons together inside the nucleus.
- **Electromagnetic** (relative strength $\sim 10^{-2}$): carried by the **photon**; acts between any charged particles.
- **Weak** (relative strength $\sim 10^{-6}$): carried by the **W and Z bosons**; responsible for processes such as beta decay.
- **Gravitational** (relative strength $\sim 10^{-38}$, by far the weakest): carried by the hypothesized (not yet directly detected) **graviton**; negligible in particle interactions but dominant at astronomical scales because of the enormous masses involved.

Almost everything known about the Standard Model's particles has been learned using **particle accelerators**: machines that use electric fields to accelerate charged particles to extremely high kinetic energies (and hence, by the de Broglie relation of E.2, to extremely short associated wavelengths capable of probing structure far smaller than a nucleus), then collide them together — either against a stationary target or head-on against a second, oppositely-directed beam. Magnetic fields are used to steer and focus the beams, and to bend them into a closed loop in ring-shaped ("synchrotron") designs, allowing particles to be accelerated over many repeated passes. The very high collision energies allow the creation of massive short-lived particles (such as the W/Z bosons, or the Higgs boson) that do not otherwise exist in ordinary matter, since by mass–energy equivalence a large enough input of kinetic energy can be converted directly into the rest mass energy of new particles.

The very different masses of the fundamental particles (from the essentially massless neutrinos to the very heavy top quark) are understood to arise from each particle's interaction with the **Higgs field**, an energy field that permeates all of space; particles that interact more strongly with this field acquire more mass, while the photon — which does not interact with the Higgs field at all — remains exactly massless, consistent with its role as carrier of the infinite-range electromagnetic force. The associated particle, the **Higgs boson**, was predicted theoretically in the 1960s and finally observed experimentally in 2012 at the Large Hadron Collider, confirming the mechanism.

When a particle meets its own antiparticle, the two can **annihilate**, converting their entire combined rest mass directly into energy — typically two photons, emitted in opposite directions to conserve the (zero) total momentum of a pair initially at rest. The reverse process, **pair production**, converts a sufficiently energetic photon into a particle–antiparticle pair (requiring at least enough photon energy to supply both particles' rest mass energy). Both processes are direct, dramatic experimental confirmations of mass–energy equivalence, $E=mc^2$ (E.4).

Ví dụ mẫu · Electron–positron annihilation

An electron and a positron, both essentially at rest, meet and annihilate, producing two photons. Find the energy of each photon.

Rest mass energy of the electron (equal to that of the positron):

$$m_e c^2 = (9.11\times10^{-31})(3.00\times10^8)^2 = 8.20\times10^{-14}\text{ J} = \frac{8.20\times10^{-14}}{1.60\times10^{-19}} = 5.12\times10^5\text{ eV} = 0.512\text{ MeV}.$$

Total energy released:

$$E_{total} = 2m_e c^2 = 2(0.512) = 1.024\text{ MeV}.$$

Since the electron and positron are (essentially) at rest, the total initial momentum is zero; conservation of momentum then requires the two photons to move in exactly opposite directions with equal magnitude momentum, and hence (since photon energy $E = pc$) equal energy:

$$E_{\text{each photon}} = \frac{E_{total}}{2} = \frac{1.024}{2} = 0.512\text{ MeV}.$$

This is the well-known 0.511 MeV annihilation line used, for example, in positron-emission tomography (PET) medical scanners.

As a synthesis of the whole chapter, beta-minus decay (E.3) can now be reinterpreted at the level of fundamental particles: inside a neutron (udd), a down quark converts into an up quark (turning the neutron, udd, into a proton, uud) by emitting a virtual W^- **boson**, which itself almost instantly decays into an electron and an electron antineutrino:

$$d \longrightarrow u + W^-, \qquad W^- \longrightarrow e^- + \bar{\nu}_e.$$

This is precisely the weak interaction at work, and it is why beta decay — unlike alpha or gamma decay — is mediated by the weak force rather than the strong or electromagnetic force.

Particle interactions obey several conservation laws beyond energy, momentum, and charge, including **baryon number** (each baryon is assigned $+1$, each antibaryon -1, each non-baryon 0; the total must be conserved in any interaction) and **lepton number** (similarly $+1$ for each lepton, -1 for each antilepton, conserved separately for each lepton family in most interactions) — both of which are automatically satisfied by the correctly balanced beta-decay equations written in E.3 (an electron, lepton number $+1$, is always accompanied by an antineutrino, lepton number -1, keeping the total lepton number at 0, unchanged from before the decay).

Ví dụ mẫu · Beta decay at the quark level

State the quark composition of a proton and a neutron. Use this to explain, at the quark level, what happens during beta-minus decay, naming the force-carrying particle involved and the fundamental interaction responsible.

Proton: uud (charge $+\frac{2}{3}+\frac{2}{3}-\frac{1}{3}=+1e$). Neutron: udd (charge $+\frac{2}{3}-\frac{1}{3}-\frac{1}{3}=0$).
In beta-minus decay, one of the neutron's down quarks (udd) converts into an up quark, turning the neutron into a proton (uud). This quark transformation happens via the emission of a virtual W^- boson, $d \to u + W^-$, which then decays as $W^- \to e^- + \bar{\nu}_e$, producing the observed beta particle and antineutrino. The process is mediated by the **weak interaction**, carried by the W boson.

VN **GIẢI THÍCH TIẾNG VIỆT**

Mô hình chuẩn: hạt vật chất cơ bản gồm **quark** (6 loại: up, down, charm, strange, top, bottom, mang điện tích phân số) và **lepton** (electron, muon, tau, cùng 3 neutrino tương ứng). Quark kết hợp thành **hadron**: **baryon** (3 quark, VD proton $= uud$, neutron $= udd$) và **meson** (1 quark + 1 phản quark). Bốn tương tác cơ bản và hạt truyền tương tác: **mạnh** (gluon, mạnh nhất), **điện**

từ (photon), **yếu** (boson W/Z, gây ra phân rã beta), **hấp dẫn** (graviton — giả thuyết, yếu nhất). Trường Higgs cho các hạt khối lượng của chúng; hạt-phản hạt gặp nhau sẽ **huỷ cặp**, biến toàn bộ khối lượng nghỉ thành năng lượng (photon). Phân rã β^- ở cấp độ quark: quark down → quark up $+W^-$, rồi $W^- \to e^- + \bar{\nu}_e$ — đây là biểu hiện của tương tác yếu.

5.6 Problem Bank

Bài 1

In the Rutherford gold-foil scattering experiment, which observation provided the strongest evidence for a tiny, dense, positively charged nucleus?

(A) Most alpha particles passed straight through the foil.

(B) A small number of alpha particles deflected through small angles.

(C) A very small fraction of alpha particles deflected through angles greater than $90°$, some almost straight back.

(D) Alpha particles were absorbed by the gold foil and did not reach the detector screen.

Lời giải chi tiết

Only a concentrated, massive positive charge could exert a force strong enough to reverse the path of a fast, positively charged alpha particle in a single close encounter; a spread-out charge distribution (as in the plum-pudding model) could never produce such large-angle scattering. **(C)**.

Bài 2

A certain isotope of sodium has 11 protons and 12 neutrons. (a) Write its nuclide notation. (b) State the mass number and the neutron number. (c) A second sodium isotope has one more neutron than this one; write its nuclide notation and explain why it is chemically identical to the first.

Lời giải chi tiết

(a) $Z = 11$ (sodium), $N = 12$, so $A = Z + N = 23$: nuclide is $^{23}_{11}$Na.
(b) Mass number $A = 23$; neutron number $N = 12$.
(c) One more neutron gives $N = 13$, $A = 24$: $^{24}_{11}$Na. Both isotopes have $Z = 11$ protons, hence 11 electrons in the neutral atom, which determines all chemical bonding behaviour — chemistry depends on electron configuration (set by Z), not on N, so the two isotopes are chemically identical though physically (in mass and possibly stability) different.

Bài 3

The emission spectrum of atomic hydrogen consists of sharp, discrete bright lines rather than a continuous rainbow of colour. This is best explained by the fact that:

(A) hydrogen atoms can only absorb energy, never emit it.

(B) electrons in the atom are confined to discrete, quantized energy levels, so only photons of specific energies (matching the gaps between levels) can be emitted.

(C) hydrogen has only one electron per atom.

(D) all the electrons in a sample of hydrogen gas have exactly the same total energy.

Lời giải chi tiết

(B). Discrete spectral lines directly reflect discrete allowed electron energy levels: a transition between two specific levels always emits a photon of one specific energy $E = hf = E_i - E_f$, so only a fixed, discrete set of photon energies (and hence wavelengths) can ever appear.

Bài 4

Calculate the energy (in eV) and the wavelength of the photon emitted when a hydrogen atom's electron falls from $n = 4$ to $n = 2$ (the second line of the Balmer series, Hβ). State the region of the electromagnetic spectrum this lies in.

Lời giải chi tiết

$$E_4 = -\frac{13.6}{4^2} = -\frac{13.6}{16} = -0.850 \text{ eV}, \qquad E_2 = -\frac{13.6}{2^2} = -3.400 \text{ eV}.$$

$$E_{photon} = E_4 - E_2 = -0.850 - (-3.400) = 2.550 \text{ eV} = 2.550 \times 1.60 \times 10^{-19} = 4.08 \times 10^{-19} \text{ J}.$$

$$\lambda = \frac{hc}{E_{photon}} = \frac{1.989 \times 10^{-25}}{4.08 \times 10^{-19}} = 4.88 \times 10^{-7} \text{ m} = 488 \text{ nm}.$$

This lies in the blue-green part of the visible spectrum — the well-known Hβ line.

Bài 5

The ionization energy of a hydrogen atom in its ground state is 13.6 eV. This means that:

(A) every possible electron transition in hydrogen releases exactly 13.6 eV.

(B) at least 13.6 eV must be supplied to completely remove the electron from the ground state ($n = 1$) to $n = \infty$.

(C) the total energy of a hydrogen atom is always $+13.6$ eV.

(D) 13.6 eV is the energy of the photon emitted in the $n = 2 \to n = 1$ transition.

Lời giải chi tiết

Ionization energy is defined as the energy needed to remove the electron completely, from its current level out to $n = \infty$ (where $E_\infty = 0$); for the ground state this is $-E_1 = 13.6$ eV. (Note that (D) is a common distractor but is numerically wrong: the $n = 2 \to n = 1$ transition releases $E_2 - E_1 = -3.4 - (-13.6) = 10.2$ eV, not 13.6 eV.) **(B)**.

Bài 6

A beam of white light is passed through a sample of cool hydrogen gas. Photons of energy 10.2 eV are strongly absorbed, producing a dark line in the transmitted spectrum. (a) Show that this photon energy corresponds to a transition from $n = 1$ to $n = 2$. (b) Explain why photons of energy 11.5 eV, which carry *more* energy than the 10.2 eV photons, are not absorbed at all.

Lời giải chi tiết

(a) $E_2 - E_1 = -3.400 - (-13.600) = 10.2$ eV, exactly matching the given photon energy — so this dark line corresponds to ground-state hydrogen atoms absorbing a photon and jumping from $n = 1$ to $n = 2$.
(b) In a cool gas, essentially all atoms start in the ground state ($n = 1$), so only photons whose

energy exactly matches an allowed transition *from* $n = 1$ can be absorbed: $E_2 - E_1 = 10.2$ eV, $E_3 - E_1 = -1.511 - (-13.6) = 12.1$ eV, and so on, up to the ionization energy of 13.6 eV (above which any excess energy becomes kinetic energy of a freed electron, so absorption becomes possible for a continuous range starting at 13.6 eV). Since 11.5 eV does not match any allowed transition energy and is also less than the 13.6 eV ionization energy, a photon of exactly 11.5 eV cannot be absorbed at all — it simply passes through the gas.

Bài 7

For a photoelectric-effect experiment using light with a frequency above the threshold frequency f_0, which change would increase the *maximum kinetic energy* of the emitted photoelectrons?

(A) Increasing the intensity of the incident light while keeping its frequency fixed.

(B) Increasing the frequency of the incident light.

(C) Decreasing the frequency of the incident light, toward f_0.

(D) Switching to a metal with a larger work function, at the same frequency.

Lời giải chi tiết

$KE_{max} = hf - \Phi$ depends only on frequency f and work function Φ, not on intensity. Increasing f increases KE_{max} directly; decreasing f or increasing Φ would decrease it; intensity affects only the number of photoelectrons per second, not their maximum energy. **(B)**.

Bài 8

A metal surface has a work function $\Phi = 4.20$ eV. Ultraviolet light of wavelength 250 nm is incident on the surface. Find (a) the energy of each incident photon, in eV; (b) the maximum kinetic energy of the emitted photoelectrons; (c) the stopping voltage; (d) the threshold wavelength of the surface.

Lời giải chi tiết

(a)

$$E = \frac{hc}{\lambda} = \frac{1.989 \times 10^{-25}}{250 \times 10^{-9}} = 7.96 \times 10^{-19} \text{ J} = \frac{7.96 \times 10^{-19}}{1.60 \times 10^{-19}} = 4.97 \text{ eV}.$$

(b)

$$KE_{max} = E - \Phi = 4.97 - 4.20 = 0.77 \text{ eV}.$$

(c) $V_s = KE_{max}/e = 0.77$ V (numerically equal to the value in eV).

(d)

$$\lambda_0 = \frac{hc}{\Phi} = \frac{1.989 \times 10^{-25}}{(4.20)(1.60 \times 10^{-19})} = \frac{1.989 \times 10^{-25}}{6.72 \times 10^{-19}} = 2.96 \times 10^{-7} \text{ m} = 296 \text{ nm}.$$

Bài 9

Which of the following observations provides evidence for the *particle* (photon) nature of light, rather than its wave nature?

(A) Diffraction of light around an obstacle.

(B) Two-source interference producing bright and dark fringes.

(C) The photoelectric effect, in which electron emission depends on frequency, not intensity.

(D) Polarization of light by a filter.

Lời giải chi tiết

Diffraction, interference, and polarization are all wave phenomena, fully explained by classical wave theory. The photoelectric effect's dependence on frequency (not intensity) for electron emission can only be explained if light delivers energy in discrete photon packets, $E = hf$. **(C).**

Bài 10

Calculate the de Broglie wavelength of (a) a proton moving at 2.0×10^6 m/s, and (b) a 0.50 kg ball moving at 20 m/s. Comment on why wave effects can, in principle, be observed for the proton but are never observed for the ball.

Lời giải chi tiết

(a)

$$p = m_p v = (1.67 \times 10^{-27})(2.0 \times 10^6) = 3.34 \times 10^{-21} \text{ kg} \cdot \text{m/s}.$$

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{3.34 \times 10^{-21}} = 2.0 \times 10^{-13} \text{ m}.$$

(b)

$$p = mv = (0.50)(20) = 10 \text{ kg} \cdot \text{m/s}.$$

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{10} = 6.63 \times 10^{-35} \text{ m}.$$

The proton's wavelength ($\sim 10^{-13}$ m) is on a subatomic scale comparable to distances relevant in particle scattering, so wave effects (diffraction) are in principle detectable with a suitable target. The ball's wavelength ($\sim 10^{-35}$ m) is inconceivably smaller than any physical aperture or object that could ever be used to diffract it — no macroscopic object ever displays observable wave behaviour.

Bài 11

Heisenberg's uncertainty principle, $\Delta x \, \Delta p \geq h/(4\pi)$, implies that:

(A) position and momentum cannot be measured simultaneously only because our instruments are not yet precise enough.

(B) there is a fundamental limit, inherent to nature itself, on how precisely position and momentum can simultaneously be known — true regardless of instrument quality.

(C) a particle's momentum is always exactly zero whenever its position is perfectly known.

(D) the principle applies only to macroscopic objects, never to subatomic particles.

Lời giải chi tiết

The uncertainty principle is a fundamental property of quantum systems (arising from wave–particle duality), not a statement about the limitations of measuring equipment. **(B).**

Bài 12

An excited nuclear state has a mean lifetime of $\Delta t = 1.0 \times 10^{-12}$ s before it decays by emitting a gamma-ray photon. Use the energy–time form of the uncertainty principle to estimate the minimum uncertainty in the energy of this state, in eV. Comment on what this implies about the emitted gamma ray.

Lời giải chi tiết

$$\Delta E \geq \frac{h}{4\pi\,\Delta t} = \frac{6.63 \times 10^{-34}}{4\pi(1.0 \times 10^{-12})} = \frac{6.63 \times 10^{-34}}{1.257 \times 10^{-11}} = 5.28 \times 10^{-23}\text{ J.}$$

$$\Delta E = \frac{5.28 \times 10^{-23}}{1.60 \times 10^{-19}} = 3.3 \times 10^{-4}\text{ eV.}$$

Because the excited state does not have a perfectly sharp, single energy, the emitted gamma-ray photons do not all have exactly the same energy either — the emission line has a small but nonzero "natural linewidth" of about 3×10^{-4} eV, a direct, measurable consequence of the finite lifetime of the excited state.

Bài 13

Which type of radioactive emission is stopped by only a few millimetres of aluminium, yet is able to pass through a sheet of paper and the outer dead layer of human skin?

(A) Alpha particles

(B) Beta particles

(C) Gamma rays

(D) X-rays

Lời giải chi tiết

Alpha particles are stopped by paper alone; gamma rays require centimetres of lead; beta particles are the emission stopped specifically by a few millimetres of aluminium. **(B)**.

Bài 14

Write balanced nuclear equations for: (a) the alpha decay of radium-226 ($Z = 88$); (b) the beta-minus decay of cobalt-60 ($Z = 27$); (c) the beta-plus decay of sodium-22 ($Z = 11$). Verify conservation of A and Z in each case.

Lời giải chi tiết

(a) Alpha decay: $A \to 226 - 4 = 222$, $Z \to 88 - 2 = 86$ (radon):

$$^{226}_{88}\text{Ra} \longrightarrow {}^{222}_{86}\text{Rn} + {}^{4}_{2}\text{He}.$$

Check: A: $226 = 222 + 4$ ✓; Z: $88 = 86 + 2$ ✓.

(b) Beta-minus decay: A unchanged $= 60$, $Z \to 27 + 1 = 28$ (nickel):

$$^{60}_{27}\text{Co} \longrightarrow {}^{60}_{28}\text{Ni} + {}^{0}_{-1}e + \bar{\nu}_e.$$

Check: A: $60 = 60$ ✓; Z: $27 = 28 - 1$ ✓.

(c) Beta-plus decay: A unchanged $= 22$, $Z \to 11 - 1 = 10$ (neon):

$$^{22}_{11}\text{Na} \longrightarrow {}^{22}_{10}\text{Ne} + {}^{0}_{+1}e + \nu_e.$$

Check: A: $22 = 22$ ✓; Z: $11 = 10 + 1$ ✓.

Bài 15

As atomic number Z increases beyond about 20, stable nuclei require an increasingly higher neutron-to-proton ratio. This trend is best explained by the fact that:

(A) neutrons become heavier as Z increases.

(B) the strong nuclear force weakens rapidly as Z increases.

(C) electrostatic (Coulomb) repulsion between protons grows with the number of protons, while extra neutrons add strong-force attraction without adding extra repulsion.

(D) protons spontaneously decay into neutrons inside large nuclei.

Lời giải chi tiết

The strong force is short-ranged (acts only between close neighbours) while the Coulomb repulsion is long-ranged (every proton repels every other proton in the nucleus). As Z grows, total repulsion grows faster than the nearest-neighbour strong-force attraction can compensate for, so extra neutrons — which add strong-force binding without adding to the repulsion — are needed to keep the nucleus bound. **(C)**.

Bài 16

A sample contains 8.0×10^{10} nuclei of a radioisotope with a half-life of 12 days. (a) How many undecayed nuclei remain after 60 days? (b) Calculate the activity of the sample at that time.

Lời giải chi tiết

(a) 60 days $= 60/12 = 5$ half-lives:

$$N = N_0 \left(\frac{1}{2}\right)^5 = \frac{8.0 \times 10^{10}}{32} = 2.5 \times 10^9 \text{ nuclei.}$$

(b) Decay constant:

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.6931}{12 \text{ days}} = 0.05776 \text{ day}^{-1} = \frac{0.05776}{86400 \text{ s}} = 6.69 \times 10^{-7} \text{ s}^{-1}.$$

$$A = \lambda N = (6.69 \times 10^{-7})(2.5 \times 10^9) = 1.7 \times 10^3 \text{ Bq.}$$

Bài 17

A radioactive isotope has a decay constant of $\lambda = 4.33 \times 10^{-4}$ s^{-1}. Calculate its half-life.

Lời giải chi tiết

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.6931}{4.33 \times 10^{-4}} = 1.60 \times 10^3 \text{ s } (\approx 26.7 \text{ minutes}).$$

Bài 18

A technician handling a sealed gamma-emitting source wants to minimize their radiation dose. Which of the following would *not* meaningfully reduce their exposure?

(A) Increasing the distance between themselves and the source.

(B) Reducing the time spent near the source.

(C) Placing a thick lead shield between themselves and the source.

(D) Wearing an ordinary cotton lab coat.

Lời giải chi tiết

Time, distance, and (dense) shielding are the three genuinely effective principles of radiation protection. Thin cotton fabric provides negligible attenuation of penetrating gamma radiation. **(D)**.

Bài 19

A wooden artifact is found to have a carbon-14 activity equal to 30.0% of that measured in a freshly cut sample of the same wood species. Using $T_{1/2} = 5730$ years for carbon-14, estimate the age of the artifact.

Lời giải chi tiết

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.6931}{5730} = 1.210 \times 10^{-4}\ \text{yr}^{-1}.$$

$$\frac{A}{A_0} = e^{-\lambda t} \Rightarrow 0.300 = e^{-\lambda t} \Rightarrow \ln(0.300) = -\lambda t \Rightarrow -1.204 = -(1.210 \times 10^{-4})t.$$

$$t = \frac{1.204}{1.210 \times 10^{-4}} = 9.95 \times 10^3\ \text{years}.$$

Bài 20

The binding energy per nucleon curve peaks near mass number $A \approx 56$ (iron/nickel). Compared with a nucleus near this peak, both very light nuclei (e.g. hydrogen, helium) and very heavy nuclei (e.g. uranium) are:

(A) more tightly bound per nucleon, which is why neither fission nor fusion releases energy.

(B) less tightly bound per nucleon, so both combining light nuclei (fusion) and splitting heavy nuclei (fission) can release energy by producing products closer to the peak.

(C) equally tightly bound per nucleon, regardless of position on the curve.

(D) unstable if and only if they are radioactive.

Lời giải chi tiết

Nuclei far from the peak (in either direction) have lower binding energy per nucleon than nuclei near $A \approx 56$; any process — fission of heavy nuclei or fusion of light nuclei — that produces daughter/product nuclei closer to the peak releases energy, since binding energy per nucleon increases. **(B)**.

Bài 21

The nucleus of carbon-12 has a measured mass of 11.9967 u. Using $m_p = 1.0060$ u and $m_n = 1.0090$ u (derived from the constants $m_p = 1.67 \times 10^{-27}$ kg, $m_n = 1.675 \times 10^{-27}$ kg, and 1 u $= 1.66 \times 10^{-27}$ kg), calculate the binding energy per nucleon of carbon-12.

Lời giải chi tiết

Carbon-12 has $Z = 6$, $N = 6$.

$$6m_p + 6m_n = 6(1.0060) + 6(1.0090) = 6.036 + 6.054 = 12.090 \text{ u}.$$

$$\Delta m = 12.090 - 11.9967 = 0.0933 \text{ u}.$$

$$E_{binding} = 0.0933 \times 931.5 = 86.9 \text{ MeV}.$$

$$\frac{E_{binding}}{A} = \frac{86.9}{12} = 7.24 \text{ MeV/nucleon}.$$

Bài 22

A sample of fissile material is described as **subcritical** when:

(A) each fission event, on average, causes more than one further fission event, so the reaction rate grows exponentially.

(B) each fission event, on average, causes exactly one further fission event, sustaining a steady reaction rate.

(C) each fission event, on average, causes fewer than one further fission event, so the chain reaction dies out.

(D) the sample contains no fissile nuclei at all.

Lời giải chi tiết

Subcritical corresponds to a neutron multiplication factor $k < 1$: on average fewer than one further fission is triggered per fission event, so the chain reaction is not self-sustaining and dies out. **(C)**.

Bài 23

State the primary function of each of the following components in a thermal nuclear fission reactor: (a) the moderator; (b) the control rods; (c) the coolant.

Lời giải chi tiết

(a) **Moderator** (e.g. water or graphite): slows down (thermalizes) the fast neutrons released by fission through repeated collisions with light nuclei, since slow neutrons are much more likely to induce further fission in U-235 than fast ones.
(b) **Control rods** (e.g. boron or cadmium): absorb a controllable fraction of the free neutrons, allowing the multiplication factor k to be held at exactly 1 for steady operation, or driven well below 1 to shut the reactor down.
(c) **Coolant** (e.g. water, gas, or liquid metal): removes the heat generated by fission from the reactor core, preventing overheating, and carries that heat away to generate steam that drives turbines to produce electricity.

Bài 24

In an alternative fission pathway, ${}^1_0n + {}^{235}_{92}\text{U} \rightarrow {}^{140}_{54}\text{Xe} + {}^{94}_{38}\text{Sr} + 2\,{}^1_0n$. Given $m(\text{U-235}) = 235.0439$ u, $m(\text{n}) = 1.0090$ u, $m(\text{Xe-140}) = 139.9216$ u, $m(\text{Sr-94}) = 93.9154$ u, calculate the energy released.

Lời giải chi tiết

Mass before:

$$235.0439 + 1.0090 = 236.0529 \text{ u}.$$

Mass after:

$$139.9216 + 93.9154 + 2(1.0090) = 139.9216 + 93.9154 + 2.0180 = 235.8550 \text{ u}.$$

Mass defect and energy:

$$\Delta m = 236.0529 - 235.8550 = 0.1979 \text{ u} \quad \Rightarrow \quad E = 0.1979 \times 931.5 = 184 \text{ MeV}.$$

Bài 25

A good moderator material in a thermal fission reactor should:

(A) have a high atomic mass, so fast neutrons bounce off with very little energy loss per collision.

(B) have a low atomic mass, so a fast neutron loses a large fraction of its kinetic energy in each collision, while rarely being absorbed.

(C) strongly absorb neutrons, so the reaction can be shut down quickly.

(D) be a good electrical conductor.

Lời giải chi tiết

Energy transfer in an elastic collision is most efficient when the target mass is close to the projectile's mass; light nuclei (such as hydrogen in water, or carbon in graphite) slow fast neutrons most effectively per collision, while having a low probability of simply absorbing the neutron. **(B)**.

Bài 26

The deuterium–deuterium fusion reaction ${}^{2}_{1}\text{H} + {}^{2}_{1}\text{H} \rightarrow {}^{3}_{2}\text{He} + {}^{1}_{0}n$ is given, with $m(\text{H-2}) = 2.0141$ u, $m(\text{He-3}) = 3.0133$ u, $m(\text{n}) = 1.0090$ u. Calculate the energy released.

Lời giải chi tiết

Mass before:

$$2(2.0141) = 4.0282 \text{ u}.$$

Mass after:

$$3.0133 + 1.0090 = 4.0223 \text{ u}.$$

Mass defect and energy:

$$\Delta m = 4.0282 - 4.0223 = 0.0059 \text{ u} \quad \Rightarrow \quad E = 0.0059 \times 931.5 = 5.50 \text{ MeV}.$$

Bài 27

Nuclear fusion between two light nuclei requires an extremely high temperature primarily because:

(A) high temperature increases the mass of each nucleus, letting the strong force act over a longer range.

(B) the nuclei need enough kinetic energy to overcome their mutual electrostatic (Coulomb) repulsion and approach close enough for the strong nuclear force to bind them.

(C) high temperature reduces the electric charge carried by each nucleus.

(D) high temperature is needed to ionize the neutrons within each nucleus.

Lời giải chi tiết

Two positively charged nuclei repel each other electrostatically as they approach; only sufficient kinetic energy (i.e. sufficiently high temperature) lets them approach closely enough for the short-range strong force to take over and bind them together. **(B)**.

Bài 28

Explain why nuclear fusion inside a sufficiently massive star's core can build up elements as heavy as iron through successive fusion stages, but cannot produce significantly heavier elements by the same process. Briefly state how elements heavier than iron are produced instead.

Lời giải chi tiết

Binding energy per nucleon increases with mass number up to iron ($A \approx 56$), so each successive fusion stage that builds toward iron still releases energy (products more tightly bound than reactants). Beyond iron, binding energy per nucleon *decreases* with increasing A, so fusing iron (or anything heavier) into a still heavier nuclide would require a net *input* of energy rather than releasing it — an ordinary stellar core, powered by energy-releasing fusion, cannot sustain this. Elements heavier than iron are instead built up mainly by rapid neutron-capture nucleosynthesis in the extreme conditions of a supernova explosion.

Bài 29

On a standard Hertzsprung–Russell diagram, a star located in the upper-right region — cool surface temperature but very high luminosity — is best classified as:

(A) a white dwarf

(B) a main-sequence star

(C) a red giant or supergiant

(D) a neutron star

Lời giải chi tiết

High luminosity despite a cool (hence low power-per-unit-area) surface can only be explained by an enormous surface area — the defining feature of a giant or supergiant star. **(C)**.

Bài 30

Outline the life cycle of a star with a mass similar to the Sun, from the main sequence to its final state. Briefly contrast this with the fate of a much more massive star.

Lời giải chi tiết

A Sun-like star spends most of its life on the main sequence, steadily fusing hydrogen into helium in its core. Once core hydrogen is exhausted, the core contracts and heats while the outer layers expand and cool, moving the star off the main sequence and into the red-giant region as it begins fusing helium (and briefly some heavier elements). Eventually the star sheds its outer layers, and the exposed hot, dense core settles as a small **white dwarf**, which then simply cools over a very long time with no further fusion. A much more massive star evolves faster, passes through a supergiant phase, builds up successively heavier fused elements in layered shells up to iron in its core, and — unable to generate further energy by fusion once the core is iron — collapses catastrophically in a **supernova** explosion, leaving behind a neutron star or (for the most massive stars) a black hole.

Bài 31

A π^+ meson consists of an up quark and a down antiquark ($u\bar{d}$). (a) State whether the π^+ is a baryon or a meson, explaining your reasoning from its quark content. (b) State the baryon number of the π^+. (c) A free neutron decays via beta-minus decay into a proton, an electron, and an antineutrino. Verify that lepton number is conserved in this decay.

Lời giải chi tiết

(a) A meson, because it consists of exactly one quark and one antiquark (not three quarks, as a baryon would).
(b) Each quark carries baryon number $+\frac{1}{3}$ and each antiquark $-\frac{1}{3}$, so the π^+ ($u\bar{d}$) has baryon number $\frac{1}{3} - \frac{1}{3} = 0$ (true of all mesons).
(c) Before decay: a neutron (not a lepton), lepton number $= 0$. After decay: a proton (lepton number 0) $+$ an electron (lepton number $+1$) $+$ an antineutrino (lepton number -1); total after $= 0 + 1 - 1 = 0$, equal to the total before. Lepton number is conserved.

Bài 32

Using $R = R_0 A^{1/3}$ with $R_0 = 1.2$ fm, calculate the radius of a lead-208 nucleus ($A = 208$), and hence estimate its density (taking its mass to be approximately 208 u). Comment on how this density compares with that calculated for uranium-238 earlier in the chapter.

Lời giải chi tiết

Radius:

$$R = (1.2 \times 10^{-15})(208)^{1/3} = (1.2 \times 10^{-15})(5.93) = 7.11 \times 10^{-15} \text{ m}.$$

Volume:

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(7.11 \times 10^{-15})^3 = 1.51 \times 10^{-42} \text{ m}^3.$$

Mass:

$$m = 208 \times 1.66 \times 10^{-27} = 3.45 \times 10^{-25} \text{ kg}.$$

Density:

$$\rho = \frac{m}{V} = \frac{3.45 \times 10^{-25}}{1.51 \times 10^{-42}} = 2.3 \times 10^{17} \text{ kg/m}^3.$$

This is essentially the same density obtained for uranium-238 despite the very different mass number, confirming that nuclear density is approximately constant across all nuclides — direct numerical support for $R \propto A^{1/3}$.

Ôn tập nhanh

- **Động học:** $v = v_0 + at$, $x = x_0 + v_0 t + \frac{1}{2}at^2$, $v^2 = v_0^2 + 2a\Delta x$, $x = x_0 + \frac{1}{2}(v_0 + v)t$. Rơi tự do: $a = g \approx 9.81$ m/s^2 hướng xuống; ném xiên: tách riêng $v_x =$ hằng số và v_y chịu gia tốc g.
- **Động lực học (Newton):** Định luật I — vật giữ nguyên trạng thái chuyển động nếu $\sum F = 0$; Định luật II — $\sum F = ma$; Định luật III — lực và phản lực bằng nhau về độ lớn, ngược chiều, tác dụng lên *hai vật khác nhau*. $W = mg$ (trọng lực); $f \le \mu N$ (ma sát); luôn vẽ sơ đồ lực trước khi viết $\sum F = ma$.
- **Động lượng:** $p = mv$; bảo toàn động lượng khi $\sum F_{ext} = 0$; xung lực $J = F\Delta t = \Delta p$. Va chạm đàn hồi: bảo toàn cả p và động năng; va chạm mềm (dính vào nhau): chỉ bảo toàn p, động năng bị mất một phần.
- **Công, năng lượng, công suất:** $W = Fd\cos\theta$; $KE = \frac{1}{2}mv^2$; $PE_{\text{hấp dẫn}} = mgh$; định lý công–động năng $W_{net} = \Delta KE$; bảo toàn cơ năng khi không có lực cản: $KE_i + PE_i = KE_f + PE_f$; công suất $P = W/t = Fv$.
- **Hấp dẫn:** $F = \dfrac{Gm_1m_2}{r^2}$; cường độ trường hấp dẫn $g = \dfrac{GM}{r^2}$; thế năng hấp dẫn $PE = -\dfrac{GMm}{r}$ (mốc 0 tại $r = \infty$); quỹ đạo tròn: $v = \sqrt{GM/r}$; định luật Kepler III: $T^2 \propto r^3$.
- **Nhiệt học:** $Q = mc\Delta T$ (nhiệt dung riêng); $Q = mL$ (nhiệt chuyển thể — nhiệt độ không đổi trong suốt quá trình chuyển thể); Định luật I nhiệt động lực học $\Delta U = Q + W$ (chú ý quy ước dấu của W); $T(\text{K}) = T(^\circ\text{C}) + 273$.
- **Chất khí lý tưởng:** $pV = nRT = Nk_BT$; động năng tịnh tiến trung bình mỗi phân tử $\overline{KE} = \frac{3}{2}k_BT$ — nhiệt độ tuyệt đối (K) tỉ lệ thuận trực tiếp với động năng trung bình phân tử, không phụ thuộc loại khí hay khối lượng mol.
- **Mạch điện:** $V = IR$; công suất $P = IV = I^2R = V^2/R$; mắc nối tiếp: $R_{\text{tổng}} = R_1 + R_2 + \cdots$ (I như nhau); mắc song song: $1/R_{\text{tổng}} = 1/R_1 + 1/R_2 + \cdots$ (V như nhau); suất điện động $\varepsilon = I(R + r)$ khi có điện trở trong r.
- **Dao động điều hoà (SHM):** $a = -\omega^2 x$ (gia tốc luôn hướng về vị trí cân bằng, tỉ lệ với li độ); con lắc lò xo $T = 2\pi\sqrt{m/k}$; con lắc đơn $T = 2\pi\sqrt{L/g}$; cơ năng toàn phần không đổi $E = \frac{1}{2}m\omega^2x_0^2$, liên tục chuyển hoá giữa động năng và thế năng.
- **Sóng:** $v = f\lambda$; sóng dọc (dao động song song phương truyền, VD âm thanh) và sóng ngang (dao động vuông góc, VD ánh sáng); giao thoa cực đại khi hiệu đường đi $= n\lambda$, cực tiểu khi $= (n + \frac{1}{2})\lambda$; sóng dừng hai đầu cố định: $L = n\lambda/2$; hiệu ứng Doppler: tần số tăng khi nguồn/người quan sát tiến lại gần, giảm khi rời xa nhau.
- **Điện trường:** định luật Coulomb $F = \dfrac{kq_1q_2}{r^2}$ (có thể hút hoặc đẩy, khác trọng lực luôn hút); cường độ điện trường $E = F/q = \dfrac{kQ}{r^2}$; thế năng điện $PE = \dfrac{kQq}{r}$; công của lực điện $W = q\Delta V$. Lực từ trên điện tích chuyển động $F = qvB\sin\theta$; trên dây dẫn mang dòng $F = BIL\sin\theta$ — dùng quy tắc bàn tay phải để xác định chiều lực.
- **Vật lý hạt nhân:** $A = Z + N$; đồng vị: cùng Z, khác N; định luật phân rã $N = N_0e^{-\lambda t}$, $T_{1/2} = \ln 2/\lambda$, hoạt độ $A_{ct} = \lambda N$; ba loại phân rã chính: α (A giảm 4, Z giảm 2), β^- (Z tăng 1), β^+ (Z giảm 1) — A, Z luôn bảo toàn ở hai vế; năng lượng liên kết $E = \Delta m\, c^2$ với Δm là độ hụt khối, $1\text{u} = 931.5$ MeV$/c^2$; đồ thị năng lượng liên kết/nucleon đạt đỉnh tại $A \approx 56$ (Fe) — cả phân hạch (hạt nặng tách) lẫn nhiệt hạch (hạt nhẹ hợp) đều toả năng lượng vì cùng đưa sản phẩm tiến gần đỉnh này hơn.
- **Vật lý lượng tử (AHL):** hiệu ứng quang điện $hf = \Phi + KE_{max}$ với $KE_{max} = eV_s$; bước sóng de Broglie $\lambda = h/p = h/(mv)$; nguyên lý bất định Heisenberg $\Delta x\,\Delta p \ge h/(4\pi)$; mức năng lượng nguyên tử hydro $E_n = -13.6$ eV$/n^2$; năng lượng photon $E = hf = hc/\lambda$.
- **Mẹo làm bài nhanh:** luôn đổi mọi đại lượng về đơn vị SI trước khi thay số; kiểm tra thứ nguyên của đáp số cuối cùng; với bài nhiều bước, liệt kê rõ đại lượng đã biết và đại lượng cần tìm trước khi chọn công thức; đọc đồ thị: diện tích dưới đồ thị vận tốc–thời gian là quãng đường đi được, độ dốc đồ thị vận tốc–thời gian là gia tốc, độ dốc đồ thị vị trí–thời gian là vận tốc.

Đồng hành cùng 8020 English

Cảm ơn bạn đã học cùng bộ giáo trình quốc tế 8020. **Điểm thật · Thầy thật · Kết quả thật** — nhiều học viên chỉ cần 1–2 khoá để đạt kết quả mong muốn. Đội ngũ 100% giáo viên đạt tối thiểu **8.0 IELTS** hoặc **1500 SAT**, gồm Giáo sư · Tiến sĩ · Thạc sĩ tốt nghiệp các trường top đầu thế giới (Carnegie Mellon — #1 Hoa Kỳ về Khoa học Máy tính).

"Bạn cần tất cả kỹ năng trên 7.5 để xét vào trường top 1 Anh Quốc về Y học (UKFPO của NHS) — và đã đạt mục tiêu." — Trần Hoàng Bảo Ngọc, IELTS Overall 8.0.

Bảng vàng học viên

AP Calculus BC — 5/5	**AP Microeconomics** — 5/5	**IB Economics** — 90/100
SAT 1590 / 1570 / 1550	**IELTS** 8.0–9.0	**Duolingo** 110 · **PTE** 90